NOTE: The solutions presented here are not the full model solutions as published. There is no commentary and where there is more than one correct response, only one such response is presented.

RET FRC Sample CBT Question

(8 points) You are given:

**Plan Provisions:**
- Retirement Benefit: 1.50% of final year’s earnings time years of service
- Employee Contributions: 4.0% of earnings, contributed at the beginning of the year
- Normal Form of Payment: Life only, payable monthly in advance
- Termination Benefit: Refund of employee contributions credited with interest at 2.0% per year
- Normal Retirement Age: Age 65
- Vesting: Immediate

**Actuarial Assumptions and Methods:**
- Discount Rate: 5.0% per year
- Salary increase rate: 2.5% per year
- Retirement age: Age 65
- Termination Decrement: 3% per year prior to age 60
- Timing of Decrements: End of year
- Actuarial Cost Method: Aggregate, level percent of pay
- Asset Valuation Method: Market value of assets

**Annuity factor:**

\[ a_{65}^{(12)} = 13.4 \]

**Financial Information**
Market value of assets as at January 1, 2019: $275,000

**Membership Data at January 1, 2019:**

<table>
<thead>
<tr>
<th></th>
<th>Member A</th>
<th>Member B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age in Years</td>
<td>60</td>
<td>56</td>
</tr>
<tr>
<td>Years of Service</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>2019 earnings</td>
<td>$70,000</td>
<td>$64,000</td>
</tr>
<tr>
<td>Member contributions with interest</td>
<td>$55,000</td>
<td>$41,000</td>
</tr>
</tbody>
</table>
(a)  
(6 points) Calculate the employer normal cost and the actuarial liability as at January 1, 2019

Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(b)  
(2 points) Compare and contrast the pattern of development over time of the normal cost under the Aggregate and Projected Unit Credit funding methods. No calculations are required.

ANSWER:
RET DAC/U Sample CBT Question

(10 points)

(a) (6 points) Compare and contrast the Traditional and Spot Rate Approaches to calculating the following U.S. Accounting Standard ASC 715 disclosures:

(i) Net Periodic Pension Cost
(ii) Accumulated Other Comprehensive Income
(iii) Projected Benefit Obligation

ANSWER:

You are given the following:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10,000</td>
<td>$10,000</td>
<td>$8,000</td>
<td>$4,000</td>
<td>$2,000</td>
</tr>
<tr>
<td>Expected Benefit Payments (end of year)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spot Interest Rate</td>
<td>2.1% per annum</td>
<td>2.0% per annum</td>
<td>1.9% per annum</td>
<td>1.8% per annum</td>
<td>1.7% per annum</td>
</tr>
</tbody>
</table>

The Traditional Approach interest rate is 1.91% per annum.

(b) (3 points) Calculate the annual Interest Cost at the beginning of year 1 using the following approaches:

(i) Traditional Approach

(ii) Spot Rate Approach

Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(c) (1 point) Critique adopting the Spot Rate Approach in an inverted yield curve environment.

No calculations required.

ANSWER:
(8 points) XZY Company sponsors a defined benefit pension plan and is reviewing its liability-driven investment strategy.

You are given:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Effective Duration</th>
<th>Key Rate Durations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Short-term D1</td>
</tr>
<tr>
<td>Total plan liabilities</td>
<td>$45,000,000</td>
<td>15</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current Portfolio</th>
<th>Market Value</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Short-term D1</td>
</tr>
<tr>
<td>Short-term bond fund</td>
<td>$0</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Medium-term bond fund</td>
<td>$37,500,000</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>Long-term bond fund</td>
<td>$0</td>
<td>25</td>
<td>0</td>
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(a) (2 points) Describe four uses of key rate durations.

ANSWER:
(b) (2 points) Calculate the effect of the following yield curve shifts on the plan’s financial position:

<table>
<thead>
<tr>
<th>Shift</th>
<th>Short-term $D_1$</th>
<th>Medium-term $D_2$</th>
<th>Long-term $D_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift 1</td>
<td>+50 bps</td>
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<td>+50 bps</td>
</tr>
<tr>
<td>Shift 2</td>
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<td>-30 bps</td>
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*The response for this part is to be provided in the Excel spreadsheet.*

(c) (6 points) Recommend changes to the current portfolio that will minimize the impact of yield curve changes on the plan’s financial position.

Justify your recommendation.

**ANSWER:**
NOTE: The solutions presented here are not the full model solutions as published. There is no commentary and where there is more than one correct response, only one such response is presented.

RET FRC Sample CBT Solution

(8 points) You are given:

**Plan Provisions:**
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(a) **(6 points)** Calculate the employer normal cost and the actuarial liability as at January 1, 2019

Show all work.

*The response for this part is to be provided in the Excel spreadsheet.*

(b) **(2 points)** Compare and contrast the pattern of development over time of the normal cost under the Aggregate and Projected Unit Credit funding methods. No calculations are required.

**ANSWER:**

**Aggregate Method:**

\[ \text{Al} = \text{MVA} \text{ since } \text{UAL} = 0. \text{ NC} = (\text{PVFB} - F - \text{PVEEC})/\text{PVFS} \times S \]

- There is no accrued liability calculation as UAL always is zero. Accrued liability is equal to plan assets.
- All deviations from experience, and even liabilities attributable prior to the plan inception, are funded over future normal costs.
- Neither AL nor NC have a definite pattern. It is highly dependent on the value of assets.

**Projected Unit Credit Method:**

\[ \text{AL} = \text{Benefit} \times \text{survival prob} \times \text{discount factor} \times \text{annuity factor}. \text{ NC} = \text{Change in benefit} \times \text{survival prob} \times \text{discount factor} \times \text{annuity factor} \]

- Cost is related to benefit accruals. NC increases with age of the member.
- Accrued benefit is calculated by projecting the retirement benefit using salary scales and then redistributing the benefit over an employee’s career to produce a hypothetical pattern of accruals that may not relate to the actual accrual pattern described by the plan.
- The rising pattern of normal costs over a career means that the fixed contribution rate cannot cover the member’s normal cost.
(10 points)

(a) (6 points) Compare and contrast the Traditional and Spot Rate Approaches to calculating the following U.S. Accounting Standard ASC 715 disclosures:

(i) Net Periodic Pension Cost

(ii) Accumulated Other Comprehensive Income

(iii) Projected Benefit Obligation

ANSWER:

Net Periodic Pension Cost

Service Cost and Interest Cost are components of the Pension Cost. Under the Traditional Approach:

- Service Cost is calculated by discounting future cash flows associated with the benefit earned, using the single weighted average discount rate.
- Interest Cost is calculated by multiplying the beginning of period benefit obligation by the single weighted average discount rate and making an adjustment for benefit payments during the period.

Under the Spot Rate Approach:

- Service Cost is determined by discounting the expected future cash flows associated with the benefit earned, using the individual spot rates for the respective period in which the future cash flow is expected to occur.
- Interest cost is determined by multiplying the present value of each future cash flow by its respective spot rate for the period.

The Service Cost calculated under Traditional Approach will be greater than the Service Cost calculated using the Spot Rate Approach when the yield curve is upward sloping.

The inverse is also true, Service Cost calculated under Traditional Approach will be less than the Service Cost calculated using the Spot Rate Approach when the yield curve is downward sloping.

Depending on the shape of the yield curve, Service Cost (and Interest Cost) may be higher or lower using Traditional Approach than the result calculated under the Spot Rate Approach.
Accumulated Other Comprehensive Income
Actuarial gains and losses are reported in the AOCI.

The benefit obligation at the beginning and end of the period is the same under both approaches, so the actuarial gains and losses will have an offsetting effect to changes in the service and interest costs.

Under the Traditional Approach the gains and losses will be smaller and service and interest costs greater when the yield curve is upward sloping.

Projected Benefits Obligation
Future cash flows are discounted back to the measurement date using the spot rate associated with the respective period in which the future cash flow is expected to occur [Spot Rate Method] or the single weighted average discount rate [Traditional Method].

Calculation of PBO at the beginning and end of the year are the same under both approaches.

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The Traditional Approach interest rate is 1.91% per annum.

(b) (3 points) Calculate the annual Interest Cost at the beginning of year 1 using the following approaches:

(i) Traditional Approach

(ii) Spot Rate Approach

Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(c) (1 point) Critique adopting the Spot Rate Approach in an inverted yield curve environment.
No calculations required.

**ANSWER:**

The Service Cost and Interest Cost for a typical pension arrangement will be greater when calculated using the Spot Rate Approach in an inverted yield curve environment.

If the goal is to reduce the Service Cost and Interest Cost and increase reported gains and losses, then the Spot Rate Approach should not be adopted in a downward sloping yield curve environment.
(8 points) XZY Company sponsors a defined benefit pension plan and is reviewing its liability-driven investment strategy.

You are given:

| Value       | Effective Duration | Key Rate Durations
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</table>

(a) (2 points) Describe four uses of key rate durations.

**ANSWER:**
1. To identify the price sensitivity of a bond to each segment of the yield curve
2. To create a replicating portfolio of a bond with embedded options using zero-coupon bonds
3. To provide valuable insights into option-embedded bond behavior
4. To provide a procedure to control interest rate risk exposure
(b) (2 points) Calculate the effect of the following yield curve shifts on the plan’s financial position:

<table>
<thead>
<tr>
<th></th>
<th>Short-term D₁</th>
<th>Medium-term D₂</th>
<th>Long-term D₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift 1</td>
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<td>+50 bps</td>
<td>+50 bps</td>
</tr>
<tr>
<td>Shift 2</td>
<td>-30 bps</td>
<td>+10 bps</td>
<td>-30 bps</td>
</tr>
</tbody>
</table>

*The response for this part is to be provided in the Excel spreadsheet.*

(c) (6 points) Recommend changes to the current portfolio that will minimize the impact of yield curve changes on the plan’s financial position.

Justify your recommendation.

**ANSWER:**

Start with the medium-term fund since it is impacted by all three shifts. The amount needed is $45M \times 8 / 15 = 24M$

The medium-term fund has exposure to short-term yields ($24M$ with duration $1$)  
Amount needed for short term fund is $(45M \times 2 - 24M \times 1) / 6 = 11M$

The medium-term fund also has exposure to long term yields of $24M$ with duration $2$.

The amount needed in the long-term fund is $(45M \times 5 - 24M \times 2) / 25 = 7.08M$

The amount needed in equities is $47.5M - 11M - 24M - 7.08M = 5.42M$