ILA LAM Model Solutions
Spring 2020

1. **Learning Objectives:**

4. The candidate will understand the basic design and function of Economic Scenario Generators and EquityLinked Insurance Models.

Learning Outcomes:

(4b) With respect to Equity-Linked models:

- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:

Investment Guarantees, Chapters 6 & 7

LAM-139-19: Simulation of a Guaranteed Minimum Annuity Benefit

Link to 139-19: Excel Model – Stochastic Simulation of a GMAB Option

Commentary on Question:

This question required candidates to understand methods & importance of hedging GMAB guarantees and to understand risks of simplified hedging strategies. It also required candidates to demonstrate how to calculate the profitability of variable annuities with and without hedging.

Solution:

(a) Describe two methods to model this GMAB guarantee.

Commentary on Question:

Most students correctly identified both methods.

Approach #1: Monte Carlo Simulation

- Take average value of guarantee under thousands of scenarios.
- This approach can provide a distribution of possible pay-offs of the guarantee.
- Usually modelling is done assuming a risk-neutral distribution of equity returns.
- Able to reflect complex policyholder behaviours and product features with no closed form.

1. Continued

Approach #2: Closed Form Solution
- GMAB can be modelled as a put option.
- Black-Scholes formula provides the closed form solution.
- Formula/solution is much more complex than simpler guarantees because of ratchet.
- This approach cannot easily reflect dynamic policyholder behaviours.

(b) Evaluate the proposed strategy on the following:

(i) Risks mitigated

(ii) Risks not mitigated

(iii) Risks created

Commentary on Question:
Most candidates successfully identified risks mitigated and risks not mitigated, but some candidates did not come up with risks created in part (iii).

(This following list is not exhaustive, and credit would still be given if candidates correctly explain the risks.)

(i) Risks Mitigated
The hedging strategy should help to mitigate some market risk because it should provide returns directionally consistent with the guarantee pay-
offs.
Rebalancing will help to mitigate some policyholder behaviour risk

(ii) Risks not Mitigated
Policyholder Behaviour Risk - If a fixed amount of put options is bought once per year, there is a risk that a different number of policyholders will lapse or die each year than estimated in which case the guarantee will be over-/under-hedged for that year.

(iii) Risks Created
Directional Risk - The charge for the guarantee will not be zero so the guarantee will pay out even on fund return of 0; strike price of put should be higher than ATM (based on management fee). This will lead to an incorrect amount of hedging and potential losses in a down-market.

Basis Risk - Since put option is based on S&P500 and not fund, fund may perform worse than the S&P500 leading to bigger payout on guarantee than put. (the term basis risk is not used in source material - naming/description of this risk may vary)

1. Continued

Volatility Risk - Put options must be purchased at future dates; if market volatility increases then put option prices will increase causing the hedging strategy to cost more than anticipated (the term volatility risk is not used in source material - naming/description of this risk may vary)

(c) Perform the following:

(i) Calculate earnings without hedging in year 3.

(ii) Calculate the hedging gain or loss in year 3.

(iii) Assess the effectiveness of the hedging strategy.

Show all work.

Commentary on Question:
Most candidates did well in part (i) but struggled with part (ii). In part (iii) most candidates failed to assess with proper reasons.

(i) Calculate earnings without hedging in year 3

Account Value

MER(t) = BOY Account Value(t)*Management Expense %
Return(t) = [BOY Account Value(t)+MER(t)]*(Fund Unit Price(t)/Fund Unit Price(t-1) – 1)
EOY Account Value(t) = BOY Account Value(t) – MER(t) + Return(t) – Lapse(t)

Year 1
BOY Account Value(1) = 10000
MER(1) = 10000*2% = 200
Return(1) = (10000-200)*(95/100-1) = -490
Lapses(1) = 0
EOY Account Value(1) = 10000 – 200 + (-490) – 0 = 9310

Year 2
BOY Account Value(2) = 9310
MER(2) = 9310*2%=186
Return(2) = (9310-186)*(105/95-1)=960
Lapses(2) = 1000
EOY Account Value(2) = 9310 – 186 + 960 -1000 = 9084

1. **Continued**

Year 3
BOY Account Value(3) = 9084
MER(3) = 9084*2%=182
Return(3) = (9084-182)*(85/105-1)=-1696
Lapses(3) = 0
EOY Account Value(3) = 9084 – 182 + (-1696) – 0 = 7207

Guarantee Value

Year 1
BOY Guarantee Value(1) = EOY Guarantee Value(1) = 10000

Year 2
Lapses(2) = BOY Guarantee Value(2)*Lapses in Account Value(2)/(BOY Account Value(2)-MER(2)+Return(2))
Lapses(2) = 10000*(-1000)/(9310-186+960) = -992
Ratchet(2) = Max(0, Fund Unit Price(2)/Fund Unit Price(1)-1)*(BOY Guarantee Value(2)-Lapses in Guarantee Value(2))
Ratchet(2) = Max (0, 105/95-1)*(10000-992) = 948
EOY Guarantee Value(2) = BOY Guarantee Value(2) – Lapses in Guarantee Value (2) + Ratchet(2)
EOY Guarantee Value(2) = 10000 – 992 + 948 = 9957

Year 3
BOY Guarantee Value(3) = EOY Guarantee Value(3) = 9957

Earning Without Hedging at Year 3
= MER(3) – Expenses (3) – Guarantee Payout(3)
= 182 – 50 – (7207 - 9957)
= -2618

(ii) Calculate the hedging gain or loss in year 3
Hedging Cost
= Notional Needed*Price Per Hundred
= 9084*0.8
= 73

Option Payout
= (Strike-Market Level)*Units
= (1100-1050)*8.3
= 413

1. Continued

Hedging P&L
= Payout - Cost
= 413 – 73
= 340

(iii) Assess the effectiveness of the hedging strategy

Cost of Hedging for all years
= 100+112+73 (from part ii)
= 284
Net Fee Income
= MER for all years – Expenses for all years
= (200+186+182) – (100+50+50)
= 368
Hence, the fees collected are sufficient to fund the hedging program

Hedge Option Payout for all years
= 1000+0+413
= 1413
Guarantee Payout
= 7207 – 9957 (from part i)
= -2750
Hedge Payoff is less than the guarantee costs due to different underlying items (fund vs S&P), and this is basis risk.

2. Learning Objectives:

3. The candidate will understand the principles of Asset-liability Management ("ALM") and Projection Models, and be able to describe and evaluate various techniques for addressing the mitigation of risk.

Learning Outcomes:

(3c) With respect to Projection Models:

- Describe and evaluate the differences in model structure, features and assumptions for models used for various actuarial purposes.
- Describe and evaluate the requirements for creating an LTC projection model from first principles.

Sources:

LAM-141-19: Case Study: LTC Insurance First Principles Modeling, Modeling

LAM-142-19: Case Study: LTC Insurance First Principles Modeling: Mortality Assumptions

LAM-143-19: Case Study: LTC Insurance First Principles Modeling: Lapse Assumptions

Commentary on Question:

The question tests the candidate's understanding of and LTC projection model from a first principles standpoint. Part (a) tests general concepts; Part (b) involves numerical calculation; Part (c) evaluates lapse assumption development based on LTC business.

Solution:

(a)

(i) Define first principles modeling.

(ii) Describe its advantages.

Commentary on Question:

Candidates performed well on this portion. Partial credit is given for incomplete answers.

- "First principles" modeling incorporates active and disabled mortality and using morbidity assumption components based on health status distinctions.
- In its most complex form, "first principles" model breaks down all major assumptions into component pieces, including disabled mortality vs. healthy life mortality, specific care setting assumptions and transfer rates, and claim termination split between recoveries and deaths.
- This is in contrast to calculating incurred claim costs outside of the projection model and using them as an input into the model to project future claims

2. Continued

Advantages of first principles modeling include:

- Detailed refinement of assumptions such that the mortality assumptions for total lives do not significantly over- or understate the combined active and disabled life assumptions
- A model that contains assumptions that are internally consistent
- A projection model that calculates paid claims and claim reserves on a more granular basis
- A projection model that includes incidence of new claims and counts of existing claims
- Refined benchmarking to be used in examining actual experience

(b) Calculate the lapse rate for each policy year.

Commentary on Question:
Many candidates failed to apply the fact of "Death occurs at mid-year"; Clear understanding of how health lives roll forward is the key to solve this question. Partial credit is given for showing the work and the thought process regardless incorrect final calculations.

Starting # of Healthy Life = prior year - # of disablements - # of death (healthy) – lapse
(6) (y) = (6) (y-1) – (8) (y-1) – (10) (y-1) – (3) (y-1)
Starting # of Disabled Life = prior year +100 - # of death from disabled
(7)(y) = (7)(y-1) + (8)(y-1) – (9)(y-1)
# of Disablements = 100 each year (given)
# of death (Disabled Life) = (Starting # of disabled lives + # disablements) * mortality rate
(9)(y) = [(7)(y) + (8)(y)] * (5)(y)
# of death (Healthy Life) = total termination - lapses - disabled death
10(y) = (4)(y) – (3)(y) – (9)(y)
Lapse Rate (Healthy Life Only) = lapse / (# healthy life Starting - # disablements - # of healthy deaths / 2)
(11)(y) = (3)(y)/[(6)(y) – (8)(y) – 10(y)/2]

(1) Year	(2) Number of Policies	(3) Lapse	(4) Total Termination	(5) Mortality Rate (Disabled Life)	(6) Starting # of Healthy Life	(7) Starting # of Disabled Life	(8) # of Disablements	(9) # of death (Disabled Life)	(10) # of death (Healthy Life)	(11) Lapse Rate (Healthy Life Only)
1	1000	35	86	1.00%	1000	0	100	1	50	4.00%
2	914	25	74	1.51%	815	99	100	3	46	3.61%
3	840	40	64	1.69%	644	196	100	5	19	7.48%

2. Continued

(c)

(i) Evaluate the use of an implied lapse assumption for this product.

(ii) Recommend actions to improve the quality of the lapse assumption.

Justify your answer.

Commentary on Question:
Most candidates didn't evaluate the "implied lapse assumption" per advantages and disadvantages. Most candidates struggled with c(ii) as they failed to retrieve the considerations from study material source. Instead, many of them discussed the comparison of implied lapse vs. results from b lapse calculation.

(i) Implied lapse assumption is one of the three approaches that are commonly used for developing first principles lapse assumptions. The approach required a projection of the runout of lives instead of being based on an experience study. The implied lapse rate is equal to the calculated implied lapse divided by healthy lives.
Benefits:

1. Overcomes lack of credible data
2. Generally creates smoother lapse rates than observed data

Drawbacks:
Using an implied lapse assumption approach fails to measure other considerations that can affect the number of lapses:

1. Fails to separate out shock lapses from the base lapse assumption. The rate increase may explain the high observed lapse rate at duration 3 compared to implied lapse rate.
2. Ignores benefit exhaustions. This skews the lapse rate especially at the older ages (possibly explaining high observed lapse rate at duration 3).

(ii) Ideas to address other lapse consideration concerns:

1. Accurate measurement of deaths: Underreporting of deaths will overstate observed lapses. This will likely cause a skew in the results by ages as there are more deaths at the older age. Recommend blend industry mortality with company own experience to achieve better measurement.
2. Impact of rate increases: Lapse rates will likely increase because of rate increase, this is commonly referred to as shock lapse. Separate out shock lapse.
3. Benefit reductions: This is a concept of how many people will reduce benefits as results of a rate increase, a similar assumption to lapse. Developing an assumption as to how many people elect benefit reduction is needed to have more accurate projections.

2. Continued

4. Benefit exhaustions: Policies that go on claim and exhaust benefits will be calculated directly by the model. Therefore, the lapse rate assumption needs to represent voluntary lapse and excludes a termination that is due to exhausting benefits.

3. Learning Objectives:

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Learning Outcomes:

(2c) Describe and evaluate best practices for actuarial model governance over process and controls

(2f) Describe issues and techniques related to sound spreadsheet model management

(2h) Describe and evaluate the guidance in the Actuarial Standards of Practice

Sources:

Actuarial Modeling Systems: How Open We WANT Them to be vs. How Closed We NEED Them to be, Modeling Platform, SoA, Nov 2017

Model Risk Mitigation and Cost Reduction Through Effective Documentation, PWC, 2013

Model Validation for Insurance Enterprise Risk and Capital Models, Stricker et. al., 2014 (exclude appendices)

Commentary on Question:

The question tests for understanding of the life cycle of model development including considerations in choosing a platform, documentation of the development, and model validation. The question synthesizes a few of the readings on these topics and has the candidate apply the concepts based on the situation in the question.

Solution:

(a) Recommend if XYZ should implement an open or closed model system based on the following considerations:

(i) Governance and Control Environment

(ii) Model Efficiency

(iii) Model Flexibility

3. Continued

Commentary on Question:
Candidates who did well could recommend either an open or a closed model, as long as they supported that recommendation. Candidates who clearly and specifically related pros/cons to XYZ earned more credit than those who did not. Candidates generally understood that governance/controls favored closed systems and flexibility favored open systems. They also understood that closed systems have professional programmers, while open systems are coded by actuaries, who are less sophisticated about run-time optimization. Many candidates discussed only one kind of system, which limited credit earned.

<u>Governance/control considerations:</u>
Closed
- (+) Locked down system will prevent or provide warnings for illogical operations or not actuarial sound calculations
- (+) Governance framework provided by vendor is industry tested, improved by customer feedback
- (-) Limited customizations

Open
- (+) Customized controls possible, work well for unique calculations and provide greater transparency into model calculations
- (-) Customized controls may not be adequate

Efficiency considerations:
Closed
- (+) Seamless automation as part of end-to-end production process
- (+) Vendor coders are professional programmers with expertise in code optimization
- (-) Customer has limited ability to make the model more efficient outside user interface

Open
- (+) Also has automation capabilities, where interaction protocols are flexible to meet model user's needs
- (-) Run time optimization is left to customers, and actuaries typically do not have deep enough understanding of code to achieve full optimization

Flexibility considerations
Closed
- (+) Vendors are open to implementing modifications to meet demands of new regulations and of new product offerings
- (-) However, flexibility may be limited to vendor set-up, so it may be difficult to fit a new product such as this one into that set-up

3. Continued

Open
- (+) Ability to customize calculations increases flexibility and allows for coding unique product features.
- (-) Open system with significant flexibility increases risk of calculation errors.

Overall, governance/control and efficiency considerations favor closed systems, while flexibility favors open systems. Because this is such an innovative product, given the above points, I recommend an open system.

(b) Draft an outline of the Model Development Documentation based on these decisions using the structure described in *Model Risk Mitigation and Cost Reduction Through Effective Documentation.*

Commentary on Question:
Candidates who simply regurgitated the five decisions under the documentation section headings earned limited credit. Candidates who discussed model limitations, risks, or risk mitigation directly related to the specific situation described in the question earned full credit. No credit was given for the "model estimation" section, since the model does not use statistical techniques.

<u>Executive Summary</u>
- General commentary about model purpose and scope
- Should identify coding simplification here as a model weakness
- Discuss model limits/risks and risk mitigation.

<u>Model Development Data</u>
- Description of process flow for internal and external data
- Explanation for use of external data, including defense of relevance and appropriateness for this model

<u>Model Theory and Approach</u>
- Details of coding simplification
- Discussion of risks of coding simplification (e.g. change in other inputs, such as model assumptions, could change impact/appropriateness of this simplification)
- Discussion of potential risk mitigants of coding simplification (e.g. periodic benchmarking of coding simplification against un-simplified model)
- Rigorous support for innovative approach to model
- Reconciliation between modeling methodologies (e.g. coding simplification, innovative approach) to industry practice

3. **Continued**

<u>Model Testing</u>

- Details of tests performed, including interest rate sensitivity and stress tests
- Measures of model uncertainty from these tests

(c) The risk department requires all new models to undergo model validation before they are used. The analyst responsible for the validation schedules a meeting to review the validation plan.

Critique the following statements made by the analyst during this meeting.

A. *Only positive testing will be conducted in which valid input parameters will be tested to ensure valid output is produced.*

B. *Materiality of input parameters should be validated through sensitivity testing.*

C. *The reporting and use of the output generated by the model is out of scope of the validation.*

D. *Validation of raw input data is not necessary as it should be covered under validations of those particular source systems.*

Commentary on Question:
Candidates performed well on this section. In order to earn full credit, candidates needed to give a full explanation and not simply state true or false.

1. False. Negative testing should be performed to verify the model will error-out when encountering invalid inputs (e.g. negative mortality rates, text input for a numeric field).
2. True. Such testing can identify the sensitivity of the model to these inputs. The greater the sensitivity of the model to a given input, the more scrutiny should be applied to that input.
3. False. Validation of output/reporting ensures that the model output is being used as intended and that the reporting does not mislead decision-makers relying on the model.
4. False. Validation should verify that system data, such as contract type or rider options, are being interpreted correctly and appropriately by the model. The validation should also verify that raw data is not being manipulated inappropriately.

4. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic models, generalized linear models ("GLMs"), multi-state, and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, applications, technical challenges, and governance.

Learning Outcomes:

(1b) With respect to generalized linear models:

- Describe and apply the basic principles of GLMs, and evaluate where GLMs might be useful in a Life Insurance context.

(1c) With respect to multi-state and transition matrix models:

- Describe and apply the methodologies for constructing multi-state and transition models
- Apply the modeling process (e.g., in LTC projection modeling and credit rating modeling)

Sources:

A Practioner's Guide to Generalized Linear Models, Towers Watson, 2007

Multi-state Transition Models with Actuarial Application, Daniel, SOA & CAS

Commentary on Question:

This question tested the candidate's ability to describe and apply the methodology for constructing multi-state and transition models.
A successful candidate should be able describe and apply the basic principles of GLMs and evaluate where GLMs might be useful in an ILA context.
Overall, students did well in this question.

Solution:

(a) Calculate the annual premium using the Equivalence Principle, assuming an annual interest rate of 10% and transitions occur at end of year. Show all work.

Commentary on Question:
Students generally did well on this part.
Common mistakes were:

- *Missing one or two transition paths in calculation of probabilities*
- *Didn't assume the correct starting transition status as Active*
- *Missing the return on premium in the calculation of the present value of benefit*

Under the Equivalence Principle,
PV of benefits = PV of premiums.

4. Continued

(1) Calculate the present value of benefits using the transition matrix
Note that policyholder can only start at State #1 at time 0.

$$Q = \begin{array}{c|cccc} & \text{Active} & \text{Lapse} & \text{Disable} & \text{Death} \\ \hline \text{Active} & 0.9 & 0.05 & 0.04 & 0.01 \\ \text{Lapse} & - & - & - & - \\ \text{Disable} & - & - & - & - \\ \text{Death} & - & - & - & - \end{array}$$

At time 1, annual benefit is paid if policyholder becomes disabled (in State #3). The probability is 0.04.

At time 2, annual benefit is paid if policyholder becomes or remains disabled.

The probability is (0.9)*(0.04) = 0.036
(0.04)*(0.6) = 0.024
which equals 0.036 + 0.024 = 0.06
Alternatively, candidates could have shown the following matrix.

$$Q * Q = \begin{array}{c|cccc} & \text{Active} & \text{Lapse} & \text{Disable} & \text{Death} \\ \hline \text{Active} & 0.9 & 0.05 & 0.04 & 0.01 \\ \text{Lapse} & - & - & - & - \\ \text{Disable} & - & - & - & - \\ \text{Death} & - & - & - & - \end{array} * \begin{bmatrix} 0.9 & 0.05 & 0.04 & 0.01 \\ 0 & 1 & 0 & 0 \\ 0.3 & 0 & 0.6 & 0.1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.822 & 0.095 & 0.06 & 0.013 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

At time 3, annual benefit is paid if the disabled person remains disabled, or if an active person becomes disabled.
The probability is 0.06 (from time 2) *(0.6) and 0.822*(0.04) which equals 0.0689.
Alternatively, students could show the following:
Alive/Active/Active/Disabled = 0.9 * 0.9 * 0.04 = 0.0324
Alive/Active/Disabled/Disabled = 0.9 * 0.04 * 0.6 = 0.0216
Alive/Disabled/Disabled/Disabled = 0.04 * 0.6 * 0.6 = 0.0144
Alive/Disabled/Recovered/Disabled = 0.04 * 0.3 * 0.04 = 0.00048
which equals 0.0689

4. Continued

$$Q * Q * Q = \begin{bmatrix} 0.822 & 0.095 & 0.06 & 0.013 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} * \begin{bmatrix} 0.9 & 0.05 & 0.04 & 0.01 \\ 0 & 1 & 0 & 0 \\ 0.3 & 0 & 0.6 & 0.1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7578 & 0.1361 & 0.0689 & 0.01422 \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

Annual Benefit is $100 if the policyholder is in State 3

Discount factors are calculated as follows:
v = 1/1.10 = 0.909
v2 = 1/(1.10)^2 = 0.826
v3 = 1/1(1.10)^3 = 0.751

The present value of benefit is [(0.04)*(100)*(0.909)
+ (0.06)*(100)*(0.826)
+ (0.06888)*(100)*(0.751)]
=13.7701

(2) Calculate the present value of premiums using the transition matrix.

The policyholder only pays premium while in State #1. The premium is assumed to be P.
The policyholder receives a refund of paid premium if he/she transitions into State #4.
The cash flow they receive depends on the amount of premium paid at the time of death.
At time 1, this is -P. At time 2, this is -2P. At time 3, this is -3P.

	Time 0	Time 1	Time 2	Time 3
	C0	C1	C2	C3
Active	1	1	1	0
Lapse	0	0	0	0
Disable	0	0	0	0
Death	0	-1	-2	-3

4. Continued

At time 0, premium paid at the beginning of the year is P.

At time 1, the probability of premium being paid is 0.9 and the probability of premium paid back to policyholder upon death is 0.01.

At time 2, premium is paid if the policyholder is active.

The transitions are if they are active in time 1 and remains active or if policyholder is disabled in time 1 and then recovers.
The probability is 0.822, which is given by the following transitional probabilities:
Alive/Active/Active = 0.9 * 0.9 = 0.81
Alive/Disabled/Recovers = 0.04 * 0.3 = 0.012

At time 2, premium paid back to policyholder upon death is 0.013, which is given by the following transitional probabilities:
Alive/Active/Dead = 0.9 * 0.01 = 0.009
Alive/Disabled/Dead = 0.04 * 0.1 = 0.004

At time 3, no premium is paid.
The probability of premium refund to the policyholder upon death is 0.0142, which is given by the following transitional probabilities:
Alive/Alive/Alive/Dead = 0.9 * 0.9 * 0.01 = (0.0081)
Alive/Alive/Disabled/Dead = 0.9 * 0.04 * 0.1 = (0.0036)
Alive/Disabled/Active/Dead = 0.04 * 0.3 * 0.01 = (0.00012)
Alive/Disabled/Disabled/Dead = 0.04 * 0.6 * 0.1 = (0.0024)

The present value of premium is [(1)*P*(1) + (0.9)*P*(0.909) + (0.01)*(-P)*(0.909) + (0.822)*(P)*(0.826) + (0.026)*(-P)*(0.826) + (0.043)*(-P)*(0.751)] = 2.4349P

Using the Equivalence Principle, 13.7701 = 2.4349P which means P = 5.6553.

(b) Contrast the benefits and limitations of using a Linear versus Generalized Linear Model to calculate the policyholder discount.

Commentary on Question:
This is a retrieval question directly from the source material.
Many students lost points for not including both the benefit and limitation for both GLM and LM by only including the benefits or limitations for one model or the other.

4. Continued

Linear Models:

- Difficult to assert Normality & constant variance for response variable.
- Values for the response variable may be restricted to be positive, violates assumption of Normality.
- If the response variable is strictly non-negative, the variance is a function of the mean.
- Many insurance risks vary multiplicatively with rating factors, not additively.

Generalized Linear Model

- GLMs consist of a wide range of models that include models as a special case.
- The LM restriction assumptions of Normality, constant variance, and additivity of effects are removed.
- The variance is permitted to vary with the mean of the distribution.
- The effect of the covariates on the response variable is assumed to be additive on a transformed scale.

(c) Determine whether a Normal or Gamma Variance Function was used to fit the model.

Commentary on Question:
Students were able to identify the correct variance function but didn't provide justifications.

Normal – assumes each observation has the same fixed variance, fits with equal weight to data points.

Gamma – assumes the variance increases with the square of the expected value of the observation.

For an average claims model, the variance is expected to increase with the size of the observation. Thus, it is inappropriate to use a normal variance model.

A Gamma variance model should be classified as appropriate.

4. Continued

(d)

(i) Define and identify the link function.

(ii) Calculate the discount a 25-year-old male with a strong credit score would receive assuming the equivalence principle. Show all work.

Commentary on Question:
Most students were able to identify the correct link function but did not provide the definition of a link function.
For the second part of the question, students generally did well. Common errors were:

- *Didn't use exponential function to calculate the average claim*
- *Didn't input the correct issue age/credit score/gender indicators in the given formula*
- *Only calculated the strong credit score scenario and failed to calculate the discount between two scenarios.*

(i) Link Function: Logarithmic
Link Function: The relationship between the random and systematic components is specified via a link function, g, that is differentiable and monotonic:

$E[U] = \mu = g^{-1}(\eta)$

(ii) Annual DI Premium (no credit check):
EXP(7 + 0.03 * 25 - 0.3 * 0 - 0.1 * 1) = 7.65
2,100.64

Annual DI Premium (strong credit):
EXP(7 + 0.03 * 25 - 0.3 * 1 - 0.1 * 1) = 7.35
1,556.20

Annual Discount:
2,100.64 - 1,556.20 = 544.45