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The $\bar{M}$-Linear Hypothesis and Varying Insurance

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Because mortality tables are generally tabulated at yearly intervals actuaries frequently must make assumptions about intermediate values in developing formulas for insurance and annuity functions.

For example by assuming a uniform distribution of deaths during each age interval one can develop the formula $\bar{A}_x = \frac{i}{\delta} A_x$.

Likewise if one assumes that D_{x+t} is linear in each age interval one can develop the formula $\bar{a}_x = \bar{a}_{x+\frac{1}{2}}$.

I propose in this paper to examine the effect on insurance and annuity formulas of assuming M_{x+t} to be linear within each interval. Consider a varying insurance benefit $(\bar{V}\bar{A})_x$ issued at age x which pays a varying benefit by if death occurs at age y.

The expected value of the benefit is given by

$$
\begin{aligned}
(\bar{V}\bar{A})_x &= \int_0^\infty b_{x+t} \cdot v^t {}_tp_x \mu_{x+t}\, dt \\
&= \sum_{k=0}^{\infty} \int_k^{k+1} b_{x+t} \cdot v^t \cdot {}_tp_x \mu_{x+t}\, dt \\
&= \sum_{k=0}^{\infty} v^k {}_kp_x \int_k^{k+1} b_{x+t} \cdot v^{t-k} {}_{t-k}p_{x+k} \mu_{x+t}\, dt \\
&= \sum_{k=0}^{\infty} {}_kE_x \int_0^1 b_{x+k+s}\, v^s \cdot {}_sp_{x+k} \cdot \mu_{x+k+s}\, ds \\
&= \sum_{k=0}^{\infty} {}_kE_x \cdot (\bar{V}\bar{A})^{1}_{x+k:\overline{1}|}
\end{aligned} \tag{1}
$$

We can therefore focus on a one year horizon and deal with

$(\bar{V}\bar{A})^{1}_{y:\overline{1}|}$.