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Fuzzy Sets Methods in Actuarial Science

This abstract is a short presentation of the work contained in the manuscript *An Investigation into Possible Applications of Fuzzy Sets Methods in Actuarial Science* (Ostaszewski, 1993) written with support of and published by the Society of Actuaries.

Fuzzy sets theory is an alternative method of modeling uncertainty. It is a part of an emerging new framework of scientific approach to the problem of uncertainty. Smithson (1989) gives a unique review of the modern theories of uncertainty and ignorance. The main characteristic differing the fuzzy sets approach from the classical methods of actuarial science is the stress placed on vagueness of phenomena modeled by fuzzy sets. Fuzzy sets are also used in dealing with problems of great complexity, for which probabilistic methods become inadequate.

Zadeh (1965) was the creator of the fuzzy sets theory. Since that historic paper appeared, the theory has grown in importance and prominence, and has become a major tool in applications. We will give the basic definition of a fuzzy set here, asking the reader to refer to the book (Ostaszewski, 1993) for the details of the study.

Let U stand for a universe of discourse; i.e., a certain set whose subsets are of interest to us. A *fuzzy subset* \tilde{E} of U is defined by its *membership function* $\mu_E: U \rightarrow [0, 1]$. For any $x \in U$, the value $\mu_E(x)$ specifies to what degree x belongs to the fuzzy set considered. We will write either (E, μ_E) for that fuzzy set or \tilde{E} , to distinguish from the notation E , generally denoting a standard set. The membership function is a generalization of a characteristic function of an ordinary set. Ordinary sets are called *crisp sets* in the fuzzy set theory, and considered a special case – a fuzzy set is crisp if, and only if, its membership function does not have fractional values.

Our book (Ostaszewski, 1993) presents the basic concepts of the fuzzy set theory, including fuzzy sets, fuzzy measures, and approximate reasoning. In all cases, references for further studies, and for investigations of more advanced concepts, are provided. Then, a review of fundamental concepts of actuarial science is given from the perspective of possible applications of fuzzy set-theoretic methods. Although careful consideration has been given to all areas where uncertainty of a fuzzy nature arises, the summary may not be exhaustive. One can most certainly imagine this topic to be a subject of further numerous studies, both of theoretical and applied nature. In fact, one of the major functions of our study is to serve as an encouragement towards such work.

References:

- Ostaszewski, K. M. (1993), *An Investigation into Possible Applications of Fuzzy Sets Methods in Actuarial Science*, Society of Actuaries, Schaumburg, Illinois.
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- Zadeh, L.A. (1965), Fuzzy sets, *Information and Control* 8, pp. 338-353.