USING CASH FLOWS TO ALLOCATE INVESTMENT INCOME

by
Charles E. Johnson

ABSTRACT

Cash flows can be used to allocate investment income among various lines of business according to their contributions to income-producing assets. This method of allocating investment income may be adapted to either an investment-year technique--whereby yearly average investment rates are maintained--or an in-force technique--whereby individual asset investment rates are recognized--according to the needs of the user.

INTRODUCTION

The purpose of this paper is to explain the method of Using Cash Flows to Allocate Investment Income. Since the income-producing assets of a life insurance company are purchased with the cash flows generated by its various lines of business, it would appear appropriate to allocate the investment income produced from these invested assets among the various lines of business according to their cash flows.

This paper will be limited to explaining the investment-year technique and the in-force technique of Using Cash Flows to Allocate Investment Income. A company may (in effect) wish to use both techniques for allocating investment income--for example, the in-force technique among various lines of business and the investment-year technique within each of the various lines of business. [The former technique allocates investment income more accurately, while the latter technique provides a means for allocating investment income among various generations of policyholders.] This paper is not concerned with the uses of, or the designs of systems to effect, these techniques but only with the concepts as developed herein.

Concepts to Be Used

The potential investment income at a point in time is the sum of the products of the book values (or costs) of invested assets in-force times their respective

59

periodic investment rates. The potential investment income over an interval of time
is the weighted average of the potential investment incomes at a number of points
on the interval. Suppose we assume that all changes in invested assets over an
interval of time occur at some particular point on the interval between two end
points from which potential investment income is measured. Then at the point of
change the previous end point's potential investment income can be used in developing
the cash flow made available for the purchase of invested assets in the interval.

Two types of cash flow are developed for allocating investment income: the cash
flow that is actually made available for the purchase of invested assets in the period
under consideration; and the cash flow that represents the period's original contribu-
tion to investments. Of course, the difference between these two cash flows is the
total contribution to new investments by prior periods. The first cash flow is the actual
contribution to new investments and is used to allocate invested assets among various
lines of business. Since the second cash flow represents the period's original cont-
ribution to investments, it is used to maintain the investment rate at which the period's
original contribution is actually invested.

Since cash flows are used to allocate invested assets among various lines of
business, it is only necessary to keep track of invested assets by period of purchase.
All periods of original contribution to invested assets will have their contributions
pass through the period of purchase. These contributions are according to the two types
of cash flow discussed in the previous paragraph. This does away with having to track
invested assets any further back than the period of purchase since the investment rate
at which a period's original contribution is actually invested can be maintained by
properly following the chains of invested asset turnovers by periods of purchase and
properly recognizing the original contributions (distinguishing the current period's
from prior periods') at each period of purchase. Also, nothing needs to be assumed
as to the average rate at which invested assets turn over since this method recognizes
the actual incidence of turnover in developing the cash flows.

In essence, what the rest of this paper does is to take these concepts--that is,
tying cash flows and purchases of invested assets to periods--and develop an allocated

cash flow model, one for each line of business, for allocating investment income.

Order of Discussion

This method of allocating investment income will be initially developed for a simplified version of the investment-year technique--with either all years select or select and ultimate years. This simplified version recognizes the fact that, for allocating investment income among various lines of business,only the average invest- ment rate for the calendar year of purchase (instead of the average investment rate for the calendar year of original contribution) needs to be adjusted on invested asset turnovers. Also, consideration will be given to the effects of the policy loans and the amortization of bonds and mortgages on the two types of cash flow developed for this technique. Since this technique as originally introduced uses a new money rate for the calendar year of purchase as the replaced investment rate on invested asset turnovers which will tend to overstate the potential investment income on the first sales of invested assets purchased in a particular calendar year (as a result of lower yielding invested assets being liquidated first), the investment-year technique will be modified to recognize the individual investment rate on each invested asset sold (or matured). Next, a second version of the investment-year technique which maintains calendar-year average investment rates for allocating investment income among various generations of policyholders will be studied. This second version recognizes that over each period of purchase, original contributions to invested assets by calendar year are according to the two types of cash flow discussed in the previous subsection. By properly recognizing the original contributions to invested assets (distinguishing the current period's from prior periods') at each period of purchase and properly following the chains of invested asset turnovers by periods of purchase, the average investment rate for each calendar year can be maintained as the average rate at which a calendar year's original contribution is actually invested.

For companies not wishing to maintain calendar-year average investment rates, the in- force technique--which dispenses with the need to maintain calendar-year average investment

rates by using, instead, actual in-force investment figures--will be developed. Next, the administration needed for investments will be discussed followed by some comments on the treatment of the home office and the electronic equipment owned by the company. Lastly, consideration will be given to improving the exposure element used in deriving potential investment income.

INVESTMENT-YEAR TECHNIQUE

This section will develop the cash flows (C^P's) that are actually made available for the purchase of invested assets, which are used to allocate invested assets purchased in a calendar year among various lines of business--thus, being the primary cash flows for this method of allocating investment income--and those cash flows (C_c's and C_K's) that represent a period's original contribution to investments, which are maintained expressly for administering the investment-year technique. Differentiating between these two types of cash flow is most important in understanding this technique as well as the basic theory underlying the method of Using Cash Flows to Allocate Investment Income.

Premises

The following premises will be required in developing the investment-year technique of Using Cash Flows to Allocate Investment Income:

First Premise: Investable assets equal assets which are cash or were cash at one time but are now invested in income-producing assets.

Second Premise: The increase in investable assets for a calendar year does not change after that calendar year; only the average investment rate changes--due to turnovers (which include scheduled

asset write-ups and write-downs)[1] of invested assets purchased
in that calendar year.

Third Premise: Invested assets purchased or sold (or matured) in a calendar
year are assumed to be transacted on some common date.

The First Premise allows the cash flow generated (made available) by a line of
business to be its potential contribution to income-producing assets. The Second
Premise makes it possible to identify contributions to income-nroducing assets by
calendar year. [Note: The above accounting is used instead of keeping track of a
calendar year's original contribution to invested assets plus investment income and
capital gains and losses earned thereon.] Since investment income is to be allocated
using cash flows which, in themselves, would require the initial allocation of invest-
ment income, the Third Premise allows the final allocation of investment income to be
obtained by only one iteration instead of a number of iterations--and would appear
to be not unreasonable. [This can be seen from the concept that if all changes in
invested assets over an interval of time occur at some particular point on the
interval between two end points from which potential investment income is measured,
then at the point of change the previous end point's potential investment income can
be used in developing the cash flow made available for the purchase of invested assets
in the interval.]

                                                                                        ı

1
 A scheduled asset write-up or write-down of an invested asset may be considered a
partial turnover of an invested asset. This scheduled asset write-up or write-down
is determined at the time of purchase and results from the investment rate (determined
by market conditions) differing from the stated rate of an invested asset. Also, for
purposes of this paper, the return of principal on mortgages will be considered a
scheduled asset write-down.

Cash Flow for a Line of Business

Using the above premises and remembering that the total cash flow for the company as a whole must be equal to the sum of the cash flows for its various lines of business, the __cash flow__ for a calendar year for a particular line of business to be used in allocating investment income may be symbolized as follows (there being a corresponding cash flow component for __each__ line of business):

Given the following:

C^P = Allocated Cash Flow made available for the purchase of invested assets in the current calendar year,

C_c = Allocated Cash Flow Made Available to Increase Investable Assets for the current calendar year, and

C_K = Allocated Cash Flow Made Available to Increase Investable Assets for calendar year K (where K may be any calendar year prior to the current calendar year including an all inclusive "prior-to" calendar year),

such that the following holds true:

C^P = $GII_A - IE_A + A - GT + NICF$,

C_c = $GII - IE + GA - GT + NICF$ (similarly for C_K), and

NICF = $P - B - CI - E5 - E6 - D - F - \Delta$,

where symbols for the insurance operations are as follows:

NICF = Net Insurance Cash Flow,

P = Premiums and Considerations (allocated by the ledger),

B = Benefits (allocated by the ledger),

CI = Commissions (allocated by the ledger),

E5 = Exhibit 5 Expenses (allocated according to the current allocation procedure),

E6 = Exhibit 6 Expenses (allocated according to the current allocation procedure),

D = Dividends (allocated by the ledger),

F = Federal Income Taxes excluding capital gains tax (allocated according to the current allocation procedure), and

Δ = Increase in Cash Required to Support Insurance Cash Flow--a discretionary value (optional) based upon equity and sound judgment (allocated according to the nature of the lines of business),

and these for the investment operations are as follows:

GII_A = Gross Investment Income--income from invested assets (allocated according to the end of the previous calendar year's allocation of potential investment income,[2] after being adjusted for scheduled asset write-ups and write-downs),

IE_A = Investment Expenses (allocated consistent with the allocation of GII_A),

GII = Gross Investment Income--true investment income (allocated according to the current calendar year's allocation of potential investment income, that is, in proportion to the GII_P's),

IE = Investment Expenses (allocated consistent with the allocation of GII),

A = Consideration from the Sale (or Maturity) of Invested Assets (allocated according to the C^P's in the calendar year of purchase),

GA = Gain from the Sale (or Maturity) of Invested Assets--consideration less book value (allocated according to the C^P's in the calendar year of purchase), and

GT = Capital Gains Tax (allocated according to the procedure used on the applicable invested assets sold),

---

[2] To allocate C^P in compliance with the Third Premise. Notice, since it is assumed that no invested asset purchases or sales (or maturities) have taken place in the current calendar year before the common date, the investment income allocation on the common date remains unchanged from that of the end of the previous calendar year, that is, it remains in proportion to the end of the previous calendar year's allocated potential investment incomes of investable assets: For each line of business, either $\sum_K (C_K \times i_{NK}) + (C_c \times i_N)$, using previous calendar year's symbols, or $\sum_K (C_K \times i_K)$, using current calendar year's symbols, (to be discussed).

and using the following:

i_N = Average Investment Rate (new money rate) for invested assets purchased in the current calendar year,

i_K = Average Investment Rate at the <u>beginning</u> of the current calendar year for invested assets purchased in calendar year K adjusted for the turnover of these invested assets,

i_{NK} = Changed Average Investment Rate for calendar year K due to the turnover in the current calendar year of invested assets purchased in calendar year K,

$(i_N)_K$ = Average Investment Rate (new money rate) for invested assets purchased in calendar year K,

A_K = Book Value (or Cost) on the Sale (or Maturity) of Invested Assets purchased in calendar year K (allocated according to the C^P's in calendar year K),

α = Weighted Average Time assumed to have elapsed from the beginning of the current calendar year to the common date for all invested asset purchases and sales (including maturities) (possibly, weighted according to annual investment income thereon), and

GII_D = Derived Gross Investment Income for the current calendar year-- the potential investment income used to allocate the true gross investment income,

the following relationship, which weights the allocated potential investment incomes of investable assets at the beginning of and at the end of the calendar year, may be derived from the Third Premise for the current calendar year:

$$(1) \quad [\ (1 - \alpha)\ (C_C^3 \times i_N)] + \sum_K\ [\ \alpha\ (C_K \times i_K) + (1 - \alpha)\ (C_K \times i_{NK})] = GII_D;$$

to be used as follows:

3
Since GII is to be allocated, substitute allocated GII_A (adjusted for scheduled asset write-ups and write-downs, to be discussed) and IE_A for allocated GII and IE, respectively, which is in compliance with the Third Premise. See Footnote 2.

$$\frac{GII_D}{\Sigma GII_D} \times [\text{total GII}] = \text{GII (allocated to each line of business)};$$

while compliance with the Second Premise requires the Average Investment Rate

for each calendar year K to be changed as follows:

(2)
$$\frac{[C_K \times i_K] - [A_K \times (i_N)_K] + [A_K \times i_N]}{C_K^4} = i_{NK}.$$

Facts About the Investment-Year Technique

The reader should be especially cognizant of the following facts about the

investment-year technique:

1. For a particular calendar year, each line of business will have a

corresponding cash flow component, as developed above, to be used in allocating

investment income. The sum of each cash flow component for all lines of business will

equal that for the company as a whole. In order to keep the notation as simple as

possible, and since the methods described herein are generally applicable to all

lines of business, the notation will not identify any one particular line of business.

In the discussion which follows, if reference is made to an allocated cash flow

component, the name of the cash flow component will be preceded by the word "allocated";

otherwise, reference is made to the unallocated cash flow component. [Notice: The

allocation of each cash flow component among various lines of business is indicated

above in parentheses.]

2. The C^P's are the Allocated Cash Flows generated (from both the insurance

operations and the investment operations) by the various lines of business which may

be used to purchase invested assets for the current calendar year. These cash flows,

one for each line of business, maintained by calendar year, are the bases for allocat-

ing invested assets among various lines of business. [Thus: The calendar year of

---

4

In the rare case where C_K equals zero, $C_K \times i_K$ would be maintained.

purchase determines which set of C^P's is used in allocating the consideration from the sale (or maturity) of invested assets, as well as the capital gain (or loss) and the capital gains tax. Also, to be discussed, the scheduled asset write-ups and write-downs would be allocated similarly.]

3. The C_c's and the C_K's are the Allocated Cash Flows Made Available to Increase Investable Assets, one for each line of business, for the applicable calendar years. They identify each calendar year's contribution to investable assets--that is, they represent the "new moneys" for each of the applicable calendar years. They differ from the corresponding C^P's by the turnover of invested assets, that is, by the allocated Book Values (or Costs) on the Sale (or Maturity) of Invested Assets (A_K's) which were purchased from previous calendar years' generated cash flows made available to increase investable assets and by the allocated scheduled asset write-ups and write-downs for the calendar year then current (to be discussed), (as well as having differing investment income allocations).[5] [Note: As will be shown, the difference between C^P and C_c resulting from allocated scheduled asset write-ups and write-downs for the calendar year then current is effected by using allocated GII_A in C^P and allocated GII in C_c.] These Allocated Cash Flows Made Available to Increase Investable Assets are the bases for calculating the Derived Gross Investment Income (GII_D) for each line of business under the investment-year technique; and the total of these allocated cash flows for all lines of business and for all prior calendar years to the present equals the total investable assets for the company as a whole.

----

[5] An example: The Net Change in Ledger Assets in Exhibit 12 of the Convention Blank, with the entries for the adjustment in book value of ledger assets reversed, represents the sum of the C_c's for the current calendar year. If this sum is adjusted for the turnover of invested assets, the sum of the C^P's is obtained.

4. GII_A differs from GII. GII_A includes all income from invested assets, before adjusting for scheduled asset write-ups and write-downs, and is allocated (after being adjusted for scheduled asset write-ups and write-downs, to be discussed) according to the end of the previous calendar year's allocation of potential investment income, since it is used in the Allocated Cash Flows made available for the purchase of invested assets (C^P's). [Notice, according to the Third Premise, all invested assets purchased or sold (or matured) in a calendar year are assumed to be transacted on some common date.] GII represents the true investment income, after recognizing scheduled asset write-ups and write-downs, and is allocated according to the current calendar year's allocation of potential investment income--that is, in projortion to the GII_D's. [Note: For all lines of business, for the calendar year, the total of the allocated GII_A's plus the total scheduled asset write-ups less the total scheduled asset write-downs equals the total of the allocated GII's. Thus, the amounts (both of which are the true investment income for the current calendar year) one of which is allocated to the C^P's and the other of which is allocated to the C_c's, respectively, according to the end of the previous calendar year's and the current calendar year's allocation of potential investment income are equal--which they should be.]

5. The fraction ⁼ and its complement are used as the exposure elements in weighting the potential investment incomes of investable assets respectively before and after invested asset purchases and sales (including maturities)--both assumed to take place on some common date, in compliance with the Third Premise. Notice, this Third Premise allows the initial allocation of investment income, which is according to the end of the previous calendar year's allocation of potential investment income, to be used in the Allocated Cash Flows made available for the purchase of invested assets (C^P's) and,thus, requires only one iteration in obtaining the final allocation of investment income.

6. The GII_D's--each of which is a weighted average of the applicable allocated potential investment incomes of investable assets at the beginning of and at the end of the calendar year--are the potential gross investment incomes for the current calendar

year derived for the various lines of business based upon the assumptions inherent in this method. Since their sum will differ from the true gross investment income, the latter will be allocated among various lines of business according to the relative amounts of these Derived Gross Investment Incomes. [Notice, in the formula for GII_D (Equation (1)), the quantity C_c has investment income (GII) allocated to it according to the end of the previous calendar year's allocation of potential investment income (which is in compliance with the Third Premise); but the quantity C_c to be used in future calendar years (then identified as C_K) will have investment income (GII) allocated to it according to the current calendar year's allocation of potential investment income, that is, in proportion to the current GII_D's.]

7. Decrements (Δ's) to the Net Insurance Cash Flows may be required because the relationship of the true cash flows generated by various lines of business may differ from that made available for the investment operations as a result of relative liquidity needs, for example, lines of business having restricted cash values versus those making cash values readily available. Also, these decrements may be used as a judgment factor to reduce the effect of the investment strategy in allocating investment income among various lines of business.

8. Notice, for each line of business, the base on which the Average Investment Rate for invested assets purchased in calendar year K adjusted for the turnover of these invested assets (i_K) is changed is C_K (Equation (2))--and not C^P for calendar year K--in order to properly reflect the contribution to Derived Gross Investment Income (GII_D) (Equation (1)) by the allocated Book Value (or Cost) on the Sale (or Maturity) of Invested Assets purchased in calendar year K (A_K) (which is the total allocated contribution to new investments by prior periods that has passed through calendar year K). [Thus, since C^P (supposedly, the sum of the allocated book values to be used in making future adjustments to a period's Average Investment Rate) is not used as the base in adjusting the Average Investment Rate, this Average Investment Rate is not, except prior to the first invested asset turnover, the average rate at which invested assets purchased in calendar year K and/or their initial turnover are/were actually invested.] The use of an Average Investment Rate for invested assets purchased in

calendar year K adjusted for the turnover of these invested assets results in a simplified version of the investment-year technique in that only the Average Investment Rates of calendar years that had invested asset purchases which turned over in the current calendar year are changed instead of the Average Investment Rates of all prior calendar years that originally contributed to the Book Values (or Costs) on the Sale (or Maturity) of Invested Assets (A_K's). [This latter version--which, for each line of business, gives an Average Investment Rate for each calendar year which is the average rate at which the original contribution to investable assets for that calendar year is actually invested--will be discussed as a second version of the investment-year technique in the subsection "Maintaining the Integrity of the Calendar-Year Average Investment Rate."] Therefore, when using the simplified version of the investment-year technique, the reader should be aware of the following:

(1) The Average Investment Rate (i_K) maintained by line of business, when associated with the Allocated Cash Flow Made Available to Increase Investable Assets (C_K) as in Equation (1), can only be used in deriving potential gross investment income for all calendar years' contributions to investable assets combined and not for each calendar year's separately--to be discussed in the next numbered paragraph. [Notice, the latter would require i_K to be the average rate at which C_K is actually invested.]

(2) This limits the use of the Average Investment Rate in that it can only be used to allocate investment income among various lines of business and cannot be used to allocate investment income among various generations of policyholders--to be discussed in a subsequent subsection.

The purchases of invested assets may be influenced by the different requirements of various lines of business. [An example of a requirement would be the scheduling of the maturities of invested assets which affects the investment rate.] This influence may be reflected in the allocation of investment income by having each calendar year a new money rate (i_N) for each line of business instead of only one overall new money rate. [The desired degree of pooling for allocating investment income will determine which set of new money rates to use.] Likewise, recognition

would also be given to this influence on the turnover of invested assets.

Comment: To properly reflect the true potential investment income of
invested assets in the Derived Gross Investment Incomes,
the Average Investment Rate (i_K) for each line of business should
also be changed in those calendar years in which there is a change
(for whatever reason) in the investment rate for an invested asset
purchased in calendar year K. This can be accomplished by sub-
stituting the new investment rate for the old investment rate using
the allocated book value (or cost) of the invested asset as the base.

9. The theory underlying the investment-year technique is as follows: In
compliance with the Second Premise, for each line of business, the Average Investment
Rate (i_K) associated with the Allocated Cash Flow Made Available to Increase Investable
Assets (C_K) for some particular calendar year K will change in each calendar year in
which there is a turnover of an invested asset purchased in calendar year K (Equation
(2)) in order to properly reflect the contribution to Derived Gross Investment Income
(GII_D) by the allocated Book Value (or Cost) on the Sale (or Maturity) of Invested
Assets purchased in calendar year K (A_K) (which is the total allocated contribution to
new investments by prior periods that has passed through calendar year K). [Note: This
Average Investment Rate will also change in each calendar year in which there is a
scheduled asset write-up or write-down of an invested asset purchased in calendar year
K--to be discussed.] After all invested assets purchased in calendar year K have turned
over, the Average Investment Rate will remain constant; and thereafter, for each line
of business, the term for the Allocated Cash Flow Made Available to Increase Investable
Assets for calendar year K in the formula for the Derived Gross Investment Income will
remain constant. Therefore, for each line of business, as income-producing assets turn
over, this procedure (which adjusts the Average Investment Rate only for the calendar
year of purchase on invested asset turnovers) results in a continual process of new
potential investment income replacing previous potential investment income for all
calendar years' contributions to investable assets combined--thus, being the underlying
theory behind the investment-year technique. It is most instructive for the reader to
trace the investable assets through a number of turnovers to see that the above process

does generate the proper potential gross investment income for investable assets at any
particular point in time. [Note: For each line of business, this technique gives the
proper Derived Gross Investment Income for all calendar years' contributions to invest-
able assets combined but not for each calendar year's separately--to be discussed in
a subsequent subsection.]

Under this procedure investment income, as well as related investment expenses,
would be allocated twice in any calendar year--(1) initially according to the end of
the previous calendar year's allocated potential investment incomes of investable
assets to be used in obtaining the Allocated Cash Flows made available for the purchase
of invested assets (C^P's), one for each line of business, which is in compliance with
the Third Premise, and (2) finally in proportion to the Derived Gross Investment
Incomes (GII_D's), one for each line of business, to obtain the current calendar year's
allocation of investment income, which is used in developing the Allocated Cash Flows
Made Available to Increase Investable Assets (C_C's and C_K's), maintained by line of
business expressly for the investment-year technique. The C^P's are the bases for
allocating invested assets among various lines of business, while the C_K's are the
bases for deriving potential gross investment income for each line of business. One
drawback to this procedure would be the negative allocated cash flow (or small positive
allocated cash flow) in the initial years of a new line of business resulting from
the small volume of in-force business to support the start-up expenses and the early-
year costs...this could be mitigated to some extent by "grading-in" expenses. [Notice,
this procedure has general application and may be used even when there are negative
allocated cash flows.]

Consideration for Policy Loans

The amount of cash flow invested in policy loans will affect the amount available
for the investment operations. Therefore, in addition to the above, any line of
business with policy loans--which earn interest at the policy loan rate(s)--should
maintain, separately by calendar year and, if necessary, by interest rate, the cash
flow and corresponding interest income applicable to policy loans...as well as identifying

their related exnenses. This information would be used to adjust the amount of cash flow to that amount available for the investment operations and to adjust the amount of the investment income to that amount to be allocated. [Notice: For a line of business, a decrease in policy loans for the current calendar year causes an increase in the amount of C^P and C_c not otherwise available for the investment operations and vice versa.]

Amortization of Bonds and Mortgages

A scheduled asset write-down or write-up of an invested asset would be analogous to a sale or purchase, respectively, at the new money rate--the constructive "sale" or "purchase" being allocated among various lines of business according to the C^P's in the calendar year of purchase. This constructive sale or purchase effectuates a redistribution of amounts between Gross Investment Income--income from invested assets (GII_A) and Consideration from the Sale (or Maturity) of Invested Assets (A) for the current calendar year cash flow. Notice, income from invested assets should be adjusted by the return of principal (amortization of premium) and the scheduled increase in invested assets (accrual of discount) before allocating what remains--that is, the true Gross Investment Income. An example of a scheduled asset write-up follows.

An Example of a Scheduled Asset Write-Up

A scheduled asset write-up of an invested asset in the current calendar year results in a redistribution of amounts between Gross Investment Income--income from invested assets (GII_A) and Consideration from the Sale (or Maturity) of Invested Assets (A) for the current calendar year cash flow, which may be analyzed for each line of business as follows (where $-A_K'$, allocated according to the C^P's in calendar year K, represents the scheduled asset write-up of an invested asset purchased in calendar year K):

1. Since the cash flow made available for the purchase of invested assets in the current calendar year cannot change, the following holds true:

$$C^P = (GII_A + A_K') - IE_A + (A - A_K') - GT + NICF,$$

which results in an increase in investment income...in conformity with Generally Accepted Accounting Principles (GAAP). Thus, C^P is decreased by the allocated scheduled

asset write-up which is then offset by the allocated adjustment to income from invested assets.

    2. Similarly, Derived Gross Investment Income is increased by $[(1 - \infty) \left\{ A_K^{'} \times (i_N)_K \right\}]$, as follows:

$$[(1 - \infty) (C^P \times i_N)] + \sum_{K-1} [(C_K \times i_K) - (1 - \infty) \left\{ A_K \times (i_N)_K \right\}]$$

$$+ [C_K \times i_K] - [(1 - \infty) \left\{ A_K \times (i_N)_K \right\}] - [(1 - \infty) \left\{ (-A_K^{'}) \times (i_N)_K \right\}] = GII_D.^6$$

    3. The Average Investment Rate for calendar year K is changed as follows:

$$\frac{[C_K \times i_K] - [A_K + (-A_K^{'})] \times (i_N)_K + [A_K + (-A_K^{'})] \times i_N}{C_K} \approx i_{NK}$$

with more assets $(A_K^{'})$ being invested at rate $(i_N)_K$, consistent with Statements 1 and 2 above.

    4. Since $C_c = C^P - \sum_K A_K - (-A_K^{'}),^7$ that is, the Allocated Cash Flow Made Available to Increase Investable Assets for the current calendar year is increased by allocated $A_K^{'}$, the following potential investment income is derived for future calendar years:

    From the current calendar year: $[A_K^{'} \times i_N]$ and

    From calendar year K: $[-A_K^{'} \times i_N] + [A_K^{'} \times (i_N)_K]$,

an increase in potential investment income of $[A_K^{'} \times (i_N)_K]$, the desired result. [Notice, for all lines of business combined, the term $-(-A_K^{'})$ in the above formula for C_c is the difference between GII_A (which is in the C^P's) and GII (which is in the C_c's).]

    Of course, this same analysis may be performed for a scheduled asset write-down by substituting $A_K^{''}$, the scheduled asset write-down of an invested asset purchased in calendar year K, for $-A_K^{'}$.

6

This form of the formula for Derived Gross Investment Income may be easily obtained by substituting in Equation (1), the formula for i_{NK}, Equation (2), assuming C_c has the same investment income allocation as C^P.

7

Except for C_c and C^P having differing investment income allocations.

Maintaining the Integrity of the Asset Investment Rate

Since the investment-year technique as discussed so far uses a new money rate for the calendar year of purchase as the replaced investment rate on invested asset turnovers which will tend to overstate the potential investment income on the first sales of invested assets purchased in a particular calendar year (as a result of lower yielding invested assets being liquidated first), it may be necessary to maintain the integrity of the asset investment rate where needed. This may be accomplished by modifying Equations (1) and (2), as follows:

In addition to previous notation, given the following (for each line of business):

$i_{K,j}$ = Investment Rate for Invested Asset j purchased in calendar year K, and

$A_{K,j}$ = Book Value (or Cost) on the Sale (or Maturity) of Invested Asset j purchased in calendar year K (allocated according to the C^P's in calendar year K),

then the following relationship, which weights the allocated potential investment incomes of investable assets at the beginning of and at the end of the calendar year, may be derived from the Third Premise for the current calendar year:

$$[(1 - \alpha)\ (C_c^8 \times i_N)] + \sum_K [\alpha\ (C_K \times i_K) + (1 - \alpha)\ (C_K \times i_{NK})] = GII_D,$$

while compliance with the Second Premise requires the Average Investment Rate for each calendar year K to be changed as follows:

$$\frac{[(C_K \times i_K) - \sum_j (A_{K,j} \times i_{K,j})] + [A_K \times i_N]^9}{C_K} = i_{NK}.$$

This modification not only eliminates the inherent discrepancies in using

---

8
See Footnote 3.

9
Using the Average Investment Rate after these invested assets have turned over is most appropriate since they will not turn over again as purchases in calendar year K--thus, no longer requiring the maintenance of the asset investment rates for these invested assets. Consideration from these invested assets will be reinvested in future calendar years for which the asset investment rates will be maintained until such invested assets are turned over.

Average Investment Rates but also uses these averages most effectively where such discrepancies will not occur--that is, when invested assets do not turn over. Thus, the exact investment rates are maintained by properly adjusting the Average Investment Rates for invested asset turnovers.

Comment: Of course, this problem of overstating the potential investment income on the first sales of invested assets purchased in a particular calendar year could be rectified in other ways. For example, instead of using a new money rate for the calendar year of purchase as the replaced investment rate, an average investment rate for all invested asset turnovers in the current calendar year could be used. This average investment rate would be the average rate at which these invested assets were yielding immediately prior to their turnover. Alternatively, this average investment rate could vary by calendar year of purchase. [Again, the desired degree of pooling for allocating investment income will determine the average investment rate(s) to be used on invested asset turnovers.]

Maintaining the Integrity of the Calendar-Year Average Investment Rate

As mentioned previously in paragraphs 8 and 9 of the subsection "Facts About the Investment-Year Technique," the Average Investment Rate for invested assets purchased in calendar year K adjusted for the turnover of these invested assets (1_K) maintained by line of business for the investment-year technique can only be used to allocate investment income among various lines of business and cannot be used to allocate investment income among various generations of policyholders. The latter would require, for each line of business, an Average Investment Rate for each calendar year K which is the average rate at which the Allocated Cash Flow Made Available to Increase Investable Assets for calendar year K (C_K) is actually invested. At first thought, this would appear to require the impossible--identifying invested assets according to each calendar year's original contribution instead of by calendar year of purchase, as under the present technique.

BUT...this problem can be resolved by adjusting the Average Investment Rates, one for each line of business, for each prior calendar year that originally contributed to the Book Values (or Costs) on the Sale (or Maturity) of Invested Assets $(A_K$'s)

according to its contribution. [Note: The Book Value (or Cost) on the Sale (or Maturity) of Invested Assets purchased in calendar year K is allocated among various lines of business according to the C^P's in calendar year K. Thus, the allocation remaining--that is, the allocation of the allocated book values (or costs) of the invested assets purchased in calendar year K which have turned over to each calendar year of original contribution for adjusting its Average Investment Rates--would be completed as explained below for each line of business.] For each calendar year in which there is a turnover of an invested asset purchased in calendar year K, the proportion of the allocated Book Value (or Cost) on the Sale (or Maturity) of an Invested Asset originally contributed by calendar year K is $[C_c/C^P]_K^{10}$, that is, for each line of business, the ratio of the Allocated Cash Flow Made Available to Increase Investable Assets for calendar year K to the Allocated Cash Flow made available for the purchase of invested assets in calendar year K; and this proportion of the allocated book value (or cost) would be used in adjusting the Average Investment Rate for calendar year K (that is, adjusting i_K to i_{NK}), for each line of business. The remaining proportion of the allocated book value (or cost), that is, for each line of business, $[(C^P - C_c)/C^P]_K$, represents the sum of the original contributions by calendar years prior to calendar year K--that had invested asset purchases which turned over in calendar year K (call these calendar years L) or were in the chain of invested asset turnovers leading up to calendar years L. Thus, this remaining proportion of the allocated book value (or cost) would be allocated to each previous calendar year L according to the ratio of the allocated Book Value (or Cost) on the Sale (or Maturity) of Invested Assets in calendar year K that were purchased in calendar year L to the total of the allocated Book Values (or Costs) on the Sale (or Maturity) of Invested Assets in calendar year K that were purchased in prior calendar years, that is, for each line of business, $[A_L/\sum_L A_L]_K$.

Now, for each previous calendar year L, this process would be repeated. Each previous calendar year L could have allocated to it the Book Value (or Cost) on the Sale (or Maturity) of Invested Assets purchased in calendar year L that have turned

---

10
  A bracketed symbol(s) with subscript K indicates the corresponding amount(s) for calendar year K.   78

over in a subsequent calendar year (similar to each calendar year K) or have allocated to it (for each line of business) a proportion of the allocated Book Value (or Cost) on the Sale (or Maturity) of Invested Assets purchased in a subsequent calendar year that have turned over--where calendar year L had invested asset purchases which turned over in this subsequent calendar year or was in the chain of invested asset turnovers leading up to this subsequent calendar year. As before, the proportion of these allocated Book Values (or Costs) on the Sale (or Maturity) of Invested Assets orig-inally contributed by calendar year L is $[C_c/C^P]_L$, for each line of business; and this proportion of the allocated book values (or costs) would be used in adjusting the Average Investment Rate for calendar year L (that is, adjusting i_L to i_{NL}), for each line of business. The remaining proportion of the allocated book values (or costs), that is, for each line of business, $[(C^P - C_c)/ C^P]_L$, represents the sum of the original contributions by calendar years prior to calendar year L--that had invested asset purch-ases which turned over in calendar year L (call these calendar years M--which are of the same kind as calendar years L) or were in the chain of invested asset turnovers leading up to calendar years M--and would be allocated to each previous calendar year M accord-ing to $[A_M/\underset{M}{\Sigma} A_M]_L$, a set of ratios for each line of business. In any current calendar year, this process (of following chains of invested asset turnovers by periods of purchase and recognizing original contributions at each period of purchase) would be continued, for each line of business, starting with the calendar years K--that had invested asset purchases which turned over in the current calendar year--and working backwards through all calendar years L, ending in an ultimate period or, if none, the earliest select period. [Note: Scheduled asset write-ups and write-downs, in the above process, would be treated as constructive purchases and sales, respectively.]

This second version of the investment-year technique, like the original version, results in a continual process of new potential investment income replacing previous potential investment income but, unlike the original version, provides the Average Investment Rates (i_K's) for each line of business which are the average rates at which the Allocated Cash Flows Made Available to Increase Investable Assets are actually invested and which can be used to allocate investment income among various

generations of policyholders. [Note: For allocating the allocated book values (or costs) of the invested assets which turn over, this second version would require, for each line of business, the additional maintenance, by calendar year, of the following K ratios: $A_K \ / \ \underset{K}{\Sigma} \ A_K.$]

IN-FORCE TECHNIQUE

If invested asset purchases and sales (including maturities) are accounted for using their exact investment rates (second preceding subsection), this causes the calendar-year Average Investment Rate to lose its significance for allocating investment income among various lines of business under the investment-year technique. But, calendar-year Average Investment Rates may still be maintained as in the preceding subsection for allocating investment income among various generations of policyholders. For those companies not wishing to maintain calendar-year Average Investment Rates, the method of Using Cash Flows to Allocate Investment Income may be effectuated through an in-force technique which uses the actual allocated book value (or cost) of invested assets as the basis for deriving the potential investment income for each line of business, as follows for a calendar year:

In addition to previous notation, given the following (for each line of business):

C_0 = Allocated Book Value (or Cost) of Invested Assets in-force at the beginning of the current calendar year (allocated according to the C^P's in the calendar year of purchase),

C_1 = Allocated Book Value (or Cost) of Invested Assets in-force at the end of the current calendar year (allocated according to the C^P's in the calendar year of purchase),

A_j = Cost on the Purchase of Invested Asset j in the current calendar year (allocated according to the C^P's in the current calendar year),

A_K' = Scheduled Asset Write-up of invested assets purchased in calendar year K (allocated according to the C^P's in calendar year K),

A_K'' = Scheduled Asset Write-down of invested assets purchased in calendar year K (allocated according to the C^P's in calendar year K),

80

i_0 = Average Investment Rate for Invested Assets in-force at the beginning of the current calendar year,

i_1 = Average Investment Rate for Invested Assets in-force at the end of the current calendar year, and

i_j = Investment Rate for Invested Asset j purchased in the current calendar year,

such that the following relationships hold true:

$$C_1 = C_0 - \sum_K A_K + \sum_j A_j + \sum_K A_K' - \sum_K A_K'',$$

$C_0 \times i_0$ = Sum of the Products of the Allocated Book Values (or Costs) of Invested Assets in-force at the beginning of the current calendar year times their respective Investment Rates,

= Allocated Potential Investment Income at the beginning of the current calendar year,

$$C_1 \times i_1 = (C_0 \times i_0) - \sum_{K,j} (A_{K,j} \times i_{K,j}) + \sum_j (A_j \times i_j)$$
$$+ \sum_{K,j} (A_{K,j}' \times i_{K,j}) - \sum_{K,j} (A_{K,j}'' \times i_{K,j}),$$

= Sum of the Products of the Allocated Book Values (or Costs) of Invested Assets in-force at the end of the current calendar year times their respective Investment Rates, and

= Allocated Potential Investment Income at the end of the current calendar year,

then the following relationship, which weights the Allocated Potential Investment Incomes at the beginning of and at the end of the calendar year, may be derived from the Third Premise for the current calendar year:

$$[\ast (C_0 \times i_0) + (1 - \ast) (C_1 \times i_1)] = GII_D,$$

where all particulars pertaining to the previous technique apply excepting for the fact that Allocated Cash Flows Made Available to Increase Investable Assets (C_c and C_K's) have been replaced by beginning and end of the current calendar year Allocated Book Value (or Cost) of Invested Assets.

Using Cash Flows to Allocate Investment Income is still maintained since all invested assets of the company are allocated among various lines of business according to the Allocated Cash Flows made available for the purchase of invested assets (C^P's) in the calendar year of purchase. Under the investment-year technique, the C_K's as well as C_c--while identifying each calendar year's contribution to investable assets-- are (in a sense) artificial numbers used, along with their respective varying Average Investment Rates, to calculate the Derived Gross Investment Income for each line of business.

Under this second technique, instead of maintaining Allocated Cash Flows Made Available to Increase Investable Assets (C_K's), for each line of business, along with their respective Average Investment Rates, for a number of calendar years, an aggregate Allocated Book Value (or Cost) of Invested Assets (C_0 or C_1), one for each line of business, along with its Average Investment Rate (i_0 or i_1), is derived for each calendar year by adjusting the previous calendar year's aggregate Allocated Book Value (or Cost) of Invested Assets, along with its Average Investment Rate, for the allocated book values (or costs) of invested asset purchases and sales (including maturities) and allocated scheduled asset write-ups and write-downs, along with their respective investment rates. Under the in-force technique, these aggregate Allocated Book Values (or Costs) of Invested Assets are the bases for calculating the Derived Gross Investment Income for each line of business. [Notice, the First and Second Premises are required only for the investment-year technique.]

ADMINISTRATION NEEDED FOR INVESTMENTS

It would appear that for invested asset purchases and sales (including maturities) in a calendar year, the following information (at most) would be required:

Asset Purchases	Asset Sales
Cost	Consideration
Investment Rate	Book Value (or Cost)
Amortization Schedule,[11] if needed	Investment Rate
	Calendar Year of Purchase

Invested assets requiring scheduled asset write-ups or write-downs could be maintained, along with their respective investment rates, on schedules showing their asset write-ups or write-downs (allocated among various lines of business according to the C^P's in the calendar year of purchase) by calendar year.

In-Force Technique

Under the in-force technique, the aggregates of allocated scheduled asset write-ups and write-downs for each calendar year's Allocated Book Values (or Costs) of Invested Assets in-force at the beginning of the calendar year (C_0's), one for each line of business, would be used in adjusting income from invested assets (cash flow), for example, payments on mortgages, coupons on bonds, etc., to obtain investment income to be allocated--according to the end of the previous calendar year's allocation of potential investment income (in compliance with the Third Premise)--to the calendar year's Allocated Cash Flows made available for the purchase of invested assets (C^P's), one for each line of business, which have been also adjusted for allocated scheduled asset write-ups and write-downs but in an opposite manner from that of income from invested assets. Also, these allocated scheduled asset write-ups and write-downs, along with

---

11

An amortization schedule on an annual basis may be developed from a recursive formula, as follows:

$$
\begin{bmatrix} \text{Book Value} \\ \text{before} \\ \text{Write-up or} \\ \text{Write-down} \end{bmatrix} \times \begin{bmatrix} 1+ \begin{pmatrix} \text{Annual} \\ \text{Effective} \\ \text{Investment} \\ \text{Rate} \end{pmatrix} \end{bmatrix} - \begin{bmatrix} \text{Total Payments} \\ \text{for the Contract} \\ \text{Year Accumulated} \\ \text{with Interest} \end{bmatrix} = \begin{bmatrix} \text{Book Value} \\ \text{after} \\ \text{Write-up or} \\ \text{Write-down} \end{bmatrix}
$$

the allocated book values (or costs) of invested asset purchases and sales (including
maturities), would be used in obtaining, for each line of business, the Allocated
Book Value (or Cost) of Invested Assets in-force at the <u>end</u> of the calendar year (C_1)
from the Allocated Book Value (or Cost) of Invested Assets in-force at the <u>beginning</u>
of the calendar year (C_0) and, as well, these allocated scheduled asset write-ups and
write-downs, along with their respective investment rates, and the allocated book values
(or costs) of invested asset purchases and sales (including maturities), along with their
respective investment rates, would be used in obtaining, for each line of business, the
Allocated Potential Investment Income at the <u>end</u> of the calendar year $(C_1 \times i_1)$ from the
Allocated Potential Investment Income at the <u>beginning</u> of the calendar year $(C_0 \times i_0)$.

Investment-Year Technique

Under the investment-year technique, the aggregates of allocated scheduled asset
write-ups and write-downs would be similarly used, as under the in-force technique, in
adjusting income from invested assets to obtain investment income to be allocated

(1)--according to the end of the previous calendar year's allocation of potential
investment income (in compliance with the Third Premise)-- to the calendar year's
Allocated Cash Flows made available for the purchase of invested assets $(C^P$'s), one for
each line of business, which have been also adjusted for allocated scheduled asset write-
ups and write-downs but in an opposite manner from that of income from invested assets,
and

(2)--according to the current calendar year's allocation of potential invest-
ment income, that is, in proportion to the GII_D's--to the calendar year's Allocated
Cash Flows Made Available to Increase Investable Assets $(C_c$'s), one for each line of
business.

But, instead of being used in obtaining end-of-the-calendar-year values from beginning-
of-the-calendar-year values for the Allocated Book Value (or Cost) of Invested Assets
and for the Allocated Potential Investment Income, these allocated scheduled asset
write-ups (taken as negative values) and write-downs (taken as positive values),

84

identified by calendar year of purchase, along with their appropriate investment rates,
together with the allocated book values (or costs) of invested assets sold (or matured),
identified by calendar year of purchase, along with their appropriate investment rates,
would be used in adjusting the Average Investment Rates (that is, adjusting i_K's to
i_{NK}'s) for previous calendar years that had invested asset purchases which turned over
in the calendar year (or and for other calendar years of original contribution to these
invested assets which turned over). For each line of business, these Average Investment
Rates (i_K's and i_{NK}'s), together with the corresponding Allocated Cash Flows Made Available
to Increase Investable Assets (C_K's), provide the contributions to Derived Gross Invest-
ment Income (GII_D) by all calendar years combined (or, if the second version of the
investment-year technique is used, by calendar year). [Of course, since the original
version of the investment-year technique derives the potential investment income for
all calendar years' contributions to investable assets combined, this derivation could
be performed by treating the C_K's as one aggregate figure.]

<div align="center">

HOME OFFICE AND ELECTRONIC EQUIPMENT

OWNED BY THE COMPANY

</div>

These two items should be treated as capitalized expenses--and not as investments.
If the company did not own them, it would have to lease their services at current costs
utilizing current cash flow. The treatment of these two items is a management decision.

If these two items were not included in the above procedure, the effect would be
to allocate--using, by implication, current market values and current costs, both of
which reflect inflation and current yields--income derived therefrom and expenses applic-
able thereto among various lines of business according to the current calendar year's
actual Allocated Cash Flows (C^P's).

Based upon certain funding analyses and cost accounting procedures with respect to
these capitalized expenses, management may want to allocate, among various lines of
business, income different from that of expenses...resulting in the C^P's being modified
cash flows--the total of which, for a particular calendar year, would equal the actual
cash flow for the company as a whole.

EXPOSURE ELEMENT

It would appear not inappropriate to introduce one other improvement in obtaining the Derived Gross Investment Income for each line of business, that is, in the exposure element of the formula: For each invested asset sold during the current calendar year, an exposure could be assigned; but then this would do away with the Third Premise-- the premise which allows the initial allocation of investment income to be used in the Allocated Cash Flows made available for the purchase of invested assets (C^P's) and, thus, requires only one iteration in obtaining the final allocation of investment income. This would obviously lead to either--

(1) the assignment of an initial allocation of investment income that requires a number of iterations (instead of only one iteration) in obtaining the final allocation of investment income, or

(2) the assignment of a set of C^P's to each invested asset purchased (instead of to each calendar year) to allocate invested assets among various lines of business. Notice, this latter alternative would provide the initial allocation of investment income to be used in each set of C^P's and, like before, would require only one iteration in obtaining the final allocation of investment income. [This can be seen from the concept that if all changes in invested assets over an interval of time occur at some particular point on the interval between two end points from which potential investment income is measured, then at the point of change the previous end point's potential investment income can be used in developing the cash flow made available for the purchase of invested assets in the interval.]

But, both alternatives would be much too cumbersome: the first alternative would require calculations too numerous to count; the second alternative would require an accounting too complex to conceive. The only possible solution for greater accuracy would be to use the above procedure, instead of on a calendar year basis, on the basis of a shorter time interval...such as on a semiannual, quarterly, or monthly basis. [For example: Suppose that for allocating investment income within a line of business, one line of business required Average Investment Rates to be maintained on a quarterly

basis, while another required Average Investment Rates to be maintained on an annual basis.
Then the second version of the investment-year technique could be performed on a
quarterly basis with the annual Average Investment Rates obtained by properly weighting
the quarterly Average Investment Rates using the Allocated Cash Flows Made Available to
Increase Investable Assets as the bases.] One drawback to this procedure would be
the variability in cash flow from period to period.

CONCLUSION

Using Cash Flows to Allocate Investment Income is an important improvement over
the conventional methods of allocating investment income in that--

(1) given a desired degree of pooling for allocating investment income, it
eliminates, at least in the in-force technique and in a modified form of the investment-
year technique to recognize the investment rate on each invested asset that turns over,
the inherent discrepancies in using average investment rates but uses these averages
most effectively where such discrepancies will not occur, that is, when invested assets
do not turn over, and

(2) since the income-producing assets of a life insurance company are purchased
with the cash flows generated by its various lines of business, it allocates the investment
income produced from these invested assets among the various lines of business according
to their cash flows.

The original version of the investment-year technique--through its maintenance of
yearly average investment rates--would appear to be a reasonable solution to the problem
of allocating investment income among various lines of business. If greater accuracy
is required, the in-force technique recognizes individual asset investment rates. The
investment-year technique modified to recognize the investment rate on each invested
asset that turns over provides the benefits of both the investment-year technique and the
in-force technique. This applies to both the original version and the second version of
the investment-year technique--but, in addition, the second version (which maintains,
by line of business, the average investment rate for each calendar year which is the
average rate at which a calendar year's original contribution is actually invested) pro-
vides the means to allocate investment income among various generations of policyholders.

A DISCUSSION OF THE PAPER

"USING CASH FLOWS TO ALLOCATE INVESTMENT INCOME"

by
Charles E. Johnson

Two questions not answered in the original paper will be answered here:
"How are segregated assets handled under this method?" and "How are capital
gains and losses allocated within a line of business?"

As to the first question, the author chose not to discuss segregated assets
because he feels that segregated assets are better handled through Separate
Accounts. For those who disagree with the author, the following comment (which
may be inserted between Paragraphs Numbered 2 and 3 of the subsection "Facts
About the Investment-Year Technique") is offered:

> If segregated assets are assumed in the General Account, then there
> would be a set of C^P's for each assumed portfolio for each calendar
> year. If a line of business did not contribute to a particular
> assumed portfolio, its C^P wo ld be zero in the appropriate set of C^P's.

The answer to the second question is as follows:

Capital gains and losses may be allocated to calendar years of original
contribution in the same manner that invested asset turnovers are allocated in
maintaining the integrity of the calendar-year average investment rate.

In closing, given a desired degree of pooling, the techniques discussed in
this paper provide the means to allocate investment income and capital gains and
losses to each dollar of original investment.*

---

*That is, these techniques provide investment rates and capital gain and loss
rates based upon periods of original contribution which may be converted to rates
based upon dollars of original contribution.