SOCIETY OF ACTUARIES

EXAM IFM INVESTMENT AND FINANCIAL MARKETS

EXAM IFM SAMPLE QUESTIONS AND SOLUTIONS

DERIVATIVES

These questions and solutions are based on the readings from McDonald and are identical to questions from the former set of sample questions for Exam MFE. The question numbers have been retained for ease of comparison.

These questions are representative of the types of questions that might be asked of candidates sitting for Exam IFM. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

In this version, standard normal distribution values are obtained by using the Cumulative Normal Distribution Calculator and Inverse CDF Calculator

For extra practice on material from Chapter 9 or later in McDonald, also see the actual Exam MFE questions and solutions from May 2007 and May 2009

May 2007: Questions 1, 3-6, 8, 10-11, 14-15, 17, and 19

Note: Questions 2, 7, 9, 12-13, 16, and 18 do not apply to the new IFM curriculum

May 2009: Questions 1-3, 12, 16-17, and 19-20

Note: Questions 4-11, 13-15, and 18 do not apply to the new IFM curriculum

Note that some of these remaining items (from May 2007 and May 2009) may refer to “stock prices following geometric Brownian motion.” In such instances, use the following phrase instead: “stock prices are lognormally distributed.”

November 2020 correction: Question 1 was edited to correct an error in the earlier version.

October 2022 correction: Questions 35 and 49 were edited to include an interest rate of zero

Introductory Derivatives Questions

1.

Determine which statement about zero-cost purchased collars is FALSE

(A) A zero-width, zero-cost collar can be created by setting both the put and call strike prices at the forward price.

(B) There are an infinite number of zero-cost collars.

(C) The put option can be at-the-money.

(D) The call option can be at-the-money.

(E) The strike price on the put option must be above the forward price.

2.

You are given the following:

- The current price to buy one share of XYZ stock is 500.
- The stock does not pay dividends.
- The continuously compounded risk-free interest rate is 6%.
- A European call option on one share of XYZ stock with a strike price of K that expires in one year costs 66.59.
- A European put option on one share of XYZ stock with a strike price of K that expires in one year costs 18.64.

Using put-call parity, calculate the strike price, K.

(A) 449

(B) 452

(C) 480

(D) 559

(E) 582

3.

Happy Jalapenos, LLC has an exclusive contract to supply jalapeno peppers to the organizers of the annual jalapeno eating contest. The contract states that the contest organizers will take delivery of 10,000 jalapenos in one year at the market price. It will cost Happy Jalapenos 1,000 to provide 10,000 jalapenos and today's market price is 0.12 for one jalapeno. The continuously compounded risk-free interest rate is 6%.

Happy Jalapenos has decided to hedge as follows:

Buy 10,000 0.12-strike put options for 84.30 and sell 10,000 0.14-stike call options for 74.80. Both options are one-year European.

Happy Jalapenos believes the market price in one year will be somewhere between 0.10 and 0.15 per jalapeno.

Determine which of the following intervals represents the range of possible profit one year from now for Happy Jalapenos.

(A) –200 to 100

(B) –110 to 190

(C) –100 to 200

(D) 190 to 390

(E) 200 to 400

4.

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5.

The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000.
- The PS index does not pay dividends.

Sam wants to lock in the ability to buy this index in one year for a price of 1,025. He can do this by buying or selling European put and call options with a strike price of 1,025.

The annual effective risk-free interest rate is 5%.

Determine which of the following gives the hedging strategy that will achieve Sam's objective and also gives the cost today of establishing this position.

(A) Buy the put and sell the call, receive 23.81

(B) Buy the put and sell the call, spend 23.81

(C) Buy the put and sell the call, no cost

(D) Buy the call and sell the put, receive 23.81

(E) Buy the call and sell the put, spend 23.81

6.

The following relates to one share of XYZ stock:

- The current price is 100.
- The forward price for delivery in one year is 105.
- P is the expected price in one year

Determine which of the following statements about P is TRUE.

(A) $P < 100$

(B) $P = 100$

(C) $100 < P < 105$

(D) $P = 105$

(E) $P > 105$

7.

A non-dividend paying stock currently sells for 100. One year from now the stock sells for 110. The continuously compounded risk-free interest rate is 6%. A trader purchases the stock in the following manner:

- The trader pays 100 today
- The trader takes possession of the stock in one year

Determine which of the following describes this arrangement.

(A) Outright purchase

(B) Fully leveraged purchase

(C) Prepaid forward contract

(D) Forward contract

(E) This arrangement is not possible due to arbitrage opportunities

8.

Joe believes that the volatility of a stock is higher than indicated by market prices for options on that stock. He wants to speculate on that belief by buying or selling at-the-money options.

Determine which of the following strategies would achieve Joe's goal.

(A) Buy a strangle

(B) Buy a straddle

(C) Sell a straddle

(D) Buy a butterfly spread

(E) Sell a butterfly spread

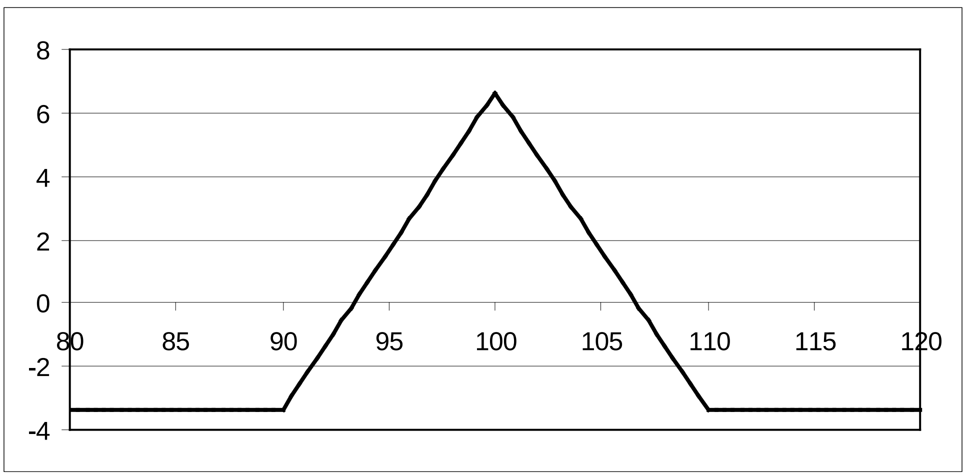
9.

Stock ABC has the following characteristics:

- The current price to buy one share is 100.
- The stock does not pay dividends.
- European options on one share expiring in one year have the following prices:

Strike Price	Call option price	Put option price
90	14.63	0.24
100	6.80	1.93
110	2.17	6.81

A butterfly spread on this stock has the following profit diagram.



The continuously compounded risk-free interest rate is 5%.

Determine which of the following will NOT produce this profit diagram.

(A) Buy a 90 put, buy a 110 put, sell two 100 puts

(B) Buy a 90 call, buy a 110 call, sell two 100 calls

(C) Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call

(D) Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts

(E) Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.

10.

Stock XYZ has a current price of 100. The forward price for delivery of this stock in 1 year is 110.

Unless otherwise indicated, the stock pays no dividends and the annual effective risk-free interest rate is 10%.

Determine which of the following statements is FALSE.

(A) The time-1 profit diagram and the time-1 payoff diagram for long positions in this forward contract are identical.

(B) The time-1 profit for a long position in this forward contract is exactly opposite to the time-1 profit for the corresponding short forward position.

(C) There is no comparative advantage to investing in the stock versus investing in the forward contract.

(D) If the 10% interest rate was continuously compounded instead of annual effective, then it would be more beneficial to invest in the stock, rather than the forward contract.

(E) If there was a dividend of 3.00 paid 6 months from now, then it would be more beneficial to invest in the stock, rather than the forward contract.

11.

Stock XYZ has the following characteristics:

- The current price is 40.
- The price of a 35-strike 1-year European call option is 9.12.
- The price of a 40-strike 1-year European call option is 6.22.
- The price of a 45-strike 1-year European call option is 4.08.

The annual effective risk-free interest rate is 8%.

Let S be the price of the stock one year from now.

All call positions being compared are long.

Determine the range for S such that the 45-strike call produce a higher profit than the 40-strike call, but a lower profit than the 35-strike call.

(A) $S < 38.13$

(B) $38.13 < S < 40.44$

(C) $40.44 < S < 42.31$

(D) $S > 42.31$

(E) The range is empty.

12.

Consider a European put option on a stock index without dividends, with 6 months to expiration and a strike price of 1,000. Suppose that the effective six-month interest rate is 2%, and that the put costs 74.20 today.

Calculate the price that the index must be in 6 months so that being long in the put would produce the same profit as being short in the put.

(A) 922.83

(B) 924.32

(C) 1,000.00

(D) 1,075.68

(E) 1,077.17

13.

A trader shorts one share of a stock index for 50 and buys a 60-strike European call option on that stock that expires in 2 years for 10. Assume the annual effective risk-free interest rate is 3%.

The stock index increases to 75 after 2 years.

Calculate the profit on your combined position, and determine an alternative name for this combined position.

	Profit	**Name**
(A)	–22.64	Floor
(B)	–17.56	Floor
(C)	–22.64	Cap
(D)	–17.56	Cap
(E)	–22.64	"Written" Covered Call

14.

The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. You are given that the price of a 35-strike call option is 3.35 higher than the price of a 40-strike call option, where both options expire in 3 months.

Calculate the amount by which the price of an otherwise equivalent 40-strike put option exceeds the price of an otherwise equivalent 35-strike put option.

(A) 1.55

(B) 1.65

(C) 1.75

(D) 3.25

(E) 3.35

15.

The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. You enter into a short position on 3 call options, each with 3 months to maturity, a strike price of 35, and an option premium of 6.13. Simultaneously, you enter into a long position on 5 call options, each with 3 months to maturity, a strike price of 40, and an option premium of 2.78.

All 8 options are held until maturity.

Calculate the maximum possible profit and the maximum possible loss for the entire option portfolio.

	Maximum Profit	Maximum Loss
(A)	3.42	4.58
(B)	4.58	10.42
(C)	Unlimited	10.42
(D)	4.58	Unlimited
(E)	Unlimited	Unlimited

16.

The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. The following table shows call and put option premiums for three-month European of various exercise prices:

Exercise Price	Call Premium	Put Premium
35	6.13	0.44
40	2.78	1.99
45	0.97	5.08

A trader interested in speculating on volatility in the stock price is considering two investment strategies. The first is a 40-strike straddle. The second is a strangle consisting of a 35-strike put and a 45-strike call.

Determine the range of stock prices in 3 months for which the strangle outperforms the straddle.

(A) The strangle never outperforms the straddle.

(B) $33.56 < S_T < 46.44$

(C) $35.13 < S_T < 44.87$

(D) $36.57 < S_T < 43.43$

(E) The strangle always outperforms the straddle.

17.

The current price for a stock index is 1,000. The following premiums exist for various options to buy or sell the stock index six months from now:

Strike Price	Call Premium	Put Premium
950	120.41	51.78
1,000	93.81	74.20
1,050	71.80	101.21

Strategy I is to buy the 1,050-strike call and to sell the 950-strike call.

Strategy II is to buy the 1,050-strike put and to sell the 950-strike put.

Strategy III is to buy the 950-strike call, sell the 1,000-strike call, sell the 950-strike put, and buy the 1,000-strike put.

Assume that the price of the stock index in 6 months will be between 950 and 1,050.

Determine which, if any, of the three strategies will have greater payoffs in six months for lower prices of the stock index than for relatively higher prices.

(A) None

(B) I and II only

(C) I and III only

(D) II and III only

(E) The correct answer is not given by (A), (B), (C), or (D)

18.

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19.

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20.

The current price of a stock is 200, and the continuously compounded risk-free interest rate is 4%. A dividend will be paid every quarter for the next 3 years, with the first dividend occurring 3 months from now. The amount of the first dividend is 1.50, but each subsequent dividend will be 1% higher than the one previously paid.

Calculate the fair price of a 3-year forward contract on this stock.

(A) 200

(B) 205

(C) 210

(D) 215

(E) 220

21.

A market maker in stock index forward contracts observes a 6-month forward price of 112 on the index. The index spot price is 110 and the continuously compounded dividend yield on the index is 2%.

The continuously compounded risk-free interest rate is 5%.

Describe actions the market maker could take to exploit an arbitrage opportunity and calculate the resulting profit (per index unit).

(A) Buy observed forward, sell synthetic forward, Profit = 0.34

(B) Buy observed forward, sell synthetic forward, Profit = 0.78

(C) Buy observed forward, sell synthetic forward, Profit = 1.35

(D) Sell observed forward, buy synthetic forward, Profit = 0.78

(E) Sell observed forward, buy synthetic forward, Profit = 0.34

22.

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23.

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24.

Determine which of the following statements is NOT a typical reason for why derivative securities are used to manage financial risk.

(A) Derivatives are used as a means of hedging.

(B) Derivatives are used to reduce the likelihood of bankruptcy.

(C) Derivatives are used to reduce transaction costs.

(D) Derivatives are used to satisfy regulatory, tax, and accounting constraints.

(E) Derivatives are used as a form of insurance.

25.

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26.

Determine which, if any, of the following positions has or have an unlimited loss potential from adverse price movement in the underlying asset, regardless of the initial premium received.

I. Short 1 forward contract

II. Short 1 call option

III. Short 1 put option

(A) None

(B) I and II only

(C) I and III only

(D) II and III only

(E) The correct answer is not given by (A), (B), (C), or (D)

27.

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28.

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29.

The dividend yield on a stock and the interest rate used to discount the stock's cash flows are both continuously compounded. The dividend yield is less than the interest rate, but both are positive.

The following table shows four methods to buy the stock and the total payment needed for each method. The payment amounts are as of the time of payment and have not been discounted to the present date.

METHOD	TOTAL PAYMENT
Outright purchase	A
Fully leveraged purchase	B
Prepaid forward contract	C
Forward contract	D

Determine which of the following is the correct ranking, from smallest to largest, for the amount of payment needed to acquire the stock.

(A) $C < A < D < B$

(B) $A < C < D < B$

(C) $D < C < A < B$

(D) $C < A < B < D$

(E) $A < C < B < D$

30.

Determine which of the following is NOT a distinguishing characteristic of futures contracts, relative to forward contracts.

(A) Contracts are settled daily, and marked-to-market.

(B) Contracts are more liquid, as one can offset an obligation by taking the opposite position.

(C) Contracts are more customized to suit the buyer's needs.

(D) Contracts are structured to minimize the effects of credit risk.

(E) Contracts have price limits, beyond which trading may be temporarily halted.

31.

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32.

Judy decides to take a short position in 20 contracts of S&P 500 futures. Each contract is for the delivery of 250 units of the index at a price of 1500 per unit, exactly one month from now. The initial margin is 5% of the notional value, and the maintenance margin is 90% of the initial margin. Judy earns a continuously compounded risk-free interest rate of 4% on her margin balance. The position is marked-to-market on a daily basis.

On the day of the first marking-to-market, the value of the index drops to 1498. On the day of the second marking-to-market, the value of the index is X and Judy is not required to add anything to the margin account.

Calculate the largest possible value of X.

(A) 1490.50

(B) 1492.50

(C) 1500.50

(D) 1505.50

(E) 1507.50

33.

Several years ago, John bought three separate 6-month options on the same stock.

- Option I was an American-style put with strike price 20.
- Option II was a Bermuda-style call with strike price 25, where exercise was allowed at any time following an initial 3-month period of call protection.
- Option III was a European-style put with strike price 30.

When the options were bought, the stock price was 20.

When the options expired, the stock price was 26.

The table below gives the maximum and minimum stock price during the 6 month period:

Time Period:	1^{st} 3 months of Option Term	2^{nd} 3 months of Option Term
Maximum Stock Price	24	28
Minimum Stock Price	18	22

John exercised each option at the optimal time.

Rank the three options, from highest to lowest payoff.

(A) I > II > III

(B) I > III > II

(C) II > I > III

(D) III > I > II

(E) III > II > I

34.

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35.

A customer buys a 50-strike put on an index when the market price of the index is also 50.

The premium for the put is 5. Assume that the option contract is for an underlying 100 units of the index. Assume an interest rate of zero.

Calculate the customer’s profit if the index declines to 45 at expiration.

(A) –1000

(B) –500

(C) 0

(D) 500

(E) 1000

36.

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37.

A one-year forward contract on a stock has a price of \$75. The stock is expected to pay a dividend of \$1.50 at two future times, six months from now and one year from now, and the annual effective risk-free interest rate is 6%.

Calculate the current stock price.

(A) 70.75

(B) 73.63

(C) 75.81

(D) 77.87

(E) 78.04

38.

The current price of a medical company's stock is 75. The expected value of the stock price in three years is 90 per share. The stock pays no dividends.

You are also given

i) The risk-free interest rate is positive.

ii) There are no transaction costs.

iii) Investors require compensation for risk.

The price of a three-year forward on a share of this stock is X, and at this price an investor is willing to enter into the forward.

Determine what can be concluded about X.

(A) $X < 75$

(B) $X = 75$

(C) $75 < X < 90$

(D) $X = 90$

(E) $90 < X$

39.

Determine which of the following strategies creates a ratio spread, assuming all options are European.

(A) Buy a one-year call, and sell a three-year call with the same strike price.

(B) Buy a one-year call, and sell a three-year call with a different strike price.

(C) Buy a one-year call, and buy three one-year calls with a different strike price.

(D) Buy a one-year call, and sell three one-year puts with a different strike price.

(E) Buy a one-year call, and sell three one-year calls with a different strike price.

40.

An investor is analyzing the costs of two-year, European options for aluminum and zinc at a particular strike price.

For each ton of aluminum, the two-year forward price is 1400, a call option costs 700, and a put option costs 550.

For each ton of zinc, the two-year forward price is 1600 and a put option costs 550.

The annual effective risk-free interest rate is 6%.

Calculate the cost of a call option per ton of zinc.

(A) 522

(B) 800

(C) 878

(D) 900

(E) 1231

41.

XYZ stock pays no dividends and its current price is 100.

Assume the put, the call and the forward on XYZ stock are available and are priced so there are no arbitrage opportunities. Also, assume there are no transaction costs.

The annual effective risk-free interest rate is 1%.

Determine which of the following strategies currently has the highest net premium.

(A) Long a six-month 100-strike put and short a six-month 100-strike call

(B) Long a six-month forward on the stock

(C) Long a six-month 101-strike put and short a six-month 101-strike call

(D) Short a six-month forward on the stock

(E) Long a six-month 105-strike put and short a six-month 105-strike call

42.

An investor purchases a non-dividend-paying stock and writes a t-year, European call option for this stock, with call premium C. The stock price at time of purchase and strike price are both K.

Assume that there are no transaction costs.

The risk-free annual force of interest is a constant r. Let S represent the stock price at time t.

$S > K$.

Determine an algebraic expression for the investor's profit at expiration.

(A) Ce^{rt}

(B) $C(1+rt)-S+K$

(C) $Ce^{rt}-S+K$

(D) $Ce^{rt}+K\left(1-e^{rt}\right)$

(E) $C(1+r)^t+K\left[1-(1+r)^t\right]$

43.

You are given:

i) An investor short-sells a non-dividend paying stock that has a current price of 44 per share.

ii) This investor also writes a collar on this stock consisting of a 40-strike European put option and a 50-strike European call option. Both options expire in one year.

iii) The prices of the options on this stock are:

Strike Price	Call option	Put option
40	8.42	2.47
50	3.86	7.42

iv) The continuously compounded risk-free interest rate is 5%.

v) Assume there are no transaction costs.

Calculate the maximum profit for the overall position at expiration.

(A) 2.61

(B) 3.37

(C) 4.79

(D) 5.21

(E) 7.39

44.

You are given the following information about two options, A and B:

i) Option A is a one-year European put with exercise price 45.

ii) Option B is a one-year American call with exercise price 55.

iii) Both options are based on the same underlying asset, a stock that pays no dividends.

iv) Both options go into effect at the same time and expire at $t = 1$.

You are also given the following information about the stock price:

i) The initial stock price is 50.

ii) The stock price at expiration is also 50.

iii) The minimum stock price (from $t = 0$ to $t = 1$) is 46.

iv) The maximum stock price (from $t = 0$ to $t = 1$) is 58.

Determine which of the following statements is true.

(A) Both options A and B are "at-the-money" at expiration.

(B) Both options A and B are "in-the-money" at expiration.

(C) Both options A and B are "out-of-the-money" throughout each option's term.

(D) Only option A is ever "in-the-money" at some time during its term.

(E) Only option B is ever "in-the-money" at some time during its term.

45.

An investor enters a long position in a futures contract on an index (F) with a notional value of $200 \times F$, expiring in one year. The index pays a continuously compounded dividend yield of 4%, and the continuously compounded risk-free interest rate is 2%.

At the time of purchase, the index price is 1100. Three months later, the investor has sustained a loss of 100. Assume the margin account earns an interest rate of 0%.

Let S be the price of the index at the end of month three.

Calculate S.

(A) 1078

(B) 1085

(C) 1094

(D) 1105

(E) 1110

46.

Determine which of the following statements about options is true.

(A) Naked writing is the practice of buying options without taking an offsetting position in the underlying asset.

(B) A covered call involves taking a long position in an asset together with a written call on the same asset.

(C) An American style option can only be exercised during specified periods, but not for the entire life of the option.

(D) A Bermudan style option allows the buyer the right to exercise at any time during the life of the option.

(E) An in-the-money option is one which would have a positive profit if exercised immediately.

47.

An investor has written a covered call.

Determine which of the following represents the investor's position.

(A) Short the call and short the stock

(B) Short the call and long the stock

(C) Short the call and no position on the stock

(D) Long the call and short the stock

(E) Long the call and long the stock

48.

For a certain stock, Investor A purchases a 45-strike call option while Investor B purchases a 135-strike put option. Both options are European with the same expiration date. Assume that there are no transaction costs.

If the final stock price at expiration is S, Investor A's payoff will be 12.

Calculate Investor B's payoff at expiration, if the final stock price is S.

(A) 0

(B) 12

(C) 36

(D) 57

(E) 78

49.

The market price of Stock A is 50. A customer buys a 50-strike put contract on Stock A for 500. The put contract is for 100 shares of A. Assume an interest rate of zero.

Calculate the customer's maximum possible loss.

(A) 0

(B) 5

(C) 50

(D) 500

(E) 5000

50.

An investor bought a 70-strike European put option on an index with six months to expiration. The premium for this option was 1.

The investor also wrote an 80-strike European put option on the same index with six months to expiration. The premium for this option was 8.

The six-month interest rate is 0%.

Calculate the index price at expiration that will allow the investor to break even.

(A) 63

(B) 73

(C) 77

(D) 80

(E) 87

51.

You are given the following information about Stock XYZ:

i) The current price of the stock is 35 per share.

ii) The expected continuously compounded rate of return is 8%.

iii) The stock pays semi-annual dividends of 0.32 per share, with the next dividend to be paid two months from now.

The continuously compounded risk-free interest rate is 4%.

Calculate the current one-year forward price for stock XYZ.

(A) 34.37

(B) 35.77

(C) 36.43

(D) 37.23

(E) 37.92

52.

The ask price for a share of ABC company is 100.50 and the bid price is 100. Suppose an investor can borrow at an annual effective rate of 3.05% and lend (i.e., save) at an annual effective rate of 3%. Assume there are no transaction costs and no dividends.

Determine which of the following strategies does not create an arbitrage opportunity.

(A) Short sell one share, and enter into a long one-year forward contract on one share with a forward price of 102.50.

(B) Short sell one share, and enter into a long one-year forward contract on one share with a forward price of 102.75.

(C) Short sell one share, and enter into a long one-year forward contract on one share with a forward price of 103.00.

(D) Purchase one share with borrowed money, and enter into a short one-year forward contract on one share with a forward price of 103.60.

(E) Purchase one share with borrowed money, and enter into a short one-year forward contract on one share with a forward price of 103.75.

53.

For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year 400-strike European call option costs 110.

The continuously compounded risk-free interest rate is 6.5%.

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

(A) 10.00

(B) 32.89

(C) 118.42

(D) 187.11

(E) 210.00

54.

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55.

Box spreads are used to guarantee a fixed cash flow in the future. Thus, they are purely a means of borrowing or lending money, and have no stock price risk.

Consider a box spread based on two distinct strike prices (K, L) that is used to lend money, so that there is a positive cost to this transaction up front, but a guaranteed positive payoff at expiration.

Determine which of the following sets of transactions is equivalent to this type of box spread.

(A) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bear spread using puts.

(B) A long position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(C) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bull spread using puts.

(D) A short position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(E) A short position in a (K, L) bull spread using calls and a short position in a(K, L) bull spread using puts.

56.

Determine which of the following positions has the same cash flows as a short stock position.

(A) Long forward and long zero-coupon bond

(B) Long forward and short forward

(C) Long forward and short zero-coupon bond

(D) Long zero-coupon bond and short forward

(E) Short forward and short zero-coupon bond

57.

DELETED

58.

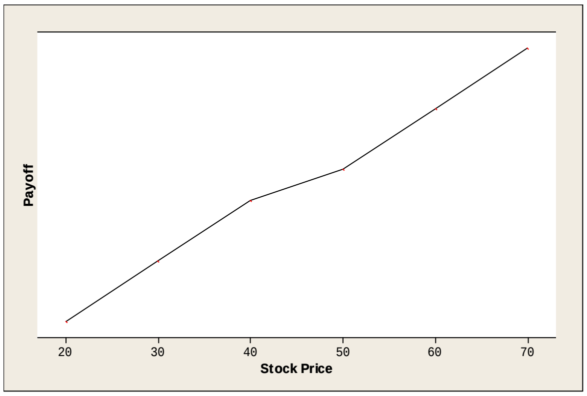
DELETED

59.

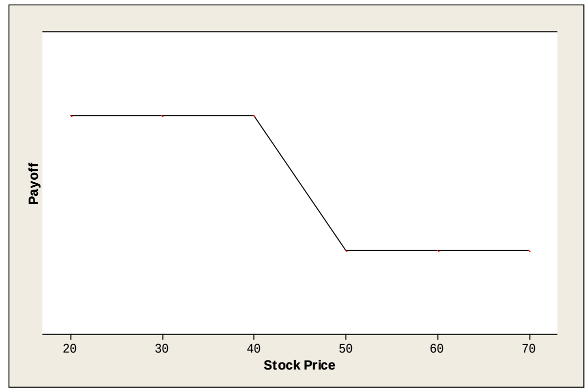
An investor has a long position in a non-dividend-paying stock, and additionally, has a long collar on this stock consisting of a 40-strike put and 50-strike call.

Determine which of these graphs represents the payoff diagram for the overall position at the time of expiration of the options.

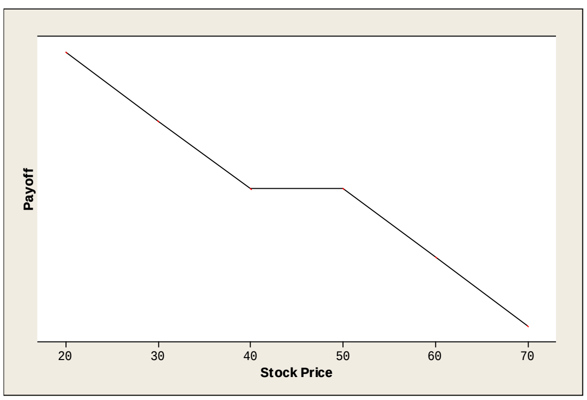
(A)



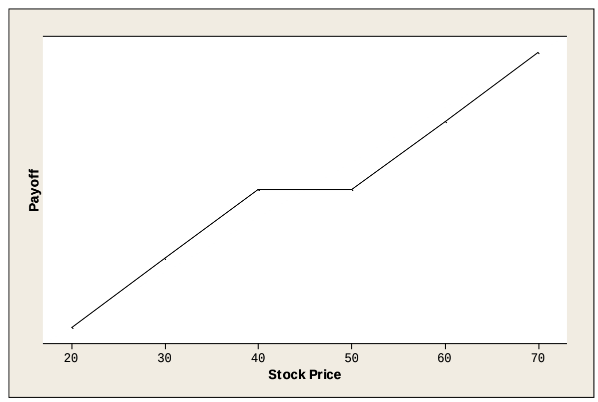
(B)



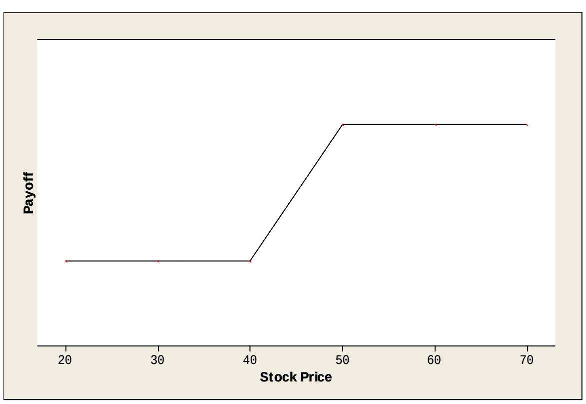
(C)



(D)



(E)



60.

Farmer Brown grows wheat, and will be selling his crop in 6 months. The current price of wheat is 8.50 per bushel. To reduce the risk of fluctuation in price, Brown wants to use derivatives with a 6-month expiration date to sell wheat between 8.60 and 8.80 per bushel. Brown also wants to minimize the cost of using derivatives.

The continuously compounded risk-free interest rate is 2%.

Which of the following strategies fulfills Farmer Brown's objectives?

(A) Short a forward contract

(B) Long a call with strike 8.70 and short a put with strike 8.70

(C) Long a call with strike 8.80 and short a put with strike 8.60

(D) Long a put with strike 8.60

(E) Long a put with strike 8.60 and short a call with strike 8.80

61.

An investor purchased Option A and Option B for a certain stock today, with strike prices 70 and 80, respectively. Both options are European one-year put options.

Determine which statement is true about the moneyness of these options, based on a particular stock price.

(A) If Option A is in-the-money, then Option B is in-the-money.

(B) If Option A is at-the-money, then Option B is out-of-the-money.

(C) If Option A is in-the-money, then Option B is out-of-the-money.

(D) If Option A is out-of-the-money, then Option B is in-the-money.

(E) If Option A is out-of-the-money, then Option B is out-of-the-money.

62.

The price of an asset will either rise by 25% or fall by 40% in 1 year, with equal probability. A European put option on this asset matures after 1 year.

Assume the following:

- Price of the asset today: 100
- Strike price of the put option: 130
- Put option premium: 7
- Annual effective risk free rate: 3%

Calculate the expected profit of the put option.

(A) 12.79

(B) 15.89

(C) 22.69

(D) 27.79

(E) 30.29

63.

DELETED

64.

DELETED

65.

Assume that a single stock is the underlying asset for a forward contract, a K-strike call option, and a K-strike put option.

Assume also that all three derivatives are evaluated at the same point in time.

Which of the following formulas represents put-call parity?

(A) Call Premium – Put Premium = Present Value (Forward Price – *K*)

(B) Call Premium – Put Premium = Present Value (Forward Price)

(C) Put Premium – Call Premium = 0

(D) Put Premium – Call Premium = Present Value (Forward Price – *K*)

(E) Put Premium – Call Premium = Present Value (Forward Price)

66.

The current price of a stock is 80. Both call and put options on this stock are available for purchase at a strike price of 65.

Determine which of the following statements about these options is true.

(A) Both the call and put options are at-the-money.

(B) Both the call and put options are in-the-money.

(C) Both the call and put options are out-of-the-money.

(D) The call option is in-the-money, but the put option is out-of-the-money.

(E) The call option is out-of-the-money, but the put option is in-the-money.

67.

Consider the following investment strategy involving put options on a stock with the same expiration date.

i) Buy one 25-strike put

ii) Sell two 30-strike puts

iii) Buy one 35-strike put

Calculate the payoffs of this strategy assuming stock prices (i.e., at the time the put options expire) of 27 and 37, respectively.

(A) –2 and 2

(B) 0 and 0

(C) 2 and 0

(D) 2 and 2

(E) 14 and 0

68.

For a non-dividend-paying stock index, the current price is 1100 and the 6-month forward price is 1150. Assume the price of the stock index in 6 months will be 1210.

Which of the following is true regarding forward positions in the stock index?

(A) Long position gains 50

(B) Long position gains 60

(C) Long position gains 110

(D) Short position gains 60

(E) Short position gains 110

69.

Determine which of the following statements about futures and forward contracts is false.

(A) Frequent marking-to-market and settlement of a futures contract can lead to pricing differences between a futures contract and an otherwise identical forward contract.

(B) Over-the-counter forward contracts can be customized to suit the buyer or seller, whereas futures contracts are standardized.

(C) Users of forward contracts are more able to minimize credit risk than are users of futures contracts.

(D) Forward contracts can be used to synthetically switch a portfolio invested in stocks into bonds.

(E) The holder of a long futures contract must place a fraction of the cost with an intermediary and provide assurances on the remaining purchase price.

70.

Investors in a certain stock demand to be compensated for risk. The current stock price is 100.

The stock pays dividends at a rate proportional to its price. The dividend yield is 2%.

The continuously compounded risk-free interest rate is 5%.

Assume there are no transaction costs.

Let X represent the expected value of the stock price 2 years from today. Assume it is known that X is a whole number.

Determine which of the following statements is true about X.

(A) The only possible value of X is 105.

(B) The largest possible value of X is 106.

(C) The smallest possible value of X is 107.

(D) The largest possible value of X is 110.

(E) The smallest possible value of X is 111.

71.

A certain stock costs 40 today and will pay an annual dividend of 6 for the next 4 years. An investor wishes to purchase a 4-year prepaid forward contract for this stock. The first dividend will be paid one year from today and the last dividend will be paid just prior to delivery of the stock. Assume an annual effective interest rate of 5%.

Calculate the price of the prepaid forward contract.

(A) 12.85

(B) 13.16

(C) 17.29

(D) 18.72

(E) 21.28

72.

CornGrower is going to sell corn in one year. In order to lock in a fixed selling price, CornGrower buys a put option and sells a call option on each bushel, each with the same strike price and the same one-year expiration date.

The current price of corn is 3.59 per bushel, and the net premium that CornGrower pays now to lock in the future price is 0.10 per bushel.

The continuously compounded risk-free interest rate is 4%.

Calculate the fixed selling price per bushel one year from now.

(A) 3.49

(B) 3.63

(C) 3.69

(D) 3.74

(E) 3.84

73.

The current price of a non-dividend-paying stock is 100. The annual effective risk-free interest rate is 4%, and there are no transaction costs.

The stock's two-year forward price is mispriced at 108, so to exploit this mispricing, an investor can short a share of the stock for 100 and simultaneously take a long position in a two-year forward contract. The investor can then invest the 100 at the risk-free rate, and finally buy back the share of stock at the forward price after two years.

Determine which term best describes this strategy.

(A) Hedging

(B) Immunization

(C) Arbitrage

(D) Paylater

(E) Diversification

74.

Consider an airline company that faces risk concerning the price of jet fuel.

Select the hedging strategy that best protects the company against an increase in the price of jet fuel.

(A) Buying calls on jet fuel

(B) Buying collars on jet fuel

(C) Buying puts on jet fuel

(D) Selling puts on jet fuel

(E) Selling calls on jet fuel

75.

Determine which of the following risk management techniques can hedge the financial risk of an oil producer arising from the price of the oil that it sells.

I. Short forward position on the price of oil

II. Long put option on the price of oil

III. Long call option on the price of oil

(A) I only

(B) II only

(C) III only

(D) I, II, and III

(E) The correct answer is not given by (A), (B), (C), or (D)

Advanced Derivatives Questions

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60.

(ii) The call option currently sells for 0.15 more than the put option.

(iii) Both the call option and put option will expire in 4 years.

(iv) Both the call option and put option have a strike price of 70.

Calculate the continuously compounded risk-free interest rate.

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

2. Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
\$40	\$11	\$3
\$50	\$6	\$8
\$55	\$3	\$11

All six options have the same expiration date.

After reviewing the information above, John tells Mary and Peter that no arbitrage opportunities can arise from these prices.

Mary disagrees with John. She argues that one could use the following portfolio to obtain arbitrage profit: Long one call option with strike price 40; short three call options with strike price 50; lend \$1; and long some calls with strike price 55.

Peter also disagrees with John. He claims that the following portfolio, which is different from Mary's, can produce arbitrage profit: Long 2 calls and short 2 puts with strike price 55; long 1 call and short 1 put with strike price 40; lend \$2; and short some calls and long the same number of puts with strike price 50.

Which of the following statements is true?

(A) Only John is correct.

(B) Only Mary is correct.

(C) Only Peter is correct.

(D) Both Mary and Peter are correct.

(E) None of them is correct.

3. An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$. The contracts offer a minimum guarantee return rate of $g\%$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\%$ is deducted by the insurance company. Thus, at the contract maturity date, T, the insurance company will pay the policyholder

$$\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T].$$

You are given the following information:

(i) The contract will mature in one year.

(ii) The minimum guarantee rate of return, $g\%$, is 3%.

(iii) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested.

(iv) $S(0) = 100$.

(v) The price of a one-year European put option, with strike price of $103, on the stock index is $15.21.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.

(A) 12.8%.

(B) 13.0%

(C) 13.2%

(D) 13.4%

(E) 13.6%.

4. For a two-period binomial model, you are given:

(i) Each period is one year.

(ii) The current price for a nondividend-paying stock is 20.

(iii) $u = 1.2840$, where u is one plus the rate of capital gain on the stock per period if the stock price goes up.

(iv) $d = 0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock price goes down.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of an American call option on the stock with a strike price of 22.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

5. Consider a 9-month dollar-denominated American put option on British pounds. You are given that:

(i) The current exchange rate is 1.43 US dollars per pound.

(ii) The strike price of the put is 1.56 US dollars per pound.

(iii) The volatility of the exchange rate is $\sigma = 0.3$.

(iv) The US dollar continuously compounded risk-free interest rate is 8%.

(v) The British pound continuously compounded risk-free interest rate is 9%.

Using a three-period binomial model, calculate the price of the put.

(A) 0.23

(B) 0.25

(C) 0.27

(D) 0.29

(E) 0.31

6. You are considering the purchase of 100 units of a 3-month 25-strike European call option on a stock.

You are given:

(i) The Black-Scholes framework holds.
(ii) The stock is currently selling for 20.
(iii) The stock's volatility is 24%.
(iv) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.
(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of the block of 100 options.

(A) 0.04

(B) 1.93

(C) 3.63

(D) 4.22

(E) 5.09

7. Company A is a U.S. international company, and Company B is a Japanese local company. Company A is negotiating with Company B to sell its operation in Tokyo to Company B. The deal will be settled in Japanese yen. To avoid a loss at the time when the deal is closed due to a sudden devaluation of yen relative to dollar, Company A has decided to buy at-the-money dollar-denominated yen put of the European type to hedge this risk.

You are given the following information:

(i) The deal will be closed 3 months from now.

(ii) The sale price of the Tokyo operation has been settled at 120 billion Japanese yen.

(iii) The continuously compounded risk-free interest rate in the U.S. is 3.5%.

(iv) The continuously compounded risk-free interest rate in Japan is 1.5%.

(v) The current exchange rate is 1 U.S. dollar = 120 Japanese yen.

(vi) The daily volatility of the yen per dollar exchange rate is 0.261712%.

(vii) 1 year = 365 days; 3 months = ¼ year.

Calculate Company A's option cost.

(A) 7.32 million

(B) 7.42 million

(C) 7.52 million

(D) 7.62 million

(E) 7.72 million

8. You are considering the purchase of a 3-month 41.5-strike American call option on a nondividend-paying stock.

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40.

(iii) The stock's volatility is 30%.

(iv) The current call option delta is 0.5.

Determine the current price of the option.

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x - 20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x - 20.453$

9. Consider the Black-Scholes framework. A market-maker, who delta-hedges, sells a three-month at-the-money European call option on a nondividend-paying stock.

You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The current stock price is 50.

(iii) The current call option delta is 0.61791.

(iv) There are 365 days in the year.

If, after one day, the market-maker has zero profit or loss, determine the stock price move over the day.

(A) 0.41

(B) 0.52

(C) 0.63

(D) 0.75

(E) 1.11

10-17. DELETED

18. A market-maker sells 1,000 1-year European gap call options, and delta-hedges the position with shares.

You are given:

(i) Each gap call option is written on 1 share of a nondividend-paying stock.

(ii) The current price of the stock is 100.

(iii) The stock's volatility is 100%.

(iv) Each gap call option has a strike price of 130.

(v) Each gap call option has a payment trigger of 100.

(vi) The risk-free interest rate is 0%.

Under the Black-Scholes framework, determine the initial number of shares in the delta-hedge.

(A) 586

(B) 594

(C) 684

(D) 692

(E) 797

19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%.

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100.

(iv) The continuously compounded risk-free interest rate is 8%.

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90

(B) 13.10

(C) 14.50

(D) 15.70

(E) 16.80

20. Assume the Black-Scholes framework. Consider a stock, and a European call option and a European put option on the stock. The current stock price, call price, and put price are 45.00, 4.45, and 1.90, respectively.

Investor A purchases two calls and one put. Investor B purchases two calls and writes three puts.

The current elasticity of Investor A's portfolio is 5.0. The current delta of Investor B's portfolio is 3.4.

Calculate the current put-option elasticity.

(A) –0.55

(B) –1.15

(C) –8.64

(D) –13.03

(E) –27.24

21-24. DELETED

25. Consider a chooser option (also known as an as-you-like-it option) on a nondividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of $100.

The chooser option price is $20 at time $t = 0$.

The stock price is $95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of $100.

You are given:

(i) The risk-free interest rate is 0.

(ii) $C(1) = \$4$.

Determine $C(3)$.

(A) $ 9

(B) $11

(C) $13

(D) $15

(E) $17

26. Consider European and American options on a nondividend-paying stock.

You are given:

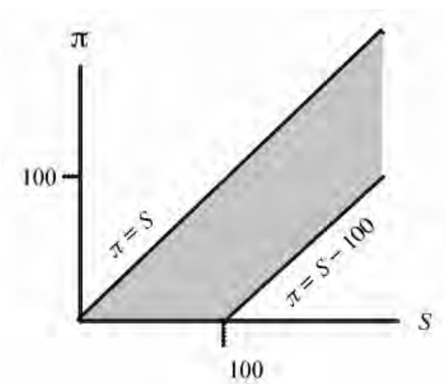
(i) All options have the same strike price of 100.

(ii) All options expire in six months.

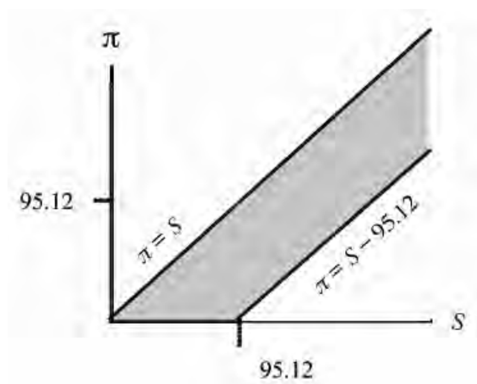
(iii) The continuously compounded risk-free interest rate is 10%.

You are interested in the graph for the price of an option as a function of the current stock price. In each of the following four charts I–IV, the horizontal axis, *S*, represents the current stock price, and the vertical axis, π, represents the price of an option.

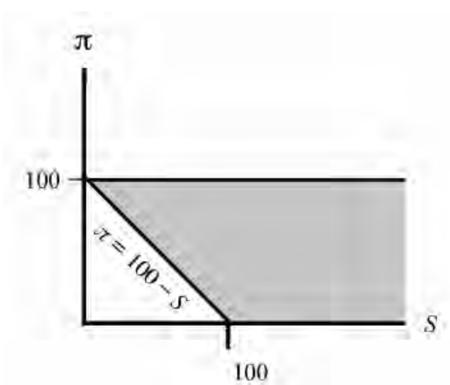
I.



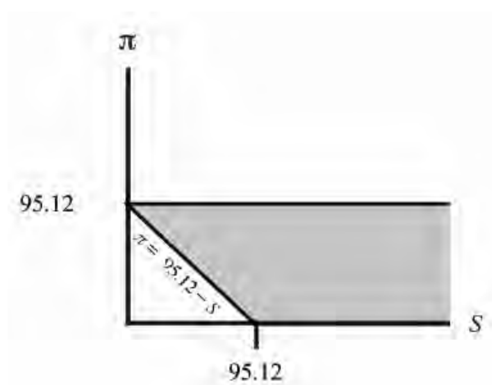
II.



III.



IV.



Match the option with the shaded region in which its graph lies. If there are two or more possibilities, choose the chart with the smallest shaded region.

	European Call	American Call	European Put	American Put
(A)	I	I	III	III
(B)	II	I	IV	III
(C)	II	I	III	III
(D)	II	II	IV	III
(E)	II	II	IV	IV

27-30. DELETED

31. You compute the current delta for a 50-60 bull spread with the following information:

(i) The continuously compounded risk-free rate is 5%.

(ii) The underlying stock pays no dividends.

(iii) The current stock price is \$50 per share.

(iv) The stock's volatility is 20%.

(iv) The time to expiration is 3 months.

How much does delta change after 1 month, if the stock price does not change?

(A) increases by 0.04

(B) increases by 0.02

(C) does not change, within rounding to 0.01

(D) decreases by 0.02

(E) decreases by 0.04

32. DELETED

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is 45.

(iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

(A) 1.59

(B) 2.24

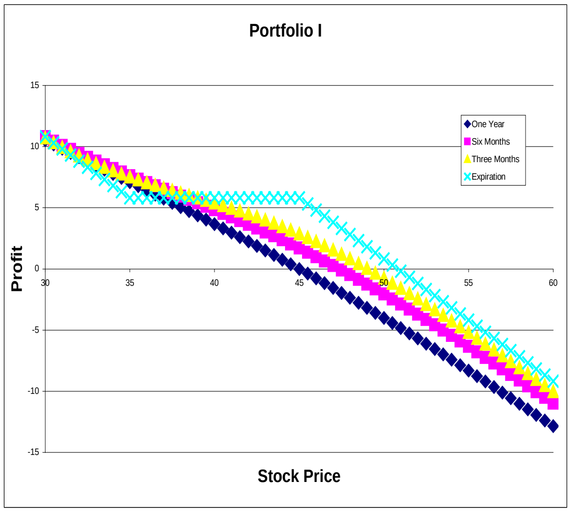
(C) 2.86

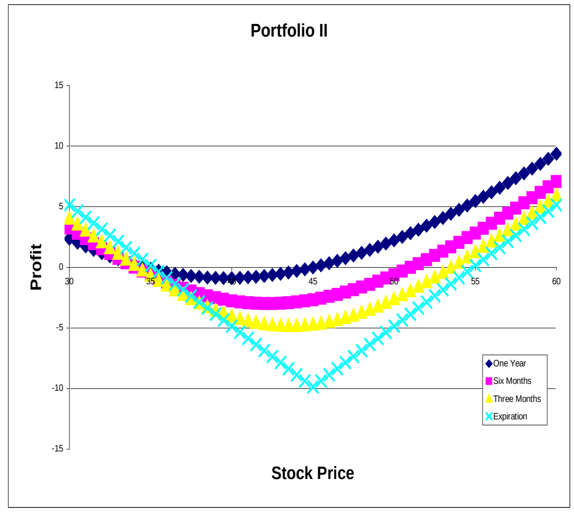
(D) .48

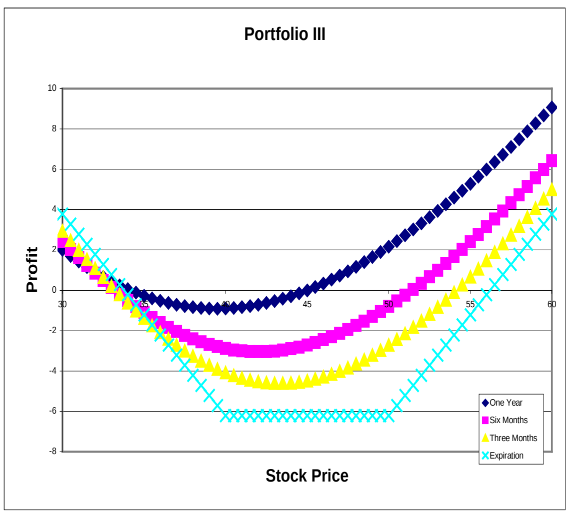
(E) 3.61

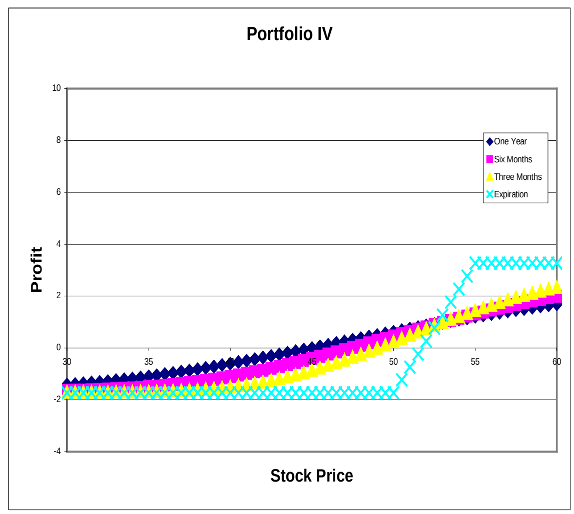
34-39. DELETED

40. The following four charts are profit diagrams for four option strategies: Bull Spread, Collar, Straddle, and Strangle. Each strategy is constructed with the purchase or sale of two 1-year European options.









Match the charts with the option strategies.

	Bull Spread	Straddle	Strangle	Collar
(A)	I	II	III	IV
(B)	I	III	II	IV
(C)	III	IV	I	II
(D)	IV	II	III	I
(E)	IV	III	II	I

41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock.

You are given:

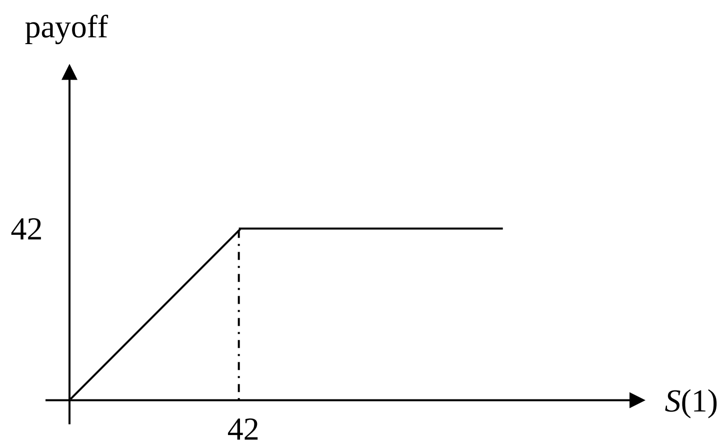
(i) The time-0 stock price is 45.

(ii) The stock's volatility is 25%.

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.

(iv) The continuously compounded risk-free interest rate is 7%.

(v) The time-1 payoff of the contingent claim is as follows:



Calculate the time-0 contingent-claim elasticity.

(A) 0.24

(B) 0.29

(C) 0.34

(D) 0.39

(E) 0.44

42. Prices for 6-month 60-strike European up-and-out call options on a stock S are available. Below is a table of option prices with respect to various H, the level of the barrier. Here, $S(0) = 50$.

H	Price of up-and-out call
60	0
70	0.1294
80	0.7583
90	1.6616
∞	4.0861

Consider a special 6-month 60-strike European "knock-in, partial knock-out" call option that knocks in at $H_1 = 70$, and "partially" knocks out at $H_2 = 80$. The strike price of the option is 60. The following table summarizes the payoff at the exercise date:

H_1 Not Hit	H_1 Hit	
	H_2 Not Hit	H_2 Hit
0	$2 \times \max[S(0.5) - 60, 0]$	$\max[S(0.5) - 60, 0]$

Calculate the price of the option.

(A) 0.6289

(B) 1.3872

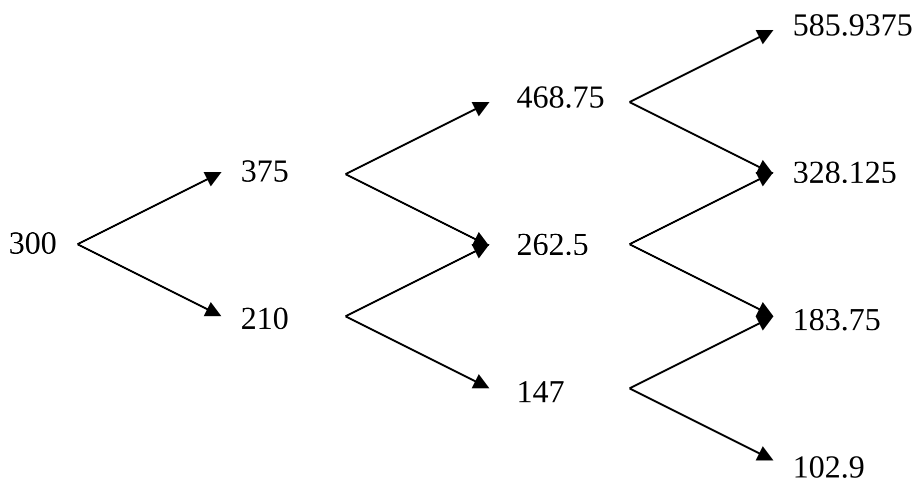
(C) 2.1455

(D) 4.5856

(E) It cannot be determined from the information given above.

43. DELETED

44. Consider the following three-period binomial tree model for a stock that pays dividends continuously at a rate proportional to its price. The length of each period is 1 year, the continuously compounded risk-free interest rate is 10%, and the continuous dividend yield on the stock is 6.5%.



Calculate the price of a 3-year at-the-money American put option on the stock.

(A) 15.86

(B) 27.40

(C) 32.60

(D) 39.73

(E) 57.49

45. DELETED

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.

You are given the following information:

(i) The risk-free interest rate is constant.

(ii)

	Several months ago	Now
Stock price	\$40.00	\$50.00
Call option price	\$ 8.88	\$14.42
Put option price	\$ 1.63	\$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.

Calculate her profit.

(A) $11

(B) $24

(C) $126

(D) $217

(E) $240

48. DELETED

49. You use the usual method in McDonald and the following information to construct a one-period binomial tree for modeling the price movements of a nondividend-paying stock. (The tree is sometimes called a forward tree).

(i) The period is 3 months.

(ii) The initial stock price is $100.

(iii) The stock's volatility is 30%.

(iv) The continuously compounded risk-free interest rate is 4%.

At the beginning of the period, an investor owns an American put option on the stock. The option expires at the end of the period.

Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.

(A) 114

(B) 115

(C) 116

(D) 117

(E) 118

50. Assume the Black-Scholes framework.

You are given the following information for a stock that pays dividends continuously at a rate proportional to its price.

(i) The current stock price is 0.25.

(ii) The stock's volatility is 0.35.

(iii) The continuously compounded expected rate of stock-price appreciation is 15%.

Calculate the upper limit of the 90% lognormal confidence interval for the price of the stock in 6 months.

(A) 0.393

(B) 0.425

(C) 0.451

(D) 0.486

(E) 0.529

51-53. DELETED

54. Assume the Black-Scholes framework. Consider two nondividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$, respectively.

You are given:

(i) $S_1(0) = 10$ and $S_2(0) = 20$.

(ii) Stock 1's volatility is 0.18.

(iii) Stock 2's volatility is 0.25.

(iv) The correlation between the continuously compounded returns of the two stocks is –0.40.

(v) The continuously compounded risk-free interest rate is 5%.

(vi) A one-year European option with payoff $\max\{\min[2S_1(1), S_2(1)] - 17, 0\}$ has a current (time-0) price of 1.632.

Consider a European option that gives its holder the right to sell either two shares of Stock 1 or one share of Stock 2 at a price of 17 one year from now.

Calculate the current (time-0) price of this option.

(A) 0.67

(B) 1.12

(C) 1.49

(D) 5.18

(E) 7.86

55. Assume the Black-Scholes framework. Consider a 9-month at-the-money European put option on a futures contract. You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The strike price of the option is 20.

(iii) The price of the put option is 1.625.

If three months later the futures price is 17.7, what is the price of the put option at that time?

(A) 2.09

(B) 2.25

(C) 2.45

(D) 2.66

(E) 2.83

56-76. DELETED

Introductory Derivatives Solutions

1. Solution: E

To construct a zero-cost collar, first set the strike price for the put option **below** the forward price. It is then possible to find a strike price above the forward price such that the call has the same premium (noting that purchased collars require the call to have a strike price that is greater than or equal to the strike price for the put).

2. Solution: C

$66.59 – 18.64 = 500 – K\exp(–0.06)$ and so $K = (500 – 66.59 + 18.64)/\exp(–0.06) = 480$.

3. Solution: D

The accumulated cost of the hedge is $(84.30 – 74.80)\exp(0.06) = 10.09$.

Let x be the market price in one year.

If $x < 0.12$ the put is in the money and the payoff is $10,000(0.12 – x) = 1,200 – 10,000x$. The sale of the jalapenos has a payoff of $10,000x – 1,000$ for a profit of $1,200 – 10,000x + 10,000x – 1,000 – 10.09 = 190$.

From 0.12 to 0.14 neither option has a payoff and the profit is $10,000x – 1,000 – 10.09 = 10,000x – 1,010$. The range is 190 to 390.

If $x > 0.14$ the call is in the money and the payoff is $–10,000(x – 0.14) = 1,400 – 10,000x$. The profit is $1,400 – 10,000x + 10,000x – 1,000 – 10.09 = 390$.

The range is 190 to 390.

4. DELETED

5. Solution: E

Consider buying the put and selling the call. Let x be the index price in one year. If $x > 1025$, the payoff is $1025 – x$. After buying the index for x you have $1,025 – 2x$ which is not the goal. It is not necessary to check buying the call and selling the put as that is the only other option. But as a check, if $x > 1025$, the payoff is $x – 1025$ and after buying the stock you have spent 1025. If $x < 1025$, the payoff is again $x – 1025$.

One way to get the cost is to note that the forward price is $1,000(1.05) = 1,050$. You want to pay 25 less and so must spend $25/1.05 = 23.81$ today.

6. Solution: E

In general, an investor should be compensated for time and risk. A forward contract has no investment, so the extra 5 represents the risk premium. Those who buy the stock expect to earn both the risk premium and the time value of their purchase and thus the expected stock value is greater than 100 + 5 = 105.

7. Solution: C

All four of answers A-D are methods of acquiring the stock. Only the prepaid forward has the payment at time 0 and the delivery at time T.

8. Solution: B

Only straddles use at-the-money options and buying is correct for this speculation.

9. Solution: D

To see that D does not produce the desired outcome, begin with the case where the stock price is S and is below 90. The payoff is $S + 0 + (110 - S) - 2(100 - S) = 2S - 90$ which is not constant and so cannot produce the given diagram. On the other hand, for example, answer E has a payoff of $S + (90 - S) + 0 - 2(0) = 90$. The cost is 100 + 0.24 + 2.17 – 2(6.80) = 88.81. With interest it is 93.36. The profit is 90 – 93.36 = –3.36 which matches the diagram.

10. Solution: D

Answer A is true because forward contracts have no initial premium.

Answer B is true because both payoffs and profits of long forwards are opposite to short forwards.

Answer C is true because to invest in the stock, one must borrow 100 at t = 0, and then pay back 110 = 100(1 + 0.1) at t = 1, which is like buying a forward at t = 1 for 110.

Answer D is false because repeating the calculation shown for Answer C, but with 10% as a continuously compounded rate, the stock investor must now pay back 100exp(0.1) = 110.52 at t = 1; this is more expensive than buying a forward at t = 1 for 110.00.

Answer E is true because the calculation would be the same as shown above for Answer C but now the stock investor gets an additional dividend of 3.00 at t = 0.5, which the forward investor does not receive (due to not owning the stock until t = 1.

11. Solution: C

The future value of the cost of the options is 9.12(1.08) = 9.85, 6.22(1.08) = 6.72, and 4.08(1.08) = 4.41 respectively.

If $S<35$ no call is in the money and the profits are –9.85, –6.72, & –4.41. The condition isn't met.

If $35 < S < 40$ the 35-strike call returns $S – 35$ and the profit is $S – 44.85$. For the 45-strike call to have a lower profit than the 35-strike call, we need $–4.41 < S – 44.85$ or $S > 40.44$. This is inconsistent with the assumption.

If $40 < S < 45$ the same condition applies for comparing the 35- and 45-strike calls and so $S > 40.44$ is needed. The 40-strike call has a profit of $S – 40 – 6.72 = S – 46.72$. For the 45-strike to exceed the 40-strike, we need $–4.41 > S – 46.72$ or $S < 42.31$.

There is no need to consider $S > 45$.

12. Solution: B

Let S be the price of the index in six months.

The put premium has future value (at t = 0.5) of 74.20[1 + 0.02] = 75.68.

The 6-month profit on a long put position is max(1,000 – S, 0) – 75.68.

The 6-month profit on a short put position is 75.68 – max(1,000 – S, 0).

0 = 75.68 – max(1,000 – S, 0).

75.68 = max(1,000 – S, 0).

75.68 = 1,000 – S. S = 924.32.

13. Solution: D

Buying a call in conjunction with a short position is a form of insurance called a cap. Answers (A) and (B) are incorrect because a floor is the purchase of a put to insure against a long position. Answer (E) is incorrect because writing a covered call is the sale of a call along with a long position in the stock, so that the investor is selling rather than buying insurance.

The profit is the payoff at time 2 less the future value of the initial cost. The stock payoff is –75 and the option payoff is 75 – 60 = 15 for a total of –60. The future value of the initial cost is (–50 + 10)(1.03)(1.03) = –42.44. the profit is –60 – (–42.44) = –17.56.

14. Solution: A

Let C be the price for the 40-strike call option. Then, C + 3.35 is the price for the 35-strike call option. Similarly, let P be the price for the 40-strike put option. Then, P – x is the price for the 35-strike put option, where x is the desired quantity. Using put-call parity, the equations for the 35-strike and 40-strike options are, respectively,

$$(C+3.35)+35e^{-0.02}-40=P-x$$

$$C+40e^{-0.02}-40=P.$$

Subtracting the first equation from the second, $5e^{-0.02}-3.35=x, \quad x=1.55.$

15. Solution: C

The initial cost to establish this position is 5(2.78) – 3(6.13) = –4.49. Thus, you are receiving 4.49 up front. This grows to 4.49exp[0.25(0.08)] = 4.58 after 3 months. Then, if S is the value of the stock at time 0.25, the profit is

5max(S –40,0) – 3max(S – 35,0) + 4.58. The following cases are relevant:

S < 35: Profit = 0 – 0 + 4.58 = 4.58.

35 < S < 40: Profit = 0 – 3(S – 35) + 4.58 = –3S + 109.58. Minimum of –10.42 is at S = 40 and maximum of 4.58 is at S = 35.

S > 40: Profit is 5(S – 40) – 3(S – 35) + 4.58 = 2S – 90.42. Minimum of –-10.42 is at S = 40 and maximum if infinity.

Thus the minimum profit is –10.42 for a maximum loss of 10.42 and the maximum profit is infinity.

16. Solution: D

The straddle consists of buying a 40-strike call and buying a 40-strike put. This costs 2.78 + 1.99 = 4.77 and grows to 4.77exp(0.02) = 4.87 at three months. The strangle consists of buying a 35-strike put and a 45-strike call. This costs 0.44 + 0.97 = 1.41 and grows to 1.41exp(0.02) = 1.44 at three months. Let S be the stock price in three months.

For $S < 40$, the straddle has a profit of $40 - S - 4.87 = 35.13 - S$.

For $S > 40$, the straddle has a profit of $S - 40 - 4.87 = S - 44.87$

For $S < 35$, the strangle has a profit of $35 - S - 1.44 = 33.56 - S$.

For $35 < S < 45$, the strangle has a profit of -1.44.

For $S > 45$, the strangle has a profit of $S - 45 - 1.44 = S - 46.44$.

For $S < 35$ the strangle underperforms the straddle.

For $35 < S < 40$, the strangle outperforms the straddle if $-1.44 > 35.13 - S$ or $S > 36.57$. At this point only Answer D can be correct.

As a check, for $40 < S < 45$, the strangle outperforms the straddle if $-1.44 > S - 44.87$ or $S < 43.43$.

For $S > 45$, the strangle outperforms the straddle if $S - 46.44 > S - 44.87$, which is not possible.

17. Solution: B

Strategy I –Yes. It is a bear spread using calls, and bear spreads perform better when the prices of the underlying asset goes down.

Strategy II – Yes. It is also a bear spread – it just uses puts instead of calls.

Strategy III – No. It is a box spread, which has no price risk; thus, the payoff is the same (1,000 – 950 = 50), no matter the price of the underlying asset.

18. DELETED

19. DELETED

20. Solution: B

We need the future value of the current stock price minus the future value of each of the 12 dividends, where the valuation date is time 3. Thus, the forward price is

$$200e^{0.04(3)} - 1.50[e^{0.04(2.75)} + e^{0.04(2.5)}1.01 + \cdots + e^{0.04(0.25)}1.01^{10} + 1.01^{11}]$$
$$= 200e^{0.12} - 1.50e^{0.11}[1 + (e^{-0.01}1.01) + \cdots + (e^{-0.01}1.01)^{10} + (e^{-0.01}1.01)^{11}]$$
$$= 200e^{0.12} - 1.50e^{0.11}\frac{1-(e^{-0.01}1.01)^{12}}{1-e^{-0.01}1.01} = 225.50 - 1.67442(11.99672) = 205.41.$$

21. Solution: E

The fair value of the forward contract is given by $S_0e^{(r-d)T} = 110e^{(0.05-0.02)0.5} = 111.66.$

This is 0.34 less than the observed price. Thus, one could exploit this arbitrage opportunity by selling the observed forward at 112 and buying a synthetic forward at 111.66, making 112 – 111.66 = 0.34 profit.

22. DELETED

23. DELETED

24. Solution: D

(A) is a reason because hedging reduces the risk of loss, which is a primary function of derivatives.

(B) is a reason because derivatives can be used the hedge some risks that could result in bankruptcy.

(C) is a reason because derivatives can provide a lower-cost way to effect a financial transaction.

(D) is not a reason because derivatives are often used to avoid these types of restrictions.

(E) is a reason because an insurance contract can be thought of as a hedge against the risk of loss.

25. DELETED

26. Solution: B

I is true. The forward seller has unlimited exposure if the underlying asset's price increases.

II is true. The call issuer has unlimited exposure if the underlying asset's price rises.

III is false. The maximum loss on selling a put is FV(put premium) – strike price.

27. DELETED

28. DELETED

29. Solution: A

The current price of the stock and the time of future settlement are not relevant, so let both be 1. Then the following payments are required:

Outright purchase, payment at time 0, amount of payment = 1.

Fully leveraged purchase, payment at time 1, amount of payment = exp(r).

Prepaid forward contract, payment at time 0, amount of payment = exp(-d).

Forward contract, payment at time 1, amount of payment = exp(r-d).

Since $r > d > 0$, $\exp(-d) < 1 < \exp(r-d) < \exp(r)$.

The correct ranking is given by (A).

30. Solution: C

(A) is a distinction. Daily marking to market is done for futures, not forwards.

(B) is a distinction. Futures are more liquid; in fact, if you use the same broker to buy and sell, your position is effectively cancelled.

(C) is not a distinction. Forwards are more customized, and futures are more standardized.

(D) is a distinction. With daily settlement, credit risk is less with futures (v. forwards).

(E) is a distinction. Futures markets, like stock exchanges, have daily price limits.

31. DELETED

32. Solution: E

The notional value of this short futures position is 1500(20)(250) = 7.5 million. The initial margin requirement is 5% of 7.5 million, or 375,000, and the maintenance margin requirement is 90% of 375,000, or 337,500. Judy has a short position, so when the index decreases/increases, her margin account would increase/decrease.

At the first marking-to-market, when the index has fallen to 1498, the margin account is:

$375{,}000 \times \exp(0.04/365) + (1500 - 1498)(20)(250) = 375{,}041.10 + 10{,}000 = 385{,}041.10.$

For Judy not to get a margin call at the second marking-to-market, the value of the index, X, would have to rise so that the account balance decreases to 337,500:

$385{,}041.10 \times \exp(0.04/365) + (1498 - X)(20)(250) = 337{,}500$

$X = 1507.52.$

33: Solution: E

Option I is American-style, and thus, it can be exercised at any time during the 6-month period. Since it is a put, the payoff is greatest when the stock price is smallest (18). The payoff is 20 – 18 = 2.

Option II is Bermuda-style, and can be exercised at any time during the 2nd 3-month period. Since it is a call, the payoff is greatest when the stock price S is largest (28). The payoff is 28 – 25 = 3.

Option III is European-style, and thus, it can be exercised only at maturity. Since it is a 30-strike put, the payoff equation is 30 – 26 = 4.

The ranking is III > II > I.

34. DELETED

35. Solution: C

If the index declined to $45 and the customer exercised the put (buying 100 shares in the market and selling it to the writer for the $50 strike price), the customer would make $500 ($5000 proceeds of sale – $4500 cost = $500). However, this would be offset by the $500 premium paid for the option. The net result would be that the customer would break even.

36. DELETED

37. Solution: B

When there are discrete dividends, the pricing formula is $S(1 + i)$ – AV(dividends), where S is the current stock price. Thus,

$$75 = S(1.06) - [1.5(1.06)^{0.5} + 1.5 = S(1.06) - 3.0443$$
$$S = 78.0433/1.06 = 73.626.$$

38. Solution: C

For stocks without dividends and in the absence of transaction costs, the stock's forward price is the future value of its spot price based on the risk-free interest rate; otherwise there would be an arbitrage opportunity. Because the risk-free interest rate is positive, the forward price must be greater than the spot price of 75.

Because these investors are risk-averse (i.e. they prefer not to take risks if the average rate of return is the same) they need to receive on the average a greater return than the risk-free interest rate on the shares they invest in this stock. In other words, they need to receive a risk premium (incentive) for taking on risk. The forward price only includes the risk-free interest rate and not the risk premium, so the forward price is less than the expected value of the future stock price, namely 90.

39. Solution: E

By definition, one way to use a 3:1 ratio spread is to buy 1 call and sell 3 calls at a different strike price, with the same 1-year maturity. (This can also be done using all puts.)

40. Solution: C

If F is the forward price, K is the strike price, C is the call option price, P is the put option price, v is the annual discounting factor at the risk-free rate, and t is the number of years, then we have the put-call parity formula $C - P = v^t\left(F - K\right)$.

Using the data for each commodity (with C being the call price zinc and K the common strike price), we have

$$\begin{aligned} 700 - 550 &= \frac{1}{(1.06)^2}\left(1400 - K\right) \\ C - 550 &= \frac{1}{(1.06)^2}\left(1600 - K\right). \end{aligned}.$$

Now we could solve for K in the top equation and then use this to solve the second equation for C, but a more efficient method is to subtract the top equation from the bottom equation to cancel out K. Therefore,

$$C - 700 = \frac{200}{1.1236} \Rightarrow C = 878.00.$$

41. Solution: E

The forwards do not have any premium. Due to put-call parity, the net premium of the remaining strategies will increase with an increasing strike price. With a 1% interest rate, the net premium for E will be positive.

42. Solution: D

Purchasing the stock results in paying K today and receiving S in t years, so the profit at expiration from this transaction is $S - Ke^{rt}$.

Selling the call results in receiving the premium C today and paying max(0, $S - K$) in t years. Because $S > K$, the profit from this transaction at expiration is $Ce^{rt} - S + K$.

The overall profit is the sum, $Ce^{rt} + K - Ke^{rt} = Ce^{rt} + K(1 - e^{rt})$.

43. Solution: C

A written collar consists of a short put option and a long call option. The initial cost of the position is –44 – 2.47 + 3.86 = –42.61.

The payoff and profit table is:

	$S_T \le 40$	$40 < S_T \le 50$	$S_T > 50$
Short stock	$-S_T$	$-S_T$	$-S_T$
Short Put	$-(40 - S_T)$	0	0
Long Call	0	0	$(S_T - 50)$
Total Payoff	–40	$-S_T$	–50
Total Profit	$-40 + 42.61e^{0.05} = 4.79$	$-S_T + 44.79$	$-50 + 44.79 = -5.21$

As shown in the table, the maximum profit is 4.79.

44. Solution: E

At expiration the price is 50 and both options are "out-of-the-money" eliminating answers (A) and (B). With a strike price of 45 and a minimum stock price of 46, option A is never in the money, eliminating answer (D). With a strike price of 55, option B will be in the money at the time the stock price is 58, eliminating answer (C) and verifying answer (E).

45. Solution: C

The change in the futures contract in the three month period is

$$200[Se^{0.75(0.02-0.04)} - 1100e^{1.0(0.02-0.04)}] = -100$$

$$197.022S - 215,643,708 = -100$$

$$S = 1094.01$$

46. Solution: B

Answer (A) is false because naked writing involves selling, not buying options.

Answers (C) and (D) are false because it is an American option that can be exercised at any time.

Answer (E) is false because being in-the-money means there is a payoff, not necessarily a profit.

47. Solution: B

Writing a covered call requires shorting the call option along with simultaneous ownership in the stock (i.e., the underlying asset).

48. Solution: E

The payoff for the 45-strike call is 12 = max(0, S – 45), so 12 = S – 45 and thus S = 57.

The payoff for the 135-strike put is max(0, 135 – S) = max(0, 135 – 57) = 78.

49. Solution: D

The customer pays 500 to purchase the options. If the option is not in the money, the investor loses the 500. If the option is in the money the investor will have a payoff and thus a loss of less than 500. Hence the maximum possible loss is 500.

50. Solution: B

The investor received 7 (bought the 70 put for 1 and sold the 80 put for 8). To break even, the investor must lose 7 on the payoff. The purchased put cannot have a negative payoff. However, if the index is at 73 upon expiration, the investor will lose 7 on the 80 put (and have no positive payoff on the 70 put).

51. Solution: B

First, find the prepaid forward price as

$$F_{0,1}^{P} = 35 - PV(divs) = 35 - 0.32(e^{-0.04*2/12} + e^{-0.04*8/12}) = 34.37 .$$

Next, the forward price is

$$F_{0,1} = F_{0,1}^{P} e^{rt} = 34.37055 * e^{0.04} = 35.77 .$$

52. Solution: C

Consider the first three answers, which are identical except for the forward price. The short sale proceeds of 100 can be lent at 3%. At time one the investor has 103. If the forward price is less than 103, the investor can buy a share for less than 103 and use that shore to close out the short position, leaving an arbitrage profit. Hence (A) and (B) represent arbitrage opportunities while (C) does.

It is not necessary to evaluate (D) and (E). However, as a check, the stock is purchased for 100.5 and with interest at time one the investor will possess one share of stock and owe 103.565. If the short forward contract requires selling the share for more than 103.565 there will be an arbitrage opportunity. Both (D) and (E) have the forwards priced higher and therefore provide arbitrage opportunities.

53. Solution: D

If F is the forward price, K is the strike price, c is the call option price, p is the put option price, v is the annual discounting factor due to risk-free interest, and t is the number of years, then we have the put-call parity formula

$$c - p = v^t \left(F - K\right).$$

Using the data for the rice commodity,

$$110 - p = \left(e^{-0.065}\right)^4 (300 - 400) \Rightarrow p = 110 + 100e^{-0.26} = 187.11. .$$

54. DELETED

55. Solution: A

This type of box spread is a long position in a synthetic forward (long call and short put) and a short position in a synthetic forward at a higher strike price (short call and long put). The payoff is the guaranteed positive difference between the strike prices. With $L > K$, the box spread is equivalent to $c(K) – p(K) – c(L) + p(L)$.

A bull spread using calls is $c(K) – c(L)$ and a bear spread using puts is $p(L) – p(K)$. To reproduce the box spread both spreads must be purchased (long position).

56. Solution: E

Cash flows like those of a short stock position are created by shorting both a forward and a zero-coupon bond.

57. DELETED

58. DELETED

59. Solution: E

The payoff table for the long stock, the long put and the short call is, where S is the stock price at expiration:

	$S <= 40$	$40 < S < 50$	$S >= 50$
Stock	S	S	S
Put	$40 - S$	0	0
Call	0	0	$-(S - 50)$
Total	40	S	50

Only Graph E is consistent with this table.

60. Solution: E

Buying a put at 8.60 and selling a call at 8.80 limits the sale price to the 8.60 -- 8.80 range. Brown also receives a premium from selling the call.

61. Solution: A

A put option is in-the-money if the current stock price is less than the strike price, at-the-money if these two prices are equal, and out-of-the-money if the current stock price is greater than the strike price.

Note that if the current stock price is less than the strike price 70, then the current stock price must be less than the strike price 80. Since option A is a 70-strike put and option B is an 80-strike put, we conclude that if option A is in-the-money, then option B must be in-the-money.

62. Solution: E

The future value of the put premium is 7(1.03) = 7.21. If the asset value falls the profit is max(0, 130 – 60) – 7.21 = 62.79. If the asset value rises, the profit is max(0, 130 – 125) – 7.21 = –2.21. The expected profit is 0.5(62.79) + 0.5(–2.21) = 30.29.

63. DELETED

64. DELETED

65. Solution: A

66. Solution: D

The current stock price, 80, is higher than the strike price, 65.

Since a call option provides the right (but not the obligation) to buy a share of the stock for only 65, a call option would have positive payoff if exercised immediately. So the option is in-the-money if it is a call option.

Since a put option provides the right (but not the obligation) to sell a share of the stock for only 65, a put option would have negative payoff if exercised immediately. So the option is out-of-the-money if it is a put option.

Therefore, the option is in-the-money if it is call option, but out-of-the-money if it is a put option.

67. Solution: C

When the stock price is 27, Payoff = -2 (30-27) + (35-27) =2.

When the stock price is 37, Payoff =0 (none of the puts would be exercised).

68. Solution: B

Long gains 1210 - 1150 = 60 (a short position will lose, not gain).

69. Solution: C

The reverse is true. Futures contracts are more useful for minimizing credit risk. This is due to the daily settlement of futures contracts.

70. Solution: C

Since the investors demand to be compensated for risk, on the average (or expected value) the stock should outperform the forward. So the expected value of the future stock price is greater than the forward price.

From the given data, the 2-year forward price of the stock is $S_0e^{(r-\delta)T} = 100e^{(0.05-0.02)(2)} = 106.18$. Since 1) *X*, the expected value of the stock price 2 years from now, exceeds the 2-year forward price, 2) *X* is assumed to be a whole number, and 3) there are no other restrictions on *X*, we conclude that the smallest possible value of *X* is 107.

71. Solution: D

$40 - 6/1.05 - 6/(1.05)^2 - 6/(1.05)^3 - 6/(1.05)^4 = 18.72$.

72. Solution; E

At time 0, CornGrower pays P to buy the put and receives C to sell the call, so the cash flow is C – P which is the premium 0.10 that CornGrower pays to set up the synthetic forward. Therefore, C – P = –0.10. By put-call parity, C – P = 3.59 – exp(–0.04)K. So, 3.59 – exp(–0.04)K = –0.10. Solving for K, we have K = 3.69exp(0.04) = 3.84059..

73. Solution: C

The term *arbitrage* refers to an opportunity for an investor to gain a riskless profit.

74. Solution: A

Buying calls allows a firm to insure against loss of profit as the price of their input increases

75. Solution: E

I is true. Entering into a short forward position means that the oil producer agrees to sell its goods for a predetermined price at a delivery time in the future, which protects the producer from drops in the goods' price.

II is true. Buying a put option allows the producer to sells its goods for a minimum price, the strike price, which protects the producer from drops in the goods' price below the strike price.

III is false. Buying a call option protects the buyer of oil, not the seller.

Thus, I and II only will hedge the producer's financial risk from the goods it sells.

Solutions to Advanced Derivatives Questions

1. Answer: (A)

The put-call parity formula (for a European call and a European put on a stock with the same strike price and maturity date) is

$$\begin{aligned} C - P &= F_{0,T}^P(S) - F_{0,T}^P(K) \\ &= F_{0,T}^P(S) - \mathrm{PV}_{0,T}(K) \\ &= F_{0,T}^P(S) - Ke^{-rT} \\ &= S_0 - Ke^{-rT}, \end{aligned}$$

because the stock pays no dividends

We are given that $C - P = 0.15$, $S_0 = 60$, $K = 70$ and $T = 4$. Then, $r = 0.039$.

Remark 1: If the stock pays n dividends of fixed amounts $D_1, D_2, \ldots, D_n$ at fixed times $t_1, t_2, \ldots, t_n$ prior to the option maturity date, T, then the put-call parity formula for European put and call options is

$$\begin{aligned} C - P &= F_{0,T}^P(S) - Ke^{-rT} \\ &= S_0 - \mathrm{PV}_{0,T}(\mathrm{Div}) - Ke^{-rT}, \end{aligned}$$

where $\mathrm{PV}_{0,T}(\mathrm{Div}) = \sum_{i=1}^{n} D_i e^{-rt_i}$ is the present value of all dividends up to time T. The difference, $S_0 - \mathrm{PV}_{0,T}(\mathrm{Div})$, is the *prepaid forward price* $F_{0,T}^P(S)$.

Remark 2: The put-call parity formula above does not hold for American put and call options. For the American case, the parity relationship becomes

$$S_0 - \mathrm{PV}_{0,T}(\mathrm{Div}) - K \le C - P \le S_0 - Ke^{-rT}.$$

This result is given in Appendix 9A of McDonald (2013) but is not required for Exam MFE. Nevertheless, you may want to try proving the inequalities as follows:

For the first inequality, consider a portfolio consisting of a European call plus an amount of cash equal to $\mathrm{PV}_{0,T}(\mathrm{Div}) + K$.

For the second inequality, consider a portfolio of an American put option plus one share of the stock.

2. Answer: (D)

The prices are not arbitrage-free. To show that Mary's portfolio yields arbitrage profit, we follow the analysis in Table 9.7 on page 285 of McDonald (2013).

	Time 0	Time T			
		$S_T < 40$	$40 \leq S_T < 50$	$50 \leq S_T < 55$	$S_T \geq 55$
Buy 1 call Strike 40	– 11	0	$S_T – 40$	$S_T – 40$	$S_T – 40$
Sell 3 calls Strike 50	+ 18	0	0	$–3(S_T – 50)$	$–3(S_T – 50)$
Lend $1	– 1	e^{rT}	e^{rT}	e^{rT}	e^{rT}
Buy 2 calls strike 55	– 6	0	0	0	$2(S_T – 55)$
Total	0	$e^{rT} > 0$	$e^{rT} + S_T – 40 > 0$	$e^{rT} + 2(55 – S_T) > 0$	$e^{rT} > 0$

Peter's portfolio makes arbitrage profit, because:

	Time-0 cash flow	Time-T cash flow
Buy 2 calls & sells 2 puts Strike 55	$2(–3 + 11) = 16$	$2(S_T – 55)$
Buy 1 call & sell 1 put Strike 40	$–11 + 3 = –8$	$S_T – 40$
Lend $2	–2	$2e^{rT}$
Sell 3 calls & buy 3 puts Strike 50	$3(6 – 8) = –6$	$3(50 – S_T)$
Total	0	$2e^{rT}$

Remarks: Note that Mary's portfolio has no put options. The call option prices are not arbitrage-free; they do not satisfy the convexity condition (9.19) on page 282 of McDonald (2013). The time-T cash flow column in Peter's portfolio is due to the identity

$$\max[0, S – K] – \max[0, K – S] = S – K.$$

In *Loss Models*, the textbook for Exam C, max[0, α] is denoted as α_+. It appears in the context of stop-loss insurance, $(S - d)_+$, with S being the claim random variable and d the deductible. The identity above is a particular case of

$$x = x_+ - (-x)_+,$$

which says that every number is the difference between its positive part and negative part.

3. Answer (C)

The payoff at the contract maturity date is

$$\begin{aligned}
&\pi \times (1 - y\%)\times\text{Max}[S(T)/S(0), (1 + g\%)^T] \\
&= \pi \times (1 - y\%)\times\text{Max}[S(1)/S(0), (1 + g\%)^1] \qquad \text{because } T = 1 \\
&= [\pi/S(0)](1 - y\%)\text{Max}[S(1), S(0)(1 + g\%)] \\
&= (\pi/100)(1 - y\%)\text{Max}[S(1), 103] \qquad \text{because } g = 3 \;\&\; S(0)=100 \\
&= (\pi/100)(1 - y\%)\{S(1) + \text{Max}[0, 103 - S(1)]\}.
\end{aligned}$$

Now, Max[0, 103 – $S(1)$] is the payoff of a one-year European put option, with strike price \$103, on the stock index; the time-0 price of this option is given to be is \$15.21. Dividends are incorporated in the stock index (i.e., $\delta = 0$); therefore, $S(0)$ is the time-0 price for a time-1 payoff of amount $S(1)$. Because of the no-arbitrage principle, the time-0 price of the contract must be

$$\begin{aligned}
&(\pi/100)(1 - y\%)\{S(0) + 15.21\} \\
&= (\pi/100)(1 - y\%) \times 115.21.
\end{aligned}$$

Therefore, the “break-even” equation is

$$\pi = (\pi/100)(1 - y\%)\times 115.21,$$

or

$$y\% = 100 \times (1 - 1/1.1521)\% = 13.202\%$$

Remarks:

(i) Many stock indexes, such as S&P500, do not incorporate dividend reinvestments. In such cases, the time-0 cost for receiving $S(1)$ at time 1 is the prepaid forward price $F_{0,1}^P(S)$, which is less than $S(0)$.

(ii) The identities

$$\text{Max}[S(T), K] = K + \text{Max}[S(T) - K, 0] = K + (S(T) - K)_+$$

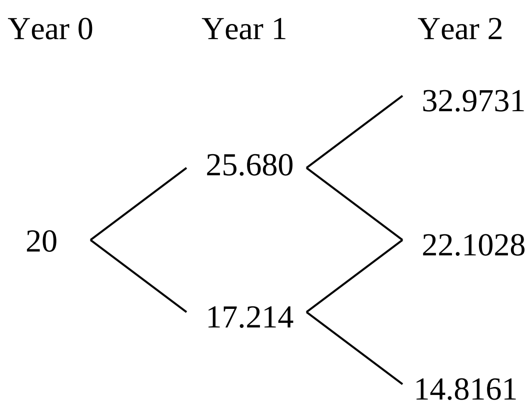
and

$$\text{Max}[S(T), K] \;=\; S(T) \;+\; \text{Max}[0, K - S(T)] \;=\; S(T) \;+\; (K - S(T))_+$$

can lead to a derivation of the put-call parity formula. Such identities are useful for understanding Section 14.6 *Exchange Options* in McDonald (2013).

4. Answer: (C)

First, we construct the two-period binomial tree for the stock price.



The calculations for the stock prices at various nodes are as follows:

$S_u = 20 \times 1.2840 = 25.680$

$S_d = 20 \times 0.8607 = 17.214$

$S_{uu} = 25.68 \times 1.2840 = 32.9731$

$S_{ud} = S_{du} = 17.214 \times 1.2840 = 22.1028$

$S_{dd} = 17.214 \times 0.8607 = 14.8161$

The risk-neutral probability for the stock price to go up is

$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{0.05} - 0.8607}{1.2840 - 0.8607} = 0.4502.$$

Thus, the risk-neutral probability for the stock price to go down is 0.5498.

If the option is exercised at time 2, the value of the call would be

$C_{uu} = (32.9731 - 22)_+ = 10.9731$

$C_{ud} = (22.1028 - 22)_+ = 0.1028$

$C_{dd} = (14.8161 - 22)_+ = 0$

If the option is European, then $C_u = e^{-0.05}[0.4502C_{uu} + 0.5498C_{ud}] = 4.7530$ and

$C_d = e^{-0.05}[0.4502C_{ud} + 0.5498C_{dd}] = 0.0440$.

But since the option is American, we should compare C_u and C_d with the value of the option if it is exercised at time 1, which is 3.68 and 0, respectively. Since 3.68 < 4.7530 and 0 < 0.0440, it is not optimal to exercise the option at time 1 whether the stock is in the up or down state. Thus the value of the option at time 1 is either 4.7530 or 0.0440.

Finally, the value of the call is

$C = e^{-0.05}[0.4502(4.7530) + 0.5498(0.0440)] = 2.0585$.

Remark: Since the stock pays no dividends, the price of an American call is the same as that of a European call. See pages 277-278 of McDonald (2013). The European option price can be calculated using the binomial probability formula. See formula (11.12) on page 335 and formula (19.2) on page 574 of McDonald (2013). The option price is

$$e^{-r(2h)}\left[\binom{2}{2}p^{*2}\,C_{uu} + \binom{2}{1}p^*(1-p^*)C_{ud} + \binom{2}{0}(1-p^*)^2C_{dd}\right]$$

$= e^{-0.1}\,[(0.4502)^2\times 10.9731 + 2\times 0.4502\times 0.5498\times 0.1028 + 0]$

$= 2.0507$

5. Answer: (A)

Each period is of length $h = 0.25$. Using the last two formulas on page 312 of McDonald (2013):

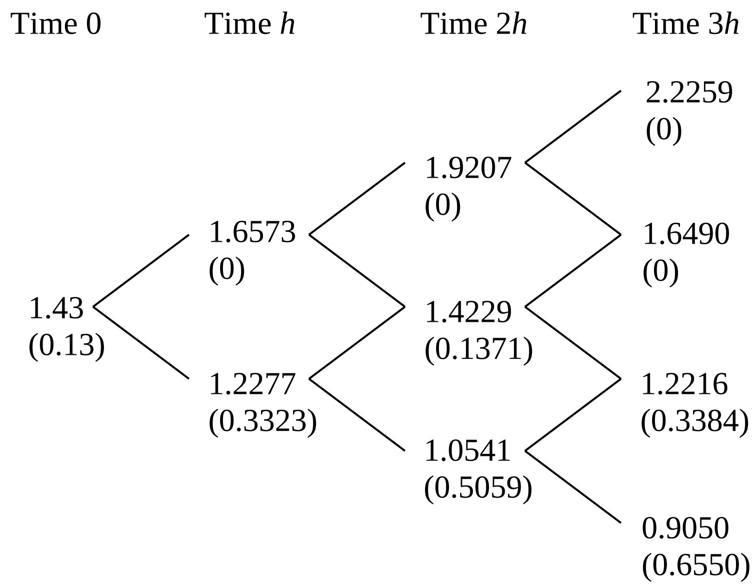
$$u = \exp[-0.01\times 0.25 + 0.3\times\sqrt{0.25}\,] = \exp(0.1475) = 1.158933,$$

$$d = \exp[-0.01\times 0.25 - 0.3\times\sqrt{0.25}\,] = \exp(-0.1525) = 0.858559.$$

Using formula (10.13), the risk-neutral probability of an up move is

$$p^* = \frac{e^{-0.01\times 0.25} - 0.858559}{1.158933 - 0.858559} = 0.4626.$$

The risk-neutral probability of a down move is thus 0.5374. The 3-period binomial tree for the exchange rate is shown below. The numbers within parentheses are the payoffs of the put option if exercised.



The payoffs of the put at maturity (at time $3h$) are

$P_{uuu} = 0$, $P_{uud} = 0$, $P_{udd} = 0.3384$ and $P_{ddd} = 0.6550$.

Now we calculate values of the put at time $2h$ for various states of the exchange rate.

If the put is European, then

$P_{uu} = 0$,

$P_{ud} = e^{-0.02}[0.4626P_{uud} + 0.5374P_{udd}] = 0.1783$,

$P_{dd} = e^{-0.02}[0.\ 4626P_{udd} + 0.5374P_{ddd}] = 0.4985$.

But since the option is American, we should compare P_{uu}, P_{ud} and P_{dd} with the values of the option if it is exercised at time $2h$, which are 0, 0.1371 and 0.5059, respectively. Since $0.4985 < 0.5059$, it is optimal to exercise the option at time $2h$ if the exchange rate has gone down two times before. Thus the values of the option at time $2h$ are $P_{uu} = 0$,

$P_{ud} = 0.1783$ and $P_{dd} = 0.5059$.

Now we calculate values of the put at time h for various states of the exchange rate.

If the put is European, then

$P_u = e^{-0.02}[0.4626P_{uu} + 0.5374P_{ud}] = 0.0939$,

$P_d = e^{-0.02}[0.4626P_{ud} + 0.5374P_{dd}] = 0.3474$.

But since the option is American, we should compare P_u and P_d with the values of the option if it is exercised at time h, which are 0 and 0.3323, respectively. Since $0.3474 > 0.3323$, it is not optimal to exercise the option at time h. Thus the values of the option at time h are $P_u = 0.0939$ and $P_d = 0.3474$.

Finally, discount and average P_u and P_d to get the time-0 price,

$$P = e^{-0.02}[0.4626P_u + 0.5374P_d] = 0.2256.$$

Since it is greater than 0.13, it is not optimal to exercise the option at time 0 and hence the price of the put is 0.2256.

Remarks:

(i) Because $\frac{e^{(r-\delta)h} - e^{(r-\delta)h-\sigma\sqrt{h}}}{e^{(r-\delta)h+\sigma\sqrt{h}} - e^{(r-\delta)h-\sigma\sqrt{h}}} = \frac{1-e^{-\sigma\sqrt{h}}}{e^{\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}}} = \frac{1}{1+e^{\sigma\sqrt{h}}}$, we can also calculate the risk-neutral probability p^* as follows:

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3\sqrt{0.25}}} = \frac{1}{1+e^{0.15}} = 0.46257.$$

(ii) $$1 - p^* = 1 - \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{e^{\sigma\sqrt{h}}}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{-\sigma\sqrt{h}}}.$$

(iii) Because $\sigma > 0$, we have the inequalities

$$p^* < ½ < 1 - p^*.$$

6. Answer: (C)

$$C(S, K, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2) \tag{12.1}$$

with

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{12.2a}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{12.2b}$$

Because S = 20, K = 25, σ = 0.24, r = 0.05, T = 3/12 = 0.25, and δ = 0.03, we have

$$d_1 = \frac{\ln(20/25) + (0.05 - 0.03 + \frac{1}{2}0.24^2)0.25}{0.24\sqrt{0.25}} = -1.75786$$

and

$$d_2 = -1.75786 - 0.24\sqrt{0.25} = -1.87786$$

Using the Cumulative Normal Distribution Calculator, we obtain $N(-1.75786) = 0.03939$ and $N(-1.87786) = 0.03020$.

Hence, formula (12.1) becomes

$$C = 20e^{-(0.03)(0.25)}(0.03939) - 25e^{-(0.05)(0.25)}(0.03020) = 0.036292362$$

Cost of the block of 100 options = 100 × 0.0363 = \$3.63.

7. Answer: (D)

Let $X(t)$ be the exchange rate of U.S. dollar per Japanese yen at time t. That is, at time t,

$$¥1 = \$X(t).$$

We are given that $X(0) = 1/120$.

At time ¼, Company A will receive ¥ 120 billion, which is exchanged to

\$[120 billion × X(¼)]. However, Company A would like to have

$$\$ \text{ Max}[1 \text{ billion},\ 120 \text{ billion} \times X(¼)],$$

which can be decomposed as

$$\$120 \text{ billion} \times X(¼) + \$ \text{ Max}[1 \text{ billion} - 120 \text{ billion} \times X(¼), 0],$$

or

$$\$120 \text{ billion} \times \{X(¼) + \text{Max}[120^{-1} - X(¼), 0]\}.$$

Thus, Company A purchases 120 billion units of a put option whose payoff three months from now is

$$\$ \text{ Max}[120^{-1} - X(¼), 0].$$

The exchange rate can be viewed as the price, in US dollar, of a traded asset, which is the Japanese yen. The continuously compounded risk-free interest rate in Japan can be interpreted as δ, the dividend yield of the asset. See also page 355 of McDonald (2013) for the *Garman-Kohlhagen model*. Then, we have

$$r = 0.035,\ \delta = 0.015,\ S = X(0) = 1/120,\ K = 1/120,\ T = ¼.$$

It remains to determine the value of σ, which is given by the equation

$$\sigma\sqrt{\frac{1}{365}} = 0.261712\ \%.$$

Hence,

$$\sigma = 0.05.$$

Therefore,

$$d_1 = \frac{(r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{(0.035 - 0.015 + 0.05^2/2)/4}{0.05\sqrt{1/4}} = 0.2125$$

and

$$d_2 = d_1 - \sigma\sqrt{T} = 0.2125 - 0.05/2 = 0.1875.$$

By (12.4) of McDonald (2013), the time-0 price of 120 billion units of the put option is

$$\$120 \text{ billion} \times [Ke^{-rT}N(-d_2) - X(0)e^{-\delta T}N(-d_1)]$$

$$= \$\,[e^{-rT}N(-d_2) - e^{-\delta T}N(-d_1)] \text{ billion} \qquad \text{because } K = X(0) = 1/120$$

Using the Cumulative Normal Distribution Calculator, we obtain $N(-0.1875) = 0.42563$ and $N(-0.2125) = 0.41586$.

Thus, Company A's option cost is

$$e^{-0.035/4}\times 0.42563 - e^{-0.015/4}\times 0.41586$$

$$= 0.007618538 \text{ billion} \approx 7.62 \text{ million}.$$

Remarks:

(i) Suppose that the problem is to be solved using options on the exchange rate of Japanese yen per US dollar, i.e., using yen-denominated options. Let

$$\$1 = ¥U(t)$$

at time t, i.e., $U(t) = 1/X(t)$.

Because Company A is worried that the dollar may increase in value with respect to the yen, it buys 1 billion units of a 3-month yen-denominated European *call* option, with exercise price ¥120. The payoff of the option at time ¼ is

$$¥ \text{Max}[U(¼) - 120, 0].$$

To apply the Black-Scholes call option formula (12.1) to determine the time-0 price in yen, use

$$r = 0.015,\ \delta = 0.035,\ S = U(0) = 120,\ K = 120,\ T = ¼, \text{ and } \sigma = 0.05.$$

Then, divide this price by 120 to get the time-0 option price in dollars. We get the same price as above, because d_1 here is $-d_2$ of above.

The above is a special case of formula (9.9) on page 275 of McDonald (2013).

(ii) There is a cheaper solution for Company A. At time 0, borrow

$$¥\ 120\times\exp(-¼\ r_{¥}) \text{ billion},$$

and immediately convert this amount to US dollars. The loan is repaid with interest at time ¼ when the deal is closed.

On the other hand, with the option purchase, Company A will benefit if the yen increases in value with respect to the dollar.

8. Answer: (D)

Since it is never optimal to exercise an American call option before maturity if the stock pays no dividends, we can price the call option using the European call option formula

$$C = SN(d_1) - Ke^{-rT}N(d_2)\,,$$

where $d_1 = \dfrac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

Because the call option delta is $N(d_1)$ and it is given to be 0.5, we have $d_1 = 0$.

Hence,

$$d_2 = -0.3\times\sqrt{0.25} = -0.15\ .$$

To find the continuously compounded risk-free interest rate, use the equation

$$d_1 = \frac{\ln(40/41.5) + (r + \frac{1}{2}\times 0.3^2)\times 0.25}{0.3\sqrt{0.25}} = 0\,,$$

which gives $r = 0.1023$.

Thus,

$C = 40N(0) - 41.5e^{-0.1023\times 0.25}N(-0.15)$

$= 20 - 40.453[1 - N(0.15)]$

$= 40.453N(0.15) - 20.453$

$= \dfrac{40.453}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{0.15} e^{-x^2/2}\mathrm{d}x - 20.453$

$= 16.138\displaystyle\int_{-\infty}^{0.15} e^{-x^2/2}\mathrm{d}x - 20.453$

9. Answer: (B)

According to the second paragraph on page 395 of McDonald (2013), such a stock price move is given by plus or minus of

$$\sigma S(0)\sqrt{h}\,,$$

where $h = 1/365$ and $S(0) = 50$. It remains to find σ.

Because the stock pays no dividends (i.e., $\delta = 0$), it follows from the bottom of page 357 that $\Delta = N(d_1)$. Thus,

$$
\begin{aligned}
d_1 &= N^{-1}(\Delta) \\
&= N^{-1}(0.61791) \\
&= 0.3
\end{aligned}
$$

by using the Inverse CDF Calculator.

Because $S = K$ and $\delta = 0$, formula (12.2a) is

$$d_1 = \frac{(r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

or

$$½\sigma^2 - \frac{d_1}{\sqrt{T}}\sigma + r = 0.$$

With $d_1 = 0.3$, $r = 0.1$, and $T = 1/4$, the quadratic equation becomes

$$½\sigma^2 - 0.6\sigma + 0.1 = 0,$$

whose roots can be found by using the quadratic formula or by factorization,

$$½(\sigma - 1)(\sigma - 0.2) = 0.$$

We reject $\sigma = 1$ because such a volatility seems too large (and none of the five answers fit). Hence,

$$\sigma S(0)\sqrt{h} = 0.2 \times 50 \times 0.052342 \approx 0.52.$$

10-17. DELETED

18. Answer: (A)

Note that, in this problem, $r = 0$ and $\delta = 0$.

By formula (14.15) in McDonald (2013), the time-0 price of the gap option is

$$C_{\text{gap}} = SN(d_1) - 130N(d_2) = [SN(d_1) - 100N(d_2)] - 30N(d_2) = C - 30N(d_2),$$

where d_1 and d_2 are calculated with $K = 100$ (and $r = \delta = 0$) and $T = 1$, and C denotes the time-0 price of the plain-vanilla call option with exercise price 100.

In the Black-Scholes framework, delta of a derivative security of a stock is the partial derivative of the security price with respect to the stock price. Thus,

$$
\begin{aligned}
\Delta_{\text{gap}} &= \frac{\partial}{\partial S}C_{\text{gap}} = \frac{\partial}{\partial S}C - 30\frac{\partial}{\partial S}N(d_2) = \Delta_C - 30N'(d_2)\frac{\partial}{\partial S}d_2 \\
&= N(d_1) - 30N'(d_2)\frac{1}{S\sigma\sqrt{T}},
\end{aligned}
$$

where $N'(x) = \dfrac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$ is the density function of the standard normal.

Now, with $S = K = 100$, $T = 1$, and $\sigma = 1$,

$$d_1 = [\ln(S/K) + \sigma^2 T/2]/(\sigma\sqrt{T}) = (\sigma^2 T/2)/(\sigma\sqrt{T}) = ½\,\sigma\sqrt{T} = ½,$$

and $d_2 = d_1 - \sigma\sqrt{T} = -½$. Hence, at time 0

$$\begin{aligned}
\Delta_{\text{gap}} &= N(d_1) - 30N'(d_2)\frac{1}{100} \\
&= N(½) - 0.3N'(-½) \\
&= N(½) - 0.3\frac{1}{\sqrt{2\pi}}\, e^{-(-½)^2/2} \\
&= 0.69146 - 0.3\frac{e^{-1/8}}{\sqrt{2\pi}} \\
&= 0.58584.
\end{aligned}$$

19. Answer: (C)

This problem is based on Exercise 14.21 on page 429 of McDonald (2013).

Let S_1 denote the stock price at the end of one year. Apply the Black-Scholes formula to calculate the price of the at-the-money call one year from today, conditioning on S_1.

$d_1 = [\ln(S_1/S_1) + (r + \sigma^2/2)T]/(\sigma\sqrt{T}) = (r + \sigma^2/2)/\sigma = 0.41667$, which turns out to be independent of S_1.

$d_2 = d_1 - \sigma\sqrt{T} = d_1 - \sigma = 0.11667$

The value of the forward start option at time 1 is

$$\begin{aligned}
C(S_1) &= S_1 N(d_1) - S_1 e^{-r} N(d_2) \\
&= S_1[N(0.41667) - e^{-0.08} N(0.11667)] \\
&= S_1[0.66154 - e^{-0.08} \times 0.54644] \\
&= 0.157112 S_1.
\end{aligned}$$

(Note that, when viewed from time 0, S_1 is a random variable.)

Thus, the time-0 price of the forward start option must be 0.157112 multiplied by the time-0 price of a security that gives S_1 as payoff at time 1, i.e., multiplied by the prepaid forward price $F_{0,1}^P(S)$. Hence, the time-0 price of the forward start option is

$$0.157112 \times F_{0,1}^P(S) = 0.157112 \times e^{-0.08} \times F_{0,1}(S) = 0.157112 \times e^{-0.08} \times 100 = 14.5033$$

Remark: A key to pricing the forward start option is that d_1 and d_2 turn out to be independent of the stock price. This is the case if the strike price of the call option will be set as a fixed percentage of the stock price at the issue date of the call option.

21. Answer: (D)

Applying the formula

$$\Delta_{\text{portfolio}} = \frac{\partial}{\partial S}\text{portfolio value}$$

to Investor B's portfolio yields

$$3.4 = 2\Delta_C - 3\Delta_P. \tag{1}$$

Applying the elasticity formula

$$\Omega_{\text{portfolio}} = \frac{\partial}{\partial \ln S}\ln[\text{portfolio value}] = \frac{S}{\text{portfolio value}} \times \frac{\partial}{\partial S}\text{portfolio value}$$

to Investor A's portfolio yields

$$5.0 = \frac{S}{2C+P}(2\Delta_C + \Delta_P) = \frac{45}{8.9+1.9}(2\Delta_C + \Delta_P),$$

or

$$1.2 = 2\Delta_C + \Delta_P. \tag{2}$$

Now, $\quad (2) - (1) \quad \Rightarrow \quad -2.2 = 4\Delta_P.$

Hence, time-0 put option elasticity $= \Omega_P = \dfrac{S}{P}\,\Delta_P = \dfrac{45}{1.9} \times -\dfrac{2.2}{4} = -13.03$, which is (D).

Remarks:

(i) If the stock pays no dividends, and if the European call and put options have the same expiration date and strike price, then $\Delta_C - \Delta_P = 1$. In this problem, the put and call do not have the same expiration date and strike price; so this relationship does not hold.

(ii) The statement on page 365 in McDonald (2013) that "[t]he elasticity of a portfolio is the *weighted average* of the elasticities of the portfolio components" may remind students, who are familiar with fixed income mathematics, the concept of *duration*. Formula (3.5.8) on page 101 of *Financial Economics*: *With Applications to Investments, Insurance and Pensions* (edited by H.H. Panjer and published by The Actuarial Foundation in 1998) shows that the so-called *Macaulay duration* is an elasticity.

(iii) In the Black-Scholes framework, the hedge ratio or delta of a portfolio is the partial derivative of the portfolio price with respect to the stock price. In other continuous-time frameworks (which are not in the syllabus of Exam MFE), the hedge ratio may not be given by a partial derivative; for an example, see formula (10.5.7) on page 478 of *Financial Economics*: *With Applications to Investments*, *Insurance and Pensions*.

21-24. DELETED

25. Answer: (B)

Let $C(S(t), t, T)$ denote the price at time-t of a European call option on the stock, with exercise date T and exercise price $K = 100$. So,

$$C(T) = C(95, 0, T).$$

Similarly, let $P(S(t), t, T)$ denote the time-t put option price.

At the choice date $t = 1$, the value of the chooser option is

$$\text{Max}[C(S(1), 1, 3), P(S(1),1, 3)],$$

which can expressed as

$$C(S(1), 1, 3) + \text{Max}[0, P(S(1),1, 3) - C(S(1), 1, 3)]. \qquad (1)$$

Because the stock pays no dividends and the interest rate is zero,

$$P(S(1),1, 3) - C(S(1), 1, 3) = K - S(1)$$

by put-call parity. Thus, the second term of (1) simplifies as

$$\text{Max}[0, K - S(1)],$$

which is the payoff of a European put option. As the time-1 value of the chooser option is

$$C(S(1), 1, 3) + \text{Max}[0, K - S(1)],$$

its time-0 price *must* be

$$C(S(0), 0, 3) + P(S(0), 0, 1),$$

which, by put-call parity, is

$$C(S(0), 0, 3) + [C(S(0), 0, 1) + K - S(0)]$$
$$= C(3) + [C(1) + 100 - 95] = C(3) + C(1) + 5.$$

Thus,

$$C(3) = 20 - (4 + 5) = 11.$$

Remark: The problem is a modification of Exercise 14.20.b.

26. Answer: (D)

$T = \frac{1}{2}$; $\text{PV}_{0,T}(K) = Ke^{-rT} = 100e^{-0.1/2} = 100e^{-0.05} = 95.1229 \approx 95.12.$

By (9.11) on page 277 of McDonald (2013), we have

$$S(0) \geq C_{Am} \geq C_{Eu} \geq \text{Max}[0,\ F_{0,T}^{P}(S) - \text{PV}_{0,T}(K)].$$

Because the stock pays no dividends, the above becomes

$$S(0) \geq C_{Am} = C_{Eu} \geq \text{Max}[0, S(0) - \text{PV}_{0,T}(K)].$$

Thus, the shaded region in II contains C_{Am} and C_{Eu}. (The shaded region in I also does, but it is a larger region.)

By (9.12) on page 277 of McDonald (2013), we have

$$K \geq P_{Am} \geq P_{Eu} \geq \text{Max}[0, \text{PV}_{0,T}(K) - F_{0,T}^{P}(S)]$$
$$= \text{Max}[0, \text{PV}_{0,T}(K) - S(0)]$$

because the stock pays no dividends. However, the region bounded above by $\pi = K$ and bounded below by $\pi = \text{Max}[0, \text{PV}_{0,T}(K) - S]$ is not given by III or IV.

Because an American option can be exercised immediately, we have a tighter lower bound for an American put,

$$P_{Am} \geq \text{Max}[0, K - S(0)].$$

Thus,

$$K \geq P_{Am} \geq \text{Max}[0, K - S(0)],$$

showing that the shaded region in III contains P_{Am}.

For a European put, we can use put-call parity and the inequality $S(0) \geq C_{Eu}$ to get a tighter upper bound,

$$\text{PV}_{0,T}(K) \geq P_{Eu}.$$

Thus,

$$\text{PV}_{0,T}(K) \geq P_{Eu} \geq \text{Max}[0, \text{PV}_{0,T}(K) - S(0)],$$

showing that the shaded region in IV contains P_{Eu}.

Remarks:

(i) It turns out that II and IV can be found on page 156 of Capiński and Zastawniak (2003) *Mathematics for Finance: An Introduction to Financial Engineering*, Springer Undergraduate Mathematics Series.

(ii) The last inequality in (9.9) can be derived as follows. By put-call parity,

$$C_{Eu} = P_{Eu} + F_{0,T}^{P}(S) - e^{-rT}K$$

$$\geq F_{0,T}^{P}(S) - e^{-rT}K \qquad \text{because } P_{Eu} \geq 0.$$

We also have

$$C_{Eu} \geq 0.$$

Thus,

$$C_{Eu} \geq \text{Max}[0,\ F_{0,T}^{P}(S) - e^{-rT}K].$$

(iii) An alternative derivation of the inequality above is to use *Jensen's Inequality* (see, in particular, page 883).

$$C_{Eu} = \text{E}^*\left[e^{-rT}\text{Max}(0, S(T) - K)\right]$$

$$\geq e^{-rT}\text{Max}(0, \text{E}^*\left[S(T) - K\right]) \text{ because of Jensen's Inequality}$$

$$= \text{Max}(0, \text{E}^*\left[e^{-rT}S(T)\right] - e^{-rT}K)$$

$$= \text{Max}(0, F_{0,T}^{P}(S) - e^{-rT}K).$$

Here, E* signifies risk-neutral expectation.

(iv) That $C_{Eu} = C_{Am}$ for nondividend-paying stocks can be shown by Jensen's Inequality.

27-30. DELETED

31. Answer: (B)

Assume that the bull spread is constructed by buying a 50-strike call and selling a 60-strike call. (You may also assume that the spread is constructed by buying a 50-strike put and selling a 60-strike put.)

Delta for the bull spread is equal to

(delta for the 50-strike call) – (delta for the 60-strike call).

(You get the same delta value, if put options are used instead of call options.)

Call option delta = $N(d_1)$, where $d_1 = \dfrac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$

50-strike call:

$$d_1 = \frac{\ln(50/50) + (0.05 + \frac{1}{2} \times 0.2^2)(3/12)}{0.2\sqrt{3/12}} = 0.175, \qquad N(0.175) = 0.56946$$

60-strike call:

$$d_1 = \frac{\ln(50/60) + (0.05 + \frac{1}{2} \times 0.2^2)(3/12)}{0.2\sqrt{3/12}} = -1.64822, \qquad N(-1.64882) = 0.04965$$

Delta of the bull spread = 0.56946 – 0.04965 = 0.51981.

After one month, 50-strike call:

$$d_1 = \frac{\ln(50/50) + (0.05 + \frac{1}{2} \times 0.2^2)(2/12)}{0.2\sqrt{2/12}} = 0.1428869 \qquad N(0.14289) = 0.55681$$

60-strike call:

$$d_1 = \frac{\ln(50/60) + (0.05 + \frac{1}{2} \times 0.2^2)(2/12)}{0.2\sqrt{2/12}} = -2.090087 \qquad N(-2.0901) = 0.01830$$

Delta of the bull spread after one month = 0.55681 – 0.01830 = 0.53851.

The change in delta = 0.53851 – 0.51981 = 0.0187 ≈ 0.02.

32. DELETED

33. Answer: (C)

The problem is a variation of Exercise 14.22, whose solution uses the concept of the forward start option in Exercise 14.21.

Let us first calculate the current price of a 3-month European put with strike price being 90% of the current stock price *S*.

With $K = 0.9 \times S$, $r = 0.08$, $\sigma = 0.3$, and $T = ¼$, we have

$$d_1 = \frac{\ln(S/0.9S) + (r + ½\sigma^2)T}{\sigma\sqrt{T}} = \frac{-\ln(0.9) + (0.08 + ½ \times 0.09) \times ¼}{0.3\sqrt{¼}} = 0.91073677$$

$d_2 = d_1 - \sigma\sqrt{T} = d_1 - 0.3\sqrt{¼} = 0.76074$

$N(-d_1) = N(-0.91074) = 0.18122$

$N(-d_2) = N(-0.76074) = 0.22341$

Put price = $Ke^{-rT}N(-d_2) - SN(-d_1) = 0.9Se^{-0.08\times0.25}\times0.22341 - S\times0.18122 = 0.015868S$

For the rolling insurance strategy, four put options are needed. Their costs are $0.015868S(0)$ at time 0, $0.015868S(¼)$ at time ¼, $0.015868S(½)$ at time ½, and $0.015868S(¾)$ at time ¾. Their total price at time 0 is the sum of their prepaid forward prices.

Since the stock pays no dividends, we have

$$F_{0,T}^{P}(S(T)) = S(0), \qquad \text{for all } T \geq 0.$$

Hence, the sum of the four prepaid forward prices is

$$0.015868S(0) \times 4 = 0.015868 \times 45 \times 4 = 2.85624 \approx 2.86.$$

34-39. DELETED

40. Answer: (D)

Profit diagrams are discussed Section 12.4 of McDonald (2013). Definitions of the option strategies can be found in the Glossary near the end of the textbook. See also Figure 3.16 on page 85.

The payoff function of a *straddle* is

$$\pi(s) = (K - s)_+ + (s - K)_+ = |s - K|.$$

The payoff function of a *strangle* is

$$\pi(s) = (K_1 - s)_+ + (s - K_2)_+$$

where $K_1 < K_2$.

The payoff function of a *collar* is

$$\pi(s) = (K_1 - s)_+ - (s - K_2)_+$$

where $K_1 < K_2$.

The payoff function of a *bull spread* is

$$\pi(s) = (s - K_1)_+ - (s - K_2)_+$$

where $K_1 < K_2$. Because $x_+ = (-x)_+ + x$, we have

$$\pi(s) = (K_1 - s)_+ - (K_2 - s)_+ + K_2 - K_1.$$

The payoff function of a *bear spread* is

$$\pi(s) = (s - K_2)_+ - (s - K_1)_+$$

where $K_1 < K_2$.

41. Answer: (C)

The payoff function of the contingent claim is

$$\pi(s) = \min(42, s) = 42 + \min(0, s - 42) = 42 - \max(0, 42 - s) = 42 - (42 - s)_+$$

The time-0 price of the contingent claim is

$$\begin{aligned} V(0) &= F_{0,1}^P[\pi(S(1))] \\ &= \text{PV}(42) - F_{0,1}^P[(42 - S(1))_+] \\ &= 42e^{-0.07} - P(45, 42, 0.25, 0.07, 1, 0.03). \end{aligned}$$

We have $d_1 = \dfrac{\ln(45/42) + (0.07 - 0.03 + ½(0.25)^2 \times 1)}{0.25\sqrt{1}} = 0.560971486$

and $d_2 = 0.310971486$. From the Cumulative Normal Distribution Calculator,

$N(-d_1) = N(-0.56097) = 0.28741$ and $N(-d_2) = N(-0.31097) = 0.37791$.

Hence, the time-0 put price is

$$P(45, 42, 0.25, 0.07, 1, 0.03) = 42e^{-0.07}(0.37791) - 45e^{-0.03}(0.28741) = 2.247951,$$

which implies $V(0) = 42e^{-0.07} - 2.247951 = 36.91259$.

$$\begin{aligned} \text{Elasticity} &= \frac{\partial \ln V}{\partial \ln S} \\ &= \frac{\partial V}{\partial S} \times \frac{S}{V} \\ &= \Delta_V \times \frac{S}{V} \\ &= -\Delta_{\text{Put}} \times \frac{S}{V}. \end{aligned}$$

$$\begin{aligned} \text{Time-0 elasticity} &= e^{-\delta T} N(-d_1) \times \frac{S(0)}{V(0)} \\ &= e^{-0.03} \times 0.28741 \times \frac{45}{36.91259} \\ &= 0.340025. \end{aligned}$$

Remark: We can also work with $\pi(s) = s - (s - 42)_+$; then

$$V(0) = 45e^{-0.03} - C(45, 42, 0.25, 0.07, 1, 0.03)$$

and

$$\frac{\partial V}{\partial S} = e^{-\delta T} - \Delta_{\text{call}} = e^{-\delta T} - e^{-\delta T} N(d_1) = e^{-\delta T} N(-d_1).$$

42. Answer: (D)

The "knock-in, knock-out" call can be thought of as a portfolio of

– buying 2 ordinary up-and-in call with strike 60 and barrier H_1,

– writing 1 ordinary up-and-in call with strike 60 and barrier H_2.

Recall also that "up-and-in" call + "up-and-out" call = ordinary call.

Let the price of the ordinary call with strike 60 be p (actually it is 4.0861),

then the price of the UIC (H_1 = 70) is p – 0.1294

and the price of the UIC (H_1 = 80) is p – 0.7583.

The price of the "knock-in, knock out" call is 2(p – 0.1294) – (p – 0.7583) = 4.5856 .

Alternative Solution:

Let $M(T) = \max_{0 \le t \le T} S(t)$ be the *running maximum* of the stock price up to time T.

Let I[.] denote the *indicator function*.

For various H, the first table gives the time-0 price of payoff of the form

$$I[H > M(½)] \times [S(½) - 60]_+ .$$

The payoff described by the second table is

$$\begin{aligned}
& I[70 \le M(½)]\{2I[80 > M(½)] + I[80 \le M(½)]\}[S(½) - 60]_+ \\
&= \{1 - I[70 > M(½)]\}\{1 + I[80 > M(½)]\}[S(½) - 60]_+ \\
&= \{1 - I[70 > M(½)] + I[80 > M(½)] - I[70 > M(½)]I[80 > M(½)]\}[S(½) - 60]_+ \\
&= \{1 - 2I[70 > M(½)] + I[80 > M(½)]\}[S(½) - 60]_+ \\
&= \{I[\infty > M(½)] - 2I[70 > M(½)] + I[80 > M(½)]\}[S(½) - 60]_+
\end{aligned}$$

Thus, the time-0 price of this payoff is $4.0861 - 2 \times 0.1294 + 0.7583 = 4.5856$.

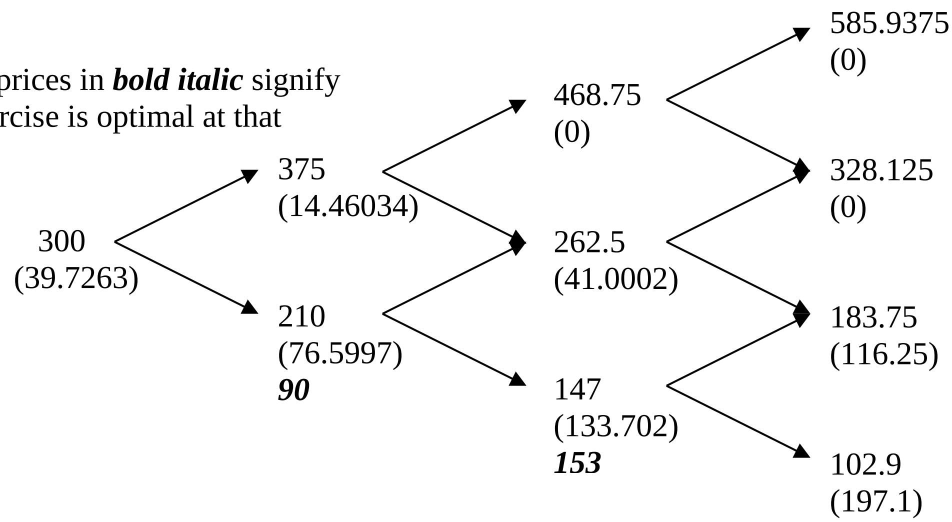
43. DELETED

44. Answer: (D)

By formula (10.5), the risk-neutral probability of an up move is

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{S_0 e^{(r-\delta)h} - S_d}{S_u - S_d} = \frac{300e^{(0.1-0.065)\times 1} - 210}{375 - 210} = 0.61022.$$

Option prices in ***bold italic*** signify that exercise is optimal at that node.



Remark

If the put option is European, not American, then the simplest method is to use the binomial formula [p. 335, (11.12); p. 574, (19.2)]:

$$e^{-r(3h)}\left[\binom{3}{3}(1-p^*)^3(300-102.9) + \binom{3}{2}p^*(1-p^*)^2(300-183.75) + 0 + 0\right]$$

$= e^{-r(3h)}(1 - p^*)^2[(1 - p^*) \times 197.1 + 3 \times p^* \times 116.25)]$

$= e^{-r(3h)}(1 - p^*)^2(197.1 + 151.65p^*)$

$= e^{-0.1 \times 3} \times 0.38978^2 \times 289.63951 = 32.5997$

45. DELETED

46. Answer: (E)

By formula (10.21), the risk-neutral probability of an up move is

$$p^* = \frac{1-d}{u-d} = \frac{1/d-1}{u/d-1}.$$

Substituting $p^* = 1/3$ and $u/d = 4/3$, we have

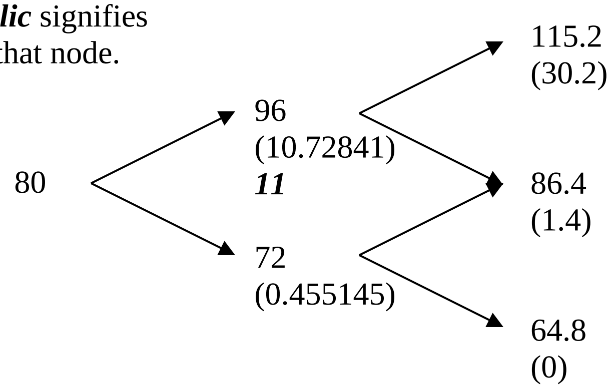
$$\frac{1}{3} = \frac{1/d-1}{4/3-1}.$$

Hence, $d = 0.9$ and $u = (4/3)\times d = 1.2$.

The two-period binomial tree for the futures price and prices of European and American options at $t = 0.5$ and $t = 1$ is given below. The calculation of the European option prices at $t = 0.5$ is given by

$$e^{-0.05\times 0.5}[30.2p^* + 1.4(1-p^*)] = 10.72841$$
$$e^{-0.05\times 0.5}[1.4p^* + 0\times(1-p^*)] = 0.455145$$

An option price in ***bold italic*** signifies that exercise is optimal at that node.



Thus, $C_{II} - C_I = e^{-0.05\times 0.5} \times (11 - 10.72841) \times p^* = 0.088.$

Remarks:

(i) $C_I = e^{-0.05\times 0.5}[10.72841p^* + 0.455145(1-p^*)] = 3.78378.$

$C_{II} = e^{-0.05\times 0.5}[11p^* + 0.455145(1-p^*)] = 3.87207.$

(ii) A futures price can be treated like a stock with $\delta = r$. With this observation, we can obtain (10.14) from (10.5),

$$p^* = \frac{e^{(r-\delta)h}-d}{u-d} = \frac{e^{(r-r)h}-d}{u-d} = \frac{1-d}{u-d}.$$

Another application is the determination of the price sensitivity of a futures option with respect to a change in the futures price. We learn from page 317 that the price sensitivity of a stock option with respect to a change in the stock price is $e^{-\delta h}\dfrac{C_u - C_d}{S(u-d)}$. Changing δ to r and S to F yields $e^{-rh}\dfrac{C_u - C_d}{F(u-d)}$, which is the same as the expression $e^{-rh}\Delta$ given in footnote 7 on page 333.

47. Answer: (B)

Let the date several months ago be 0. Let the current date be t.

Delta-hedging at time 0 means that the investor's cash position at time 0 was

$$100[C(0) - \Delta_C(0)S(0)].$$

After closing out all positions at time t, her profit is

$$100\{[C(0) - \Delta_C(0)S(0)]e^{rt} - [C(t) - \Delta_C(0)S(t)]\}.$$

To find the accumulation factor e^{rt}, we can use put-call parity:

$$C(0) - P(0) = S(0) - Ke^{-rT},$$

$$C(t) - P(t) = S(t) - Ke^{-r(T-t)},$$

where T is the option expiration date. Then,

$$e^{rt} = \frac{S(t) - C(t) + P(t)}{S(0) - C(0) + P(0)} = \frac{50 - 14.42 + 0.26}{40 - 8.88 + 1.63} = \frac{35.84}{32.75} = 1.0943511.$$

Thus, her profit is

$$100\{[C(0) - \Delta_C(0)S(0)]e^{rt} - [C(t) - \Delta_C(0)S(t)]\}$$

$$= 100\{[8.88 - 0.794 \times 40] \times 1.09435 - [14.42 - 0.794 \times 50]\}$$

$$= 24.13 \approx 24$$

Alternative Solution: Consider profit as the sum of (i) capital gain and (ii) interest:

(i) capital gain = $100\{[C(0) - C(t)] - \Delta_C(0)[S(0) - S(t)]\}$

(ii) interest = $100[C(0) - \Delta_C(0)S(0)](e^{rt} - 1)$.

Now,

$$\begin{aligned}\text{capital gain} &= 100\{[C(0) - C(t)] - \Delta_C(0)[S(0) - S(t)]\}\\ &= 100\{[8.88 - 14.42] - 0.794[40 - 50]\}\\ &= 100\{-5.54 + 7.94\} = 240.00.\end{aligned}$$

To determine the amount of interest, we first calculate her cash position at time 0:

$$\begin{aligned}100[C(0) - \Delta_C(0)S(0)] &= 100[8.88 - 40\times 0.794]\\ &= 100[8.88 - 31.76] = -2288.00.\end{aligned}$$

Hence,

$$\text{interest} = -2288\times(1.09435 - 1) = -215.87.$$

Thus, the investor's profit is 240.00 – 215.87 = 24.13 ≈ 24.

Third Solution: Use the table format in Section 13.3 of McDonald (2013).

Position	Cost at time 0	Value at time t
Short 100 calls	$-100 \times 8.88 = -888$	$-100 \times 14.42 = -1442$
100Δ shares of stock	$100 \times 0.794 \times 40 = 3176$	$100 \times 0.794 \times 50 = 3970$
Borrowing	$3176 - 888 = 2288$	$2288e^{rt} = 2503.8753$
Overall	0	24.13

Remark: The problem can still be solved if the short-rate is **deterministic** (but not necessarily constant). Then, the accumulation factor e^{rt} is replaced by $\exp[\int_0^t r(s)\mathrm{d}s]$, which can be determined using the put-call parity formulas

$$C(0) - P(0) = S(0) - K\exp[-\int_0^T r(s)\mathrm{d}s],$$

$$C(t) - P(t) = S(t) - K\exp[-\int_t^T r(s)\mathrm{d}s].$$

If interest rates are stochastic, the problem as stated cannot be solved.

48. DELETED

49. Answer: (B)

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{rh+\sigma\sqrt{h}} = e^{(0.04/4)+(0.3/2)} = e^{0.16} = 1.173511$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{rh-\sigma\sqrt{h}} = e^{(0.04/4)-(0.3/2)} = e^{-0.14} = 0.869358$$

S = initial stock price = 100

The problem is to find the smallest integer K satisfying

$$K - S > e^{-rh}[p^* \times \text{Max}(K - Su, 0) + (1 - p^*) \times \text{Max}(K - Sd, 0)]. \qquad (1)$$

Because the RHS of (1) is nonnegative (the payoff of an option is nonnegative), we have the condition

$$K - S > 0. \qquad (2)$$

As $d < 1$, it follows from condition (2) that

$$\text{Max}(K - Sd, 0) = K - Sd,$$

and inequality (1) becomes

$$K - S > e^{-rh}[p^* \times \text{Max}(K - Su, 0) + (1 - p^*) \times (K - Sd)]. \tag{3}$$

If $K \geq Su$, the right-hand side of (3) is

$$\begin{aligned} &e^{-rh}[p^* \times (K - Su) + (1 - p^*) \times (K - Sd)] \\ &= e^{-rh}K - e^{-\delta h}S \\ &= e^{-rh}K - S, \end{aligned}$$

because the stock pays no dividends. Thus, if $K \geq Su$, inequality (3) always holds, and the put option is exercised early.

We now investigate whether there is any K, $S < K < Su$, such that inequality (3) holds. If $Su > K$, then $\text{Max}(K - Su, 0) = 0$ and inequality (3) simplifies as

$$K - S > e^{-rh} \times (1 - p^*) \times (K - Sd),$$

or

$$K > \frac{1 - e^{-rh}(1 - p^*)d}{1 - e^{-rh}(1 - p^*)} S. \tag{4}$$

The fraction $\dfrac{1 - e^{-rh}(1 - p^*)d}{1 - e^{-rh}(1 - p^*)}$ can be simplified as follows, but this step is not necessary. In McDonald's forward-tree model,

$$1 - p^* = p^* \times e^{\sigma\sqrt{h}},$$

from which we obtain

$$1 - p^* = \frac{1}{1 + e^{-\sigma\sqrt{h}}}.$$

Hence,
$$\begin{aligned} \frac{1 - e^{-rh}(1 - p^*)d}{1 - e^{-rh}(1 - p^*)} &= \frac{1 + e^{-\sigma\sqrt{h}} - e^{-rh}d}{1 + e^{-\sigma\sqrt{h}} - e^{-rh}} \\ &= \frac{1 + e^{-\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}}}{1 + e^{-\sigma\sqrt{h}} - e^{-rh}} \qquad \text{because } \delta = 0 \\ &= \frac{1}{1 + e^{-\sigma\sqrt{h}} - e^{-rh}}. \end{aligned}$$

Therefore, inequality (4) becomes

$$K > \frac{1}{1+e^{-\sigma\sqrt{h}} - e^{-rh}} S$$

$$= \frac{1}{1+e^{-0.15} - e^{-0.01}} S = 1.148556 \times 100 = 114.8556.$$

Thus, the answer to the problem is $\lceil 114.8556 \rceil = 115$, which is (B).

Alternative Solution:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{rh+\sigma\sqrt{h}} = e^{(0.04/4)+(0.3/2)} = e^{0.16} = 1.173511$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{rh-\sigma\sqrt{h}} = e^{(0.04/4)-(0.3/2)} = e^{-0.14} = 0.869358$$

S = initial stock price = 100

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.3/2}} = \frac{1}{1+e^{0.15}} = \frac{1}{1+1.1618} = 0.46257.$$

Then, inequality (1) is

$$K - 100 > e^{-0.01}[0.4626 \times (K - 117.35)_+ + 0.5374 \times (K - 86.94)_+], \quad (5)$$

and we check three cases: $K \le 86.94$, $K \ge 117.35$, and $86.94 < K < 117.35$.

For $K \le 86.94$, inequality (5) cannot hold, because its LHS < 0 and its RHS = 0.

For $K \ge 117.35$, (5) always holds, because its LHS = $K - 100$ while its RHS = $e^{-0.01}K - 100$.

For $86.94 < K < 117.35$, inequality (5) becomes

$$K - 100 > e^{-0.01} \times 0.5374 \times (K - 86.94),$$

or

$$K > \frac{100 - e^{-0.01} \times 0.5374 \times 86.94}{1 - e^{-0.01} \times 0.5374} = 114.85.$$

Third Solution: Use the method of trial and error. For $K = 114, 115, \ldots$, check whether inequality (5) holds.

Remark: An American call option on a nondividend-paying stock is never exercised early. This problem shows that the corresponding statement for American puts is not true.

50. Answer: (A)

This problem is a modification of #4 in the May 2007 Exam C.

The conditions given are:

(i) $S_0 = 0.25$,

(ii) $\sigma = 0.35$,

(iii) $\alpha - \delta = 0.15$.

We are to seek the number $S_{0.5}^U$ such that $\Pr(S_{0.5} < S_{0.5}^U) = 0.95$.

The random variable $\ln(S_{0.5} / 0.25)$ is normally distributed with

$$\text{mean} = (0.15 - ½ \times 0.35^2) \times 0.5 = 0.044375,$$

$$\text{standard deviation} = 0.35 \times \sqrt{0.5} = 0.24749.$$

Because $N^{-1}(0.95) = 1.64485$, we have

$$0.044375 + 0.24749 N^{-1}(0.95) = 0.451458927.$$

Thus,

$$S_{0.5}^U = 0.25 \times e^{0.45146} = 0.39265.$$

Remark The term "confidence interval" as used in Section 18.4 McDonald (2013) seems incorrect, because S_t is a random variable, not an unknown, but constant, parameter. The expression

$$\Pr(S_t^L < S_t < S_t^U) = 1 - p$$

gives the probability that the random variable S_t is between S_t^L and S_t^U, not the "confidence" for S_t to be between S_t^L and S_t^U.

51-53. DELETED

54. Answer: (A)

At the option-exercise date, the option holder will sell two shares of Stock 1 or one share of Stock 2, depending on which trade is of lower cost. Thus, the time-1 payoff of the option is

$$\max\{17 - \min[2S_1(1), S_2(1)], 0\},$$

which is the payoff of a 17-strike put on $\min[2S_1(1), S_2(1)]$. Define

$$M(T) = \min[2S_1(T), S_2(T)].$$

Consider put-call parity with respect to $M(T)$:

$$c(K, T) - p(K, T) = F_{0,T}^{P}(M) - Ke^{-rT}.$$

Here, $K = 17$ and $T = 1$. It is given in (vi) that $c(17, 1) = 1.632$. $F_{0,1}^{P}(M)$ is the time-0 price of the security with time-1 payoff

$$M(1) = \min[2S_1(1), S_2(1)] = 2S_1(1) - \max[2S_1(1) - S_2(1), 0].$$

Since $\max[2S_1(1) - S_2(1), 0]$ is the payoff of an exchange option, its price can be obtained using (14.16) and (14.17):

$$\sigma = \sqrt{0.18^2 + 0.25^2 - 2(-0.4)(0.18)(0.25)} = 0.361801$$

$$d_1 = \frac{\ln[2S_1(0)/S_2(0)] + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} = \tfrac{1}{2}\sigma\sqrt{T} = 0.18090, \quad N(d_1) = 0.57178$$

$$d_2 = d_1 - \sigma\sqrt{T} = -\tfrac{1}{2}\sigma\sqrt{T} = -0.18090, \quad N(d_2) = 0.42822$$

Price of the exchange option = $2S_1(0)N(d_1) - S_2(0)N(d_2) = 20N(d_1) - 20N(d_2) = 2.8712$

Thus,

$$F_{0,1}^{P}(M) = 2F_{0,1}^{P}(S_1) - 2.8712 = 2\times 10 - 2.8712 = 17.1288$$

and

$$p(17, 1) = 1.632 - 17.1288 + 17e^{-0.05} = 0.6741.$$

Remarks: (i) The exchange option above is an "at-the-money" exchange option because $2S_1(0) = S_2(0)$. See also Example 14.3 of McDonald (2013).

(ii) Further discussion on exchange options can be found in Section 23.6, which is not part of the MFE syllabus. Q and S in Section 23.6 correspond to $2S_1$ and S_2 in this problem.

55. Answer: (D)

By (12.7), the price of the put option is

$$P = e^{-rT}[KN(-d_2) - FN(-d_1)],$$

where $d_1 = \dfrac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$.

With $F = K$, we have $\ln(F / K) = 0$, $d_1 = \dfrac{½\sigma^2 T}{\sigma\sqrt{T}} = ½\sigma\sqrt{T}$, $d_2 = -½\sigma\sqrt{T}$, and

$$P = Fe^{-rT}[N(½\sigma\sqrt{T}) - N(-½\sigma\sqrt{T})] = Fe^{-rT}[2N(½\sigma\sqrt{T}) - 1].$$

Putting $P = 1.6$, $r = 0.1$, $T = 0.75$, and $F = 20$, we get

$$1.625 = 20e^{-0.1\times 0.75}[2N(½\sigma\sqrt{0.75}) - 1]$$
$$N(½\sigma\sqrt{0.75}) = 0.54379$$
$$½\sigma\sqrt{0.75} = 0.10999$$
$$\sigma = 0.254011$$

After 3 months, we have $F = 17.7$ and $T = 0.5$; hence

$$d_1 = \frac{\ln(F / K) + ½\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln(17.7 / 20) + ½ \times 0.254^2 \times 0.5}{0.254\sqrt{0.5}} = -0.59040$$

$N(-d_1) = N(-0.59040) = 0.72254$

$d_2 = d_1 - \sigma\sqrt{T} = -0.59040 - 0.254\sqrt{0.5} = -0.77000$

$N(-d_2) = N(0.77000) = 0.77935$

The put price at that time is

$$P = e^{-rT}[KN(-d_2) - FN(-d_1)]$$
$$= e^{-0.1 \times 0.5}[20 \times 0.77935 - 17.7 \times 0.72254]$$
$$= 2.66158$$

Remarks:

(i) A somewhat related problem is #8 in the May 2007 MFE exam. Also see the box on page 282 and the one on page 560 of McDonald (2013).

(ii) For European call and put options on a futures contract with the same exercise date, the call price and put price are the same if and only if both are at-the-money options. The result follows from put-call parity. See the first equation in Table 9.9 on page 287 of McDonald (2013).

(iii) The point above can be generalized. It follows from the identity

$$[S_1(T) - S_2(T)]_+ + S_2(T) = [S_2(T) - S_1(T)]_+ + S_1(T)$$

that

$$F_{0,T}^P((S_1 - S_2)_+) + F_{0,T}^P(S_2) = F_{0,T}^P((S_2 - S_1)_+) + F_{0,T}^P(S_1).$$

(See also formula 9.8 on page 271.) Note that $F_{0,T}^{P}((S_1 - S_2)_+)$ and $F_{0,T}^{P}((S_2 - S_1)_+)$ are time-0 prices of exchange options. The two exchange options have the same price if and only if the two prepaid forward prices, $F_{0,T}^{P}(S_1)$ and $F_{0,T}^{P}(S_2)$, are the same.

56-76. DELETED