SOCIETY OF ACTUARIES

Issue No. 46 • February 2005

RISKS AND REWARDS

THE NEWSLETTER OF THE INVESTMENT SECTION

PUBLISHED IN SCHAUMBURG, ILL. BY THE SOCIETY OF ACTUARIES

Econophysics: Making Money before Doomsday

by Shane Whelan, PhD.

A version of this article appeared in The Actuary *(UK) in August 2004. Reprinted with permission.*

Lost Money

In these days of frustrating markets, it is consoling to ponder the very weak—arguably nonexistent—relationship between intelligence and stock-picking. Isaac Newton, the greatest scientist of all, notoriously sold early in the South Sea Bubble of 1720 after doubling his investment and remarking, somewhat smugly, that he could "calculate the motions of the heavenly bodies but not the madness of people." But he was tempted back in again a few months later when he saw the market continue to climb exponentially. He bought at the top of what was probably the worst stock-market crash on record, losing a fortune of £20,000.[1]

It Could Be the End of the World

The challenge of modeling the madness of crowds has been taken up by many disciplines in science and by even more pseudo-sciences over the intervening centuries. Physicists, though, have only started to study this phenomenon but are quickly catching up, judging by some notable successes and worrying predictions. Didier Sornette, one of the leaders in the emerging discipline of econophysics, claims to detect log-periodic oscillations decorating a super-exponential trend in key long-term demographic, economic and financial series that, when extrapolated, explode to infinity in about the year 2050.[2] In short, he predicts the end of the world in or about the year 2050. Remarkably, this date coincides with Newton's conclusion from studying the Bible, when he settled on the year 2050 as the starting date for the everlasting reign of the Saints of the Most High.[3]

turn to page 4

---

1) *Kindleberger, C.P. (1996) Manias, Panics, and Crashes: A History of Financial Crises. Wiley.*

2) *Sornette, D. (2003) Why Stock Markets Crash: Critical Events in Complex Financial Systems. Princeton University Press*

3) *Robinson, A.B. (1991) Introduction to Observations upon the Prophecies of Daniel and the Apocalypse of St John by Sir Isaac Newton (London, 1733). The Oregon Institute of Science and Medicine, Oregon, US.*

Inside...

SOCIETY OF ACTUARIES

RISKS AND REWARDS

Issue Number 46 • February 2005

Published by the Investment Section
of the Society of Actuaries

475 N. Martingale Road, Suite 600
Schaumburg, IL 60173-2226

Phone: 847-706-3500
Fax: 847-706-3599
World Wide Web: www.soa.org

This newsletter is free to section members. A subscription is $15.00 for nonmembers. Current-year issues are available from the communications department. Back issues of section newsletters have been placed in the SOA library and on the SOA Web site: (*www.soa.org*). Photocopies of back issues may be requested for a nominal fee.

Outgoing Chairperson's Corner

by Mark W. Bursinger

My term as Investment Section Chairperson went by quickly as did my three years as a council member. My final chairperson's corner will address upcoming changes for the Investment Section and how the membership will be called upon to help.

The current mission of the Investment Section is to facilitate the professional development of our members in regard to the investment of institutional funds, especially insurance company and pension fund assets and in the measurement and management of those assets in relation to the institution's liabilities. I've mentioned this in the past and I reiterate it here to reinforce the main purpose of our section is for the "development of our members."

The recent governance audit unveiled that the sections were an effective structure to build upon largely due to the grassroots nature of our activities. We all know what happens to people who do a good job—they get more to do! Let's pause for a moment and thank our council members for their efforts and a job well done. I'd like to give special mention to our exiting council members Joe Koltisko and Larry Rubin for their numerous contributions over the past three years. Continuing on with the council is Mike O'Connor (Chairperson), Steve Easson (Secretary), Bryan Boudreau, Martin le Roux, Sean Casey and Steve Stone. Congratulations should be extended to the newly elected council members Nancy Bennett, Ellen Cooper and Catherine Ehrlich.

Looking ahead, there will be significant changes for this year's council and section members. The sections are going to be taking on more responsibility as the practice area structure is dissolved. Practice area leadership, section councils and SOA staff have been working together to plan the transition and there remain many details to be worked out.

Here is a preview of what you should expect to see. Sections will be spending more time thinking about how to create and deliver membership value. This requires an ability to identify key professional issues and member needs. We will need input from our membership to make this happen. The sections will be more closely involved with the Board of Governors to ensure the SOA strategic direction is in sync with these needs. You should expect to hear more about activities relating to:

- Emerging issues,
- Advocating for actuaries,
- Maintaining relationships with external organizations, and
- Providing thorough leadership.

turn to page 23

Stochastic Simulation for C3 Risk: A Statistical Review

by Richard Q. Wendt

In 2003, the American Academy of Actuaries Life Capital Adequacy Subcommittee (LCAS) published a set of 10,000 stochastic scenarios that was intended to be used to support the publication, "Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Products with Guarantees (Excluding Index Guarantees)." Any practitioner may download the prepackaged scenarios from the AAA Web site.[1] The prepackaged scenarios include 360 months of simulated data for three interest rates and nine asset classes. The public availability of this data provides a unique opportunity for analysis.

The primary purpose of this article is to illustrate statistical measures that can be used to evaluate stochastic simulations. Given the public availability of the data, readers may download the data and replicate the calculations. The secondary purpose of this article is to use those measures to compare the pre-packaged scenarios to current forecasting practice.

The prepackaged scenarios were created to satisfy the recommendations of the LCAS for simulations with tails that satisfy specified distributional requirements. Those requirements were based on Mary Hardy's regime-switching lognormal stochastic model (RSLN2) for the S&P 500. The LCAS model was developed by extending the RSLN2 model from a single asset class model to a multiple asset class model. While the Hardy model was described in an extensive paper in the NAAJ and an accompanying Excel spreadsheet, the LCAS did not publish comparable documentation on the extension of the model to multiple asset classes. Due to the complexity of the Hardy model and the LCAS extension, this writer was not able to determine if the LCAS model extension has the same degree of validation as the original Hardy model. Nevertheless, it is possible to analyze the prepackaged scenarios as a set of data observations.

Description of the Data

The prepackaged scenarios consist of 10,000 scenarios of 360 monthly returns for nine asset classes and 361 beginning-of-month yields for three interest rate categories.

turn to page 7

1) *The url for the files and supporting information is on the Academy Web site at* http://www.actuary.org/life/phase2.html.

From Page 1

Econophysics

Sornette is part of a movement of physicists modeling economic systems using techniques and concepts developed in studying the out-of-equilibrium dynamics of complex systems. The movement was named 'econophysics' in 1997 by H. Eugene Stanley, but can be dated from 1991 when a leading physics journal, Physica A, began publishing papers on this topic. A sub-group of these econophysicists specialize in studying capital markets (a sub-discipline that has come to be called 'phynance,' which has maintained its own dedicated journal from 2001, Quantitative Finance) and along with Sornette and his research team, other centers of excellence in phynance have sprung up about Stanley, Sorin Solomon, Rosario Mantegna and Doyne Farmer (all of whom maintain excellent Web sites). Some have even given the research a commercial edge with companies such as the Olsen Group, Science & Finance and The Prediction Company developing practical trading or risk control models to exploit the perceived opportunities.

Econophysicists, in contrast to financial economists, begin with data—huge quantities of data.

In The Beginning, There Is Data

Econophysicists, in contrast to financial economists, begin with data—huge quantities of data. Their studies into financial markets typically analyze several million price changes—capturing, say, every price change every minute over the last couple of decades or every bargain on every equity over a couple of years. Several empirical regularities in the price formation process are now documented that shed light on the way speculative prices evolve (see box). These empirical regularities or stylized facts are observed in markets as diverse as commodity markets, currency markets, cash, bond, equity, and property markets and seem to be present no matter how frequently or infrequently prices are sampled. That is, the same patterns observed in asset price returns measured over every ten minutes appear when returns are measured in months.

The empirical regularities can be used to characterize the evolution of asset prices or, equivalently, the returns from capital assets. We know that active trading leads to these patterns in all capital markets and so the detail of the dealing structure must be irrelevant. Further, the same regularities are observed irrespective of the time interval between prices, so the institutional structure of the traders must also be irrelevant. Taking a short leap, we might conclude that, as the resultant patterns are the same, the forces giving rise to the patterns must also be very similar. That is, pension funds investing in equities over decades are participating in essentially the same game as intra-day traders acting on minute movements of the dollar-yen market—the principal difference being the former is played out in excruciatingly slow slow-motion.

Agent modeling

So what is common to all the different capital markets over any time period and characterizes the trading process? John Maynard Keynes, no mean investor himself, described it well: "The actual, private object of the most skilled investment today is to 'beat the gun,' as the Americans so well express it, to outwit the crowd and to pass the bad, or depreciating, half-crown to the other fellow.[4]" So the game of professional investment is a "battle of wits to anticipate the basis of conventional valuation a few months hence…For it is, so to speak, a game of Snap, of Old Maid, of Musical Chairs. …" Physicists take this metaphor rather literally and have modeled markets as a game played by similar players ('agents') that can only be won by a minority of the players ('minority game').

This sort of modeling invites parallels with the Boltzmann-Maxwell reduction of thermodynamics to elementary mechanics, modeling thermodynamic properties as the simple aggregate of many simple collisions between many similar billiard-ball molecules. And just as Boltzmann was lead to the surprising Second Law of Thermodynamics—the irreversibility of time—when contemplating the aggregate of these time-reversible collisions, the

---

4) *Keynes, J.M. (1936) The General Theory of Employment, Interest and Money. MacMillan & Cambridge University Press.*

econophysicists are reporting some surprising consequences of agent modeling in minority games.

First, such agent models can replicate many of the 'stylized facts' above that characterize asset price evolution. Second, they suggest that (as J. P. Morgan memorably remarked when asked what the market will do) the market will fluctuate—the equilibrium they reach is dynamic as the price is expected to change even in the absence of new information. Third, when markets reach what looks like a dynamic equilibrium, there remain exploitable patterns.[5]

This latter argument is wonderfully general. Let us say all agents record the last m changes in price as simply up (1) or down (0). Now a trading strategy is a mapping from the set of all m-tuples of 1 or 0 into the indicator set 1 (meaning next trade is a buy as expect upward movement) or 0 (meaning next trade is a sell as expect downward movement). There are 2^m elements in the domain, and each element can be mapped to either a 1 or 0. Accordingly, there are such 2^{2^m} mapping. Each agent selects from a pool of n strategies and, say, there are A agents in total. So there are somewhat less than n.A strategies actually being played while the total universe of strategies is of the order of 2^{2^m}. Now, for any plausible numbers assigned to m, n and A, we find that 2^{2^m} is several orders of magnitude greater than n.A. (For instance, with m=12, $2^{2^{12}} >> 10^{1200} >> 10^{1000}.10^{10}$ which is significantly greater than the current best estimate of the number of elementary particles in the universe times the number of humans alive at the moment.) Hence the actual number of strategies being played is a negligible proportion of the total number of all strategies. Finally, put in operation some evolutionary mechanism that ensures the population of successful agents prosper while the unsuccessful ones perish, and we find that the evolutionary mechanism emphasizes some strategies more than others, leading to small biases in the original population being magnified in the surviving population. These biases create patterns in the future evolution of the price, induced by the not-so-random surviving trading strategies.

More speculative agent models are reporting that trend following rules induce trends but with an oscillatory feature, which favors different trend, following rules and, surprisingly, not all value strategies push market values closer to fundamental value.

Self-Organized Criticality

Agent modeling is just one approach the econophysicists have brought to a new level of sophistication. It could not, though, forecast the end of the world. Sornette takes another approach. Rather than drawing parallels between the stock market and games, he finds parallels with many natural phenomena—specifically those phenomena with a large number of interacting parts with feedback, which typically can self-organize and perhaps make a sudden transition to a new state or phase (e.g., evolution, epidemics, earthquakes, ferromagnetism, weather, ecology, ruptures). He attempts to forecast these points of 'self-organized criticality.' In attempting to estimate the point of rupture of pressure tanks in rockets, he claims to have detected some tell-tale signs of the approaching rupture—log-periodic oscillations about an underlying trend—that throws the trend into sharper relief, thus allowing it to be extrapolated. Sornette has applied this approach to stock market indices and demographic, economic and other time series to detect a trend and make predictions. True, this is making a rather heroic generalization but, as pointed out by Maury Osborne (who, with Louis Bachelier and Benoit Mandelbrot, is one of the great forerunners of the econophysics movement), speculation in science is always in the best tradition of Chicken Little.[6] Inevitably, not all Sornette's forecasts have proved correct, but, unlike Chicken Little, he can claim some notable successes—in January 1990

Agent modeling is just one approach the econophysicists have brought to a new level of sophistication.

Sornette forecast that the Nikkei would rise 50 percent by the end of the year (it rose just over 49 percent) and he also forecast the NASDAQ would crash in April 2000. Maybe the sky is falling.

The econophysicist's approach in general, and Sornette's in particular, see speculative markets as just another instance of a much more general phenomenon—game-playing or some complex natural phenomenon. This fresh perspective already

turn to page 6

5) *Farmer, J.D. (1999) Physicists attempt to scale the ivory towers of finance. Computing in Science & Engineering, Nov./Dec, 26-39.*

6) *Osborne, M.F.M. (1977) The Stock Market and Finance from a Physicist's Viewpoint. Crossgar Press, Minneapolis.*

From Page 5

adds value. Their empirical emphasis has squeezed some universal regularities out of the process of price evolution that have helped characterize the process of speculation. Some econophysicists, such as Bertrand Roehner, have taken to collect data on such related markets as regional wheat prices over previous centuries and on prices of collectables such as rare books, coins, stamps and baseball cards.[7] Sornette and others claim data on many natural catastrophes are relevant to predicting stock market crashes or bubbles—being just a different manifestation of the same underlying phenomenon. More data, and more novel ways to analyze it, must accelerate the growth of our knowledge of the perplexing behavior of assets.

Doomsday 2050

We are perhaps nowadays more disposed to Sornette's rationale for doomsday in 2050 than to Newton's. But both physicists will be right if the world as we know it ends in or around 2050—if anyone then cares. And, arguably, both could claim to be right for the right reasons: Newton would doubtlessly have expected no more from the final generations than to use knowledge of doomsday to increase their material wealth.

Shane F. Whelan, FLA, PHD, FFA, FSA, FSAI, is the head of the Department of Statistics and Actuarial Science at University College Dublin, Ireland. He can be reached at shane.whelan@ucd.ie.

EMPIRICAL REGULARITIES DETECTED IN RETURNS ON CAPITAL ASSETS [8]

(1) Return series are non-stationary. Past returns are really not a guide to future returns and all those stationary models (e.g., the ARMA and ARCH models) will eventually fail.

(2) There is little or no correlation between successive returns.

(3) Returns come from a heavy-tailed distribution, where the variance exists but the kurtosis (4th moment) does not. Further, even when volatility clustering is removed, the declustered residuals still exhibit heavy tails (although somewhat less heavy than the original returns). Volatility tends to cluster in time, and the decay from high bouts of volatility tends to follow a characteristic power-law.

(4) Others, for example:
   a. The correlation of the current return to future volatility is negative, decaying to zero as time increases.
   b. The correlation between volume traded and volatility is high.
   c. There is an asymmetry between large positive and negative movement, with the latter more frequent.

---

7) Roehner, B.M. (2002) *Patterns of Speculation: A Study in Observational Econophysics.* Cambridge University Press.

8) Cont, R. (2001) *Empirical properties of asset returns: stylized facts and statistical issues.* Quantitative Finance, Vol 1, 223-236.

From Page 3

The modeled asset classes are:

Asset Class	Market Proxy
Money Market	3-month Treasury Returns
Intermediate Term Government Bonds (U.S. IT GVT)	U.S. Intermediate Term Government Bonds
Long-Term Corporate Bonds (U.S. LT CORP)	U.S. Long-Term Corporate Bonds
Diversified Fixed Income	65% USITGVT + 35% USLTCORP
Diversified Balanced	60% U.S. Equity + 40% Fixed Income
Diversified U.S. Equity	S&P 500 Total Return Index
International Equity	MSCI-EAFE $USD Total Return Index
Intermediate Risk	U.S. Small Capitalization Index
Aggressive or Specialized Equity	Emerging Markets, Hang Seng

Interest Rate	Market Proxy
Short-Term Rates	3-month U.S. T-Bill Yields
Medium-Term Rates	7-year U.S. Treasury Yields
Long-Term Rates	10-year U.S. Treasury Yields

For the purposes of this analysis, the asset returns have been converted to annual returns for 30 years and the yields have been converted to beginning-of-year yields for the initial date and the beginning of each subsequent year. As a 30-year annual simulation, the converted simulation is consistent with the common practice of expressing simulation assumptions and historical statistics in annual terms. The statistics referenced by LCAS are in annual or multi-year terms.

Since the simulation was based on a monthly model, translation to annual statistics tests the connections among the months, as well as the monthly simulation.[2] The data is available in csv files; since current versions of Excel are able to read only 256 of the 361 columns, the author used APL2 for the calculations and Excel to create the graphs.

turn to page 8

2) *Please refer to my article, "Time Track: Analyzing Historical Returns" in the September 2000 issue of* Risks and Rewards *for examples of the differences between monthly and annual returns.*

From Page 7

Statistics For Stochastic Forecasts

Readers are undoubtedly familiar with the commonly used statistics that are used for economic modeling: mean returns, standard deviation of returns and correlation of returns. Less well known is that there are at least two variations of "mean," and seven variations of both "standard deviation" and "correlation." These variations exist because the simulation consists of a matrix of 10,000 scenarios, unlike history, which is a single scenario. The added dimensionality of the simulation creates the opportunity for these alternative measures, which are described in the following paragraphs. In each case, the statistics are calculated for an N year time horizon, where N can be as large as the length of the simulation—30 years for the pre-packaged scenarios.

The year-by-year standard deviations can be used to determine whether the average volatility changes over time.

Mean return: Mean return can be expressed either as arithmetic mean or annualized compound mean. The annualized compound mean is also known as "geometric mean" or "compound mean." Arithmetic mean return for the N year horizon is the simple average of the N x 10,000 observations. For annualized compound mean, the annualized compound return is calculated for each scenario and then the arithmetic mean of the 10,000 annualized compound returns is calculated. (Note that the compound mean is actually the arithmetic mean of the geometric means for all scenarios.)

Arithmetic Mean = average (10,000 x N observations, taken individually)

Compound Mean = average (Annualized Compound Return of each scenario, for 10,000 scenarios)

Mean Cumulative Return: Mean cumulative return is calculated by taking the arithmetic average return for each scenario and then taking the average of the 10,000 averages. The mean cumulative return equals the arithmetic mean return for the same time horizon. The standard deviation of the mean cumulative return will approximate the standard deviation of compound returns.

Mean Cumulative Return = average (arithmetic average of each scenario, for 10,000 scenarios)

Mean Wealth:[3] Return can also be expressed as the total of initial principal and growth, or accumulated value, with investment growth reinvested. This is equal to 1.0 plus the non-annualized compound return. Note that annualizing is non-linear and the Nth root of mean wealth is generally not equal to the mean compound return.

Mean Wealth = average (accumulated value of each scenario, for 10,000 scenarios)

Standard Deviation: There are three variations of standard deviation that relate to annual returns and one variation that relates to compound returns. The simplest measure is the global standard deviation, which is the standard deviation[4] of the N x 10,000 observations, taken individually. Two other measures access the simulation data as a two dimensional matrix. The longitudinal standard deviation represents the average standard deviation over N years (however, it is calculated by averaging variances); the cross sectional standard deviation represents the average standard deviation over the 10,000 simulations. Where the simulation data is a result of an i.i.d. process, all three annual measures will be approximately equal. When there are trends or other connections among the years, the statistics may vary. Since the standard deviation of the N years of a scenario is conceptually identical to the standard deviation of an N year historical period, the longitudinal standard deviation is typically considered to be the best comparator to history. Note that the longitudinal standard deviation uses the standard deviation

3) The distribution of wealth, particularly at the tails, is the focus of the RBC requirements.

4) Opinions differ as to usage of the sample or population variations of the standard deviation. The sample standard deviation has the benefit that the global, cross sectional and longitudinal standard deviations would all be equal for data consisting of white noise.

or variance of each scenario, and it is also possible to calculate the standard deviation of the standard deviations. Ideally, the distribution of standard deviations in the simulation scenarios would be comparable to the range of standard deviations seen in history. The year-by-year standard deviations (i.e., the basis of the cross sectional standard deviation) can be used to determine whether the average volatility changes over time. (Although means are linear, readers should note that standard deviations are not.)

The standard deviation of compound returns takes the standard deviation of the annualized compound return in each scenario. If the observations are i.i.d, then the standard deviation of compound returns would be approximately equal to the global standard deviation of annual returns, divided by the square root of N, the time horizon. In the presence of positive (negative) serial correlation, the standard deviation of compound returns will be higher (lower) than that approximation.

Global Standard Deviation = Standard Deviation (N x 10,000 observations, taken individually)

Longitudinal Standard Deviation =
(average (variance of each scenario for 10,000 scenarios))$^{.5}$

Cross sectional Standard Deviation =
(average (variance of each year for N years))$^{.5}$

Standard Deviation of Compound Returns =
standard deviation (compound return of each scenario, for 10,000 scenarios)

Standard Deviation of Wealth: The standard deviation of wealth is the standard deviation of the accumulated value, over 10,000 scenarios.

Correlations: The variations of the correlation measure are parallel to the standard deviation measure. There are three measures of annual correlation and one measure of compound correlation. Of course, the correlation matrix is computed for each pair of asset classes, with parallel observations for each asset class —global, longitudinal, cross sectional or compound.

In the case of returns with i.i.d., correlation measures would be approximately equal. As with standard deviations, the longitudinal correlation is most comparable to historical statistics.

Global Correlation = correlation (N x 10,000 observations, taken individually)

Longitudinal Correlation = average (correlation for each scenario, for 10,000 scenarios)

Cross Sectional Correlation = average (correlation for each year, for N years)

Compound Correlation = correlation (compound return of each scenario, for 10,000 scenarios)

Serial Correlation: The serial correlation of annual returns may also be calculated. Typically, the global form of the statistic is calculated.

Serial Correlation = correlation (each return to the return of the prior year, for all 290,000 feasible observations)

Percentile Ranking of Results: It is very common for stochastic results to be presented in the form of percentile. For example, the 10th, 25th, 50th, 75th and 90th percentile of the return in each year or the compound return can be calculated. Percentiles are typically calculated on a global, cross sectional basis or compound basis; the longitudinal variant is rarely seen.

The standard deviation of compound returns takes the standard deviation of the annualized compound return in each scenario.

Examples of Detailed Statistics for the U.S. Equity Class

Table 1 shows the year-by-year distribution of U.S. equity returns in the prepackaged scenarios. The reported statistics are the mean return in each year, the standard deviation of the returns in each year, the minimum and maximum return in each year and the percentile of the return in each year. The statistics in the bottom row of the table are the arithmetic averages of the annual statistics. The average statistics for the minimum, maximum and percentile are informational, but possibly ambiguous, e.g., the average of

turn to page 10

From Page 9

each year's minimum return is not a particularly useful statistic. It does, however, provide one measure of the overall distribution. The bottom portion of the table reports three standard deviation measures for the 30-year time horizon. All three statistics are very similar, which implies that there are minimal trends or serial correlation in the underlying data.

Table 2 on page 11 shows the distribution of annualized compound returns over various time horizons. For example, the average 30-year compound return is 11.13 percent and the standard deviation of the 30-year returns (10,000 observations) is 3.49 percent. Some practitioners believe that the standard deviation of compound returns provides a misleading portrayal of risk. Others, including the author, believe that it is one measure of risk that should be evaluated along with the other measures of risk. The fact that the standard deviation of compound return is 3.49 percent should not be interpreted to mean that the asset

Table 1: Distribution of Simulated U.S. Equity Returns – Annual Returns

				Percentiles							
Year	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
1	12.50	17.50	-57.16	-31.41	-9.96	1.65	12.74	23.61	34.64	53.49	91.03
2	12.91	17.35	-51.41	-30.04	-8.72	1.98	12.94	23.94	34.30	54.03	91.82
3	12.43	17.58	-50.03	-30.62	-9.86	1.27	12.59	23.85	34.29	53.78	94.71
4	12.96	17.49	-53.82	-29.09	-9.12	1.90	13.02	23.97	34.73	55.90	97.21
5	12.82	17.18	-51.48	-30.97	-8.54	2.05	12.85	24.03	34.32	54.38	87.85
6	12.36	17.65	-60.15	-32.81	-9.54	1.55	12.58	23.49	34.21	53.49	91.68
7	12.28	17.52	-50.28	-30.37	-9.70	1.27	12.45	23.61	34.20	54.46	94.85
8	12.54	17.52	-50.29	-31.25	-9.72	1.68	12.62	23.68	34.12	54.37	85.12
9	12.42	17.42	-54.83	-30.50	-9.52	1.37	12.51	23.70	34.09	54.33	87.77
10	12.30	17.40	-55.73	-30.84	-9.69	1.28	12.40	23.40	33.94	54.81	84.05
15	12.68	17.76	-54.87	-31.50	-9.94	1.43	12.90	24.17	34.89	54.85	83.18
20	12.01	17.56	-64.27	-30.84	-9.96	1.39	12.01	23.43	34.89	53.80	96.03
25	12.42	17.49	-55.23	-30.26	-9.92	1.36	12.67	23.84	33.48	53.47	87.52
30	12.24	17.72	-51.36	-31.78	-10.21	1.10	12.39	23.32	34.40	55.53	90.33
Avg	12.49	17.52	-53.99	-30.80	-9.60	1.52	12.59	23.78	34.33	54.22	95.33

Summary	30-Year Standard Deviation
Cross Sectional	17.52
Longitudinal	17.49
Global	17.52

class has low risk. In fact, Table 1 shows that the annual standard deviation is approximately 17.5 percent. The statistical measures of volatility are completely consistent, as long as they are appropriately compared.

Inspection of Table 2 indicates that the mean compound return decreases with time horizon, even though the year-by-year data shown in Table 1 is reasonably constant. This has often confused new practitioners, as the decreasing trend seems contrary to the stated assumptions. In fact, it is a simple artifact of a non-tended simulation. The mean compound return can be approximated from the arithmetic return by subtracting half of the variance. In this example,

Estimated mean compound return = $.1249 - .5^{*}.1752^{2}$
= .1096,

which compares favorably to the actual 11.13 percent.

turn to page 12

Table 2: Distribution of Simulated U.S. Equity Returns – Compound Returns

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
1	12.50	17.50	-57.16	-31.41	-9.96	1.65	12.74	23.61	34.64	53.49	91.03
2	12.05	12.74	-38.54	-21.15	-4.36	4.17	12.50	20.52	27.72	40.58	61.95
3	11.72	10.62	-33.40	-15.14	-2.32	4.96	12.12	19.03	24.85	35.03	50.79
4	11.68	9.26	-24.98	-12.01	-0.50	5.84	12.03	18.03	22.91	32.42	49.05
5	11.63	8.28	-20.63	-9.26	1.02	6.30	12.06	17.28	21.85	30.05	40.94
6	11.52	7.62	-19.65	-7.95	1.63	6.61	11.81	16.80	21.00	28.49	40.30
7	11.42	7.08	-16.24	-6.16	2.14	6.81	11.71	16.30	20.23	27.06	41.09
8	11.39	6.64	-18.14	-4.89	2.63	7.12	11.59	16.02	19.61	25.70	37.90
9	11.35	6.30	-16.97	-4.44	2.99	7.20	11.57	15.73	19.19	25.23	36.08
10	11.30	6.00	-14.48	-3.64	3.54	7.40	11.51	15.41	18.72	24.67	35.03
15	11.21	4.93	-8.35	-0.91	4.74	7.96	11.38	14.58	17.36	21.98	31.22
20	11.13	4.27	-7.62	0.78	5.57	8.36	11.23	14.01	16.53	20.69	28.18
25	11.14	3.82	-4.96	1.77	6.20	8.64	11.27	13.75	15.93	19.67	26.62
30	11.13	3.49	-2.40	2.84	6.60	8.81	11.16	13.50	15.56	18.96	22.75

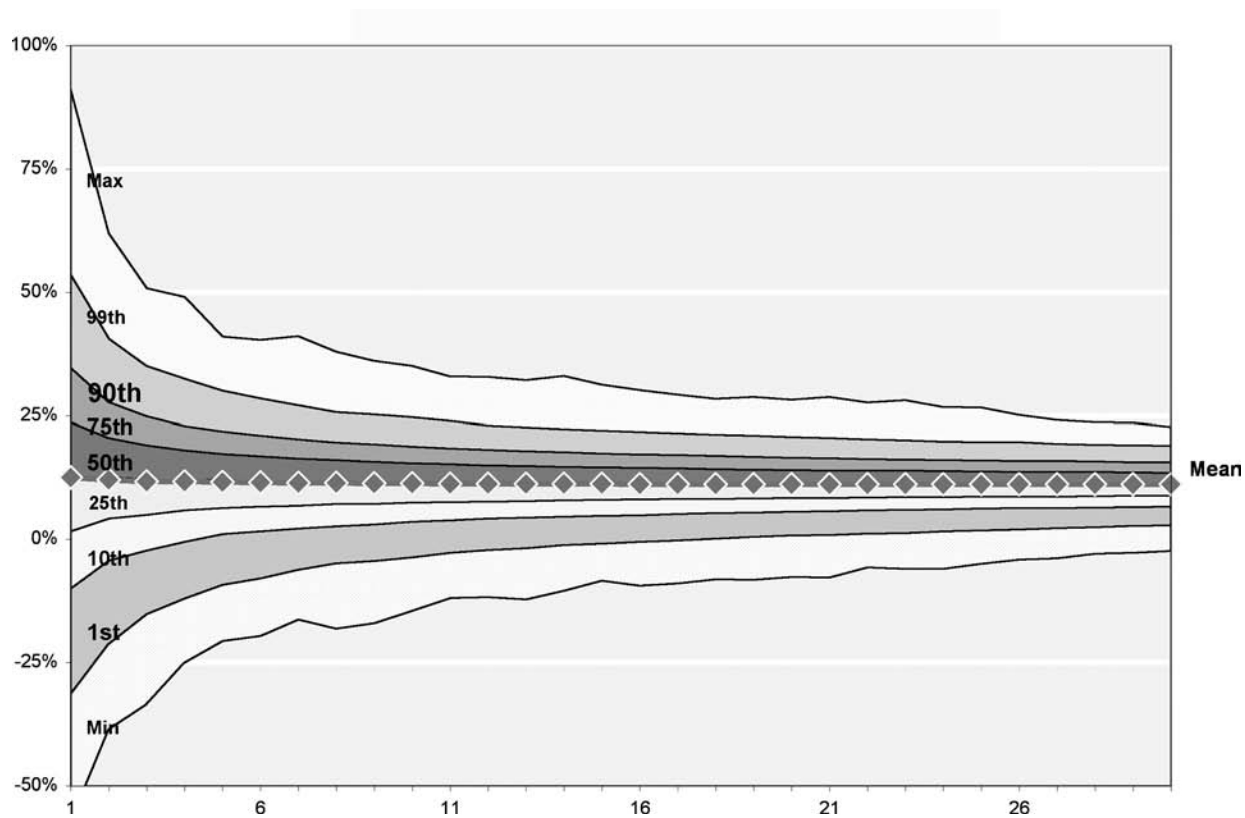
From Page 11

Chart 1 provides a graphic presentation of the distribution of compound returns for all years out to the 30th year. The chart shows that there is a fairly wide gap between the 99th percentile and the maximum observation and also between the first percentile and the minimum observation at each time horizon.

Table 3 illustrates the Cumulative Arithmetic Average Return. This statistic is similar to the compound return, except that the summation and division operations replace the product and root operations. The mean cumulative average return for each time horizon is exactly equal to the arithmetic average return for that time horizon. (For example, the 30-year average of 12.49 percent matches the 30-year average in Table 1.) The standard deviation of cumulative average return, on the other hand, is very close to the standard deviation of compound returns.

The next table on page 13 shows the distribution of accumulated wealth, which is the non-annualized

Chart 1: Distribution of U.S. Equity Returns Compound Returns



compound return. Note that the 10th root of the 10-year Mean Wealth is 12.70 percent, while the Mean 10-year Compound Return is 11.30 percent. This illustrates the principle that the average of the root is not equal to the root of the average, since the 10th root is a non-linear operation. On the other hand, the percentile of wealth and compound return line up one-one in rank order. Therefore, the 10th root of the median 10-year Wealth is 11.51 percent, which equals the median 10-year compound return. That relationship holds for all percentile.

The standard deviation of the 10-year Compound Return is 6.00 percent, while the global standard deviation in the 10th year (17.46 percent) divided by the square root of 10 is 5.52 percent. The fact that the standard deviation of compound return is greater than the estimate indicates the presence of a small amount of positive serial correlation.

turn to page 14

Table 3: Distribution of Simulated U.S. Equity Returns – Cumulative Arithmetic Average Return

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
10	12.55	5.82	-12.68	-1.61	5.09	8.75	12.70	16.51	19.79	25.78	36.16
15	12.52	4.78	-6.17	0.97	6.31	9.36	12.68	15.76	18.49	23.16	32.30
20	12.47	4.13	-5.80	2.75	7.13	9.72	12.57	15.24	17.72	21.83	29.15
25	12.50	3.70	-1.63	3.57	7.72	10.06	12.58	15.00	17.17	20.84	27.91
30	12.49	3.38	-0.47	4.50	8.12	10.26	12.49	14.80	16.81	20.15	24.30

From Page 13

Table 4: Distribution of Simulated U.S. Equity Returns – Wealth (Initial Value = 100)

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
1	112.5	17.5	42.8	68.6	90.0	101.7	112.7	123.6	134.6	153.5	191.0
2	127.2	28.3	37.8	62.2	91.5	108.5	126.6	145.2	163.1	197.6	262.3
3	143.2	39.6	29.5	61.1	93.2	115.6	141.0	168.6	194.6	246.2	342.9
4	161.9	52.2	31.7	60.0	98.0	125.5	157.5	194.0	228.2	307.5	493.5
5	182.8	65.8	31.5	61.5	105.2	135.7	176.7	221.9	268.6	372.0	556.1
6	205.7	82.2	26.9	60.9	110.2	146.8	195.4	253.9	313.8	450.0	762.7
7	231.3	100.6	28.9	64.1	116.0	158.6	217.0	287.8	363.2	534.6	1112.8
8	260.6	121.5	20.2	67.0	123.1	173.4	240.4	328.2	418.9	623.4	1307.4
9	293.7	147.1	18.7	66.5	130.4	187.0	267.9	372.3	485.4	757.7	1599.9
10	330.4	176.6	20.9	69.0	141.6	204.3	297.2	419.1	556.3	907.2	2014.9
15	598.3	406.5	27.0	87.2	200.3	315.6	503.6	770.1	1103.3	1968.6	5890.9
20	1076.7	876.7	20.5	116.9	295.4	498.4	840.1	1376.0	2133.4	4301.7	14334.9
25	1961.1	1861.2	28.0	155.1	450.0	793.6	1444.5	2505.6	4028.6	8911.2	36519.8
30	3556.5	3754.6	48.2	231.4	681.2	1260.2	2392.5	4463.0	7658.1	18269.0	46817.8

Chart 2: Distribution of Simulated U.S. Equity Returns Wealth

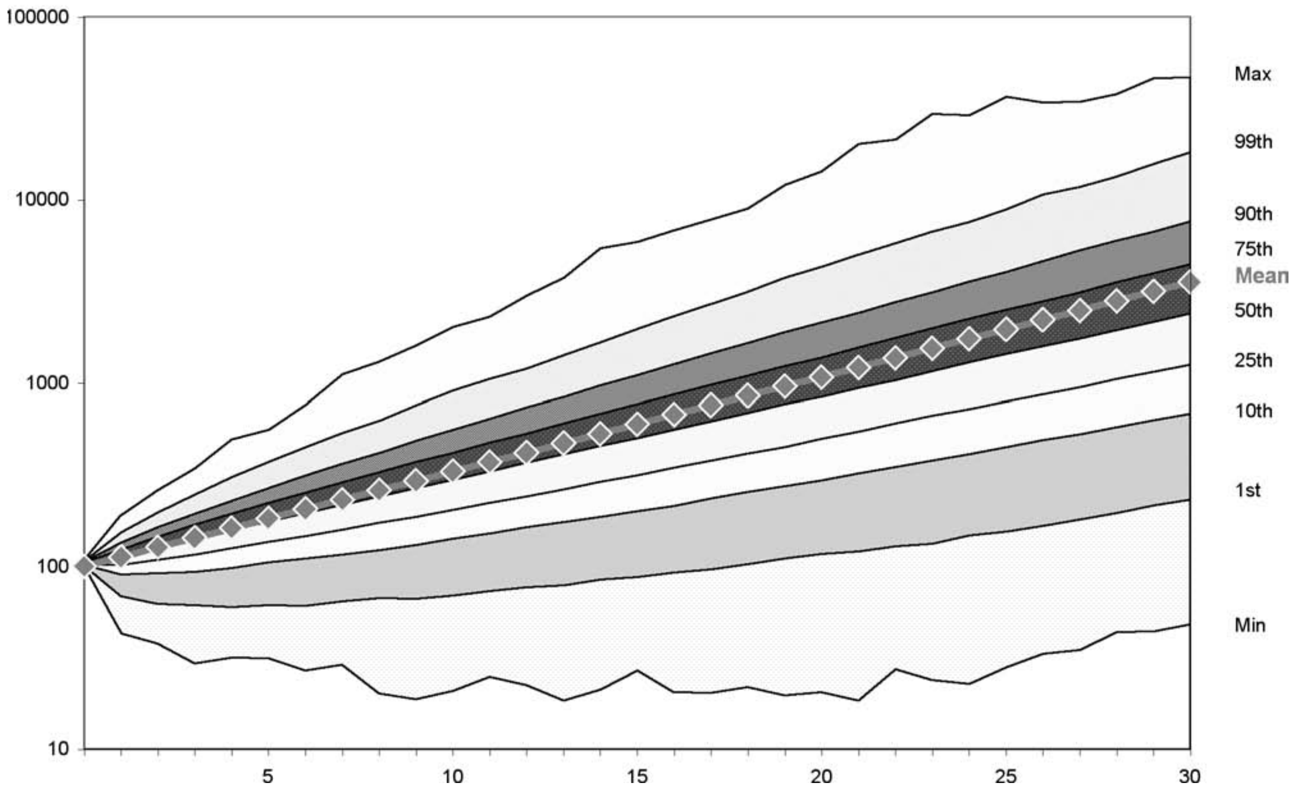


Chart 2 above illustrates the distributions of wealth for each time horizon.

As discussed in the definition of standard deviation, the standard deviation of the returns in each scenario can be calculated. For example, Table 5 shows that, over a 30-year horizon, the simulated standard deviations range from a minimum of 9.0 percent to a maximum of 27.7 percent. It is not surprising that a sample of 10,000 scenarios could have such a wide distribution; the 10th to the 90th percentile provide a more reasonable estimate of the range of the scenarios—from 14.3 percent to 20.9 percent. That is a relatively narrow range around the average of 17.5 percent. This table can be seen on page 16.

turn to page 16

From Page 15

Table 5: Distribution of Simulated U.S. Equity Returns – Longitudinal Standard Deviations

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
10	16.74	4.45	3.99	7.95	11.61	13.92	16.77	19.93	22.96	28.89	37.25
15	17.15	3.62	5.78	9.81	12.85	14.86	17.22	19.72	22.12	26.54	32.56
20	17.33	3.13	7.57	10.81	13.58	15.28	17.39	19.53	21.59	25.27	29.31
25	17.42	2.80	8.20	11.53	14.01	15.59	17.47	19.38	21.16	24.52	28.99
30	17.49	2.57	9.00	11.99	14.34	15.81	17.52	19.30	20.93	23.84	27.66

Table 6: Distribution of Simulated U.S. Equity Returns – Correlations to Long Corporate Bonds

Horizon	Annual	Cross Sectional	Longitudinal	Global	Cumulative Average	Compound	Wealth
10	-0.02	-0.03	-0.02	-0.03	-0.14	-0.13	-0.13
15	-0.02	-0.03	-0.02	-0.03	-0.18	-0.17	-0.16
20	-0.04	-0.03	-0.03	-0.03	-0.20	-0.19	-0.16
25	-0.04	-0.03	-0.03	-0.03	-0.21	-0.20	-0.16
30	-0.02	-0.03	-0.03	-0.03	-0.21	-0.21	-0.15

Table 6 shows seven measures of correlation between U.S. Equity and Long Corporate Bonds. While the four correlations of annual returns are close to 0.0, the three correlations of compound returns are negative.

Comparison of Prepackaged Scenarios to Current Forecasting Practice

This section compares the summary statistics for the prepackaged scenarios to assumptions commonly used in ALM studies for pension plans and other institutional investors. The purpose of the prepackaged scenarios is to evaluate the risk for annuity guarantees at the extreme tails of the distribution, while pension plans and institutional investors are more concerned about the middle of the distribution. Therefore, it is not surprising that there are some incompatibilities between the prepackaged scenarios and assumptions typically used for pension forecasting. This analysis does not comment on the utility of the scenarios for their intended purpose.

Table 7 below shows the 10-year means and standard deviations for the seven basic asset classes in the prepackaged scenarios:

The average U.S. Equity returns are much higher than current mainstream forecasting assumptions and the fixed income returns are somewhat lower than current practice. The spread between the equity and fixed-income returns, which represents the equity risk premium, is much higher than current practice.

turn to page 18

Table 7: 10-Year Means and Standard Deviations

Asset Class	Arithmetic Average Return	Longitudinal Standard Deviation	Compound Return
U.S. Equity	12.6%	16.5%	11.3%
Intermediate Risk Equity	15.3%	22.8%	12.9%
Aggressive Equity	17.6%	28.3%	13.9%
International Equity	12.7%	18.7%	11.1%
Intermediate Term Government Bonds	4.0%	3.6%	3.9%
Long-Term Corp Bonds	4.2%	5.4%	4.0%
Money Market	2.6%	1.0%	2.6%

From Page 17

On December 31, 2003, 30-year Treasury yields were about 5 percent. One reasonable approach for setting asset class assumptions would be to estimate 10-year long Treasury returns at the initial yield level. One would then expect long corporate bonds to be higher and intermediate term government bonds to be lower than the long Treasury return. Reasonable mainstream assumptions might be 4.5 percent for intermediate government bond returns and 5.5 percent for long corporate bond returns. Those estimates would indicate that simulated intermediate government returns are about 0.6 percent too low and that simulated long corporate bond returns are about 1.5 percent too low.

The risk premium for large cap stock over long T-Bond returns has typically been in the 2 percent to 4 percent range for the last 45 years. A common estimate for today's forecasting practice is about at the 3 percent level. The simulated U.S. Equity class has a spread of 7.4 percent and 7.3 percent over the intermediate government bond class and the long corporate bond class, respectively. That degree of spread is significantly above current forecasting practice. In nominal terms, expectations for large cap stocks are commonly in the 8 percent to 9 percent range; again significantly below the simulated nominal returns.

The historical standard deviation of long corporate bonds has ranged from about 1 percent in the 1940s to 19 percent in the 1980s. Volatility assumptions for long corporate bonds in current practice range from 7 to 13 percent, as compared to 5.4 percent for the prepackaged scenarios.

Table 8 shows the 10-year longitudinal correlations for the seven basic asset classes.

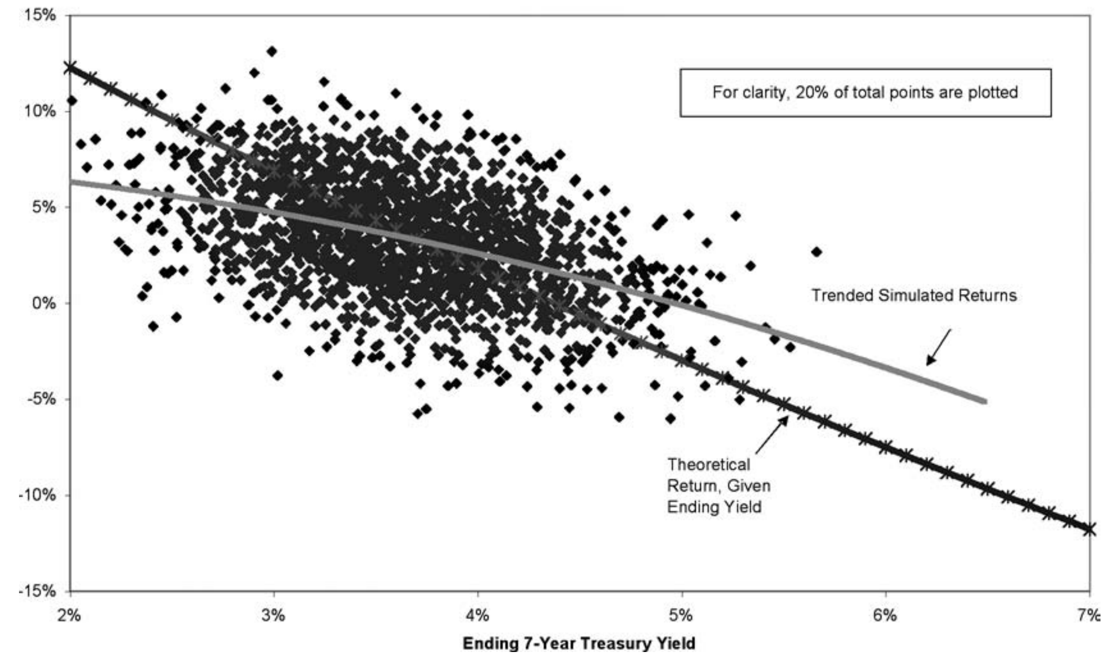
The correlation between U.S. Equity and the bond categories is significantly lower than current practice. For example, the prepackaged scenarios have a correlation of 0.00 between U.S. Equity and long corporate bonds, indicating a lack of correlation. However, most practitioners are using correlations in the 0.30 to 0.50 range, which indicates a lower degree of diversification than in the simulation.

The final analysis is to test the relationship of bond yields to bond returns. In most pension ALM

Table 8: 10-Year Longitudinal Correlations

Longitudinal 10-Year Correlations	U.S. Equity	Inter Risk Equity	Aggr Equity	Intl Equity	Inter Term Govt	Long Term Corp	Money Market
U.S. Equity	1.00						
Inter Risk Equity	0.75	1.00					
Aggressive Equity	0.65	0.65	1.00				
International Equity	0.60	0.50	0.55	1.00			
Intermediate Term Government Bond	0.10	0.10	0.05	0.15	1.00		
Long Term Corporate Bond	0.00	0.00	0.00	0.05	0.80	1.00	
Money Market	0.00	(0.05)	0.00	(0.05)	0.15	0.10	1.00

Chart 3: Bond Returns vs. Ending Yield
Year 1



forecasts, the liability discount rate is modeled by referencing a specified bond yield. If bond yields and bond returns are not synchronized with each other, then it is likely that the liabilities will not be synchronized with asset returns. This relationship is of utmost concern in a pension ALM forecast, as well as for other types of investors.

Chart 3 is an XY scatter chart of the first year simulated Intermediate Government Bond Return versus the simulated 7-Year Treasury yield at the end of the first year. Given the starting yield of 3.63 percent, it is possible to make reasonable estimates of the theoretical return for the various levels of ending yield.[7] In the prepackaged scenarios, the simulated bond returns have significant differences from the theoretical returns and the trend line has a concave shape instead of the desired slightly convex shape. Consequently, it does not appear that the yields should be used to model pension liabilities that act like bonds. This is not surprising, since the yields were designed for models of account balances and not for taking present values.

Summary

This article has illustrated and defined the multiple statistics that provide a summary of a stochastic forecast and has provided the actual results for the set of 10,000 prepackaged scenarios prepared by the LCAS. In addition, the article compared the simulation results to current practice for ALM studies for pension plans and other institutional investors and found that there were a number of differences. The expected return for U.S. Equity, the spread of U.S. Equity returns over bond returns and the correlation between U.S. Equity and bonds were the most notable differences. We also found that the simulated yields should not be used to model liability discount rates.

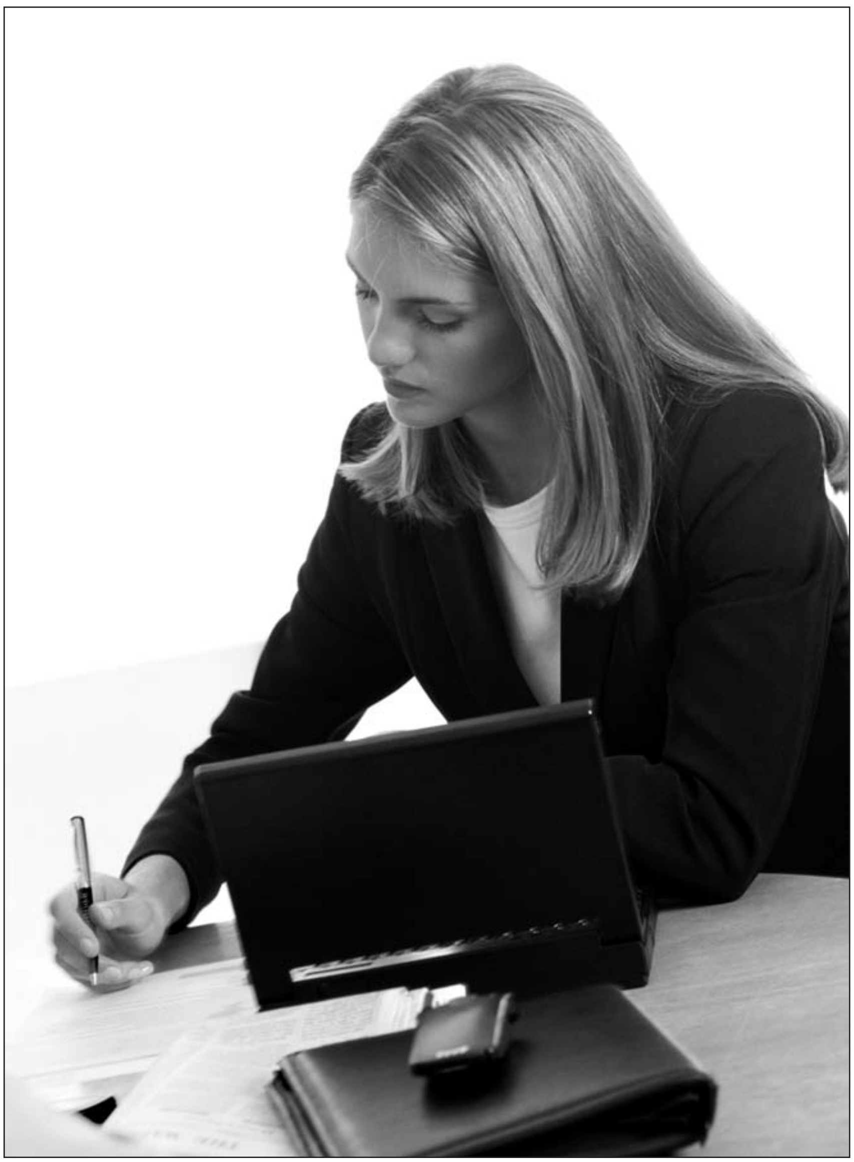
Richard Q. Wendt, FSA, MAAA, FCA, is principal at Towers Perrin in Philadelphia, Pa. He is also a member of the Society of Actuaries' Board of Governors. He can be reached at Richard.Q.Wendt@towersperrin.com.

*7) The theoretical yield is estimated by the equation (1+.25 * (0.363+ending yield)^2 * (Price of a six-year bond with coupon of 3.63 percent and yield equal to the ending yeld/100) - 1.*

Taking Stock: Will Hedge Funds Change Investment Thinking?

by Nino A. Boezio

Hedge funds have grown tremendously over the last few years and will continue to grow, with invested assets currently being estimated at well over $1 trillion worldwide. Hedge funds often boast the ability to make money in both rising and falling markets (even though they may not always be able to do so). If equity markets continue to go sideways for the next several years, hedge funds could provide an attractive alternative to other investment strategies, or can become a good compliment to existing asset classes. But regardless of what one feels about hedge funds, what role will they ultimately play in the investment industry? How are they going to influence our views on investing as we move into the future? Have they already influenced our views on investing, perhaps inappropriately?



Traditional Investment Management

Anyone who has read traditional investment literature has been taught various principles and guidelines on investing, which may include the following:

- Abiding by an investment policy is normally the best determinant of investment performance, not stock picking;
- Diversification mitigates risk;
- Stay invested—do not go into cash for there will be times when you likely will exit the market at the wrong time;
- Market timing is virtually impossible, at least for most, and even if some claim they can do it, they cannot exercise it correctly all the time;
- Most investment managers do not necessarily have skill, but can at least achieve returns close to the benchmark. However, it is often hard to identify the skilled managers in advance, and not even the mediocre investment managers will ever admit they have no skill;
- Good prudence for a portfolio is to have one dollar of investment assets supporting each dollar invested by the client (i.e. no leverage);
- Do not short securities—this is normally a loser's game, and the gain from the downside is limited while the loss from the upside is unlimited;
- Some even discourage the use of options and other derivatives in a portfolio, feeling that it produces unnecessary risk versus the reward.

You may not agree with all of the above points, but in general, these are some of the general beliefs underlying traditional portfolio management. These also often form much of the underlying basis as to why investment policies are written the way they are, and why certain investment activities are restricted or forbidden.

Are Hedge Funds Doing Anything Special Or Just Claiming So?

Considering the traditional views of portfolio management identified above, there are a number of claims that hedge funds make, which can include one or more of the following:

- No Need for an Actual Investment Policy Statement and/or to Abide By It—some hedge funds may engage in a number of strategies at the same time, whichever strategy is felt to be most appropriate for the market conditions that are present. Hence the hedge fund may stop doing merger arbitrage and engage in shorting if the stock market turns down and the economy changes direction. A specific investment policy reduces the flexibility of a fund to take the appropriate strategy as the markets evolve and fluctuate, and due to this lack of flexibility, reduces potential return.
- Concentration—the hedge fund need not diversify as broadly, but may focus a large part of the portfolio in a few limited investments. Concentration helps achieve value-added returns, especially when the research strongly supports the current view. Diversification unfortunately can dilute investment performance to mediocrity in some cases, even if some of the bets were "very" right.
- Raising Cash—there will be times when it pays not to be invested. Hence raising cash is a viable investment strategy when either opportunities do not exist, or there is considerable market uncertainty.
- Market Timing—hedge funds claim they can alter the asset mix successfully to anticipate market conditions, which may include major tactical shifts in particular market environments
- Special Skill or Knowledge—the managers of the hedge fund have particular talent or knowledge that is inaccessible to the average portfolio manager. It may employ computer models in its analyses, which no one else has. However, there are so many hedge funds out there, you would tend to think there is a considerable amount of skill everywhere (so who is the dumb investor taking the opposite side of each of the hedge funds trades?)
- Use of Leverage—some hedge funds employ a leverage factor of as much as 1.5x invested assets, claiming that not all of the invested monies are required for the selected security positions. The hedge fund organization may use its credit standing to borrow the extra leverage. The hedge fund also requires leverage in order to promise the higher than normally expected returns suggested from traditional investment management, while at the same time possibly promising a return of principal at worst, at the end of a specified period.
- Ability to Short—Hedge funds claim they can identify overpriced securities and can short them successfully;
- Derivatives—the hedge fund will claim the ability to employ derivatives effectively and easily, to produce added value performance.

... raising cash is a viable investment strategy when either opportunities do not exist, or there is considerable market uncertainty.

If you notice this latter list of items, they are the opposite views of what I itemized above for traditional investment management. Which views are right?

Can A Case Be Made For Special Skill?

I had a notable conversation with a "value" portfolio manager several years ago. The Canadian tech stock darling Nortel, was trading in the $60-70 price range at the time, and it had been trading at over $100, less than six months earlier (Canadian dollars). I was not following the stock, but I figured that it might be a buying opportunity for many portfolio managers, given that it already was off quite a bit from its recent highs. However, she told me that according to her valuations, the stock was not even worth $30. I was surprised. I did not think that such a widely followed and well-publicized stock could be so grossly mispriced. Of course, I never looked at the financial statements and never tried to do my own valuation.

Now in hindsight, as we know, this portfolio manager was right. The stock had tumbled to trade well under $1 per share, and has now rebounded to trade at about $5 (Canadian). However, this portfolio manager, with all her wisdom (and she was a 1st quartile manager, achieving positive returns during the market decline of 2000-2002) did not even make a penny on her insight. She could only buy stocks "long" in her portfolio, and thus could not profit from her knowledge that a certain stock was grossly overvalued. She could not short, since that was against her investment mandate and the investment policy guidelines.

So a case can be made that some of the ideas supporting traditional investment management are flawed, given that various restrictions are placed on investment managers, and whether through

turn to page 22

From Page 21

investment policy or regulation, an investment manager is inhibited from making additional choices. However, I do think traditional investment management has got it right—there may be a few portfolio managers out there who have the skill, wisdom and the foresight to enhance returns if they were not too unduly restricted, but in the main, relaxing such restrictions may help some achieve better returns while allowing most others to mess it up—now finding more ways to get it wrong and destroy a portfolio's performance. I have noted from my own experiences that skill exists among the few, not the many, even though the many will claim they have the skill. What may help some investment managers to improve performance will hurt most investment managers who now will try to do more things, and will even become more distracted from what they are able to do best, simply because they may want to compete with hedge funds. I also do not believe too many hedge funds in general have got the skilled management to produce the returns they promised, even though they all claim to do so. Interestingly, many hedge funds do not last two years, while the good ones tend to close since they get too much money, which is an admission that the hedge fund label does not imply that skill commonly exists. But as it was with Internet stocks not too long ago, some see the hedge fund label as a new strategy (or should I say new fad) to get additional returns.

Hedge funds do provide a good diversification tool and do allow certain pools such as pension funds to get into strategies and markets...

Nino Boezio, FSA, FCIA, is chief editor of this issue and is a consulting actuary at Matheis Associates in Pickering, Ontario. He can be reached at nboezio@sympatico.ca.

Hedge Funds Will Put Pressure on Traditional Investment Management

Regardless of my comments above, as long as hedge funds continue to dominate the investment marketplace and continue to take in substantial inflows of cash, and as this cash continues to be drained away from traditional investment managers, there will be pressure on certain traditional investment managers to ask their clients to relax any restrictions they are now operating under. For some managers this may be a good thing, but for most if they also get such relief, it could be a bad thing—and they may just be asking for this relaxation simply in order to try and play on a level playing field with hedge funds. I feel that hedge funds are doing a disservice to the investment industry in general (unless the investment public becomes well-educated on the matter, which normally never happens) as certain hedge funds appear to suggest that they can generate returns under all market conditions.

Hedge funds have been opening the door for those without skill to endeavour at selling services and strategies that they are not capable of adding value in, and such people are also being lured by the glitzy image that hedge funds have these days, and the promises of higher compensation than is otherwise available through traditional investment management. We therefore have to be wary that the common sense and perhaps conservatively drafted principles of traditional investment management are still sound and valid principles of prudence have been time tested and still have not been proven wrong. Traditional investment management may seem boring, its returns considered more subdued, and its returns will still have its ups and down, but at the same time such principles cannot be abandoned for the "free lunch" sometimes implied by hedge funds. Hedge funds do provide a good diversification tool and do allow certain pools such as pension funds to get into strategies and markets that they otherwise may not have gotten into before, but at the same time, hedge funds have to be analyzed carefully.

I do strongly believe that the contradictory philosophies of traditional investment management versus hedge funds have to be carefully considered, reflected upon and evaluated, since they form important determinants in the returns that will ultimately be achieved, and will provide useful questions that need to be addressed in assessing the various skill sets and novel techniques that hedge funds claim to have and are employing. Only then should you be able to sleep at night knowing that your portfolio is positioned in such a way that it would not get killed by some unforeseen investment twist—we may not always have a little dog named Toto to pull the drapes away for us, from the venerable Wizard of Oz who is trying to dazzle with his gadgetry—so we have to do our own homework, and not believe what we hear or see is what we really will get.

Call For Articles

Risks & Rewards is your newsletter! Other practitioners are wrestling with the same investment-related questions you are, and they want to hear your ideas. This is the forum for new ideas on interest rates, equities, options, futures, MBS, real estate, trading and hedging strategies—as well as asset/liability management.

What is special about *R&R*? It speaks to the actuaries who are actively involved with investment decisions. Yes, there are complex developments in financial reporting; yes, there are new products; yes, there is a Risk Management Section, which opens the door to operational and enterprise risk management developments. But, investment expertise is fundamental and securities markets are changing all the time. *R&R* helps you keep up and contribute at a higher level.

So share your modeling acumen and vast experience by submitting an article. Or, if you notice an article worth sharing, let us know about it for reprint. The next publication deadline is :

May 30, 2005 for August publication

Or, contact our editors to discuss how you can develop an idea into a front-page article:

Nino Boezio
nboezio@sympatico.ca

Joe Koltisko
joseph.koltisko@aig.com

Dick Wendt
Richard.q.wendt@towersperrin.com

From Page 2

The sections will also become more involved with some traditional responsibilities including:

- Basic and continuing education,
- Research initiatives, and
- Experience studies.

This is not an all-inclusive list and the level of involvement in these activities will vary greatly from section to section. Much thought is being put into how the sections will be able to accomplish this work and in the end, each section will have the ability to structure itself as needed to meet its goals. One common theme is the need for help beyond the traditional nine section council members and SOA staff.

This is where you come in. The council is going to need help from the membership. Yes, if you're reading this, that's you. The success of our section and the Society of Actuaries as a whole has been and will continue to be dependent on participation from our members. This can occur in a variety of ways and much of this is yet to be defined, but no need to wait. I'd encourage you to contact Mike O'Connor (incoming chairperson) and express your willingness to get involved.

I personally found many advantages to being involved with the Investment Section Council, including developing a network of contacts with other experts in the field and learning from their perspectives, improved organizational and communication skills, and the opportunity to develop leadership skills. Most importantly I gave back to the organization which is responsible for creating value for our profession.

My time is up. Best wishes to this year's council—the future is bright!

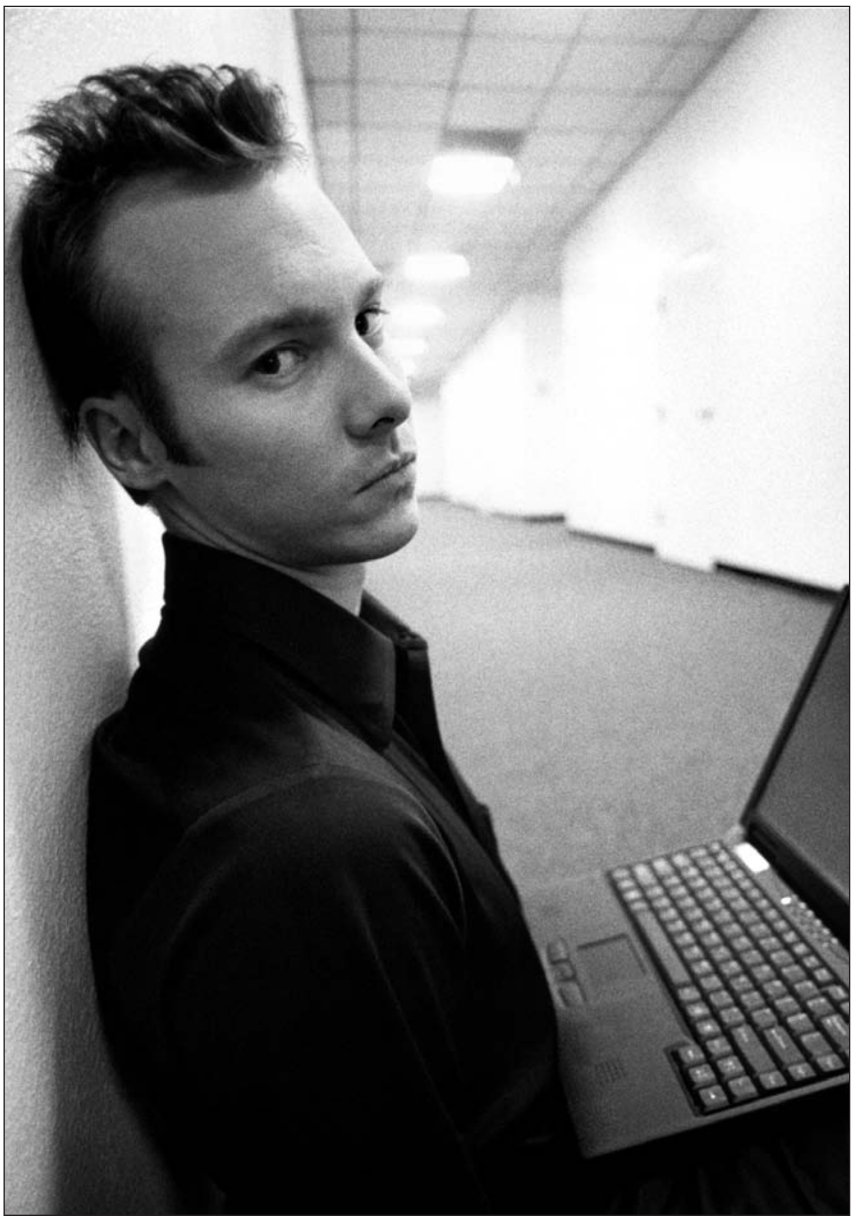
Mark W. Bursinger, FSA, MAAA, is vice president, risk management AEGON USA, Investment Management, LLC in Cedar Rapids, Iowa. He can be reached at mbursinger@aegonusa.com.

Another Perspective on Black-Scholes Option Formulas

by Mark D. Evans

This article shows a different form of the Black-Scholes formula for European calls and puts under risk-neutral assumptions, that permits a comfortable verbal interpretation. The derivation of this alternative form appears at the end of the article.

Consider the Black-Scholes equation for a European call option on a stock with initial price 1, no dividend, strike X, time to expiry t, and volatility σ. Let r be the risk-free rate. Let C be the value of a call option and P the value of a put option.



Let $a = [\ln(X) - (r - \sigma^2/2)t]/(\sigma\sqrt{t})$ and let $b = a - \sigma\sqrt{t}$.

First, consider the case $X = \exp(rt)$, i.e. the call is struck at the money forward. Then later we will show that:

$$C = (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon .$$

Now $a - b = \sigma\sqrt{t}$ and $(1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon$ is the probability under the normal distribution of a value being between b and a standard deviations. Then the area represented by the above integral has width equal to one standard deviation (adjusted for time) and height corresponding to the normal distribution frequency. In other words, we are integrating across the probability function by one standard deviation of the stock price, or the option price is equal to a one standard deviation move times the probability of a move that size. This has a nice intuitive feel.

Returning to the more general case where X can take any value, we will show that the equation can be written as,

$$C = (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon - (X - \exp(rt))\exp(-rt)(1/\sqrt{2\pi})\int_a^{-\infty} \exp(-\varepsilon^2/2)d\varepsilon .$$

The first term is equivalent to the result we received when $X = \exp(rt)$. The second term can be described as the difference between the strike and the at-the-money forward price times the probability that the option pays off. So the option price is a one standard deviation move times the probability of a move that size with an adjustment term adjusting for the difference between the actual strike and the at-the-money forward price.

A similar analysis exists for a European put option of price P.

Once again, consider the case $X = \exp(rt)$, i.e. the put is struck at-the-money forward. Then we will show that

$$P = (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon.$$

Returning to the more general case where X can take any value, we will show that the equation can be written as,

$$P = (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon - (X - \exp(rt))\exp(-rt)$$
$$(1/\sqrt{2\pi})\int_{-\infty}^a \exp(-\varepsilon^2/2)d\varepsilon.$$

These results have analogous interpretations to the call results.

The remainder of this article shows how the above formulas are developed.

Under the lognormal assumption for stock price change we can represent the value of the stock at time *t* as:

$$\exp[(r - \sigma^2/2)t + \varepsilon\sigma\sqrt{t}\,]$$

Then we can write the value of the call option, as:

$$C = (1/\sqrt{2\pi})\int_a^\infty \{\exp[(r - \sigma^2/2)t + \varepsilon\sigma\sqrt{t}\,] - X\}$$
$$\exp(-rt)\exp(-\varepsilon^2/2)d\varepsilon.$$

Manipulating the equation for C,

$$C = (1/\sqrt{2\pi})\int_a^\infty \exp[-(\varepsilon^2 - 2\varepsilon\sigma\sqrt{t} + \sigma^2 t)/2]d\varepsilon$$
$$- \exp(-rt)\,(X/\sqrt{2\pi})\int_a^\infty \exp(-\varepsilon^2/2)d\varepsilon.$$

If we make the substitution $y = \varepsilon - \sigma\sqrt{t}$ into the first integral above, then we have:

$$C = (1/\sqrt{2\pi})\int_b^\infty \exp[-y^2/2]dy - \exp(-rt)\,(X/2\sqrt{\pi})$$
$$\int_a^\infty \exp(-\varepsilon^2/2)d\varepsilon.$$

Next we substitute $\varepsilon = y$ to facilitate further development which yields,

$$C = (1/\sqrt{2\pi})\int_b^\infty \exp[-\varepsilon^2/2]d\varepsilon - \exp(-rt)\,(X/\sqrt{2\pi})$$
$$\int_a^\infty \exp(-\varepsilon^2/2)d\varepsilon.$$

Further manipulation leads to the usual expression for the Black-Scholes equation for a call option, but heading in a different direction allows this formula to be viewed as discussed above.

First, consider the case $X = \exp(rt)$. Then,

$$C = (1/\sqrt{2\pi})\int_b^\infty \exp[-\varepsilon^2/2]d\varepsilon - (1/\sqrt{2\pi})\int_a^\infty \exp$$
$$(-\varepsilon^2/2)d\varepsilon.$$

$$= (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon.$$

Returning to the more general case where X can take any value, the equation can be written as,

$$C = (1/\sqrt{2\pi})\int_b^\infty \exp[-\varepsilon^2/2]d\varepsilon - (X - \exp(rt))\exp(-rt)$$
$$(1/\sqrt{2\pi})\int_a^\infty \exp(-\varepsilon^2/2)d\varepsilon - \exp(rt))\exp(-rt)\,(1/\sqrt{2\pi})\int_a^\infty$$
$$\exp(-\varepsilon^2/2)d\varepsilon.$$

$$= (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon - (X - \exp(rt))\exp(-rt)\,(1/\sqrt{2\pi})$$
$$\int_a^\infty \exp(-\varepsilon^2/2)d\varepsilon.$$

A similar derivation exists for a European put option of price P.

$$P = (1/\sqrt{2\pi})\int_{-\infty}^a \{X - \exp[(r - \sigma^2/2)t + \varepsilon\sigma\sqrt{t}\,]\}\exp(-rt)$$
$$\exp(-\varepsilon^2/2)d\varepsilon.$$
$$= \exp(-rt)\,(X/\sqrt{2\pi})\int_{-\infty}^a \exp(-\varepsilon^2/2)d\varepsilon$$
$$- (1/\sqrt{2\pi})\int_{-\infty}^a \exp[-(\varepsilon^2 - 2\varepsilon\sigma\sqrt{t} + \sigma^2 t)/2]d\varepsilon.$$

$$= \exp(-rt)\,(X/\sqrt{2\pi})\int_{-\infty}^a \exp(-\varepsilon^2/2)d\varepsilon - (1/\sqrt{2\pi})$$
$$\int_{-\infty}^b \exp(-\varepsilon^2/2)d\varepsilon.$$

Once again, consider the case $X = \exp(rt)$. Then,

$$P = (1/\sqrt{2\pi})\int_{-\infty}^a \exp[-\varepsilon^2/2]d\varepsilon - (1/\sqrt{2\pi})\int_{-\infty}^b \exp(-\varepsilon^2/2)d\varepsilon.$$

$$= (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon.$$

This is the same result we obtained for the call option.

Returning to the more general case,

$$P = (X - \exp(rt))\exp(-rt)(1/\sqrt{2\pi})\int_{-\infty}^a \exp[-\varepsilon^2/2]d\varepsilon$$
$$+ \exp(rt))\exp(-rt)(1/\sqrt{2\pi})\int_{-\infty}^a \exp(-\varepsilon^2/2)d\varepsilon$$
$$- (1/\sqrt{2\pi})\int_{-\infty}^b \exp(-\varepsilon^2/2)d\varepsilon.$$

$$= (1/\sqrt{2\pi})\int_b^a \exp(-\varepsilon^2/2)d\varepsilon + (X - \exp(rt))\exp(-rt)$$
$$(1/\sqrt{2\pi})\int_{-\infty}^a \exp(-\varepsilon^2/2)d\varepsilon.$$

Mark D. Evans, FSA, MAAA, is vice president and actuary at AEGON USA Inc. in Louisville, Ky. He can be reached at 502-560-2338 or mevans@aegonusa.com.

Information Alerts for Investment Section Members

by Richard Q. Wendt

Section members should be aware of some recent investment-related happenings in the "outside world." The first happening occurred in November, with the Academy and the SOA Joint Task Force on Financial Economics and the Actuarial Model presenting a roundtable discussion on financial economics for pension plans. The second happening occurred in December, when the Morris Review released its interim report on the UK actuarial profession. Both of these events touch on investment issues.

The financial economics roundtable was held on Nov. 18 in New York City, with approximately 60 attendees from the pension consulting, academic, regulatory and accounting realms. Jeremy Gold, co-author of the paper that initiated the financial economics challenge to the actuarial model, was the principal presenter. Although much of the discussion focused on accounting for pension plans, asset allocation was a strong undercurrent to the session. Based on financial economics theory, Gold and others proposed that pension plans should reduce or eliminate their equity exposure and use bonds to support their plan liabilities. Several reasons were cited in support of this proposition:

- The availability of tax arbitrage for the plan sponsor with pension funds allocated to bonds
- The reduction in risk to plan sponsors who could better match liabilities with bonds
- Shareholders would prefer not to invest in pension risk, but would prefer to select their own risks
- If plan sponsors were to buy back their own common stock, issue bonds and transfer pension holdings into bonds, capital markets would be more transparent and more efficient.

Further support for the proposition that plan sponsors should move to a bond portfolio was offered by the example of Boots, Inc., a UK plan sponsor that had moved its entire pension portfolio into bonds. Although Gold agreed that movement to a bond portfolio would likely result in an increase in pension costs, he argued that the higher costs represented the true cost of the benefits.

Interested readers may obtain further information on financial economics at the Pension Finance Resources page at the SOA Web site (*http://www.soa.org/ccm/content/areas-of-practice/special-interest-sections/pension/pension-finance/pension-finance-resources/)*

The second event was the release of the Morris Review Interim Report *(http://www.hm-treasury.gov.uk/independent_reviews/morris_review/review_morris_index.cfm)*. The Morris Review was established in the wake of the Equitable (UK) scandal, in which substantial criticism was leveled at actuaries. Lord Morris was directed to "focus on considering how best to modernise the profession and see that high standards are delivered in a more open, challenging and accountable professional culture."

The Executive Summary to the Interim Report includes the following statements:

10. Actuaries have been criticised in this context on three main grounds. These are: first, failing to allow adequately for the persistently downward path of inflation and interest rates in the 1990's; second, failing to allow adequately for the subsequent precipitate fall in the stock market; and third, more generally, for not questioning sufficiently the prevailing orthodoxy at the time that high equity returns could be expected to provide healthy long-term returns but with a degree of confidence only appropriate to bond investments. These criticisms may be thought all the more telling in that actuaries were relied upon as being particularly well qualified to assess such matters.

11. Against this, actuarial expertise must not be confused with an ability to forecast the future. Moreover, an actuary who, in the early 1990s, persisted with forecasts of inflation and interest rates that in the event turned out to be correct would at that time have lost a substantial amount of credibility. But actuarial work emphasising such outcomes as (rather remote) possibilities would have been unlikely to have much impact.

12. A more sustainable view, however, is that actuaries, as the relevant experts, were too slow to adjust to the changing circumstances; were, with some exceptions, too inflexible to consider or reflect sufficiently on the likelihood or the consequences of large adverse movements; and thereby provided, explicitly or implicitly, considerably more assurance to customers and consumers than was warranted.

In this writer's opinion, The Morris Review is setting a high, perhaps impossibly high, bar for actuarial performance in the investment arena. Not only should actuaries conservatively evaluate the risks and rewards in the market, they should strongly convey their views to other professionals and the general public. Of course, if investors did become more conservative, market values would likely suffer. It seems that Lord Morris not only wants actuaries to be smarter than the market, he wants actuaries somehow to inject caution into the market without affecting market movements.

As the Morris Review affects the global actuarial profession, several North American actuarial organizations are planning to submit comments.

Richard Q. Wendt, FLA, MAAA, FCA, is principal at Towers Perrin in Philadelphia, Pa. He is also a member of the Society of Actuaries' Board of Governors. He can be reached at Richard.Q.Wendt@towersperrin.com.

Annual Meeting Photos from New York City

Clockwise from mid-left: Mike O'Connor, Dick Wendt (standing), Lis Chinnock, Steve Stone, Ellen Cooper and Martin Le Roux all look on, as Dick Wendt leads the discussion at the SOA Annual Meeting back in October 2004 held in New York.

Retiring Sections Manager, Lois Chinnock, presents a gift to Joe Koltisko.

Annual Meeting Photos from New York City

Top left: Mark Bursinger (left) presents Doug George the "Risks and Rewards" best article prize for 2004.

Bottom right: members of the Investment Section pose for a group shot in New York. Standing: Mike O'Connor, Martin Le Roux, Steve Easson, Sean Casey, Nancy Bennett and Steve Stone. Seated: Ellen Cooper, Mark Bursinger, Cathy Ehrlich and Bryan Boudreau.

Investment Section breakfast speaker, Jim Ware, addresses the audience during the council meeting.

Above: Mark Bursinger and Steve Easson enjoy a laugh during a break at the meeting.

Bottom left: Dick Wendt (standing) addresses the council members. Clockwise: Lois Chinnock, Steve Stone, Mark Bursinger, Steve Easson and Mike O'Connor.