GIRR Model Solutions
Spring 2020

1.   **Learning Objectives:**
2.   The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3.   The candidate will know how to calculate and evaluate projected ultimate values.

5.   The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:
(2a)   Create development triangles of claims and counts from detailed claim transaction data.
(2b)   Describe the different types of exposures used for conducting actuarial work.
(3f)   Demonstrate knowledge of good practice related to projecting ultimate values.
(5b)   Identify the time periods associated with trending procedures.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 10, 11, 14, and 25.

Commentary on Question:
This question tests the candidate's understanding of certain fundamental concepts relating to development triangles, credibility, policy earnings, and trending periods.

Solution:
(a)   The development triangle is an important tool for analyzing historical relationships and for projecting similar relationships into the future.

      True
      - Development triangles are fundamental to some of the most frequently used methods for projecting ultimate claims and counts.
      - Development triangles can be used for numerous types of data and with various intervals for the experience period and maturity age.

1. Continued

(b) It is not appropriate to combine indemnity and ALAE payments in the same development method triangle if they have different development patterns.

False
- Paid indemnity and ALAE do not need similar patterns to combine in a development triangle.
- The data may be combined as long as the relationship for each one is internally consistent, and the relationship between them is consistent.

(c) Credibility of the data is typically not considered when selecting age-to-age factors in the development method.

False
Credibility is judgmentally considered in selecting an appropriate experience period for age-to-age factors and selected averages.

(d) It is appropriate to assume that policies are earned evenly throughout the year when conducting a ratemaking analysis on aggregate stop-loss coverage.

False
This is valid for most GI polices but there are examples where it is not valid (e.g., hurricanes, hail coverages, or aggregate stop-loss).

(e) Given the same experience period, the claim trend period for reserving with the expected method is generally not the same as the claim trend period for ratemaking.

True
For reserves, claims are trended to the average date in the latest year (e.g., average accident date for accident year), For ratemaking, claims are trended to the average accident date in the future rating period (likely well after the reserving date).

2. **Learning Objectives:**

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.
(2d) Adjust historical earned premiums to current rate levels.
(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11, 12, and 16.

Commentary on Question:

This question tests the candidate's understanding of earned premium, unearned premium and adjusting earned premiums to current rate level. The candidate also needs to calculate the expected claim ratio.
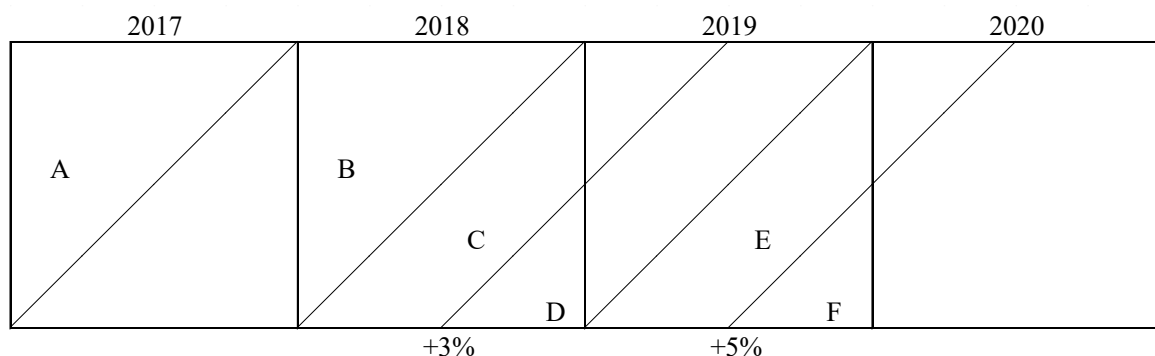
Solution:

(a) Calculate the unearned premium as of December 31, 2016.

$1/2 \times 500 \times 900 = 225,000$.

(b) Calculate the earned premium for:

    (i) Calendar year 2017, and

    (ii) Calendar year 2018.

2. **Continued**

For calendar year 2017:
- Section A = the portion of unearned premium @ Dec 31, 2016
- Section B = the portion of the 2,000 written policies in 2017 that are earned in 2017, or 1/2 of 2,000

Section	(1) Area	(2) # of Polices Written	(3) Average Premium	(4) = (1)(2)(3) Earned Premium
A	50.0%	900	500	225,000
B	50.0%	2,000	500	500,000
Total	100.0%			725,000

For calendar year 2018:
- Section B = the portion of the 2,000 written policies in 2017 that are earned in 2018, or 1/2 of 2,000
- Section C = the portion of the policies written in 2018 at the 500 premium level
- Section D = the portion of the policies written in 2018 at the 500×1.03 premium level

Section	(1) Area	(2) # of Polices Written	(3) Average Premium	(4) = (1)(2)(3) Earned Premium
B	50.0%	2,000	500	500,000
C	37.5%	2,200	500	412,500
D	12.5%	2,200	515	141,625
Total	100.0%			1,054,125

2. Continued

(c) Calculate the on-level earned premium at the 2019 average rate level for the purpose of projecting expected claim ratios for reserving, for:

  (i) Calendar year 2017, and

  (ii) Calendar year 2018.

As policies are written and earned uniformly throughout the year, the parallelogram approximation can be used to estimate the on-level factors.
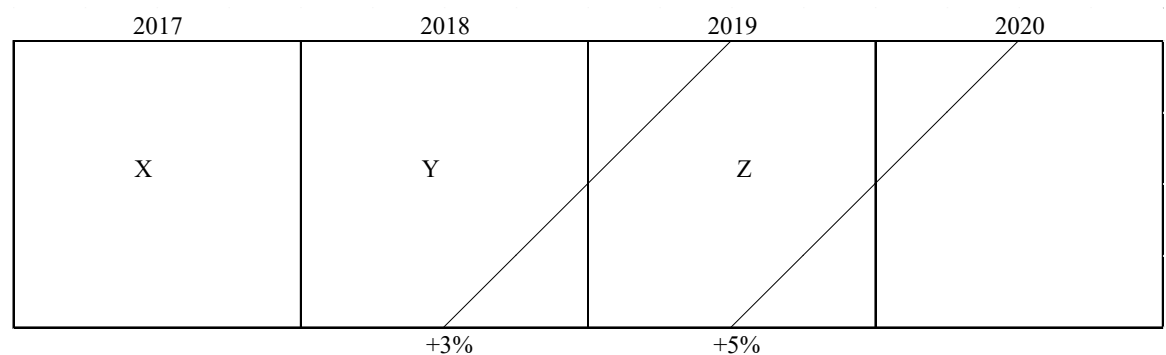
Rate Level	Rate Level Relative Value	Area in Calendar Year		
		2017	2018	2019
X	1.0000	100.0%	87.5%	12.5%
Y	1.0300		12.5%	75.0%
Z	1.0815			12.5%
Average Rate Level:		1.00000	1.00375	1.03269
Premium On-Level Factor		1.03269	1.02883	1.00000

e.g., $1.00375 = (1×0.875) + (1.03×0.125)$
$1.02883 = 1.03269 / 1.00375$

2017 on-level earned premium $= 725,000 × 1.03269 = 748,700$

2018 on-level earned premium $= 1,054,125 × 1.02883 = 1,084,515$

2. **Continued**

(d)   Calculate the 2019 cost level expected claim ratio, using the average of all years.

	(1)	(2)	(3)	(4)	(5) = (2)(3)(4)	(6) = (5)/(1)
Accident Year	On-Level Earned Premiums	Paid Claims	Development to Ultimate Factors	Claim Trend @3%	Ultimate Claims	Claim Ratio
2017	748,700	360,000	1.2	1.0609	458,309	61.2%
2018	1,084,515	450,000	1.5	1.0300	695,250	64.1%
2019	1,214,172	410,000	1.8	1.0000	738,000	60.8%
Average:						62.0%

3. **Learning Objectives:**
3.   The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:
(3c)   Identify the types of development triangles that can be used for investigative testing.
(3d)   Analyze development triangles for investigative testing.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 13.

Commentary on Question:
This question tests the candidate's understanding of various diagnostic tests on development triangles.

Solution:
(a)   Describe the expected pattern in a stable environment for each of:

      (i)   Data triangle A

      (ii)   Data triangle B

      (i)   The ratios are not expected to change materially from one accident year to another (i.e., looking down the values in each column).

      (ii)   The values are expected to be relatively consistent at each maturity age, with changes down each column (from accident year to accident year) limited to the rate of trend only.

(b)   Describe a limitation when analyzing data triangle A.

      Changes in the settlement of claims, which could affect the numerator (paid claims), can be offset by changes in claim processes related to case estimates, which could affect the denominator (reported claims).

(c)   Explain why it is preferable to use paid claims on closed counts when analyzing data triangle C.

      Commentary on Question:
      Other answers are possible.

      Paid claims on closed counts ensure there is a precise match between the payments in the numerator and the counts in the denominator.

3. **Continued**

    (d)   Describe what an unstable pattern could indicate for each of:

        (i)   Data triangle A

        (ii)   Data triangle B

        (i)   An unstable pattern could indicate a change in the settlement patterns or adequacy of case estimates over the experience period.

        (ii)   An unstable pattern could indicate a possible claims processing backlog.

    (e)   Describe one possible cause of this anomaly in data triangle E.

        Commentary on Question:
        Other answers are possible.

        An increase in case reserve adequacy.

4. **Learning Objectives:**
  3. The candidate will know how to calculate and evaluate projected ultimate values.

  4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

  8. The candidate will be able to define an approach for actuarial analyses supporting financial reporting and ratemaking analyses under various real-life scenarios. The candidates will be able to define funding requirements for self-insurers.

Learning Outcomes:
(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
(4f) Calculate claim liabilities.
(8d) Estimate ultimate values.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 16, and 23 .

Commentary on Question:
This question tests determining expected claims and unpaid claims using the expected method.

Solution:
(a) State a key assumption of the development method which is not fulfilled for XYZ Insurance.

  The activity observed to date is relevant for projecting future activity.

(b) Describe why earned vehicles are a better exposure base than earned premiums for XYZ Insurance.

  Earned premiums require adjustments for rate changes over the experience period. However, the number of vehicles is an example of an exposure base that requires no adjustment. All other things being equal, it is best to choose an exposure base that requires the fewest adjustments.

4. Continued

(c) Calculate expected claims for accident years 2018 and 2019.

Accident years 2014 to 2017 are fully developed and can be used for determining pure premiums. AY 2018 and 2019 should not be used due to the change caused by the restructuring of the finance team.

Accident Year (AY)	(1) Earned Vehicles	(2) Paid Claims	(3) % Claims Paid	(4) = (2)/(3) Ultimate Claims (000)	(5) Claim Trend @1.5%	(6) = (4)(5)/(1) Trended Pure Premiums
2014	76,000	10,900	100%	10,900	1.077	154.46
2015	76,000	11,400	100%	11,400	1.061	159.15
2016	81,000	12,100	100%	12,100	1.046	156.25
2017	81,000	12,700	100%	12,700	1.030	161.49
2018	*85,000*	*11,000*	*88%*	*12,500*	*1.015*	*149.26*
2019	*89,000*	*2,200*	*17%*	*12,941*	*1.000*	*145.40*

Average of 2014 through 2017 trended pure premium at AY 2019 cost level = 157.84.

Pure premium at AY 2018 cost level: 157.84 / 1.015 =   155.51
Expected claims for AY 2018: 155.51 × 85,000 =   13,218,350

Pure Premium at AY 2019 cost level: 157.84
Expected claims for AY 2019: 157.84 × 89,000 =   14,047,760

(d) Calculate the total unpaid claims as of December 31, 2019.

AY	Unpaid Claims	
2018	13,218,350 – 11,000,000 =	2,218,350
2019	14,047,760 – 2,200,000 =	11,847,760
Total		14,066,110

5. **Learning Objectives:**

3. The candidate will know how to calculate and evaluate projected ultimate values.

8. The candidate will be able to define an approach for actuarial analyses supporting financial reporting and ratemaking analyses under various real-life scenarios. The candidates will be able to define funding requirements for self-insurers.

Learning Outcomes:
(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
(3g) Estimate ultimate values using the methods cited in (3e).
(8d) Estimate ultimate values.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 15, 17, and 18.

Commentary on Question:
This question tests the candidate's understanding of various methods of estimating ultimate claims, as well as estimating ultimate claims using the Generalized Cape Cod method.

Solution:
(a) Compare the application of professional judgment in the Cape Cod method and the Bornhuetter Ferguson method.

Professional judgment can be used when selecting the expected value used in the Bornhuetter Ferguson method, whereas the expected value used in the Cape Cod method is determined by a formula and as such does not typically play a role.

5. Continued

(b) Provide one situation where the Generalized Cape Cod method is preferred over the Cape Cod method.

Commentary on Question:
Other answers are possible.

Any of the following is acceptable:
- Actuary wants to give different weights to years in the experience period (via a judgmental decay factor).
- Actuary wants to assume a distinct expected claim ratio for each year in the experience period. (The Cape Cod method assumes all years have the same expected claim ratio).
- Actuary wants to ensure that excessive weight is not given to years that are out of date relative to origin year.

(c) Provide one situation where the frequency-severity method is preferred over the Cape Cod method.

The frequency-severity method allows you to analyze frequency & severity separately.

(d) Calculate the accident year 2018 expected claims using the Generalized Cape Cod method and a decay factor of 90%.

Accident Year	(1) Earned Premiums (000)	(2) Premium On-Level Factor	(3) = (1)(2) On-Level Earned Premiums (000)	(4) Expected % Reported	(5) = (3)(4) Used-Up On-Level Earned Premiums (000)
2017	1,460	1.037	1,514	72.0%	1,090
2018	1,390	1.018	1,415	60.0%	849
2019	1,500	1.000	1,500	34.0%	510

Accident Year	(6) Actual Reported Claims (000) at Dec. 31, 2019	(7) Claim Trend Factor @1.5%	(8) = (6)(7) Adjusted Claims (000) at Dec. 31, 2019	(9) = (8)/(5) Claim Ratio
2017	800	1.030	824	75.6%
2018	630	1.015	639	75.3%
2019	410	1.000	410	80.4%

5. **Continued**

Accident Year	(10) Decay Factors by Accident Year 2019	(11) Decay Factors by Accident Year 2018	(12) Decay Factors by Accident Year 2017
2017	81.0%	90.0%	100.0%
2018	90.0%	100.0%	90.0%
2019	100.0%	90.0%	81.0%

AY 2018 expected claim ratio 2019 cost level =
{Σ column(5) × column(9) × column (11)} / {Σ column(5) × column(11)}

$$= \frac{1,090 \times 0.756 \times 0.9 + 849 \times 0.753 + 510 \times 0.804 \times 0.9}{1,090 \times 0.9 + 849 + 510 \times 0.9} = 76.5\%$$

Expected claims at 2018 cost level:
= 76.5% × AY2018 On-Level Earned Premium / AY2018 Claim Trend Factor
= 76.5% × 1,415 / 1.015 = 1,066

(e)   Calculate the accident year 2018 ultimate claims using the result from part (d).

Expected % unreported for AY2018 = 1 – 60.0% = 40.0%

AY2018 ultimate claims = 630 + 1,066 × 40.0% = 1,056

(f)   Re-calculate the accident year 2018 ultimate claims using a decay factor of 0%.

A decay factor of 0% would result in the development method.

Development method ultimate claims for AY2018 = 630 / 0.600 = 1,050

6. **Learning Objectives:**
1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

Learning Outcomes:
(1s) Analyze and describe the types of reinsurance.
(1t) Understand important reinsurance contract provisions that potentially affect actuarial work.

Sources:
Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix H.

Commentary on Question:
This question tests the candidate's understanding of the variability of reinsurance claims experience as well as certain details of reinsurance contracts.

Solution:
(a) State three reasons why a reinsurer often experiences greater variability in claims than a primary insurer.

Any three of the following are acceptable:
- Longer lags common with the reporting and settlement of reinsured claims
- Severity of large claims above high attachment points
- Lower frequency at higher reinsured layers
- Greater variability inherent in the types of claims typically covered by reinsurance
- The fact that case estimates are set by numerous claims adjusters working at many different primary insurers, each with its own processes and philosophies about claim management
- Limitations in the extent of detailed information that is available for reinsured claims

(b) Describe one other reinsurance contract provision that can result in ceded claims that vary based on experience.

Commentary on Question:
Other provisions are acceptable.

The reinsurance agreement may contain a negotiated profit-sharing (contingent) commission paid to the insurer at the end of the treaty year if the reinsurer's profits are greater than expected.

6. Continued

(c)   Calculate the amount paid by PPP and RRR for each claim.

Claim	PPP	RRR	
1,000,000	1,000,000	0	
5,000,000	1,000,000	4,000,000	
6,000,000	1,000,000	5,000,000	
12,000,000	7,000,000	5,000,000	*
5,000,000	0	5,000,000	
1,000,000	0	1,000,000	
Total	10,000,000	20,000,000	

* Primary company reaches AAD limit here so no further amounts for PPP

7.

Learning Objectives:
3.   The candidate will know how to calculate and evaluate projected ultimate values.

4.   The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3g)   Estimate ultimate values using the methods cited in (3e).
(3k)   Estimate ultimate claims by layer using common methods.
(3l)   Understand the differences in development patterns and trends for various claim layers.
(4d)   Demonstrate knowledge of good practice related to actuarial work supporting financial reporting.
(4f)   Calculate claim liabilities.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14 and 23.

Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix I.

Commentary on Question:
This question tests the calculation of claims excess of a limit using various methods.

Solution:
(a)   Calculate AY 2019 IBNR for claims excess of 500,000 as of December 31, 2019 using the following approaches:

  (i)   Development method applied to excess claims

  (ii)   Theoretically-derived development approach using Siewert's severity relativity formulas

  (iii)   The increased limits factor

  (i)   Ultimate claims excess of 500,000 = 400,000×7.084 = 2,833,600
           IBNR excess of 500,000 = 2,833,600 – 400,000 = 2,433,600

  (ii)   Age-to-ultimate development factor excess of 500,000
                   = 1.889×(1 – 0.830)/(1 – 0.950) = 6.423
           Ultimate claims excess of 500,000 = 400,000×6.423 = 2,569,200
           IBNR excess of 500,000 = 2,569,200 – 400,000 = 2,169,200

7. **Continued**

    (iii)   Total limits estimated ultimate claims
        =   Basic limits ultimate claims × Increased limit factor (ILF) for claims
            excess of 500,000
        = 10,360,000×1.185 = 12,276,600
        Ultimate claims excess of 500,000 = 12,276,600 – 10,360,000 = 1,916,600
        IBNR excess of 500,000 = 1,916,600 – 400,000 = 1,516,600

(b)   Recommend an AY 2019 IBNR reserve for claims excess of 500,000 as of December 31, 2019 using your results from part (a). Justify your recommendation.

    Recommend the IBNR of 1,516,600 from ILF method. The ILF method is recommended because it does not incorporate the actual excess reported claims at 12 months which is too immature and volatile to be relied upon.

8.   **Learning Objectives:**
2.   The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3.   The candidate will know how to calculate and evaluate projected ultimate values.

4.   The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(2a)   Create development triangles of claims and counts from detailed claim transaction data.
(3g)   Estimate ultimate values using the methods cited in (3e).
(4b)   Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 10, 19, and 22.

Commentary on Question:
This question tests the candidate's understanding of the classical paid-to-paid method for determining the ULAE ratio. In addition, this question tests the estimation of pure IBNR and unpaid ULAE using the refined approach to the paid-to-paid method.

Solution:
(a)   Determine the ULAE ratio using the classical paid-to-paid method.

Paid claims in calendar year 2017 = 59 + 129 + 90 = 278
Paid claims in calendar year 2018 = 62 + 138 + 95 + 38 = 333
Paid claims in calendar year 2019 = 64 + 151 + 106 + 41 + 24 = 386

ULAE ratio 2017 = 32 / 278 = 11.5%
ULAE ratio 2018 = 39 / 333 = 11.7%
ULAE ratio 2019 = 44 / 386 = 11.4%

With no apparent trend, the selected ULAE ratio is based on the average ULAE ratio = 11.5%

8. **Continued**

(b)   Estimate the pure IBNR *counts* as of December 31, 2019 for the automobile business.

Accident Year	(1) Reported Counts	(2) Closed Counts	(3) Disposal Ratio	(4) = (2)/(3) Ultimate Counts	(5) = (4) − (1) IBNR Counts
2018	1,132	1,085	0.864	1,256	124
2019	1,079	766	0.587	1,305	226

(c)   Estimate the pure IBNR *claims* as of December 31, 2019 for the automobile business.

Accident Year	(6) Ultimate Claims (000)	(7) = 1000×(6)/(4) Ultimate Severity	(8) = (5)(7)/1000 Pure IBNR Claims (000)
2018	12,130	9,658	1,198
2019	13,866	10,625	2,401
Total			3,599

(d)   Estimate unpaid ULAE for the automobile business as of December 31, 2019 using the ULAE ratio determined in part (a), a multiplier of 50%, and the pure IBNR claims from part (c).

Unpaid ULAE = (ULAE ratio × pure IBNR) + [ULAE ratio × multiplier × (case estimates + development on case estimates)]
   = 11.5%×(1,198 + 2,401) + [11.5%×50%×(12,130 + 13,866 − 8,473 − 7,215 − 1,198 − 2,401)] = 800.

9. **Learning Objectives:**
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 29.

Commentary on Question:
This question tests the candidate's understanding of expenses used for ratemaking.

Solution:
(a) Describe two approaches an actuary may take to address potential misestimation from using historical expenses for ratemaking purposes.

- Apply trending procedures to historical expenses to reflect anticipated changes between the experience period and the forecast period
- Rely on budgeted or planned expenses

(b) Describe two ways an actuary can assess the reasonableness of the future expenses used in ratemaking.

Any two of the following are acceptable:
- Consider how past experience compares to past forecasts
- Consider how past experience compares to current forecast (i.e., budget)
- Consider if projection is consistent with industry experience

(c) Describe a situation that would result in an inadequate expense provision if all expenses are treated as variable.

Fixed expenses will be underestimated if there is a rate decrease.

(d) Calculate the commission expense ratio. Justify your approach.

Expenses incurred at the inception of a policy are typically expressed as a % of written premiums.

Commission ratio = Total Commissions / Total Written Premiums
= 1,000 / 42,750 = 2.3%

9.   Continued

(e)   Calculate the operational expense ratio. Justify your approach.

Expenses incurred throughout the policy period are typically expressed as a % of earned premiums.

Historical operational expenses are trended from the average incurred date in each calendar year (i.e., July 1) to the average incurred date in the future rating period (i.e., July 1, 2021).

(1)	(2) = $1.02^{(1)}$	(3)	(4) = (2)(3)	(5)	(6) = (4)/(5)
Calendar Year	Trend Period (years)	Trend factor @ 2%	Operational Expenses	Trended Operational Expenses	Trended Earned Premiums at Current Rate Level	Expense Ratio
2017	4	1.082	850	919.70	12,250	7.5%
2018	3	1.061	975	1,034.48	13,930	7.4%
2019	2	1.040	1,050	1,092.00	15,000	7.3%
Average						7.4%

There are no significant outliers in the ratios, so the average is used.

10. **Learning Objectives:**

    6.   The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

    9.   The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:
(6g)   Calculate loadings for catastrophes and large claims.
(9c)   Describe the advantages and limitations of catastrophe models.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 30.

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:
This question tests the candidate's understanding of catastrophe models used in ratemaking.

Solution:
(a)   State two advantages of using catastrophe models in ratemaking.

    Commentary on Question:
    Other advantages are possible.

    Catastrophe simulation models use a database of scenario events that are designed to be comprehensive and realistic. The frequency of each event is calibrated to reflect the scientific view of the likelihood of that event.

    Catastrophe models allow users to import and analyze the current exposure and settlement terms, therefore avoiding the pitfalls in adjusting historical experience to reflect changes in the number, types, and values of structures exposed to the hazard.

10. Continued

(b)   Calculate the catastrophe loading expressed as a claim ratio, based on catastrophe model results.

Peril	(1) In-Force Date Reflected in Cat Model	(2) Modeled Expected Claims as of July 1, 2020	(3) Annual Exposure Trend	(4) Annual Severity Trend
Tornado	Nov. 1, 2019	36,000	1%	2%
Earthquake	Nov. 1, 2019	141,000	1%	6%

Peril	(5) Exposure Trend to July 1, 2020	(6) Severity Trend to January 1, 2022
	Trend Period in Months	
Tornado	8	18
Earthquake	8	18

Peril	$(7) = [1+(3)]^{[(5)/12]}$ Exposure Trend	$(8) = [1+(4)]^{[(6)/12]}$ Severity Trend	$(9) = (2)(7)(8)$ Trended Modeled Claims
Tornado	1.0067	1.0301	37,332
Earthquake	1.0067	1.0913	154,904
Total			192,236

Earned premium at current rate level =   11,290,000
Catastrophe loading = 192,236 / 11,290,000 =   1.70%

(c)   Recommend an approach for selecting the catastrophe load. Justify your recommendation.

Recommend using a credibility weighting between the catastrophe model and the experience-based loading of 0.5%. More weight should be given to the catastrophe model, as the actual event is infrequent.

11. **Learning Objectives:**

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 31.

Commentary on Question:

This question tests the calculation of overall rate indications using a pure premium approach.

Solution:

(a) Recommend the number of years to include in the weighted average pure premium for the ratemaking analysis. Justify your recommendation.

Three years gets to the full credibility standard, which balances responsiveness and stability.

(b) Calculate the weighted average pure premium.

Commentary on Question:

Other weights can be used.

The weighted average is calculated over the most recent three years of experience, using earned exposures as the weights.

Accident Year	Earned Vehicles	Ultimate Counts	Trended Ultimate Pure Premium	Weights Using Earned Vehicles
2017	58,000	1,460	255	30.7%
2018	67,000	1,490	220	35.4%
2019	64,000	1,550	240	33.9%
Total	189,000			237.53

(c) Calculate the indicated rate.

First need to determine $1 - V - Q$ using formula 31.5: $PCR = \dfrac{1 - V - Q}{1 + G}$

$1 - V - Q = PCR(1 + G) = 0.75 \times 1.05 = 0.7875$

Using formula 31.1: $R_I = \dfrac{PP + F}{1 - V - Q} = \dfrac{237.53 + 45}{0.7875} = 358.77$

11. Continued

(d)   Calculate the experience claim ratio.

Using formula 31.4: $ECR = \dfrac{C}{E \times R_C}$ and the most recent three years:

$C = (58{,}000{\times}255) + (67{,}000{\times}220) + (64{,}000{\times}240) = 44{,}890{,}000$

$$ECR = \frac{C}{E \times R_C} = \frac{44{,}890{,}000}{189{,}000{\times}325} = 73.08\%$$

12. **Learning Objectives:**
6.   The candidate will understand how to apply the fundamental ratemaking
        techniques of general insurance.

Learning Outcomes:
(6m)   Describe key considerations in the analysis of deductible factors and increased
        limits factors.
(6n)   Calculate deductible factors and increased limits factors.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

Commentary on Question:
*This question tests the candidate's understanding of how deductibles can affect claim
severity as well as determining deductible factors.*

Solution:
(a)   State two ways a deductible can reduce the claims paid by insurers.

        Any two of the following are acceptable:
        - Reduction of moral/morale hazard
        - Motivating risk control
        - Elimination of processing costs associated with small claims
        - Reduction of exposure to catastrophic claims

(b)   Determine a deductible amount that will *decrease* the insurer's claim severity.

        Commentary on Question:
        Other examples are possible.

        If deductible = 0, then the average severity would be (200+1000)/2 = 600.
        If deductible = 100, then the average severity would be (100+900)/2=500, which
        is a decrease compared to no deductible.

(c)   Determine a deductible amount that will *increase* the insurer's claim severity.

        If the deductible = 300, then the average severity would be (700)/1 = 700, which
        is an increase compared to no deductible.

12. Continued

(d)   Calculate the indicated deductible factors for deductibles of 2,000 and 4,000 relative to the base deductible.

Claim eliminated at 1,000 deductible
$$= 300,000 + 1,000 \times (280 + 100 + 20) = 700,000$$
Total claims at 1,000 deductible $= 1,500,000 - 700,000 = 800,000$

Claim eliminated at 2,000 deductible
$$= 300,000 + 400,000 + 2,000 \times (100 + 20) = 940,000$$
Total claims at 2,000 deductible $= 1,500,000 - 940,000 = 560,000$

Claim eliminated at 4,000 deductible
$$= 300,000 + 400,000 + 300,000 + 4,000 \times 20 = 1,080,000$$
Total claims at 4,000 deductible $= 1,500,000 - 1,080,000 = 420,000$

Indicated factor for 2,000 deductible $= 560,000 / 800,000 = 0.700$
Indicated factor for 4,000 deductible $= 420,000 / 800,000 = 0.525$

(e)   Recommend a factor for a deductible of 3,000. Justify your recommendation.

Deductible	Deductible Factor
1,000	1.000
2,000	0.700
3,000	x
4,000	0.525

First, the factor for a deductible of 3,000 needs to be between 0.525 and 0.700, and we can use the consistency test to find the appropriate range for a factor.

For the 1,000 to 2,000 deductible compared to the 2,000 to 3,000 deductible:
$(1 - 0.700) / (2000 - 1000) > (0.700 - x) / (3000 - 2000)$, which solves for $x > 0.400$

For the 2,000 to 3,000 deductible compared to the 3,000 to 4,000 deductible:
$(0.700 - x) / (3000 - 2000) > (x - 0.525) / (4000 - 3000)$, which solves for $x < 0.6125$

Therefore, any deductible factor which satisfies $0.525 < x < 0.6125$ is acceptable.

13. **Learning Objectives:**
6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(6q) Distinguish occurrence-based and claims-made based coverage.
(6r) Calculate rates for claims-made coverage as well as claims-made maturity and tail factors.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:
This question tests the candidate's understanding of claims-made ratemaking.

Solution:
(a) Calculate the premium for a report year 2020 mature claims-made policy effective January 1, 2020.

Premium = (60,000 + 5,000) / (1 – 0.18 – 0.04) = 83,333

(b) Calculate the tail factor following the mature claims-made coverage from part (a).

Accident Year (AY) by Report Year (RY) Matrix of Ultimate Claims

AY Lag	% from prior years	2020	2021	2022	2023
0	50%	30,000	33,000	36,300	39,930
1	30%	18,000	19,800	21,780	23,958
2	20%	12,000	13,200	14,520	15,972

e.g., RY 2020, AY Lag 1 = 30%×60,000×1.1^2

Numerator: 19,800 + 14,520 + 13,200 =   47,520
Denominator: 30,000 + 18,000 + 12,000 =   60,000
Tail factor = 47,520 / 60,000 =   0.792

(c) Calculate the premium for the 2021 occurrence policy.

2021 occurrence policy expected claims = 33,000 + 21,780 + 15,972 = 70,752
2021 fixed expenses = 5,000 × (1 + 0.02) = 5,100
Premium = (70,752 + 5,100) / (1 – 0.18 – 0.04) = 97,246

13. Continued

(d)   Explain whether or not ABC Services would need a tail policy.

A tail policy covers claims that are reported after a claims-made policy has expired or been cancelled but that is not covered by the occurrence policy.

14.

Learning Objectives:

3.   The candidate will know how to calculate and evaluate projected ultimate values.

4.   The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(3e)   Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g)   Estimate ultimate values using the methods cited in (3e).

(4f)   Calculate claim liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14 and 23.

Commentary on Question:

This question tests the calculation of IBNR using the development method when there is a legislative change that reduces claim costs. This question also tests the candidate's understanding of tail factors.

Solution:

(a)   Calculate reported claim development factors using a volume-weighted average.

Accident	Adjusted Incremental Paid Claims			
Year	12	24	36	48
2016	*3,650*	*3,460*	1,040	600
2017	*4,050*	3,300	1,000	
2018	3,150	2,900		
2019	3,900			

Note: Claims prior to calendar year 2018 are multiplied by 90% to reflect the 10% reduction in claims costs due to the legislative change (e.g., 4,056×0.9 = 3,650).

Accident	Adjusted Cumulative Paid Claims			
Year	12	24	36	48
2016	*3,650*	*7,110*	8,150	8,750
2017	*4,050*	7,350	8,350	
2018	3,150	6,050		
2019	3,900			

14. **Continued**

Accident	Adjusted Case Estimates			
Year	12	24	36	48
2016	*2,850*	*1,890*	1,800	1,400
2017	*3,600*	2,000	1,900	
2018	3,650	2,650		
2019	3,400			

Note: Case estimates prior to calendar year 2018 are multiplied by 90% to reflect the 10% reduction in claims costs due to the legislative change (e.g., 3,167×0.9 = 2,850).

Accident	Adjusted Reported Claims			
Year	12	24	36	48
2016	*6,500*	*9,000*	9,950	10,150
2017	*7,650*	9,350	10,250	
2018	6,800	8,700		
2019	7,300			

Adjusted reported claims = Adjusted cumulative paid claims + Adjusted case estimates

Volume-weighted average reported claims development factors:
12-24: (9,000 + 9,350 + 8,700) / (6,500 + 7,650 + 6,800) = 1.291
24-36: (9,950 + 10,250) / (9,000 + 9,350) = 1.101
36-48: 10,150 / 9,950 = 1.020

(b) Calculate the accident year 2019 IBNR reserve as of December 31, 2019 using the results from part (a).

	(1)	(2)	(3)	(4) = (1)(3)	(5) = (4) – (1)
		Age-to-Age	Age-to-Ultimate		
Accident	Reported	Development	Development	Ultimate	
Year	Claims	Factors	Factors	Claims	IBNR
2016	10,150	1.030	1.030	10,455	305
2017	10,250	1.020	1.051	10,773	523
2018	8,700	1.101	1.157	10,066	1,366
2019	7,300	1.291	1.493	10,899	3,599
Total	36,400			42,193	5,793

e.g., 1.493 = 1.030×1.020×1.101×1.291

14. Continued

(c)   Identify two other approaches that could be used to determine a tail factor for this coverage.

Any two of the following are acceptable:
- Bondy method
- Algebraic method
- Modified Bondy method
- Curve fitting methods

(d)   Critique each approach identified in part (c).

Commentary on Question:
Only the critiques to the first two from part (c) are provided here.

The Bondy method repeats the last observed development factor as the tail factor. This method is simple but could be appropriate in this case as this is short-tailed.

The algebraic method is used to determine a paid tail based on the relationship between the paid and reported triangles. This is not appropriate for determining a tail factor for reported claims.

15.

Learning Objectives:

3.   The candidate will know how to calculate and evaluate projected ultimate values.

7.   The candidate will understand the need for monitoring results.

Learning Outcomes:

(3e)   Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3f)   Demonstrate knowledge of good practice related to projecting ultimate values.

(3g)   Estimate ultimate values using the methods cited in (3e).

(7b)   Analyze actual claims experience relative to expectations.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 17, and 36 .

Commentary on Question:

This question tests the calculation of ultimate claims using the development and Bornhuetter Ferguson methods. This question also tests the candidate's understanding of monitoring actual versus expected reported claims.

Solution:

(a)   Provide two situations where the BF method is preferred over the development method.

Any two of the following are acceptable:
- immature experience
- new product introduced
- new geographical area
- if there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

15. Continued

(b) Calculate the ultimate claims as of December 31, 2018 for each accident year using the development method.

Accident Year	(1) Reported Claims as of Dec. 31, 2018	(2) Unreported Claim Percentages	(3) = 1 / [1 − (2)] Age-to-Ultimate Development Factor	(4) = (1)(3) Development Method Ultimate Claims
2016	300,000	10%	1.1111	333,330
2017	240,000	30%	1.4286	342,864
2018	166,000	50%	2.0000	332,000

(c) Calculate the ultimate claims as of December 31, 2018 for each accident year using the BF method.

Accident Year	(5) Earned Premium	(6) Premium On-Level Factors	(7) = (5)(6)×0.60 Expected Claims	(8) = (1) + (2)(7) BF Method Ultimate Claims
2016	500,000	1.035	310,500	331,050
2017	520,000	1.015	316,680	335,004
2018	560,000	1.000	336,000	334,000

(d) Calculate the actual versus expected reported claims for calendar year 2019, using the BF method.

Accident Year	(9) Reported Claims as of Dec. 31, 2019	(10) = (9) − (1) Actual Reported Claims in 2019	(11) Expected Percent Claims Reported in 2019	(12) = (7)(11) Expected Reported Claims in 2019	(13) = (10) − (12) Actual − Expected Reported Claims
2016	335,000	35,000	10%	31,050	3,950
2017	308,000	68,000	20%	63,336	4,664
2018	238,000	72,000	20%	67,200	4,800

Column (11) expected % claims reported in 2019:
- AY2016: between 36 and 48 months development = 10%
- AY2017: between 24 and 36 months development = 30% – 10% = 20%
- AY2018: between 12 and 24 months development = 50% – 30% = 20%

15. Continued

(e)   Critique the appropriateness of the selected assumptions as of December 31, 2018, using the results from part (d).

Actual reported claims are higher than expected for all three accident years. Therefore, the initial expected claim ratio is likely inappropriate and recommend increasing it.

16.

Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(3g) Estimate ultimate values using the methods cited in (3e).
(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 15 and 25.

Commentary on Question:

This question tests the development-based frequency-severity method for estimating ultimate claims.

Solution:

(a) Describe two alternative sources for trend if an insurer's own claim experience is not sufficiently credible.

Any two of the following are acceptable:
- Industry data if it is applicable
- Combine with regional or countrywide experience, but review differences in regulatory/legal environment such as statutes of limitations, caps on damages
- Other affiliated insurers, so long as there are similar policies with respect to underwriting, claim management and reinsurance

16. Continued

(b) Recommend an annual severity trend to use for the frequency-severity method. Justify your recommendation.

Commentary on Question:
Candidates need to analyze severity (claims / counts) and not pure premium (claims / exposures) for severity trend. Other options for selected trend are possible.

Accident Year	(1) Ultimate Claims	(2) Ultimate Counts	(3) Earned Exposures	(4) = (1)/(2) Severity	(5) = $(4)_i/(4)_{i-1} - 1$ Change in Severity
2015	4,600,000	500	6,500	9,200	
2016	5,000,000	520	6,700	9,615	4.51%
2017	5,250,000	540	6,900	9,722	1.11%
2018	5,550,000	545	6,900	10,183	4.74%
2019	6,550,000	585	7,300	11,197	9.95%

The change in severity is erratic and 2019 might be influenced by highly leveraged development factors. A trend between 4% and 5% appears reasonable, so an annual trend of 4.5% is recommended.

(c) Calculate the accident year 2018 ultimate claims using the development-based frequency-severity method.

Accident Year	(6) Severity Trend at 4.5%	(7) = (4)(6) Trended Severity	(8) = (2)/(3) Indicated Frequency
2015	1.1925	10,971	0.0769
2016	1.1412	10,973	0.0776
2017	1.0920	10,616	0.0783
2018	1.0450	10,641	0.0790
2019	1.0000	11,197	0.0801

Selected severity at 2019 level = 10,800 (average excluding 2019)
Selected frequency at 2019 level = 0.078 (average excluding 2019)

AY 2018 ultimate severity = 10,801 / 1.045 = 10,335
AY 2018 ultimate counts = 0.078×6,900 = 538
AY 2018 ultimate claims = 10,335×538 = 5,560,230

17.

Learning Objectives:

2.   The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

4.   The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(2c)   Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

(4g)   Describe the components of premium liabilities in the context of financial reporting.

(4h)   Evaluate premium liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11 and 24 .

Commentary on Question:

This question tests the candidate's understanding of premium liabilities.

Solution:

(a)   Define premium liabilities.

Premium liabilities are the value of claim and expense payments to be made after the accounting date related to insured events occurring after the accounting date arising from policies effective on or before the accounting date.

(b)   Calculate the unearned premium reserves as of September 30, 2020, gross and net of reinsurance.

Gross of reinsurance = Gross written premium × 3/12
        = 1,600,000 × 3/12 = 400,000
Net of reinsurance = (Gross written premium – Reinsurance premium) × 3/12
        = (1,600,000 – 240,000) × 3/12 = 340,000

17. Continued

(c) Calculate the expected claims including ALAE for valuation of premium liabilities as of September 30, 2020, gross and net of reinsurance.

(1)	(2)	(3)	(4) = (2) + (3)	(5) = (4)×10%
		Expected Windstorm Claims		
Probability	Catastrophe Claims	Non Catastrophe Claims	Total	ALAE
2%	10,000,000	150,000	10,150,000	1,015,000
98%	0	150,000	150,000	15,000
Total			350,000	35,000

(1)	(6) = (4) + (5)	(7)	(8) = (6) − (7)
	Total Including		
Probability	ALAE	Retained	Reinsured
2%	11,165,000	5,000,000	6,165,000
98%	165,000	165,000	0
Total	385,000		123,300

Notes:
- Probabilities use the quarter 4 expected distribution of windstorm claims
- Expected windstorm claims excluding catastrophe claims = 3×50,000
- Totals are the sumproduct of the probabilities and the amounts
  - e.g., 0.02×10,150,000 + 0.98×150,000 = 350,000

Gross expected claims including ALAE = 350,000 + 35,000 = 385,000
Net expected claims including ALAE = 350,000 + 35,000 − 123,300 = 261,700

(d) Calculate the premium liabilities as of September 30, 2020, gross and net of reinsurance.

ULAE = 5% of gross expected claims = 0.05×385,000 = 17,500
Policy administration expenses = 3% of gross earned premium
= 0.03×400,000 = 12,000

Gross premium liabilities = 385,000 + 17,500 + 12,000 = 414,500
Net premium liabilities = 261,700 + 17,500 + 12,000 = 291,200

17. Continued

(e)   Determine the equity (or premium deficiency) in gross and net unearned premium as of September 30, 2020, labeling your answers as equity (or premium deficiency), as applicable.

	Gross of Reinsurance	Net of Reinsurance
Unearned premium reserve	400,000	340,000
Less: Policy Liabilities	(414,500)	(291,200)
Equity / (Deficiency)	(14,500)	48,800

18.

Learning Objectives:

3.   The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3h)   Explain the effect of changing conditions on the projection methods cited in (3e).

(3i)   Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 20.

Commentary on Question:

This question tests the candidate's understanding of how claims data is affected by various changing conditions.

Solution:

(a)   Describe how a change in third party claim administrator will affect each of the following:

   (i)   Indemnity, ALAE, ULAE, or combination

   (ii)   The triangle row, column, diagonal, or other effect

   (iii)   Paid or reported data, or both

   (i)   Likely result in a change in case reserve adequacy for indemnity and ALAE.

   (ii)   Case reserve adequacy change would affect diagonal in a triangle.

   (iii)   Case reserve adequacy change affects reported data only.

18. Continued

    (b)   Describe how a change from in-house legal staff to external legal consultants will affect each of the following:

        (i)   Indemnity, ALAE, ULAE, or combination

        (ii)   The triangle row, column, diagonal, or other effect

        (iii)   Paid or reported data, or both

        (i)   Likely result is a change in the level of ALAE costs.

        (ii)   Change would affect cost on the diagonal of a triangle.

        (iii)   Change in claims and/or ALAE costs would affect paid data.

    (c)   Describe how a change in the system used for processing claim payments will affect each of the following:

        (i)   Indemnity, ALAE, ULAE, or combination

        (ii)   The triangle row, column, diagonal, or other effect

        (iii)   Paid or reported data, or both

        (i)   Likely result is a change in settlement pattern affecting indemnity and ALAE.

        (ii)   Change in settlement pattern would affect diagonal in a triangle.

        (iii)   Change in settlement pattern primarily affects paid data.

18. Continued

(d) Describe how a new tort reform law capping punitive damages for all claims occurring on or after January 1, 2016, will affect each of the following:

   (i)   Indemnity, ALAE, ULAE, or combination

   (ii)   The triangle row, column, diagonal, or other effect

   (iii) Paid or reported data, or both

   (i)   Likely result is a change in indemnity.

   (ii)   Change in claims would affect rows in triangle for 2016 and subsequent.

   (iii) Change in claims affects both paid and reported data.

19.

Learning Objectives:

9.   The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:

(9a)   Describe the structure and modules of catastrophe models.

(9c)   Describe the advantages and limitations of catastrophe models.

(9d)   Understand and apply common risk metrics associated with catastrophe modeling results.

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a)   Provide an example of the hazard intensity component of a catastrophe model for the following events:

      (i)   Hurricane

      (ii)   Earthquake

      (i)   Local intensity of the event; what conditions are inside the event footprint.

      (ii)   Inundation depth of a flood, wind speed of a hurricane, or ground movement accelerations of an earthquake.

(b)   Recommend whether you should use catastrophe modeling for each of the four perils. Justify your recommendations.

      I.   Fire: No
             Historical fire events are frequent, adequate, and credibility for analysis.

      II.   Flood: Yes
             Infrequent past flood events with volatile severity. Affecting a lot of risk all at once in certain area, meeting the definition of a catastrophe.

      III.   Hail: No
             Historical hail events are frequent, adequate, and credible for analysis.

      IV.   Tornado: Yes
             Infrequent past tornado events with volatile severity. This affects a lot of risk all at once in certain area, meeting the definition of a catastrophe.

19. Continued

(c)   Explain why the probable maximum loss (PML) for all causes of loss combined is less than the sum of the PMLs from each cause of loss.

To illustrate why PMLs are not additive, consider the probability that a one in 100-year event occurs for each cause of loss. The probability that all causes have a one in 100-year event in the same year is much less than 1 percent. Therefore, the sum of the one in 100-year PMLs is associated with a much longer return period.

20. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.

(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).

(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

(5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 26.

Commentary on Question:

This question tests the candidate's understanding of trend on premiums for ratemaking purposes.

Solution:

(a) Recommend the annual premium trend to account for changes in the proportion of homeowners policyholders with the 10% discount. Justify your recommendation.

Commentary on Question:

The justification needs to include the reason why 2017 is removed from the recommendation.

Calendar Year	(1) Percent of Policyholders with Discount	(2) Average Discount Factor	(3) = $(2)_i/(2)_{i-1} - 1$ Annual Change
2015	15%	0.985	
2016	17%	0.983	–0.203%
2017	23%	0.977	–0.610%
2018	25%	0.975	–0.205%
2019	28%	0.972	–0.308%

e.g., $2015 = 0.15 \times (1 - 0.10) + (1 - 0.15) \times 1 = 0.985$

- Ignore the change in 2017 due to the one-time shift
- Recommend average of all years excluding 2017 = –0.24%
- Justification: the years other than 2017 reflect a reasonably stable trend and need to remove 2017 due to the one-time shift that would not be expected again in the future

20. Continued

(b)   Calculate the 2015 earned premium at current rate level trended for ratemaking.

Need to split the trend calculation into 2 pieces due to the one-time shift in 2017: from 2015 to 2019 (current trend), and from 2019 to future rating period (forecast trend).

Current trend to 2019: Use the average discount factors from part (a) to trend from average earned date in 2015 to average earned date in 2019 = 0.972 / 0.985 = 0.9868

Forecast trend from 2019:
- Rates effective from October 1, 2020 to April 1, 2022, or 18 months.
- Therefore, average earned date = 9 months after October 1, 2020, or July 1, 2021.
- Trend period from July 1, 2019 to July 1, 2021 or 24 months (2 years).

Trend factor = $(1 - 0.0024)^2 = 0.9952$

2015 on-level premium trended to the future rating period = 540,000 × 0.9868 × 0.9952 = 530,314

21. **Learning Objectives:**
6.   The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(6l)   Calculate risk classification changes.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

Commentary on Question:
This question tests the candidate's understanding of risk classification.

Solution:
(a)   Describe how credibility and homogeneity can present conflicting objectives in the actuarial work supporting risk classification systems.

Risk classes should be sufficiently large to maintain credibility yet dividing a group into more risk classes may improve homogeneity.

(b)   Calculate the indicated class relativities for the territory risk characteristic.

(1)	(2)	(3)	(4) = (3)/(2)	(5) = (4)$_i$/(4)$_{Total}$
Territory	Trended Earned Premium at Current Rate Level	Trended Ultimate Claims	Claim Ratio	Indicated Relativity
1	12,500,000	7,500,000	60.0%	0.901
2	4,500,000	3,150,000	70.0%	1.051
3	5,000,000	4,000,000	80.0%	1.201
Total	22,000,000	14,650,000	66.6%	1.000

(1)	(6)	(7)	(8)	(9) = (5)(7) + [1 − (7)](8)
Territory	Ultimate Counts	Credibility	Complement of Credibility	Credibility Weighted Indicated Relativity
1	3,000	100.0%	1.0	0.901
2	810	90.0%	1.0	1.046
3	250	50.0%	1.0	1.101
Total	4,060			

Note: Credibility = $\sqrt{(Ultimate\ Counts)/1,000}$ to a maximum of 1.0.

21. Continued

(c)   Calculate the new base rate.

(1)	(2)	(9)	(10) = (9)$_i$/(9)$_1$
	Trended Earned Premium at Current	Credibility Weighted Indicated	Relativity to Base
Territory	Rate Level	Relativity	Territory
1	12,500,000	0.901	1.000
2	4,500,000	1.046	1.161
3	5,000,000	1.101	1.222
Total	22,000,000		1.083

Note: 1.083 = Sumproduct[(2),(10)] / 22,000,000

New base rate = 500 / 1.083 = 461.68

(d)   Provide an assumption that is required to obtain a unique set of estimates of parameters $\lambda, \alpha_1, \alpha_2$, and α_3.

Either of the following is acceptable:
- Set one of the α values equal to zero, which is equivalent to setting that risk class as the base class, as the multiplicative factor for that class will be 1.
- Force the α values to add to zero. This ensures exp(λ) is the overall pure premium.

(e)   State the implication of rejecting the null hypothesis.

Rejecting the null hypothesis that all the α values are zero provides statistical evidence that the factor does have discriminatory power.