AN OVERVIEW OF THE BALL STATE CONFERENCE

by

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Introduction

Most actuaries have heard the criticism that their basic intellectual tool, the mathematics of life contingencies, solidified sometime in the last century and not much has happened since. Often this general criticism is followed by a series of statements of which the following are typical. Life contingencies, with its stress on expected values, misses the whole point that time until death is a random variable. The entire risk dimension is left out of most treatments of life contingencies. Actuaries don't seem to realize that the construction of a life table is a statistical estimation problem. As if all of this is not bad enough, life contingencies traditionally uses fixed interest rates and thereby neglects another important component of risk faced in the management of an insurance enterprise. Then, when it comes to computation, actuaries developed the commutation functions approximately two centuries ago. They persist in stating their basic formulas in terms of these functions, although they no longer represent efficient computing formulations. In many ways, the Ball State Conference was directed to answering the critics who have expressed these views.

Individual Risk Theory

For over a century it has been known that the entire mathematics of life contingencies may be built on a stochastic foundation. That is, when time until death is viewed as a random variable, with its distribution given by the life table, premiums and reserves may be defined in terms of expected values of appropriate conditional loss functions. This approach permits the development of all of the traditional results and yields a dividend in the ease by which the variances of future losses may be computed and used as a measure of mortality risk. In the keynote presentation at the Conference, Aaron Tennenbein summarized this individual risk theory approach to life contingencies.

Once the fundamental observation that time until death is a random variable and that premiums and reserves are expected values is made, the possibility of developing interrelations among life contingencies and other probability models exists. Hans Gerber developed several of these relationships in his paper. Some difficult results in life contingencies become very easy when placed in a more general setting.

Individual Risk and Hazy Distributions

If actuarial values are in fact expected values, it becomes important to consider the certainty with which the basic distributions are known. Since life tables are estimated from a sample of lives, there is a sampling distribution for the estimate of each q_x. These distributions will induce distributions for functions derived from the life table estimate. Stuart Klugman showed how the distributions of the estimates of q_x may be used to find the variance in the estimate of a life annuity value (a_x), induced by sampling variation in the estimate of the mortality probabilities.

Arnold Shapiro's contribution to the conference extended the traditional individual risk theory model, which assumes that time until termination and cause of termination (death, disability, withdrawal, retirement) are two random variables, to incorporate uncertainty about the basic distribution assumptions. This uncertainty is not due to sampling error in the estimation of the basic probabilities, as in the Klugman contribution, but rather is fundamental uncertainty about the environment in which a pension system will evolve. By an ingenious use of Bayesian analysis, he incorporates both sources of uncertainty in making stochastic pension projections.

William Bailey is also concerned about uncertainty about basic mortality probabilities. He too uses Bayesian theories to incorporate this uncertainty. His application involves the computation of stop-loss insurance premiums.

Estimating the Distribution

In any approach to life contingencies, the life table is a key tool. If one stresses that the survival function one uses is an estimate of the distribution of time until death, new insights into life table construction are possible. Several contributions to the conference developed these insights.

Stuart Klugman considered the problem of graduating observed values of q_x. He adopted the general framework of Whittaker-Henderson. That is, he measured departures from perfect fit and smoothness in the form of a function F+kS. However, rather than using squared deviations as a measure of fit, following Whittaker, or absolute deviations, as done by Donald R. Schuette in a current TSA paper, he adopted a blended measure of fit. The blend has the effect of using squared deviations in measuring small deviations and absolute deviations when measuring large deviations. This idea, adopted from the field of "robust statistics" is designed to reduce the impact on graduated values of extreme observations.

The actuarial method of constructing life tables is based on the idea of estimating a sequence of conditional probabilities of death (q_x), with the intervals of age fixed. Of course, the main problem with this approach is the existence of incomplete observations, lives not observed for the entire age interval. Actuaries developed the exposure concept to solve this problem. In its traditional form, the exposure idea uses the Balducci assumption, ${}_{1-t}q_{x+t} = (1-t)q_x$. T. N. E. Greville explored some of the implications on exposure formulas of using the alternative constant force of deaths assumption. His contribution elicited discussion from Robert Batten and Hendrik Boom.

In the field of statistics, most estimation problems start with the assumption of a family of distributions. Then sample information is used to estimate the parameters of the distribution. With the notable exception of Gompertz and Makeham distributions, actuaries have seldom used this approach. However, the exception is important, and William Wetterstrand presented an exhaustive study of the variation in Makeham constants among sets of recent mortality data. He also explored modifications of the basic Makeham model. In reviewing the results, one is impressed by how well the Makeham function fits beyond young ages. Perhaps the exponential function for the force of mortality is about as close as we will come to a universal law in biology.

Random Interest

In recent years several research minded actuaries have worked on the development of models for life insurance and annuities where both time until death and the interest rate earned on invested assets are random variables. At the Ball State Conference, D. R. Bellhouse and H. H. Panjer presented a major advance in this development. Their model builds on the life table model for random time until death. They augment this model with a time series model for the interest rate. The analysis is simplified by an ingenious use of moment generating functions. The conclusion is that statistical properties of actuarial functions may be determined from combined models for a broad group of time series models for interest.

Computation

Computation has undergone a technological and economic revolution in the past twenty five years. Inexpensive hand-held computers now possess programming capabilities. This ability means that recursive relations may be easily used with small computers to calculate actuarial values. Difference equations, which in classical life contingencies led to insights but were seldom used directly in computation, have renewed importance. William Roach presented a paper which illustrated how a small, inexpensive computer may be programmed to produce a variety of actuarial functions.

Extension of Classical Models

Several papers presented at the Conference illustrate that, despite statements to the contrary, the traditional deterministic life contingencies model can lead to new results and deeper insights. Cecil Nesbitt used a dynamic pension model with population and salary growth to explore the implications of a pension system in which the portion of the ultimate pension, accrued to date under the individual funding method used, is vested. A variety of interesting results were derived. Because of the interest of many pension reformers in improved vesting and portability features of private pension plans, the results are of more than academic interest.

Daniel Jesionowski examined the stationary population model. He studied an apparent paradox in the application of three dimensional Lexis type diagrams to the portrayal of stationary population problems. The paradox was resolved by constructing a revised model.

Richard Ziock examined Lidstone's theorem, proved within classical life contingencies by the manipulation and interpretation of the difference equations that generate reserves. By shifting to the three factor contribution dividend formula, he produced a new, informal, demonstration of Lidstone's theorem.

Public Policy

Although it has seldom been clearly articulated, it appears that most theories for the determination of individual premiums assume that a class of insureds is a group such that the deviations in losses (actual-expected) behave as random noise. If there are non-random aspects of these deviations, opportunities for anti-selection exist and equity will be served by the creation of additional classes. Several recent court decisions, legislative proposals, and administrative directives seem to imply that random behavior of actual minus expected losses is not a satisfactory criterion for insurance classification from the public's view. Public policy considerations may require consideration of new principles of premium determination. Neil Vance contributed a paper in which he attempted to build mathematical models for new concepts of equity as they appear to be emerging in recent actions.

Educational Response

The response of the Society of Actuaries to the developments in life contingencies, as represented by the papers presented at the Ball State Conference, was the subject of a panel headed by Warren Adams, Director of Education of the Society. Adams described how the Society's Education and Examination Committee developed plans for new educational materials reflecting the ideas presented at the Conference. The team of authors (Bowers, Gerber, Hickman, Jones, Nesbitt), recently selected to carry out the project, outlined some of the ideas that they hope to incorporate into the new textbooks.