INVESTMENT SECTION
"A KNOWLEDGE COMMUNITY FOR THE SOCIETY OF ACTUARIES"
Issue No. 50 • August 2007

RISKS AND REWARDS

THE NEWSLETTER OF THE INVESTMENT SECTION

PUBLISHED IN SCHAUMBURG, ILL. BY THE SOCIETY OF ACTUARIES

Understanding the Black-Scholes Equation

By Steve Stone

A rigorous derivation of the Black-Scholes option pricing equation requires advanced mathematical techniques, such as using Ito's Lemma to solve a Stochastic Differential Equation.[1] Once the equation is derived, though, it is in a fairly simple functional form that can easily be programmed in Excel or other software. Unfortunately, because of its ease of use it has become a plug-and-play application. Often little time is devoted to understanding the equation and it is easy to fall into the trap of applying it without a solid knowledge of its fundamental properties. This article will attempt to provide some insights into the Black-Scholes equation using an informal, non-rigorous approach.

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[1] The equation can also be derived as the solution of a Partial Differential Equation.

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SOCIETY OF ACTUARIES

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Published by the Investment Section
of the Society of Actuaries

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Chairperson's Corner

by Cathy Ehrlich

Each year, the Investment Section Council seeks out current investment topics that are of interest to our members. These topics serve as a basis for sessions at SOA meetings, symposia, research and newsletter articles. The meetings at which we determine our list of topics can be wonderful opportunities to take a break from some of the more mundane aspects of my job and address some of the larger issues facing the industry. Instead, the meetings often make me feel overwhelmed by the number of and complexity of issues that I should be following, understanding and proactively managing. At other times I am unhappily reminded that the dizzyingly important issue that I may be losing sleep over is of little or no importance to most investment actuaries.

At these meetings, we also make a point to search for topics that are of interest to pension actuaries. Why is it that there seems to be so little overlap between topics that interest pension actuaries and topics that interest investment actuaries at insurance companies? True, the regulatory environment differs for pension plans and insurance companies. But if both types of entities are investing to fund long-term benefit payments, shouldn't there be strong similarities in the investment strategies? As a council, it is important for us to find the common ground for insurance company and pension plan investment actuaries. Finding this common ground will enable us to provide content on subjects that are of interest to all of our membership. If we address these topics in a way that engages our insurance company members as well as our pension members, we have a unique opportunity to add insight and further the development of the topic.

This year, we included Liability Driven Investing (LDI) on our list of important topics that are of interest to pension actuaries. For those of you who are not in the pension field, LDI is a portfolio management approach being adopted by pension plans that focuses on the risk relative to the liabilities when allocating assets. The approach recognizes that the ultimate measure of success of the pension plan is its ability to fund its future obligations. LDI strives to bring transparency to the measurement and management of the plan's investment risk.

The need to fund future obligations is just as important for insurance companies as it is for pension plans. Although transparency in both the measurement and management of risk is not currently a hallmark of insurance company processes, it certainly is a desirable goal. Clearly, LDI could be equally applicable to insurance companies.

The Investment Section will be promoting the discussion of LDI for both pension plans and insurance companies through all the avenues open to us. We hope this will pave the way for more topics of interest to all.

In 2008, the Investment Symposium will continue this effort by offering topics that are designed to be of interest to pension actuaries. We hope to offer these topics in a way that will also be useful to those outside the pension field. Since the agenda for this symposium has been quite full, we are exploring ways of offering the additional topics without detracting from the tracks we now have.

Nicola Barrett will be chairing the 2008 Investment Symposium and Doug Andrews will be chairing the pension track. We are beginning the planning process now, so please contact them if you have thoughts on this topic or would like to get involved.

Cathy E. Ehrlich, FSA, MAAA, is an investment actuary with Swiss Re Life & Health America in Armonk, N.Y. She can be reached at Catherine_Ehrlich@swissre.com.

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The Black-Scholes Equation

The Black-Scholes equation for the value of a call option on a stock or stock index,[2] using the symbols defined in Appendix 1, is:

$$(1)\ C = S_0 \times e^{-qT} \times \Phi(d_1) - K \times e^{-rT} \times \Phi(d_2), \text{ where}$$

$$(2)\ d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2) \times T}{\sigma \times \sqrt{T}} \text{ and}$$

$$(3)\ d_2 = d_1 - \sigma \times \sqrt{T} = \frac{\ln(S_0/K) + (r - q - \sigma^2/2) \times T}{\sigma \times \sqrt{T}}.$$

All of the variables in equations (1), (2) and (3) have simple physical interpretations with the exception of the two cumulative normal distributions, $\Phi(d_1)$ and $\Phi(d_2)$. As cumulative normals these are probabilities, but of what? The key to understanding the Black-Scholes equation, it would seem, is to gain an understanding of these cumulative normal distributions and to provide a physical interpretation of them. Once these probabilities have been explained, we will also be able to gain an understanding of the key greeks, Δ and ρ, which also rely on the same probabilities.

$$(4)\ \Delta = e^{-qt} \times \Phi(d_1)$$

and

$$(5)\ \rho = T \times K \times e^{-rt} \times \Phi(d_2).$$

Far from being additional information, understanding these two greeks provides the basis for understanding the Black-Scholes equation.

Laying the Foundation

Our approach is to start with some premises that we will assume to be true and for which we will not provide proof. Each of the premises is reasonably straightforward, but to formally prove them would require the advanced mathematics we are trying to avoid.

The first two premises are direct results of the risk neutral valuation assumption.

Premise 1: ***All present values are calculated using the risk-free interest rate***.

Premise 2: ***The market price of all contingent cash flows is their expected value.***

In order to calculate expected values we need some sort of assumption of the distribution of stock prices at maturity of the option. This leads to the next premise.

Premise 3: ***Stock prices have a lognormal distribution.*** More details about this assumption are contained in Appendix 2.

Breaking the Black-Scholes Equation into Two

We now have the tools to start analyzing the Black-Scholes equation. In order to simplify the analysis we break the equation into its two component parts and analyze each separately. The two terms on the right hand side of the equation correspond to the two sides of the transaction that will take place if the option finishes in-the-money[3] and is physically settled.[4] The term containing S,

$$(6)\ S_0 \times \Phi(d_1) \times e^{-qT}$$

is related to the delivery of stock to the owner of the call[5] at maturity if the call is in-the-money. The term containing K

$$(7)\ - K \times \Phi(d_2) \times e^{-rT}$$

is related to the payment of the strike price by the owner of the call at maturity if the call is in-the-

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[2] Throughout the article I will refer to the option being on a stock, though most actuarial applications apply to stock indices.

[3] In-the-money means that $S_T>K$ for a call option. When the option is in-the-money at maturity it will be exercised.

[4] Index options are cash settled, so payments upon exercise of the option are nettled versus each other for a single payment of $Max(S_T-K,0)$. For physical settled options, such as listed options on individual stocks, both legs of the transaction occur upon exercise of the option as the owner of the call must pay K in order to have the stock delivered.

[5] All of the discussion in the article is from the point of view of the owner of the call, or the long position. All of the results are easily extended to the writer of the call, the short position, as well as to puts.

money. Each leg of the transaction can be evaluated separately to determine what $\Phi(d_1)$ and $\Phi(d_2)$ are.

Interpreting $-K \times \Phi(d_2) \times e^{-rT}$

We start our analysis with equation (7), the term involving K. Since K is a fixed amount and S_T is a random variable, equation (7) is easier to analyze and interpret than equation (6).

Our premises imply that this term should be the present value of the expected value of the payment of K by the owner of the option. The present value factor, e^{-rT}, is clearly identifiable in equation (7). If it is removed from equation (7), in order to get the expected value of the payment K at the maturity of the option, we get:

(8) $-K \times \Phi(d_2)$.

The expected value is a simple calculation because the payment of K will be made if the option finishes in-the-money and 0 otherwise, so the expected value is just –K[6] times the probability that the option finishes in-the-money. Based upon this understanding it is obvious upon inspection that $\Phi(d_2)$ is the probability of the option finishing in-the-money. This result is derived somewhat more formally in Appendix 3.

The owner of the option has implicitly borrowed $K x \Phi(d_2)$. This is the current expectation of the amount he will have to pay to purchase S at time T. Based on this understanding it is possible to interpret equation (7) as the price of a zero coupon bond with a notional amount equal to the expected payment, equation (8). If we calculate the duration of a zero coupon bond that has a fixed notional amount equal to $-K x \Phi(d_2)$[7] we get:

(9) $T \times K \times e^{-rT} \times \Phi(d_2)$

which is the same as the ρ of the option equation (5).[8] This demonstrates that the ρ of the option is not due to the risk-free rate's impact on the expected growth rate of the stock. The impact of the interest rate on the price of the option is entirely due to the impact on the implicit borrowing costs for the expected payment of the strike by the owner of the call.[9]

Interpreting $S_0 \times \Phi(d_1) \times e^{-qT}$

Complicating the understanding of equation (6) is that there is no visible discount factor and $\Phi(d_1)$ cannot be a simple probability. Since S_T is a random variable $\Phi(d_1)$ must be contributing to the calculation of an average value of S_T delivered to the owner of the call upon maturity of the option.

Let's back out the discount factor to determine expected value of S_T upon exercise of the call. Dividing equation (6) by the discount factor e^{-rT} results in:

(10) $e^{(r-q) \times T} \times S_0 \times \Phi(d_1)$.

We would like to verify that this is the expected value of S_T upon exercise of the call. In order to verify this we need to calculate the partial expectation[10] of S_T, which is a lognormal random variable, with respect to the threshold K. Fortunately, the formula for the partial expectation of a lognormal random variable with respect to a threshold is available. Appendix 4 provides full proof that equation (10) is the partial expectation of S_T with respect to the threshold K. In equation form we have:

(11) $E(S_T; S_T > K) = S_0 \times \exp^{((r-q) \times T)} \times \Phi(d_1)$.

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[6] The value is –K since it is a payment.

[7] This differs from a zero coupon bond with a notional equal to $K \times \Phi(d_2)$. Since we are holding the dollar amount of the notional fixed, the impact of a change in r on d_2 is not included.

[8] Our trick of fixing the dollar amount of the notional only works for the first order effect, which is ρ or the duration of the option. The convexity of the option does include an impact from a change in d_2 due to a change in r.

[9] The impact of the interest rate change on equation (6) is exactly offset by the impact of the change in r on d_2 in equation (7), leaving only the first order effect on equation (7) mentioned above.

[10] Partial expectations are extremely useful in option valuation. It differs from the better known concept of a conditional expectation. Mathematically they are related. The partial expectation is the conditional expectation times the probability of the event occurring.

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In summary we have concluded that $\Phi(d_1)$ has no direct interpretation as a probability, but it is a part of a partial expectation calculation. So equation (6) is the present value of the expected value of the stock that will be delivered to the owner of the call option if the option is in-the-money.

We have seen that the price of the option is just the present value of the expected value of each leg of the transaction that will occur if the option finishes in-the-money. These amounts are also the amount of cash needed to conduct the transactions necessary to hedge both Δ and ρ. … Since the combined option and hedge position requires zero cash to establish, it is referred to as a self-financing position. …

The relationship between equation (6) and Δ is even more direct than the relationship between equation (7) and ρ. As a matter of fact, equation (6) is the same as equation (4), the equation for Δ, except that it is multiplied by S_0. So Δ is the present value of the expected value of the number of shares of stock that will be delivered to the owner of the call option if the option is in-the-money. Multiplying by S_0 translates from the number of shares to a dollar value of the Δ hedge.

Insights into Price

We have seen that the price of the option is just the present value of the expected value of each leg of the transaction that will occur if the option finishes in-the-money. These amounts are also the amount of cash needed to conduct the transactions necessary to hedge both Δ and ρ. In equation form we get:

(12) Call Price = Value of Δ Hedge – Value of ρ Hedge

An owner of the call option would need to sell stock short to be Δ hedged and would need to purchase bonds to be ρ hedged. The funds generated by the sale of stock would exceed the funds used to purchase the zero coupon bond, leaving him with exactly enough money to purchase the call. Since the combined option and hedge position requires zero cash to establish, it is referred to as a self-financing position, which is an important concept in derivatives valuation.

We can rewrite equation (1) as:

(13) $C = S_0 \times \Delta + \rho/T = f(S_0, T, \Delta, \rho)$.

It is interesting that the value of an option is a function of ρ even though interest rates are assumed to be fixed though it is not a function of ν when σ is also assumed to be fixed. The key difference is that ρ quantifies the interest rate risk on the implicit borrowing that is needed to finance the expected payment of K made by the owner of the option, so it is directly related to one of the legs of the settlement of the option. σ only enters the equation indirectly as a parameter in the calculation of probabilities and expectations.

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Hull, John C. 2006. ***Options, Futures, and Other Derivatives***. Pearson Prentice Hall. 6th Edition.

Kwok, Yue-Kuen. 1999. ***Mathematical Models of Financial Derivatives.*** Springer Finance. 1st Edition.

Appendix 1

Inputs to and Outputs from Black-Scholes Equation	
Symbol	Input to Black-Scholes/*Output from Black-Scholes*
S_0	Current (Initial) Index Level
K	Strike Price
r	Interest Rate
q	Dividend Yield
σ	Volatility
T	Time until Maturity
S_T	Index Level at Option Maturity
C	*Call Price*
Δ	*Delta*
ρ	*Rho*
ν	*Vega*

Table of Functions	
Symbol	Function
$\phi(x)$	Standard Normal Probability Distribution
$\Phi(x)$	Standard Cumulative Normal Distribution
$\ln(x)$	Natural Logarithm
e^x	Exponential
$E(x; x>y)$	The Partial Expectation of x with respect to a threshold y

Appendix 2

The Lognormal Property of Stock Prices

The assumption that the distribution of stock prices is lognormal is equivalent to the assumption that the distribution of the logarithms of stock prices is normal. So substituting in $X=\ln(S_T)$ in order to keep the notation simpler we get:

(A1) $\ln(S_T) = X \sim \phi(\mu_X, \sigma_X)$.

In a risk-neutral valuation framework the expected return of all assets is the risk-free rate of return, r. This, in combination with the definition of a lognormal random variable's mean, gives:

(A2) $\mu_X = \ln(S_0) + (r - q - \frac{\sigma^2}{2}) \times T$

which is the average of the logarithms of stock prices at maturity. Also

(A3) $\sigma_X = \sigma \times \sqrt{T}$

is the standard deviation of the logarithms of stock prices at maturity.

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Appendix 3

The Probability of Finishing In-The-Money

The probability of finishing in-the-money can be calculated as the number of standard deviations the strike price is from the mean stock price at maturity. This calculation is easier to do using the logarithms of stock prices, as opposed to the stock prices themselves, due to the ability to use the normal distribution instead of the lognormal distribution. So,

(A4) Probability of finishing in-the-money = $\Phi(\frac{\mu_X - \ln(K)}{\sigma_X})$.

Substituting μ_X and σ_X as defined in (A2) and (A3) results in

(A5) Probability of finishing in-the-money = $\Phi(\frac{\ln(S_0) + (r - q - \frac{\sigma^2}{2}) \times T - \ln(K)}{\sigma \times \sqrt{T}})$

which simplifies to

(A6) Probability of finishing in-the-money = $\Phi(\frac{\ln(\frac{S_0}{K}) + (r - q - \frac{\sigma^2}{2}) \times T}{\sigma \times \sqrt{T}}) = \Phi(d_2)$

So d_2 is the number of standard deviations the strike price is from the mean and $\Phi(d_2)$ is the probability of the option finishing in-the-money.

Appendix 4

The Partial Expectation of the Stock Price

Based on (A1) we can write the equation for the partial expectation of the stock price, which is the equation for the partial expectation of a lognormal random variable, as:

(A7) $E(S_T; S_T > K) = \exp(\mu_x + \frac{\sigma_x^2}{2}) \times \Phi(\frac{\mu_x - \ln(K)}{\sigma_x} + \sigma_x)$

Substituting (A2) and (A3) into (A7) gives

(A8) $E(S_T; S_T > K) = \exp(\ln(S_0) + (r - q - \frac{\sigma^2}{2}) \times T + \frac{\sigma^2 \times T}{2}) \times \Phi(\frac{\ln(S_0) + (r - q - \frac{\sigma^2}{2}) \times T - \ln(K)}{\sigma \times \sqrt{T}} + \sigma \times \sqrt{T})$.

Simplifying, results in

(A9) $E(S_T; S_T > K) = S_0 \times \exp((r - q) \times T) \times \Phi(\frac{\ln(S_0/K) + (r - q + \frac{\sigma^2}{2}) \times T}{\sigma \times \sqrt{T}})$

and

(A10) $E(S_T; S_T > K) = S_0 \times \exp^{((r-q) \times T)} \times \Phi(d_1)$.

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Editor's Note

by Joe Koltisko

Dear Market Participant

Chances are, someone will be asking you, "how would bond traders deal with this pesky persistency and morbidity risk thing?" Indeed, what does the market think? Well, short of gathering the department together with the Ouija Board to pose the question directly to the shades of Charles Dow, Joe Kennedy, Sr. or Fischer Black, it's your chance to opine on the big questions. When the chance comes, be humble in speaking for that force of nature, the financial market. Recall that in ancient Phoenician mythology, hubris was punished by being condemned for an eternity locked in a conference room with a white board and a fresh, young ALM consultant eliciting responses to, "What is Risk?"

While there are market prices to work from for some estimates, that doesn't mean the market thinks at all. As Steve Scoles describes in this excellent issue of *Risks and Rewards*, guessing how many jelly beans are kept in a jar is a different kind of estimate from guessing how to fly an airplane. So, we bring you articles to help you fly right. This issue contains insight from Steve Stone on how the Black-Scholes formula works. We have Dick Mattison showing us how to apply deflators and Extreme Value Distribution to price options. Nicola Barrett and Valdimar Armann describe inflation swaps, a useful tool for managing pension portfolios. Aaron Meder helps us follow developments with the Pension Fitness Tracker. All great stuff!

What can you do with Fair Value? Well, when the stock market bubble sent AOL's share price through the roof management used it to buy Time Warner. The pre-merger market price of AOL's shares was not a good forecast of long-term fundamental earnings, or an assurance of the ability to deliver benefits to pensioners and policyholders 50 years from now. Using market prices does not mean, "Leave your brain at home."

FASB has done everyone a great service by helping define what is not a fair value measurement. FAS 157 points out a fair value measurement is what you would pay to transfer an obligation to a party that is actually able to assume it. Not to an investment bank! Not to your sister-in-law with the hot stock tips! So, in the reinsurance buyout market, which are the real market participants we need to think about in applying fair value techniques to insurance and pensions—what do real people really do?

FASB has done everyone a great service by helping define what is not a fair value measurement.

Real market participants make some allowance for earning a portion of the risk premium on a feasible risky benchmark portfolio, over the 50 years that funds are accumulated to pay benefits. The liabilities are illiquid and long term. Real market participants allow for spreads over LIBOR in their prices, and we see it every day. This contradicts a belief that God decided fair value means the liability discount rate equals the swap rate flat. Why is it "economic" to pretend we won't earn spread margins?

When the opportunity comes, use it for a pragmatic discussion about what the liability discount rate should be—so that on a fair value basis, earnings emerge as a signal of new information. It would be unfortunate if fair value measurements turn into another form of stat accounting, so that we need to coach the boss on why a loss on a fair value basis is still a good business decision.

Joe Koltisko, FSA, MAAA

The Wisdom of Crowds—A Better Way to Think About the Markets

by Steve Scoles

The stock market has fascinated me for a long time. As a youngster, I would often scour the financial pages. There were stocks whose prices fluctuated dramatically in very short periods of time. Vast sums of money were made and lost with seemingly little effort (but with much euphoria and pain). And there was no shortage of market commentators with a wide range of analysis and market predictions.

At the tender age of 15, with some trepidation, I ventured into my very first stock market investment. That was September of 1987. A month later, on October 19th, stock markets around the world had their largest one day drop ever with the Dow Jones Industrial Average falling 23.7 percent. It would later be called Black Monday. I still have vivid memories of watching a newscast that day with a scene of the floor traders desperately trading shares.

I was baffled. What the heck is this stock market? How do presumably smart adults participate in and create this crazy situation? And why did I just lose 30 percent on my investment?

That experience was almost 20 years ago. Since that time I have studied finance in university, wrote my actuarial exams (taking the investment track), and even taken some CFA exams. To be honest, with all of that study, I don't think any of that has really helped me get a deep understanding of the financial markets. And I'm not talking about the technical aspects of the markets such as specialists, market makers, clearinghouses and the like. No, I'm talking about the strange market behavior, the crazy swings, the sudden changes, and why it seems so difficult to beat the markets, but some investors can. The Efficient Markets Hypothesis and behavioral finance ideas have been useful, but I needed something more to get a big picture feel for the stock market.

A Better Way

Looking outside the usual mainstream financial writings, I have come across what I think is a far better way of thinking about the financial markets and the stock market in particular. A better framework of how markets work and don't work.

The basic idea here is recognizing that the stock market is really just a special case of a larger field of study—how collective systems work. Understanding the conditions required for these systems to function well and why they breakdown can tell you a lot about the behavior of the stock market.

What follows is part multiple book review and part overview of the ideas the authors present. The catchy name used for these ideas is: the wisdom of crowds.

Efficient Market Hypothesis

Before going into the wisdom of crowds ideas, it is helpful to review the Efficient Markets Hypothesis (EMH). The basic idea of the EMH is that rational, profit-maximizing parties competing in the market place will drive all available information into securities prices quickly. The prices that emerge are unbiased predictors of the future performance; it is not stating that the prices will turn out to be correct, just that the future variations will be random (and follow a normal distribution). And that it is not possible to beat the market on a risk-adjusted basis.

I feel the EMH is a very useful idea. However, I would suggest that it is only an approximation for market behavior; a reasonable starting point for explaining the markets. It does not give you a good feel for Black Monday or the tech bubble of the late 1990s or a myriad of other unusual happenings in the stock markets.

The Wisdom of Crowds

The *Wisdom of Crowds* is actually the title of a book by James Surowiecki that highlights how crowds, or collectives, can solve certain types of problems much better than individuals and experts. It covers the conditions under which collective thinking works well and when it doesn't. The book is an excellent introduction to these ideas.

(The title of the book is a play on the classic book, *Extraordinary Popular Delusions and the Madness of Crowds*, written by Charles Mackay in 1841. That book covers a wide range of examples of where crowds have gone astray, including the Tulip mania in 17th century Holland.)

Before jumping into the details of the *Wisdom of Crowds*, a simple experiment described in Surowiecki's book is useful to understand the principles. An insightful comment from another author, Nassim Taleb, points out that part of the problem with understanding financial markets is that it is an area of investigation where there is an enormous amount of data, but no ability to conduct true experiments like in physics or other disciplines. So we'll venture out of the financial markets for this experiment.

Jelly Beans

A useful example that helps illustrate the wisdom of crowds is the classic jelly-beans-in-a-jar contest. In this experiment, participants make guesses on the number of jelly beans in the jar, with the closest answer getting a prize (typically the filled jar). I have seen the results of this contest/experiment many times. Each time, the average guess is incredibly accurate, even though the individual guesses vary widely. In fact, the average guess is typically better than almost all of the individual guesses.

The idea here is that the collective answer, the average in this case, can be quite accurate even without experts solving the problem. The individual errors essentially cancel out and an answer emerges that is better than those of the vast majority of the individuals.

Interestingly, when this experiment is conducted by giving additional direction to the participants, results start to deteriorate. In part of the experiment described in the book, participants were told to notice the fact there was air at the top of the jar or that the jar was made of plastic and thus could hold more than a glass jar. Under these conditions, the group's average guess started to vary dramatically from the correct answer.

The point here is that when people were led to think a certain way about a problem, the collective problem solving ability worsened significantly. Or more specifically, it demonstrates that diversity in thinking was an integral component for the collective to be able to solve the problem.

On a personal level, I have tried a variation of the jelly bean experiment with similar results. Occasionally, my work department has sports betting pools. Analyzing past pool's results, I could see that the collective answer performed markedly better than most individual bets. I even tried to monetize this concept in future pools by making open bets to any participant who wanted to match their guesses against the collective answer. After awhile, seeing the proportion of outcomes in my favor, no one wanted to continue with this bet. They felt it was an unfair bet, but didn't understand why!

Wisdom of Crowds Hypothesis

Author Michael Mauboussin, in his book, *More Than You Know*, and in many articles, further refines the wisdom of crowds ideas and how they apply to stock markets. He describes very succinctly what I'll call the wisdom of crowds hypothesis.

The basic premise of this hypothesis is that collectives can solve certain types of problems better than the vast majority of the individuals within the collective under the following conditions:

1) **Diversity**—the individuals within the collective must have cognitive diversity, i.e., different approaches and information for solving the problem.
2) **Aggregation mechanism exists**—an ability exists to turn individual judgments into a collective answer; there must be a way to aggregate all of the disparate views.
3) **Incentives**—collectives work much better when there is an incentive to be right.

A fundamental point here is that the success of the collective depends on diversity. When diversity is not present or breaks down, the collective's ability suffers. Diversity breakdowns thus offer opportunities for individuals to outdo the collective.

It's also important to note the types of problems that collectives are good at solving. Examples of the problems that collectives are good at are:

1) Estimating current states—such as the jelly bean contest.
2) Predictions about the future.

A type of problem that collectives are *not* good at is a system requiring very specific rules for success

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such as flying an airplane. An experienced pilot is going to do a lot better than the collective problem solving ability of the plane's passengers (even though the passengers would have a strong incentive to be right!).

Stock markets generally reflect the wisdom of crowds conditions. Diversity is typically present with the various strategies used: growth versus value, technical versus fundamental, and short-term versus long-term investment horizons. The market itself is the aggregation mechanism. And there are huge financial incentives for success. Thus, as long as these conditions are present, stock markets should be quite effective at figuring things out.

Also, it is interesting to see that this hypothesis does not have rationality of the individuals as a required condition. As long as the thinking is diverse, irrational agents can produce rational outcomes. Sometimes behavioral finance theorists use irrationality of individual investors to show that markets can be irrational. However, as the hypothesis postulates, irrational individuals don't necessarily make for irrational, or inefficient, markets.

Prediction Markets

In recent years, prediction markets have started to flourish. Prediction markets are essentially financial markets for the predictions of the outcomes of events such as elections and sports.

An example of a prediction market is Intrade (*www.intrade.com*). Participants can buy or sell contracts on event outcomes such as Hilary Clinton becoming the Democrats' nominee for the 2008 presidential election. If the outcome becomes true, the contract will have a value of $100; otherwise it will have a value of zero. With this type of mechanism, the prices that emerge while the contract is still active are the collective estimates of the probability of the outcome.

Surowiecki and Mauboussin describe a number of prediction markets and how uncanny their predictive abilities have been. Also, the February edition of *Risks and Rewards* newsletter had an interesting article describing the accuracy of forecasts emerging from the economic derivatives markets. The success of prediction markets demonstrates how effective collectives are at synthesizing massive amounts of information into accurate predictions.

It is important to note that these prediction markets are not simply online polls where people pick who they want to win. Rather, this is where people invest money on who they think will win. The big difference here is the incentives, which is one of the conditions of the wisdom of crowds.

In comparing prediction markets to stock markets, there is one important difference. Prediction markets have very defined time horizons and outcomes; while stock markets are continuous and generally do not reach a final outcome. Essentially, prediction markets have more boundaries that limit their potential for the speculative excesses that develop in stock markets.

Complex Adaptive Systems

Now for something a little more complex. Many collectives fall into the field of study called complex adaptive systems. A complex adaptive system is a system that emerges from the interactions of lots of different agents. The agents use a variety of decision rules, take information from the environment, and adapt their behavior accordingly. These changes cause others to react and adapt their behavior and so on.

Examples of complex adaptive systems include ant colonies, bee hives, traffic flow during rush hour, and stock markets.

Mauboussin covers this field more in-depth than what I can in this article. However, there are three key points about complex adaptive systems that are very useful when it comes to understanding the stock markets.

First, in complex adaptive systems, the whole is greater than the sum of the parts. The system that emerges is much bigger than what the various individuals are trying to accomplish. Think of Adam Smith's invisible hand analogy for market economies.

Second, unusual things happen to these systems when diversity breaks down. When a system lacks diversity, the systems become fragile and susceptible to dramatic changes—the so-called fat tails.

The third key point about these systems is that they tend to have a lot of non-linearity—the magnitude of the outcome can be significantly disproportionate to the incremental input. Humans have a strong desire to link cause and effect. In complex systems, cause and effect can be extremely hard to relate.

To illustrate the last point, think of grains of sand falling onto the ground. As the sand pile builds, it eventually hits a state where the next few grains of sand will all of a sudden cause an avalanche. Even though the incremental grains are like all of the others already fallen, their effect is far more dramatic.

Mauboussin describes how this last characteristic appears in the financial markets with the following statement, "Sometimes a piece of information barely moves the market. At other times, seemingly similar information causes a big move."

Viewing the stock market as a complex adaptive system is useful to see that large dramatic shifts, although rare, should be expected occasionally. The demise of the Long Term Capital Management hedge fund in 1998 was an example of financial models not considering rare large scale changes. Normal distributions in their analysis did not work well when the system was in its extreme.

Breakdowns in Diversity

The ideas presented here focus on the importance of diversity to an effective system. One example from social psychology on how diversity breakdowns can happen in human systems is the concept of herding (also called social proof). Psychologists have found that when a situation is ambiguous and there is much uncertainty, we are prone to accepting the actions of others as *proof* of what we should do. That is, when we are unsure, we tend to imitate the actions of others, rather than thinking independently.

This type of diversity breakdown may manifest itself in the stock market by individuals buying certain stocks simply because their prices have been going up.

For further reading on the importance of diversity to an effective system, I recommend Scott Page's *The Difference*. The important contribution Page makes to these ideas is that he defines the underlying logic and mathematics of diversity. To paraphrase a popular quote, he substitutes demonstrations for impressions.

Some Implications for Stock Markets

I think the wisdom of crowds/complex adaptive system approach lays a great foundation for understanding the behavior of the stock market. Unfortunately, it does not offer a simple formula for getting rich. However, in any competitive endeavor, understanding your opponent is often critical to victory, and deciding if you want to try to compete in the first place!

Here is a summary of some of the useful points that I have learned from this approach to understanding the stock market:

1) Markets are very powerful mechanisms for figuring out answers. Most of the time they will be very accurate and difficult to beat. In this sense, I view the efficient markets hypothesis to be a good approximation of how markets work.
2) Any risk management system should expect rare, but very large price changes—normal distributions are not enough to describe market behavior.
3) You need to relax your inherent desire to link cause and effect in markets.
4) Diversity is a critical component for the success of the markets. Opportunities to outdo the markets likely require a diversity breakdown.

Looking outside of the financial arena has definitely helped me get a better grasp of the behavior of the stock market. Stock markets will often be unpredictable in the details, but no longer mysterious to me.

Going back to the market crash of 1987, I found a quote from Eugene Fama, considered by many to be the father of the efficient markets theory. Commenting on Black Monday and other crashes, he said, "People all of a sudden become very risk averse and then you get a crash." Now *that* sounds like a diversity breakdown.

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Credit Risk Management For An Annuity Fund

by Daniel Blamont, Pierre Hauviller and David Prieul

We consider the case of a fixed rate annuity fund, backed by assets of similar duration as liabilities, with a large credit risk exposure. Given the absence of a significant interest rate duration mismatch, the largest contributor to the economic capital (EC) requirement (estimated as the one-year 0.5 percentile of the net asset value (NAV) distribution of the fund) is credit risk, and more specifically the credit spread widening risk.

Credit derivatives (e.g., CDS, CDO, etc.) may be used to mitigate specific types of credit risk such as default risk or spread risk. In the context of a reduction of the economic capital requirement, we find that the most cost effective family of risk management solutions is senior tranches of CDOs, typically with 12 percent—22 percent attachment-detachment points, owing to their convexity properties.

Credit Risk Measurement

We calculate the EC of the annuity fund as well as the standalone contribution of the credit asset class. For this purpose, we perform 10,000 simulations of an interest rate and credit stochastic model to obtain market scenarios for various state variables on a one-year horizon.

We assume a typical asset allocation of 85 percent credit risk assets, 10 percent Treasuries and five percent other assets. The initial NAV of the fund is USD 100 million for total assets of USD 1,000 million.

The standalone EC for credit is USD 41.5 million, similar to the requirement for the whole fund (USD 45.0 million). The other sources of risk for this annuity fund are the Treasury swap spread (the liabilities being valued at a swap yield curve less 25bp) and a small duration mismatch. Their contribution to the EC sums to USD 10.0 million, which is nearly cancelled out by the diversification benefits.

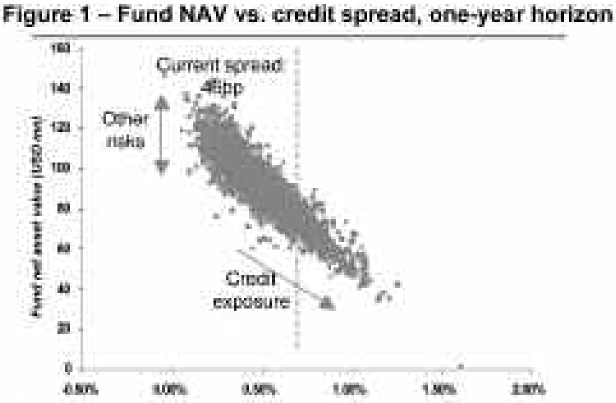
Displaying the NAV of the annuity fund versus credit spreads confirms that credit is the main factor driving the value of the fund. The 0.5 percent worst scenario corresponds to a credit spread (to swaps) widening of 71bp, from 46bp (which implies an expect loss of 3.7 percent over 10 years) to 117bp.

Credit Risk Management

A CDO structure can be used to mitigate the credit risk in terms of default and spread (market-to-market or MTM) risk. The final value of a CDO will depend on the realised losses, while on a mark-to-market basis, the CDO price will depend on realized credit spreads (i.e., implied future losses).

Junior tranches are a possible vehicle to hedge against realised losses as senior tranches are unlikely to be subject to defaults. However, we see in the table below that senior tranches are more efficient when dealing with credit spread widening.

Figure 1 – Fund NAV vs. credit spread, one-year horizon



Source: Deutsche Bank

Table 1 - 10-year CDO tranches MTM spread sensitivity

	0-3%	3-6%	6-9%	9-12%	12-22%
PV protection cost	64.27%	27.94%	12.92%	6.16%	1.56%
17bp increase in portfolio spread (25% of EC spread widening*)					
MtM gain	6.40%	7.91%	5.64%	3.52%	1.20%
MtM gain / cost	0.10	0.28	0.44	0.57	0.77
35bp increase in portfolio spread (50% EC*)					
MtM gain	11.19%	15.03%	11.36%	7.46%	2.74%
MtM gain / cost	0.17	0.54	0.88	1.21	1.75
71bp increase in portfolio spread (100% EC*)					
MtM gain	17.67%	26.76%	22.26%	15.92%	6.70%
MtM gain / cost	0.28	0.96	1.72	2.59	4.29

** The EC spread widening corresponds to the 0.5% worst scenario*

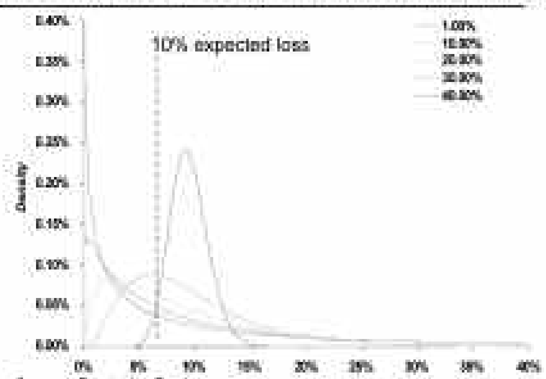
Our pool of credit assets is modeled under the homogeneous pool assumption, meaning that each name has identical characteristics in terms of credit spread, default risk, expected loss and pool weighting with a single pair-wise correlation. Specifically we assume an average credit spread of 46bp for each name and a pair-wise default correlation between all obligors of 20 percent.

The pair-wise correlation will then be the main driver of the value of the portfolio. At one extreme, if the pair wise default correlation is high, default outcomes are more likely to be clustered. With 100 percent correlation, we face only two possible scenarios: either no bond defaults or all bond default. The probability of the latter case is given by the expected loss of each bond. Only such a high correlation environment would lead to a risk for senior tranches (low probability of a high number of realised defaults). So correlation increases the price of protection on senior tranches. Similarly it lowers the price of protection on equity tranches as it increases the probability of having no defaults.

At the other extreme, if correlation is low, defaults will be more evenly distributed; actual pool defaults are likely to be closer to expected bond defaults. As a consequence, junior tranches are expected to be greatly impacted in case of low correlation (high probability of a small, but non-zero, number of realised default).

For mezzanine tranches, the effect of correlation is less clear cut and depends on the level of the attachment/detachment points relative to the expected pool default rate. In other words, the effect on mezzanine tranches will depend on how junior or senior the tranche is relative to the pool.

Figure 2 - Sensitivity of loss distribution to correlation



Source: *Deutsche Bank*

Table 2 - 10-year CDO tranches cost / benefit analysis

	0-3%	3-6%	6-9%	9-12%	12-22%
Tranche notional (USD mn)	85	94	128	187	425
Cost p.a.	14.75%	4.10%	1.73%	0.80%	0.20%
Cost p.a. (USD mn)	12.5	3.8	2.2	1.5	0.8
EC reduction	15	15	15	15	15
Portfolio spread net of cost (to swaps)	-101bp	1bp	20bp	28bp	36bp

We investigate the possibility of buying protection on CDO tranches as a way to mitigate the credit spread risk and hence reduce the EC. Developing on our previous example, we assume a CDO based on a pool identical to the credit portfolio. We look at the cost of different strategies with the same objectives: reducing the EC capital by USD 15 million to USD 30 million (a reduction of one-third).

The super senior tranche (12- to 22-percent) proves to be efficient in hedging against spread widening risk. Indeed, the cost of reducing the EC by USD 15 million is only USD 0.8 million per annum, which is equivalent to reducing the spread on the bond portfolio by 10bp. The adequacy of using a senior tranche is due to its convexity, which means that the tranche has a greater impact in a high credit spread environment. The equity tranche in contrast also protects against defaults, and therefore is an expensive hedge against credit spread widening risk. The rationale for hedging versus reserving can then be assessed by comparing the hedging cost with the hurdle rate on capital.

The super senior tranche (12- to 22-percent) proves to be efficient in hedging against spread widening risk.

Combining multiple CDO tranches may prove to be an efficient way to reduce the hedging cost. Protection is only required against substantial spreads increase, so protection against small spread increases can be sold away. Buying protection on a senior tranche and selling protection on a junior makes use of the varying convexity of the different tranches. With our EC reduction target of USD 15 million in mind, we consider two strategies:

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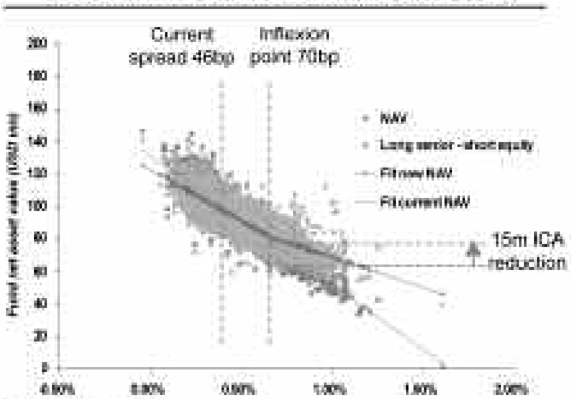
Table 3 - 10-year CDO tranches cost / benefit analysis

	Short Equity	Long Senior	Total	Short Mezz	Long Senior	Total
Tranche	0-3%	12-22%		3-6%	12-22%	
Notional	-9	468		-10	468	
Cost p.a.	14.75%	0.20%		4.10%	0.20%	
Cost p.a. (USD mn)	-1.32	0.93	-0.39	-0.42	0.93	0.51
EC reduction			15			15
Portfolio spread net of cost (to swaps)			51bp			40bp

super senior versus equity and super senior versus mezzanine. This is similar to a put spread in the equity world.

The first strategy leads to a positive carry while the second one reduces the cost of protection. On the EC criteria only, the first strategy is the most efficient, achieving the same relief target for a cheaper cost. However, this strategy has also more exposure to default risk and this feature should be taken into account when selecting a specific strategy.

Figure 3 - Long super senior, short equity



Source: Deutsche Bank

Conclusion

We considered a typical annuity fund with a high allocation to credit risk assets. The fund is exposed to credit spread widening risk, and to default risk. This leads to significant economic capital requirements.

Typical instruments with linear payoffs (e.g., CDS) can be inefficient at mitigating this risk as the reduction in downside risk is mirrored by a reduction in upside risk.

On the other hand, CDO tranches, senior ones in particular, offer the required convexity to reduce downside risk whilst retaining a significant upside risk exposure. Furthermore, combining CDO tranches gives access to further payoff functions and may reduce the cost of protection.

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Defined Benefit Pension Funds Longevity and Inflation Risks

by Nicola Barrett and Valdimar Armann

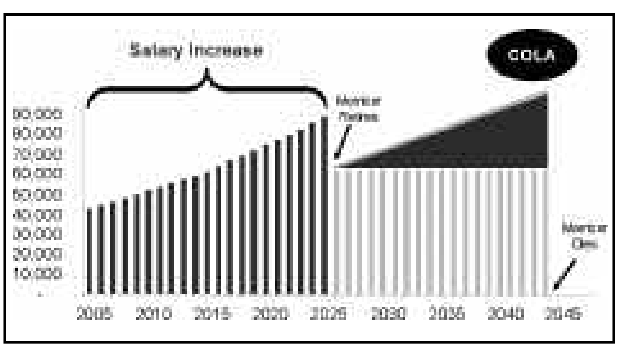
Capital market risks have always been a big concern for defined benefit pension plans. Along with duration and longevity is inflation risk, which can grow quite fast in plans where benefits are tied to wages and even faster if the plan offers inflation protection or COLA (cost of living adjustment) on the retirement payout. In 2003, 354 S&P 500 companies offered their employees defined benefit plans and a handful of those offered COLA. At the end of 2005, private defined benefit plans held about $1.8 trillion in assets.

How can one estimate these long-term unknown liabilities? Currently, actuaries use prospective inflation projections to calculate future pension obligations, but with the flourishing inflation market in the United States, that is no longer necessary; there is a liquid market that trades on future inflation expectations making future inflation no longer an unknown number.

The Global Risk of Defined Benefit Plans

Many defined benefit plans are exposed to inflation either through the final salary component or through COLA on payout or both. The final salary (often an average of the previous three to five years' salaries) of an employee, determines the payment he will receive from the plan. In a COLA plan this payment is then escalated every year with a cost of living index. The different exposure is drawn in figure 1.

Figure 1: Pension plan exposure to inflation



Source: ABN AMRO.

For a true assessment, the inflation risk of a defined benefit pension plan should be put into context with all the other risks that a pension fund faces, such as investment risk (e.g., asset values, equity returns), interest rate (e.g., rates used to discount future pension payments) and demographic risk (e.g., longevity, disability and survivor risks). In a defined benefit plan, the employer generally assumes all risks (unlike a defined contribution plan that normally passes all risks to employees) and inflation seems to be a common component of each type of risk.

Typically these risks are different in size and in terms of predictability, but, importantly, financial products exist that can hedge some of these risks such as interest rate derivatives and immunized fixed income portfolios to hedge duration mismatch. There is even a market emerging in longevity risks. Even though Social Security's normal retirement age is gradually increasing from 65 to 67, private section pension plans must start paying benefits by age 65 to eligible participants who are no longer working. Therefore, as the retiree population lives longer and longer, benefits must be paid out for many more years than originally anticipated. Interestingly and of topical importance is the fact that inflation risk compounds on the longevity risk. For instance, a 1 percent yearly improvement in mortality starting at age 60 might increase pension plan liabilities by 5 percent, while a 1 percent increase in inflation might increase them by 16 percent for a defined benefit pension plan.

When analyzing the risks together, one might conclude that the appropriate usage of the risk allocation of the fund is perhaps not best spent on inflation risk. Is the inflation risk sufficiently well rewarded? It might be more efficient to hedge the inflation risk (partly or fully) and be able to allocate resources to other risks that offer better rewards. However, very few plans in the United States, if any, have hedged their inflation exposure.

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Pension Accounting and Funding Rule Changes

New pension accounting rules, which are effective for public companies as of the end of the fiscal year ending after Dec. 15, 2006, require that the overfunded or underfunded status of a defined benefit pension plan (measured as the difference between the pension plan's assets (at fair value) and its projected benefit obligations) be recognized as an asset or liability on the balance sheet with an offset to other comprehensive income (OCI) which is a component of equity. Any future changes in the funded status will be recognized through OCI as well. This change will cause balance sheet volatility as any change in the funded status will be recorded in equity. Additionally, new pension funding rules, which are effective in 2008, impose stricter funding requirements and generally require that underfunded plans make up any shortfalls in seven years.

Taken together, the new pension accounting and funding rules are a strong incentive for companies to manage all risks that could impact the funded status of their pension plans, including inflation risk.

Hedging Inflation Risk

Defined benefit plans have traditionally allocated a significant percentage of their assets to equities. Normally, this could be anywhere from 50- to 70-percent of total plan assets, but has been roughly 60- to 65- percent from 1999 to 2004.[1] Equity real return can be positive as well as negative depending on how equity and inflation perform separately and even though equity is likely to outperform inflation on average, there is evidence that the long considered relationship between equities and inflation-related liabilities does not hold. Stocks have been shown to be a poor hedge against inflation and it has even sometimes shown negative correlation.[2] Residential real estate has probably shown the most effective inflation hedge over the long term, but that asset class is certainly not always liquid and it can be difficult to manage large pools of these assets. Furthermore, this asset class has shown it is subject to bubbles, thereby making the inflation hedge imperfect. With the new inflation market, it is now possible to buy future inflation directly and there is no need to make any approximations. This is effectively the only real way to hedge inflation risk.

Estimation of Future Inflation

The U.S. government started issuing inflation linked Treasuries (TIPS) in 1997 and the inflation swap market started to develop around 2000.[3] The main building block of the inflation swap market is the Zero-Coupon Break-Even Inflation swap (ZC BEI swap).[4] At the maturity date of the swap, it pays the actual accumulated inflation[5] from start date to maturity date on one leg versus compounded fixed rate inflation on the other leg. The fixed rate is called Break-Even Inflation or BEI, and represents the market's expectations for average inflation for the duration of the swap. These swaps range in maturity from one to 30 years and can extend even further.

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1 Goldman Sachs Global Strategy Research, June 2005.

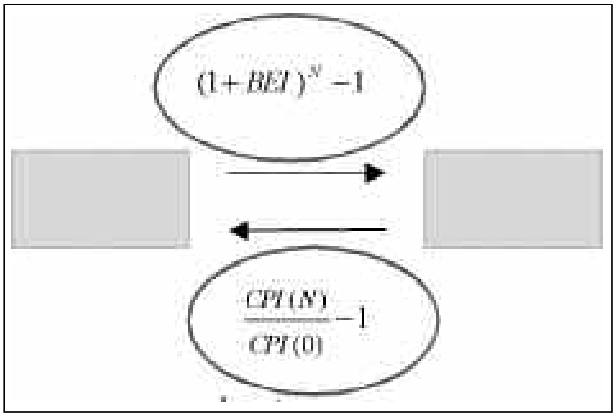
2 Fama & Schwert 1977.

3 For further information about inflation products and the market, please see "Inflation-Linked Products: A Guide for Investors and Asset & Liability Managers," published by RISK in association with ABN AMRO in 2005.

4 The inflation market also offers the inflation protection in other formats. One frequently used is the Year-over-Year (YoY) inflation swap where counterparties exchange periodically (e.g., annually or monthly) the annualised inflation rate versus a fixed rate.

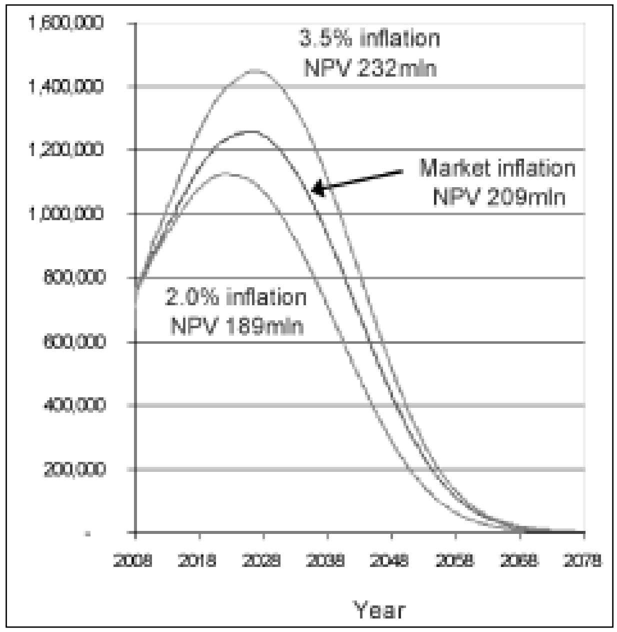
5 The swaps are indexed on US CPI Urban Consumers All Items non-seasonally adjusted that is published monthly by BLS.

Figure 2:
The Zero-Coupon Break-Even Inflation swap



Source: ABN AMRO.

Figure 3: Pension plan liabilities assuming different inflation



Source: ABN AMRO.

This swap is ideal for a pension fund as its exposure is accumulated inflation. It can be customized to be a string of zero-coupon swaps that every year would pay the inflation accumulation from start date until payment date versus the fixed compounding. This, to an extent, is like an inflation-linked annuity and can be used to hedge the liability profile that a defined benefit pension plan shows.

With the inflation swap instrument as explained above, one can estimate future inflation linked liabilities. Figure 3 shows the net present value of a pension plan's expected payout given three different inflation assumptions. The first assumption is annual 3.5 percent inflation on the payout, the second is 2.0 percent and the last is the so called market inflation, which is derived from the inflation swap market. The last payout is the only one that is actually hedgeable and gives the best evaluation of future obligations. The other two assumptions might understate or overstate the expected payout resulting in an inaccurate funded status. In this example, the 3.5 percent inflation assumption results in overstatement by $23 million, i.e., the difference between estimated NPV using 3.5 percent inflation and estimate NPV using market inflation. This would allow the pension fund first of all to lower their estimated liabilities by $23 million and secondly, to lock that gain in by actually hedging the future inflation.

Emergence of Hybrid Products

The newly emerging inflation option market enables conventional inflation products to be combined with other financial instruments. These new "hybrid" products satisfy the requirements of inflation income protection today with exposure to additional upside tomorrow. One such structure is the so-called "best of" product that offers purchasing power protection with potential high return. The payoff could, for example, be the highest of inflation growth, equity growth or fixed coupon growth from start date to end date of the product. This product is available on the market today and can potentially be extended to create an annuity stream of payments. Each year, the product (either in a note or swap format) would pay a cash-flow, which would increase with inflation or other selected indexes, whichever is greatest. In the future we envision this product being combined with longevity risk to offer a fully fledged annuity solution.

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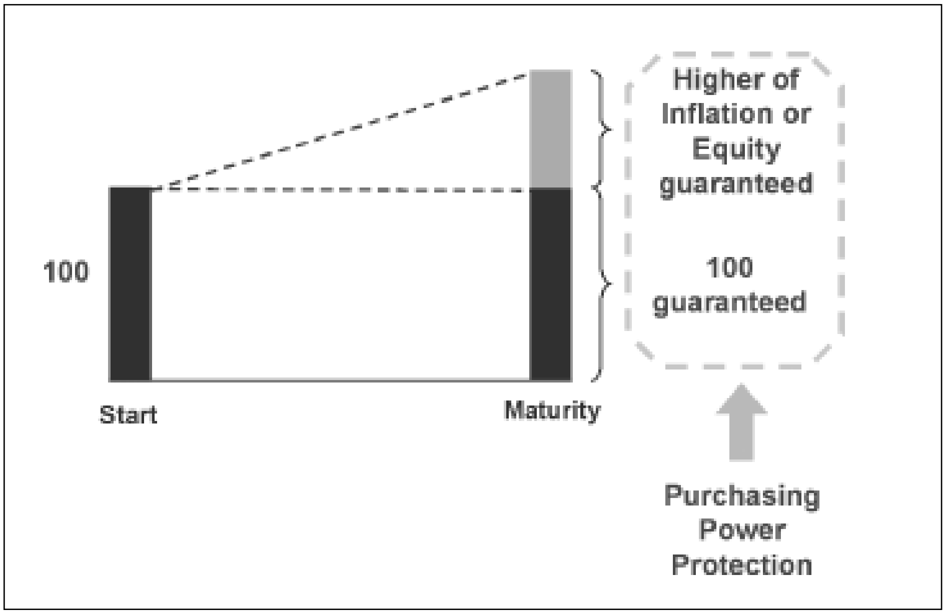
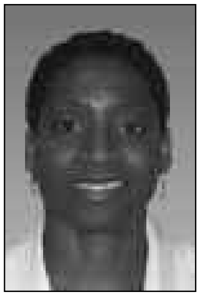
Figure 4: "Best-of" structure guarantees purchasing power



Source: ABN AMRO.



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Conclusion

Actuaries face the problem of forecasting future pension fund liabilities. Investment managers face the task of providing pension fund assets that meet or exceed those liabilities. Inflation is the newest generation of swappable entities and the market now offers a fixed rate replacement for something currently unknown. The next step for providers of inflation protected pension benefits or inflation linked insurance products is to estimate their future liabilities using the market inflation curve. The final step would be to manage the inflation risk and hedge out unwanted and non-rewarding risks by using the inflation products available.

Investors buying traditional structures face the task of predicting future inflation rates and thus predicting the real rates of return on their investments. No financial instrument today provides investors with a reliable inflation hedge, except a product that is directly linked to the inflation rate. Aging populations and growing life expectancy will increase pressure on pension funds and life insurance companies to supply retirement products that ensure stable income that adjust with the cost of living. The correct investment choices will be crucial in order to rise to these challenges. Inflation hedging plays an increasingly important role in achieving these goals—from the use of inflation-linked bonds and swaps to hedge inflation risk, to the use of hybrid and "best of" products to supply a stable income with potential upside. Inflation structures come as close as possible to offering a risk-free financial product.

Option Pricing Using Natural Deflators

by Richard S. Mattison

State prices and state price deflators can be very useful in pricing a variety of options (state or path dependent cash flows) that we encounter in actuarial practice. One implementation of state price deflators that I have used is $(1+x)^{-1}$, where x represents a particular state return on a portfolio. I have used this as a "natural" deflator for pricing future state dependent cash flows (such as future contributions for a defined benefit pension plan) when using return probability distributions other than Lognormal. Using this approach, a current price of future state dependent cash flows can be calculated as a present value of the form:

$$\textbf{\textit{Price}} = \int \frac{C(x)\,f(x)\,dx}{(1+x)}$$

This pricing methodology using the natural deflator $(1+x)^{-1}$ is appealing for a number of reasons including:

1. The ability to calculate prices using natural probabilities, the P-measure, instead of converting to risk-neutral probabilities, the Q-measure, and
2. The methodology provides an easy method for discounting scenario specific cash flows from stochastic simulations within an ALM model.

This can be very useful where it may be difficult (or impossible) to calculate a risk-free rate and corresponding risk-neutral probabilities, e.g., under a long-term asset optimization problem where liability cash flows extend beyond the available term structure and portfolios are composed of stocks and bonds.

Option Pricing Equations

Historical time series data show that the Extreme Value Distribution (EVD) is a better fitting model for actual annual stock returns than the Lognormal Distribution. Hence, stock option pricing using EVD may be more accurate than option pricing using a traditional Black-Scholes approach. Assume that you seek to price a stock option on a non-dividend-paying stock using the EVD.

Definitions:
r = one year risk-free spot rate

μ = assumed annual mean return for large cap stocks

σ = assumed standard deviation of returns for large cap stocks

$a = -\mu - 0.5772b$, the mode of the EVD

$b = \sigma\sqrt{6}\,/\,\pi$, the scale parameter for the EVD

$f(x) = (1/b)\exp[-(-x-a)/b] \times \exp\{-\exp[-(-x-a)/b]\}$,

the probability density function for the EVD [note this form of EVD uses "-x" on the rhs as we want the smallest extremes]

S_0 = current asset price

K^*= the ratio of the strike price for the option to the current asset price, with $0 < K^* \leq \infty$.

Let Call(K^*) equal the price today for a European option with strike price ratio K^* payable in one year. Then:

$$\text{Call}(K^*) \;=\; S_0\Big(\int_{K^*-1}^{\infty} \frac{[(1+x)-K^*]f(x)dx}{(1+x)} \Big)$$

Rewriting we have:

$$\text{Call}(K^*) \;=\; S_0\Big(\int_{K^*-1}^{\infty} f(x)dx - K^* \int_{K^*-1}^{\infty} \frac{f(x)dx}{(1+x)} \Big)$$

Similarly, the Put option can be valued as:

$$\text{Put}(K^*) \;=\; S_0\Big(K^* \int_{-1^+}^{K^*-1} \frac{f(x)dx}{(1+x)} - \int_{-\infty}^{K^*-1} f(x)dx \Big)$$

We can calculate the price of the implied risk-free asset paying S_0K^* in one year as:

$$S_0 + \text{Put}(K^*) - \text{Call}(K^*) \;=\; S_0\Big(K^* \int_{-1^+}^{\infty} \frac{f(x)dx}{(1+x)} \Big)$$

Hence our implied risk-free rate is $(S_0K^*)/[S_0+\text{Put}(K^*) - \text{Call}(K^*)] - 1$. The EVD model can be

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calibrated to the desired risk-free return by adjusting the assumed values for μ and σ.

Option Pricing Example

Using the EVD, let μ = 10%, σ = 18%, then a = -0.181, b = 0.140. Assuming S_0=1 and setting $K^* = 1.1$, we have:

Call(1.1) = 0.054505,

Put(1.1) = 0.122425, and

r = 0.03.

More generally, any asset with state dependent cash flows payable in one year can be priced using a variant of:

$$\text{Price} = \int \frac{C(x)f(x)dx}{(1+x)}$$

Similarly, prices (present values on a "market" pricing basis) can be calculated for state varying cash flows using state returns (portfolio returns) in the deflator within a discrete ALM stochastic model.

Proof of Concept

We know that

$$\int_{-\infty}^{\infty} \frac{(1+x)f(x)dx}{(1+x)} = \int_{-\infty}^{\infty} f(x)dx = 1$$

Set μ and σ such that

$$\int_{-1^+}^{\infty} \frac{f(x)dx}{(1+x)} = \frac{1}{(1+r)}$$

And set the strike price ratio K^* to the risk-free return,

$$K^* = \frac{(1+r)}{1}$$

Then we should have Put(K^*) = Call(K^*)
Or

$$S_0\left(K^* \int_{-1^+}^{r} \frac{f(x)dx}{(1+x)} - \int_{-\infty}^{r} f(x)dx\right) =$$

$$S_0\left(\int_{r}^{\infty} f(x)dx - K^* \int_{r}^{\infty} \frac{f(x)dx}{(1+x)}\right)$$

Grouping terms on one side we have:

$$\left(K^* \int_{-1^+}^{r} \frac{f(x)dx}{(1+x)} - \int_{-\infty}^{r} f(x)dx\right) -$$

$$\left(\int_{r}^{\infty} f(x)dx - K^* \int_{r}^{\infty} \frac{f(x)dx}{(1+x)}\right) = 0$$

Rearranging terms we have:

$$K^* \int_{-1^+}^{\infty} \frac{f(x)dx}{(1+x)} - \int_{-\infty}^{\infty} f(x)dx = \frac{K^*}{(1+r)} - 1 = 0$$

Which is the result we required.

More generally, let K^*= (1+Z), for $0<Z<\infty$. We should have r= $(S_0K^*)/[S_0$+Put(K^*)–Call(K^*)]–1. Rearranging terms and using the above equations we have:

$$(S_0 K^*)/[\,S_0 + \text{Put}(K^*) - \text{Call}(K^*)] =$$

$$(S_0 K^*) / \Big[S_0 + S_0\left(K^* \int_{-1^+}^{Z} \frac{f(x)dx}{(1+x)} - \int_{-\infty}^{Z} f(x)dx\right) -$$

$$S_0\left(\int_{Z}^{\infty} f(x)dx - K^* \int_{Z}^{\infty} \frac{f(x)dx}{(1+x)}\right)\Big] =$$

$$K^* / [1 + K^*/(1+r) - 1] = (1+r).$$

Hence Put Call parity holds under this natural deflator methodology for any K* = (1+Z), with $0 < Z < \infty$.

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The Funding Ratio Rollercoaster

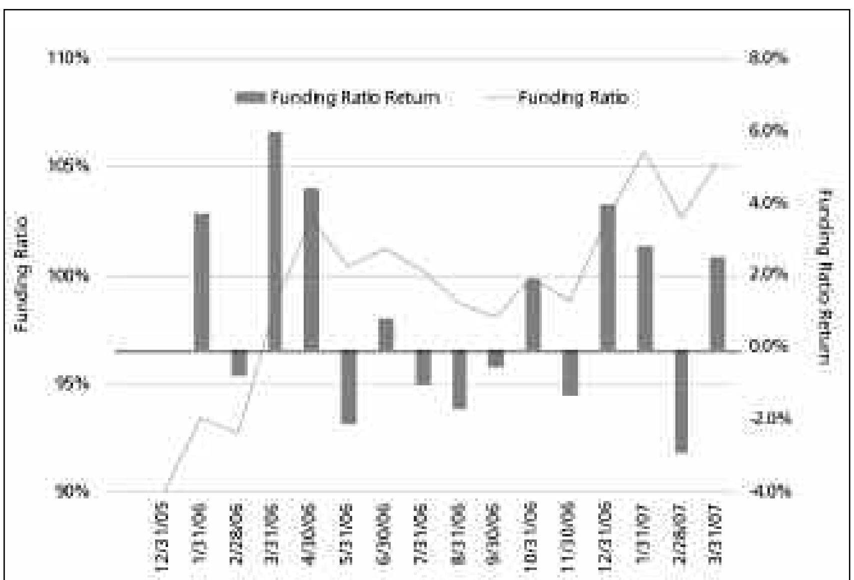
by Aaron Meder

The UBS Global Asset Management U.S. Pension Fund Fitness Tracker is a quarterly estimate of the overall health of the typical U.S. defined benefit pension plan. This report, issued quarterly, also includes our estimate of the fair value of pension liabilities.

Our U.S. Pension Fund Fitness Tracker shows the wild ride pension plan funding ratios took through the first quarter of 2007 due to volatility in both equity markets and interest rates. The typical pension fund started the year with a funding ratio of approximately 103 percent, then edged forward in the first quarter of 2007 to a slightly improved funding ratio of approximately 105 percent. The ride there, however, was anything but smooth.

The funding ratio measures a pension fund's ability to meet future payouts to plan participants. The main factors impacting the funding ratio of a typical U.S. defined benefit plan are equity market returns, which grow (or shrink) the asset pool from which plan participants' benefits are paid; and liability returns, which move inversely to interest rates.

Chart 1—Wild ride in first quarter caused by volatile interest rates and equity markets; UBS U.S. Pension Fund Fitness Tracker of typical U.S. corporate plan funding ratio



Source: UBS Global Asset Management, Bear Stearns, International Index Company

Much like in 2006, the improved health of the typical U.S. defined benefit pension plan in the first quarter was due largely to strong equity market returns. We estimate that the typical large corporate defined benefit plan saw an increase in assets for the quarter of almost 2 percent, outperforming liabilities, which were roughly flat for the quarter, as measured by the iBoxx U.S. Pension Liability indices.

Despite the overall positive numbers, the first quarter of 2007 points out an area of concern for pension managers: namely, significant funding ratio volatility due to interest rate swings, and plans' over-reliance on equity market risk. While a brief interruption in the equity market advance caused some of the funding ratio volatility, a greater contribution by far resulted from the teeter-tottering of interest rates in the quarter. For example, liabilities were down 2 percent in January as interest rates moved higher, but regained all and more of that decline, increasing by almost 4 percent in February as rates reversed course.

Of most interesting note was the single day volatility in the funding ratio experienced on Feb. 27th. On that day, equity markets gave up about 2 percent, with interest rates falling at the same time. This has been a recurring phenomenon in recent years, and in this case resulted in funding ratios deteriorating by almost 4 percent in a single day. This single day volatility should make pension managers take heed.

While many plans are currently exposed to this risk, there are alternative investment approaches that can help to better align assets and liabilities. Plans can implement liability-driven strategies that significantly reduce the uncertainty in their future pension contributions, often without reducing expected plan returns. We believe the improvement in overall pension health in 2006 and the first quarter of 2007 provides many with an excellent opportunity to hedge some of their liability risk.

A Note on the Fair Value of Liabilities and Hedging Liabilities

The main capital market drivers of pension liability valuations are interest rates. Interest rate volatility can have a dramatic impact on overall funding ratios, as it did throughout the first quarter of 2007. Many plan sponsors are beginning to consider liability hedging strategies, to manage this risk. We believe this is a prudent course, and have developed a fair-value methodology for valuing liabilities to assist sponsors with the liability hedging decision.

Chart 2 below shows UBS Global Asset Management's estimate of the price/value discrepancy of pension liabilities. Liability fair value is derived by comparing our estimated fair value yield curve and fair value spreads to currently observed market levels. The liability price is based on the iBoxx U.S. Pension Liability indices. When the line is at zero, liabilities are estimated to be at fair value. With the line above zero, pension liabilities, in our view, are "overvalued." This means that we expect liability values to decrease as interest rates move higher over time, toward more sustainable equilibrium levels.

Many plan sponsors are hesitant to implement significant liability hedging positions, against an uncertain interest rate backdrop. Our liability fair value framework was designed to assist plan sponsors in making this decision. For example, as interest rates rose during January of 2007, our model signaled that liabilities were close to fair value. If a plan had begun to implement a liability hedge at mid-year 2006, we believe it would have resulted in an even greater improvement in the funding ratio during the first quarter of 2007.

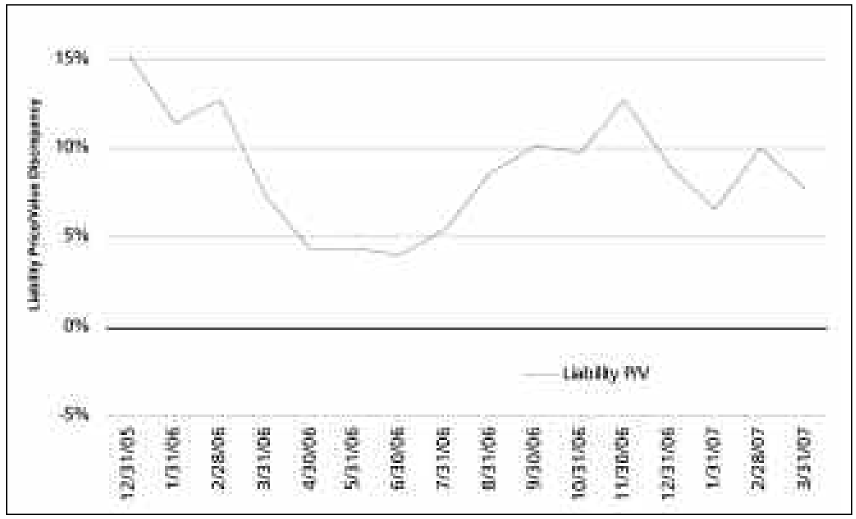
Liability Indices: Methodology

The iBoxx U.S. Pension Liability Index—Aggregate mimics the overall performance of a model defined benefit plan in the United States, taking into consideration the passage of time and changes in the term structure of interest rates. The index is based on actual liability profiles, and mimics the investment grade yield curve. It is therefore more appropriate than most existing indices for measuring the performance of defined benefit plans. This index, (along with its related active member and retired member indices) is published daily, using the LIBOR interest rate swap curve as the discount curve, a highly liquid universe. This provides the flexibility to use combinations of the indices in order to accurately represent customized liability profiles based on a plan's specific participant population.

Asset Index: Methodology

UBS Global Asset Management approximates the return for the "typical" U.S. defined benefit plan using the reported asset allocation of the corporate plan subset of the Pension & Investments 1000.

Chart 2: Pension liability remains overvalued by approximately 8 percent; Estimated liability price/value discrepancy



Source: UBS Global Asset Management, International Index Company

The series is constructed using the reported asset allocation weightings and publicly available benchmark information; with geometrically linked monthly total returns.

Additional Information about the Indices

The iBoxx U.S. Pension Liability Indices include data provided by International Index Company (IIC). The information and opinions contained in this document have been compiled or arrived at based upon information obtained from sources believed to be reliable and in good faith. All such information and opinions are subject to change without notice. A number of the comments in this document are based on current expectations and are considered "forward-looking statements." Actual future results, however, may prove to be different from expectations. The opinions expressed are a reflection of UBS Global Asset Management's best judgment at the time this release is compiled, and any obligation to update or alter forward-looking statement as a result of new information, future events, or otherwise is disclaimed.

The views expressed are those of UBS Global Asset Management as of March 31, 2007.

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2007 SOA Annual Meeting—Great Lineup of Sessions in Washington, D.C.

By Marc Altschull, Investment Section representative on the 2007 Annual Meeting Planning Committee

The Investment Section is planning an exciting program at the 2007 Society of Actuaries Annual Meeting that will be held at the Marriott Wardman Park, Oct. 14-17. The showcase session on life settlements is being jointly sponsored with the Life Product Development Section and will be preceeded by an introductory session on this topic. Please plan to join us for a hot breakfast on Oct. 15, a jointly sponsored reception with the Financial Reporting section that same evening, and to enjoy the following sessions:

Investment Section Hot Breakfast (October 15 at 7 a.m.)

Join members of the Investment Section for a great networking and learning opportunity. We are sponsoring a hot breakfast that will feature David Merkel, FSA, CFA, who is a senior investment analyst at Hovde Capital, responsible for analysis and valuation of investment opportunities for the FIP funds, particularly of companies in the insurance industry. He is a leading commentator at the excellent investment Web site RealMoney.com (*http://www.alephblog.com/wp-admin/www.RealMoney.com*). Back in 2003, after several years of correspondence, James Cramer invited David to write for the site, and write he does—on equity and bond portfolio management, macroeconomics, derivatives, quantitative strategies, insurance issues, corporate governance and more. His specialty is looking at the interlinkages in the markets in order to better understand individual markets. David will be explaining his macro and micro views of where he thinks the global markets are heading. The section council will also be updating you on all our exciting plans for the upcoming year.

Life Settlements 101 (October 15 at 10:30 a.m.)

Originating with the "death pools" in Victorian England, life settlements are gaining in popularity. As might be expected, this popularity is accompanied by abuses, risks and opportunities. Do you know about the interrelationship of insurable interest and life settlement transactions? What is driving the emergence of the secondary markets? What role can actuaries play in the future of life settlements? Actuaries working with life settlements will present this primer on the secondary markets as an educational introduction to the debate to be presented in the session titled: "Betting the Over/Under on Death with Life Settlements."

Betting the Over/Under on Death with Life Settlements (October 15 at 2 p.m.)

The line is set at seven, and no, this isn't the spread on the Michigan/Notre Dame game. Rather this is the life expectancy that a life settlement medical underwriter has reported for a 65-year-old, high net-worth male with certain medical impairments and who is considering selling his life insurance policy. With the "line" set, the mortality game is played out to determine the winners and losers in the life settlement business. A life settlement provider will square off against an insurance company actuary in a point/counterpoint debate surrounding ethics and issues and discuss whether life settlements are good for the insurance industry.

Liability-Driven Investing—Best Practice, Buzzword, or Babe with a Future? (October 16 at 8:30 a.m.)

Is Liability-Driven Investing (LDI) asset-liability management with a new name? Is it fully developed to a best practice status that could be used in any company or pension plan? Or is LDI in its infancy with exciting developments ahead that will change how we think about investing and the investment products required? In this session, a panel of international actuaries will describe what LDI is, its uses and limitations, and will report on its stage of development in their industry.

Capital Market Solutions (October 16 at 10:30 a.m.)

In 2006, life insurance companies executed nine insurance linked securitizations with over $4.5 billion of issuance. Other companies used securitization technology to implement alternative structures to help finance XXX and AXXX reserves. Presenters will discuss the latest transactions, alternative public and private capital markets solutions and the continued convergence of the capital markets and insurance.

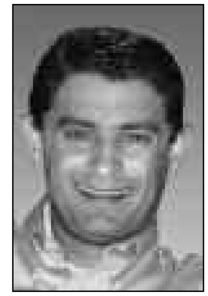
Role-reversal Debate between Portfolio Management and Actuaries (October 16 at 2:30 p.m.)

This role-reversal debate between a portfolio manager playing the role of the actuary and an actuary playing the role of the portfolio manager will show attendees how portfolio managers view the actuarial world as well as provide some insights into how actuaries view portfolio managers. The discussion will be opened to the audience to participate in asking questions of the panel who will remain in character for their responses.

Financial Research in the Insurance Industry (October 17 at 9 a.m.)

In addition to their daily lives, qualified actuaries and academics conduct financial research on an ongoing basis. Two of the latest research reports include topics that are at the forefront of today's insurance companies: *Interest Rate Hedging on Traditional Health and Life Products and Pension Risk Management*. Speakers will discuss their recent research projects, data, methodology and analysis. Most importantly, presenters will discuss the implications of the results for the future of the insurance marketplace, as well as the opportunities presented to us as insurance and consulting actuaries.

We hope you'll all agree that the program that's planned by the Investment Section has a lot to offer. If you have any interest in speaking at one of these sessions or have a recommendation for a speaker, please contact Marc Altschull at *Marc.Altschull@PacificLife.com*. We look forward to seeing you in D.C.!



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