GIRR Model Solutions
Spring 2025

1. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3c) Identify the types of development triangles that can be used for investigative testing.

(3d) Analyze development triangles for investigative testing.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 14.

Commentary on Question:

This question tests investigative analysis of various development triangles.

Solution:

(a) State two purposes for conducting investigative testing with development triangles.

Any two of the following are acceptable (others are possible):

- to review the reasonableness of management's assertions regarding company operations
- to determine if the qualitative information gathered is consistent with patterns observed in the quantitative data.
- help identify the need for additional data and information.
- provides an excellent means for the documentation required about the data and information gathering phase of actuarial work

(b) Describe the pattern (row or column) you would expect to observe in each of these triangles if the line of business is in a stable environment.

For ratio of paid to reported claims, expect ratios to be consistent (i.e., the same value) down each column.

For ratio of closed to reported counts, expect ratios to be consistent (i.e., the same value) down each column.

1. Continued

(c) Evaluate each triangle to determine if this line of business likely does or does not represent a stable line of business.

Accident	Change in Ratios of Paid to Reported Claims			
Year	12	24	36	48
2020-2021	0.8%	0.7%	–0.5%	–1.1%
2021-2022	2.4%	–0.7%	–2.8%	
2022-2023	–2.9%	–7.1%		
2023-2024	–7.4%			

Conclusion: A possible change in the most recent 1 (or 2) diagonals, therefore this diagnostic test likely does indicate an unstable line of business.

Accident	Change in Ratios of Closed to Reported Counts			
Year	12	24	36	48
2020-2021	1.7%	0.8%	–0.4%	–0.1%
2021-2022	–2.5%	–1.0%	0.1%	
2022-2023	1.9%	0.8%		
2023-2024	–1.0%			

Conclusion: There is no clear indication of anything unstable, so this diagnostic likely suggests a stable line of business.

(d) State what a triangle of ratios of closed counts with no payments to total closed counts would help identify.

This triangle can help give insight into potential changes in settlement processes.

(e) Describe a data adjustment needed to ensure this test would provide consistency between the payments in the numerator and the counts in the denominator.

Commentary on Question:
Many candidates misunderstood this question and incorrectly answered that counts with no payments should be removed from the denominator. The inconsistency in these ratios comes from the numerator including payments on open and closed claims and the denominator only includes counts of claims that have closed.

The partial payments would need to be removed from the numerator.

2. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(2a) Create development triangles of claims and counts from detailed claim transaction data.

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 11 and 15.

Commentary on Question:

This question tests the candidate's understanding of calendar year paid and case estimates, as well as calculating ultimate claims using the development method. This question also tests the algebraic method for determining a tail factor.

Solution:

(a) Calculate the following:

i) Total claims paid in calendar year (CY) 2024

= (2,078,097 + 4,117,247 + 5,583,814 + 7,217,780 + 8,106,189 + 8,885,445 + 9,269,896) – (1,851,044 + 3,648,224 + 5,345,638 + 6,755,984 + 7,837,333 + 8,616,782) = 11,203,463

ii) Total change in case estimates in CY 2024

CY 2024 incremental reported claims
= (4,585,964 + 6,281,172 + 7,498,246 + 8,653,058 + 9,086,519 + 9,423,176 + 9,391,874) – (4,185,696 + 5,725,217 + 7,115,668 + 8,119,190 + 8,746,908 + 9,097,359) = 11,929,971

Total change in case estimates in CY 2024
= 11,929,971 – 11,203,463 = 726,508

2. Continued

(b) Describe one reason why the change in case estimates is expected to be negative for each accident year prior to 2024.

As claim files close, case estimates are released, which decreases the outstanding amounts creating a negative change.

(c) Describe one reason why a change in case estimates could be positive for any accident year prior to 2024.

Open claim files could increase their case estimates for an amount that is greater than any release of case estimates from closing claim files or from payments made on open files.

(d) Calculate projected ultimate claims using cumulative paid claims. Justify ***all*** selections and use the algebraic method for the tail factor.

	Age-to-age development factors					
AY	12-24	24-36	36-48	48-60	60-72	72-84
2018	2.017	1.592	1.332	1.238	1.179	1.076
2019	2.198	1.494	1.369	1.213	1.134	
2020	2.069	1.575	1.324	1.200		
2021	2.140	1.541	1.350			
2022	1.813	1.531				
2023	2.224					
2024						
Simple 3	2.059	1.549	1.348	1.217	1.156	1.076
Simple All	2.077	1.547	1.344	1.217	1.156	1.076
Vol Wtd 5	2.078	1.545	1.344	1.216	1.156	1.076
Vol Wtd All	2.069	1.545	1.344	1.216	1.156	1.076
Medial All	2.106	1.549				
Selected	2.106	1.549	1.344	1.217	1.156	1.076
Justification:	For 12-24 and 24-36 months, use medial average to remove the effect of the outlier. For all other development periods, use simple all years average due to fewer observations.					
Age-to-84:	6.636	3.151	2.034	1.514	1.244	1.076

2. Continued

Algebraic Method for Paid Claims Tail Factor:

AY	Ultimate Rep. Claims	Paid Claims	Paid Dev. Fac. to 84	Paid Claims Dev. to 84	Implied Tail Factor
2018	9,695,924	9,269,896	1.000	9,269,896	1.046
2019	10,043,178	8,885,445	1.076	9,558,922	1.051
2020	10,613,552	8,106,189	1.244	10,084,994	1.052
Average					1.050
Selected					1.050

Justification: Not much volatility, so average of all 3 years is reasonable.

	12-ult	24-ult	36-ult	48-ult	60-ult	72-ult	84-ult
Age-to-ult:	6.966	3.307	2.136	1.589	1.306	1.129	1.050

Accident Year	Paid Claims	Age-to-ult. Factor	Ultimate Claims from Paid Claims
2018	9,269,896	1.050	9,730,389
2019	8,885,445	1.129	10,033,774
2020	8,106,189	1.306	10,585,978
2021	7,217,780	1.589	11,470,175
2022	5,583,814	2.136	11,924,313
2023	4,117,247	3.307	13,617,700
2024	2,078,097	6.966	14,475,614
Total	45,258,468		81,837,943

3. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

Commentary on Question:

This question tests the candidate's understanding of expenses used in ratemaking.

Solution:

Critique your co-worker's recommendation.

	General Expenses			
Calendar Year	Variable	As a % of Earned Premiums	Commission and Premium Tax Expense Ratio	Total Variable Expense Ratio
2021	170,394	2.86%	11.00%	13.86%
2022	177,146	3.02%	11.00%	14.02%
2023	182,448	3.20%	11.00%	14.20%
2024	187,905	3.14%	11.00%	14.14%
2025 Budget	199,500	3.47%	10.00%	13.47%

Co-worker recommendation: 14.06%

- Commission ratios are generally based on budgets because of the prospective nature of ratemaking.
- All years average for variable general expenses likely understates what is expected in the future (older years much lower and budget ratio is higher).
- Exposures much lower for 2025 budget, so should give more consideration for those ratios as there could be a change in the line of business that the past might not capture.

Therefore, the co-worker's recommendation is not preferred.

4. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

(4f) Calculate claim liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 15, 17, 18, and 24.

Commentary on Question:

This question tests the estimation of ultimate allocated loss adjustment expenses (ALAE) using the expected method and the Bornhuetter Ferguson method. This question also tests the estimation of IBNR for ALAE.

Solution:

(a) State two situations where ultimate indemnity and ultimate ALAE should be estimated separately.

Any two of the following are acceptable:

- The reporting and payment patterns are changing for indemnity and ALAE
- Inconsistent (or changes) in the volume relationship to one another over time
- Changes in the insurer's practices for setting case estimates (for indemnity or ALAE)
- ALAE is a significant portion of the claim, e.g., ALAE can be larger than indemnity in legal liability claims

4. Continued

(b) Calculate the projected AY 2024 ultimate ALAE using the expected method.

Accident Year (AY)	(1) Earned Exposures	(2) Projected Ultimate ALAE Based on Reported Development Method	(3) = (2) / (1) Pure Premium	(4) Trend Factors	(5) = (3)(4) Trended Pure Premium
2020	8,433	1,086,216	128.81	1.2388	159.57
2021	8,637	1,123,621	130.09	1.1742	152.76
2022	8,570	1,213,024	141.54	1.1130	157.54
2023	8,728	1,281,322	146.81	1.0550	154.88
2024	8,808	1,380,962	156.78	1.0000	156.78
Average (2020-2023)					156.19
2024 ultimate ALAE = 8,808 × 156.19 =					1,375,700

(c) Calculate the projected AY 2024 ratio of ultimate ALAE to ultimate claims using the Bornhuetter Ferguson method and your results from part (b).

AY 2024 expected ultimate ALAE to ultimate claim ratio = 1,375,700 / 13,809,620 =	0.100
IBNR Factor = 1 – 1/1.020 =	0.020
AY 2024 Bornhuetter Ferguson ultimate ALAE ratio = 0.095 + 0.100×0.020 =	0.09695

(d) Evaluate the reasonableness of the inputs for the Bornhuetter Ferguson method in part (c) by comparing the actual reported ALAE ratio to the expected ALAE ratio.

Actual reported ALAE ratio =	0.095
Expected ultimate ALAE ratio =	0.100
Expected percent reported = 1 / 1.020 =	0.980
Expected reported ALAE ratio = 0.100 × 0.980 =	0.098
Actual minus expected difference = 0.095 – 0.098 =	–0.003
Difference as a percent of expected ultimate ALAE ratio = –0.003 / 0.100 =	–2.7%

4. Continued

(e) Calculate the AY 2024 IBNR for ALAE using your results from part (c).

AY 2024 expected ultimate claims =	13,809,620
AY 2024 ult ALAE ratio from BF method =	0.0970
AY 2024 Ultimate ALAE = 13,809,620×0.0970 =	1,338,888
AY 2024 Actual Reported ALAE = 3,318,135×0.095 =	315,223
AY 2024 ALAE IBNR = 1,338,888 – 315,223 =	1,023,665

5. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

Learning Outcomes:

(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

(2d) Adjust historical earned premiums to current rate levels.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 12 and 13.

Commentary on Question:

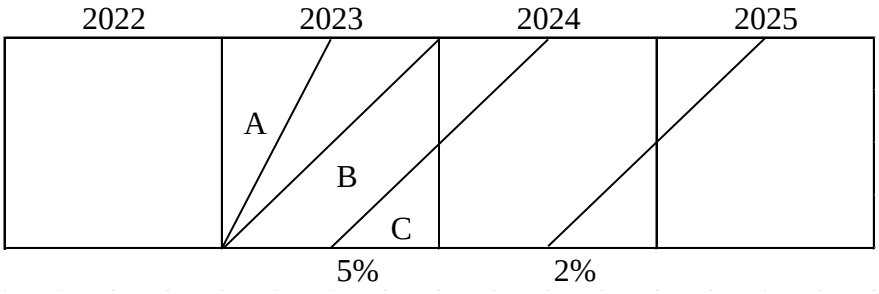
This question tests the candidate's understanding of earned premiums and adjusting earned premiums to current rate levels for ratemaking purposes.

Solution:

(a) Calculate the CY 2023 written premiums for the policies that renewed in 2023.

Commentary on Question:

Candidates are not required to draw the diagram to answer this question, but the correct diagram is helpful in getting this question correct.



Policies written in area B in 2023 (these were policies expiring between Jan 1/23 and June 30/23 as these policies were all 6-month policy terms):

Percent of policies that renew:	90%
Annual premium for all policies that renew:	1,200
Written premiums = 90% × 4,200 × 1,200 =	4,536,000

Note: The premium increase takes effect on July 1, and the last policy to expire in 2023 renews on June 30, 2023.

5. Continued

(b) Calculate the CY 2023 earned premiums for the policies that renewed in 2023.

Policies earned in area B in 2023:

Written premiums (from part (a)):	4,536,000
Total area of parallelogram B in 2023 & 2024:	50%
Part of parallelogram B in 2023 only:	37.5%
% of B that is in 2023 only = 37.5% / 50% =	75%
Earned premium: 4,536,000 × 75% =	3,402,000

Note: The renewals for these policies occurred from January 1, 2023 through June 30, 2023, so the premium increase on July 1did not affect these policies.

(c) Calculate the CY 2023 earned premiums for the *new* policies written in CY 2023.

Policies written from January 1, 2023 through June 30, 2023:	
Written premiums = 1,560 × 1,200 × 0.5 =	936,000
Precent earned in 2023	75%
Earned premiums: 936,000 × 75% =	702,000
Policies written from July 1, 2023 through December 31, 2023:	
Written premiums = 1,560 × 1,200 × 0.5 × (1 + 0.05) =	982,800
Percent earned in 2023 = (0.5 × 0.5 × 0.5) / 0.5 =	25%
Earned premiums: 982,800 × 25% =	245,700
Total earned premiums:	947,700

5. Continued

(d) Calculate the total unearned premiums as of December 31, 2023.

Unearned premiums from renewal policies:	
Written premiums	4,536,000
% unearned as of Dec. 31, 2023	25%
Unearned premiums as of Dec. 31, 2023	1,134,000
Unearned premiums from new policies written in 2023:	
Written premiums before the rate change	936,000
% unearned as of Dec. 31, 2023	25%
Unearned premiums as of Dec. 31, 2023	234,000
Written premiums after the rate change	982,800
% unearned as of Dec. 31, 2023	75%
Unearned premiums as of Dec. 31, 2023	737,100
Total unearned premiums as of Dec. 31, 2023	2,105,100

5. Continued

(e) Calculate the CY 2023 earned premiums at current rate levels.

Commentary on Question:
Some candidates misunderstood that the parallelogram approach is an approximation, and it assumes that policies are written and earned evenly throughout the year. CY 2023 had both 6-month and 12-month policies earned as well as uneven written policies and therefore did not have policies written and earned evenly throughout the calendar year. As a result the parallelogram approach is not an accurate approximation in this case. However, an approximation is not needed in this case as the CY 2023 earned premiums can be directly calculated for each earned premium component by multiplying by the subsequent rate changes.

Policies earned in 2023 from the policies in force and renewed in 2023:
Need both rate changes to adjust to current levels.
$(1{,}260{,}000 + 3{,}402{,}000) \times (1 + 0.05) \times (1 + 0.02) =$ 4,993,002
{need to include the UEP @ Dec. 31, 2022 in the 2023 EP}

Newly written policies in 2023 written before July 1, 2023:
Need both rate changes to adjust to current levels.
$702{,}000 \times (1 + 0.05) \times (1 + 0.02) =$ 751,842

Newly written policies in 2023 written on or after July 1, 2023:
Need just the 2024 the rate change to adjust to current levels.
$245{,}700 \times (1 + 0.02) =$ 250,614

Total 2023 earned premiums at current rate levels: 5,995,458

6. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3h) Explain the effect of changing conditions on the projection methods cited in (3e).
(3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 21.

Commentary on Question:

This question tests the candidate's understanding of ultimate claims when conditions are changing.

Solution:

(a) State which data should be chosen when the rate of growth of earned exposures changes markedly within the year.

Substitute accident quarter for accident year data.

(b) Describe the data distortion that could occur when the rate of growth of earned exposures changes markedly within the year.

This situation could cause a distortion in development factors due to significant shifts in the average accident date within each exposure period.

(c) Describe two potential scenarios, *that do not involve actions of the insurer,* which could cause the changing condition of a shift in policy limits.

1. Demands from policyholders that are related to economic and judicial considerations can lead to changes in policy limits.
2. Change in regulation that affects policy limits (e.g., auto no fault limits).

(d) Explain how the estimated ultimate claims from the development method based on reported claims will be affected by the case adequacy change.

- Because the average of all years is used in development factor selections, the development factors will be affected very little with the decrease.
- However, since these factors will be applied to the latest diagonal, which is much lower due to the case change, the ultimate values will be understated.

6. Continued

(e) Describe why the frequency-severity method could be a reasonable approach for this line of business.

The frequency-severity method is often used if there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future.

(f) Describe how the expected method could be affected by the case adequacy change in this situation.

If the claim ratio (or pure premium) selection is based on reported claims, then the expected claims could be affected (i.e., should use claim ratio or pure premiums based on paid claims only).

7. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.

(4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

(4f) Calculate claim liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 23 and 24.

Commentary on Question:

This question tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method.

Solution:

(a) Estimate unpaid ULAE as of December 31, 2024 using the classical paid-to-paid method.

Commentary on Question:

Candidates need to adjust for the one-time cost in CY2022. This can be done by either removing from paid ULAE in the ratios or ignore that year's ratio in selecting the ratio to use. There are various answers that are reasonable for the selected ratio, but candidates should give consideration to the increasing trend.

Calendar Year (CY)	Paid ULAE	Paid Claims	Ratio
2021	538,680	5,670,300	0.095
2022	*527,220*	5,669,000	0.093
2023	622,000	6,282,800	0.099
2024	732,130	7,108,100	0.103
Total	2,420,030	24,730,200	0.098
Selected ratio:			0.101

Rationale for selected ratio: Increasing ratios, so the average of the most recent two years has been used.

Note: Paid ULAE for CY2022: 527,220 = 579,220 – 52,000

7. Continued

Case = 36,861,900 – 34,514,400 =	2,347,500
Total IBNR = 42,514,600 – 36,861,900 =	5,652,700
IBNYR	1,243,600
IBNER = 5,652,700 – 1,243,600 =	4,409,100

Component	Value as of Dec. 31, 2024	Multiplier	Ratio	Unpaid ULAE
Case	2,347,500	60%	0.101	142,258
IBNYR	1,243,600	100%	0.101	125,604
IBNER	4,409,100	60%	0.101	267,191
Total				535,053

(b) Identify the weakness in the classical paid-to-paid method according to Kittel.

The classical paid-to-paid method overestimates the unpaid ULAE for a growing company in an inflationary environment.

(c) Explain why the weakness identified in part (b) occurs.

The weakness in part (b) occurs because the numerator of the paid-to-paid ratio is more reactive to the increasing exposure than the denominator.

(d) State two circumstances where the Mango and Allen smoothing adjustment is particularly valuable.

Any two of the following are acceptable:

- Long-Tail lines of business
- Changing exposure volume
- When large claims result in significant distortions to the CY paid and reported claims from year to year
- Sparse or volatile data
- A relatively new insurer who does not have a significant volume of credible paid or reported claims

8. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.
(5e) Calculate trend factors for claims and exposures.
(6f) Explain the requirements for loadings for catastrophes and large claims in ratemaking.
(6g) Calculate loadings for catastrophes and large claims.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 27 and 31.

Commentary on Question:

This question tests the candidate's understanding of catastrophe models used in ratemaking.

Solution:

(a) State one way that noninsurance data can be used in catastrophe models.

To estimate the overall frequency of these events, as well as the frequency of the key defining characteristics of these events.

(b) State one reason why catastrophe claims might trend at a rate materially different than non-catastrophe claims.

There could be post-even inflation or demand surge that causes the catastrophe trend to be higher.

8. Continued

(c) Calculate the pure premium for the earthquake endorsement effective September 1, 2025.

Midpoint of future rating period:	Sep. 1, 2026
1. Trend the modeled catastrophe claims from the date of in-force exposures to the midpoint of the future rating period:	
Exposure trend period (months): Nov. 1, 2024 to Mar. 1, 2025	4
Exposure trend = $(1.020^{(4/12)})$ =	1.00662
Severity trend period (months): Mar. 1, 2025 to Sep. 1, 2026	18
Severity trend = $(1.10^{(18/12)})$ =	1.15469
Trended modeled catastrophe claims = 2,112,000×1.00662×1.15469 =	2,452,730
2. Trend exposures from the date of in-force exposures to the midpoint of the future rating period:	
Exposure trend period (months): November 1, 2024 to September 1, 2026	22
Trended exposures = $19{,}700\times(1 + 0.020)^{(22/12)}$ =	20,428.35
3. Pure premium = 2,452,729.56 / 20,428.35 =	120.07

9. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(3g) Estimate ultimate values using the methods cited in (3e).
(4f) Calculate claim liabilities.
(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16, 24, and 26.

Commentary on Question:

This question tests the candidate's understanding of the development method and the frequency-severity claim closure method for estimating unpaid claims. In addition, this question tests the candidate's understanding of calculating claim trend.

Solution:

(a) Describe two alternative sources for trend, other than industry data, if an insurer's own claim experience in LOB A for state X is not sufficiently credible.

Commentary on Question:

Other alternatives are possible.

- Combine with regional or countrywide experience, but review differences in regulatory/legal environment such as statutes of limitations, caps on damages etc.
- Other affiliated insurers, so long as there are similar policies with respect to underwriting, claim management and reinsurance.

9. Continued

(b) Recommend a severity trend based on the industry data. Justify your recommendation.

Accident Year	Industry Projected Ultimate Severity	Year-to-year change
2019	5,030	
2020	5,467	8.69%
2021	5,718	4.59%
2022	6,098	6.65%
2023	6,620	8.56%
2024	6,789	2.55%
Average (all years)		6.21%
Fitted:		6.30%
Selected:		6.21%

Justification: Due to volatility, select average of all years.

(c) Assess whether your company's severity trend for LOB A is expected to be lower, equal, or higher than the industry severity trend if the company's policy limit for LOB A is 300,000.

- Lower limits would have a lower severity trend
- This is because some claims between 300,000 and 500,000 would get no increase for the company book of business while they would get higher than 0% trend with industry data.

(d) Calculate the incremental severity at the 2024 cost level for all development ages 12 through 72 using a simple all year's average and your recommended severity trend from part (b).

Accident	Industry Incremental Paid Severity at 2024 Cost Level					
Year	12	24	36	48	60	72
2019	3,597	6,527	7,937	8,488	8,615	9,720
2020	3,686	6,297	8,445	8,867	8,969	
2021	3,860	6,123*	8,381	8,393		
2022	3,850	6,619	7,656			
2023	3,711	6,571				
2024	3,653					
Average:	3,726	6,428	8,105	8,583	8,792	9,720

* e.g., AY2021 @ 24 months: $6{,}123 = 5{,}111 \times 1.0621^2$

9. Continued

(e) Calculate the company's total unpaid claims estimate as of December 31, 2024 for LOB A using the frequency-severity closure method.

Need to first estimate ultimate counts for LOB A using industry age-to-ultimate factors:

AY	Closed Counts to Date	Age-to-Ult Factor	Ultimate Counts
2022	162	1.499	243
2023	275	2.105	579
2024	203	4.004	813

LOB A Incremental Closed Counts

AY	12	24	36	48	60	72	84	Ultimate
2022	*60*	*54*	*48*	50	23	5	3	243
2023	*145*	*130*	113[(1)]	118	54	13	7	579
2024	*203*	185	157	164	76	18	9	813

Incremental Paid Severity

AY	12	24	36	48	60	72	84
2022				7,609	7,794	8,617	8,617
2023			7,631[(2)]	8,081	8,278	9,152	9,152
2024		6,428	8,105	8,583	8,792	9,720	9,720

Incremental Paid Claims

AY	12	24	36	48	60	72	84	Total Unpaid Claims
2022				378,895	178,560	45,695	24,390	627,540
2023			860,298[(3)]	951,491	448,404	114,751	61,248	2,436,192
2024		1,191,550	1,276,188	1,411,466	665,173	170,225	90,856	4,805,458
								7,869,190

Notes:

(1) 113 = 0.371×(579 – 145 – 130)
(2) 7,631 = 8,105 / 1.0621
(3) 860,298 = 113×7,631

10. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3f) Demonstrate knowledge of good practice related to projecting ultimate values.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 20.

Commentary on Question:

This question tests the candidate's understanding of Berquist-Sherman adjustments to data triangles.

Solution:

(a) Describe your general approach to estimating ultimate claims based on the frequency-severity method applied to reported claims. (You do not need to outline specific steps).

Adjust the reported claims triangle for change in case adequacy and use as input for the frequency-severity method.

(b) Describe the steps to follow when adjusting reported claims for both large claims and the change in the adequacy of case estimates.

- Exclude the large claims from the average case estimates triangle and paid claims triangle
- Adjusted for the change in case adequacy
- Calculate adjusted reported claims triangle
- Add back in the reported value of the large claims

(c) Provide two approaches for selecting a tail factor in such a situation.

Any two of the following are acceptable:

- assume a reduction in the tail factor
- use an external source of information
- professional judgement
- review the relationship of paid to reported claims for the most mature years in the insurer's experience period

11. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(1g) Identify different types of data used for actuarial work.
(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 4 and 19.

Commentary on Question:

This question tests the estimating of ultimate claims using the Cape Cod method.

Solution:

(a) State two desirable characteristics of an exposure base.

- The expected claims should be directly proportional to the exposure base.
- The exposure base should be easy to measure.

(b) Calculate projected ultimate claims for all accident years using the Cape Cod method with a pure premium based on an experience period of 2018 through 2024.

Commentary on Question:

Inflation sensitive exposures work the same way as the claim ratio that adjusts premiums to on-level.

Pure Premium Trend: $(1 + 0.063)(1 – 0.006) – 1 = 5.662\%$

11. Continued

Accident Year	(1) Earned Exposures	(2) Reported Claims as of Dec. 31, 2024	(3) Cumulative Development Factors	(4) = 1 / (3) Expected % Developed	(5) Exposure Trend	(6) = (4)(5) Used-Up Earned Exposures
2018	24,036	9,304,916	1.055	94.79%	1.0870	24,765
2019	24,429	9,343,805	1.114	89.77%	1.0720	23,508
2020	24,934	9,119,571	1.224	81.70%	1.0572	21,536
2021	25,042	8,456,780	1.421	70.37%	1.0426	18,373
2022	25,370	7,273,955	1.752	57.08%	1.0282	14,889
2023	25,914	5,693,605	2.357	42.43%	1.0140	11,148
2024	26,312	3,976,374	3.661	27.31%	1.0000	7,187
Total						121,406

Accident Year	(7) Pure Premium Trend	(8) = (2)(7) Adjusted Claims	(9) = (A)(1)(5)/(7) Expected Claims	(10) = (2) + (9)[1 – 1/(3)] Projected Ultimate Claims
2018	1.3916	12,948,825	10,007,224	9,826,620
2019	1.3170	12,306,145	10,598,363	10,428,377
2020	1.2465	11,367,187	11,272,150	11,182,448
2021	1.1797	9,976,173	11,796,835	11,951,831
2022	1.1165	8,121,007	12,453,708	12,619,382
2023	1.0566	6,015,988	13,255,446	13,325,188
2024	1.0000	3,976,374	14,024,760	14,170,279
Total		64,711,699		83,504,125

Adjusted expected pure premium:	533.02 (A)

Note: (A): 533.02 = 64,711,699 / 121,406

11. Continued

(c) Calculate projected ultimate claims for all accident years using the Cape Cod method with a pure premium based on an experience period of 2021 through 2024.

Accident Year	(6) Used-Up Earned Exposures	(8) Adjusted Claims	(11) = (B)(1)(5)/(7) Expected Claims	(12) = (2) + (11)[1 – 1/(3)] Projected Ultimate Claims
2018			10,220,820	9,837,755
2019			10,824,577	10,451,527
2020			11,512,745	11,226,479
2021	18,373	9,976,173	12,048,630	12,026,430
2022	14,889	8,121,007	12,719,522	12,733,476
2023	11,148	6,015,988	13,538,373	13,488,078
2024	7,187	3,976,374	14,324,108	14,387,860
Total	51,598	28,089,543		84,151,604
Adjusted expected pure premium:		544.39 (B)		

Note: (B): 544.39 = 28,089,543 / 51,598

12. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(2d) Adjust historical earned premiums to current rate levels.
(5e) Calculate trend factors for claims and exposures.
(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13, 26, 27, and 32.

Commentary on Question:

This question tests the candidate's understanding of basic ratemaking, including making claim and premium trend adjustments.

Solution:

(a) Calculate the forecasted profit for CY 2027 for LOB A.

		Trend		2027
(1)	Average annual premium	1%	$1{,}600.00\times(1.01)^3 =$	1,648.48
(2)	Forecasted earned exposures			9,180
(3)	Projected total premium: (1) × (2)			15,133,061
(4)	Pure premium	4.94%	$1{,}104.00\times(1.0494)^3 =$	1,275.83
(5)	Projected total claims & ALAE = (2) × (4)			11,712,104
(6)	Projected total claims, ALAE, & ULAE = (5)[1 + 7%]			12,531,952
(7)	Fixed expenses = 75 × (2)			688,500
(8)	Variable expenses = 0.15 × (3)			2,269,959
(9)	Profit = (3) – (6) – (7) – (8)			–357,350

12. Continued

(b) Calculate the forecasted profit for CY 2027 for LOB B.

Commentary on Question:
Starting with the current rates (i.e., the 2024 rate level) being a factor of 1, earned premiums for new policies written on or after the rate change date will be at the higher premium. Therefore, this works just like an on-level calculation, where the average rate level for CY2027 is multiplied by the 2027 earned premiums that exclude any rate adjustment.

Should also note how candidates kept calculating the % at rate level wrong (i.e., .5(8/12)^2). Also, show the diagram so they can see it.*

# of months remaining in 2026: 4

	Rate Level	% at rate level in CY 2027
	1.000	5.56%
	1.050	94.44%
Average:	1.04722	

Variable expense ratio: 975,000 / 7,500,000 = 13%

		2027
(1)	Projected total premium: 7,500,000 × 1.04722	7,854,167
(2)	Projected total claims and LAE	6,190,000
(3)	Fixed expenses	475,000
(4)	Variable expenses = 13% × (1)	1,021,042
(5)	Profit = (1) – (2) – (3) – (4)	168,125
(6)	As a % of premium: (5) / (1)	2.14%

(c) Critique your colleague's proposal.

Reducing exposures does not impact the combined ratio, but it will reduce the underwriting loss.

13. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 22.

Commentary on Question:

This question tests the estimation of ultimate claims using the development method, the Bornhuetter Ferguson method, and the expected method.

Solution:

Critique the appropriateness of each of the following estimates as a potential selection of ultimate claims:

(i) Paid development method for AY 2021

(ii) Reported development method for AY 2021

(iii) Paid Bornhuetter Ferguson method for AY 2023

(iv) Expected method for AY 2024

Commentary on question:

Candidates need to indicate for each selection whether it is appropriate or not.

(i) Paid development method for AY2021:

- The estimate is stable and there is low leverage and it is not affected by the case change
- However, AY2021 is not appropriate because it is missing the large claim

(ii) Reported development method for AY2021:

- This method accounts for the large claim
- However, it is not appropriate because method is affected by case change

13. Continued

(iii) Paid Bornhuetter Ferguson method for AY2023:

- This method not affected by case change
- It has somewhat high leverage, but that is stabilized by using BF method
- Overall, this method is appropriate

(iv) Expected method for AY2024:

- This method is not affected by case change
- This method is not affected by large claim
- Overall, it is an appropriate method

14. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.
(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
(5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 27.

Commentary on Question:

This question tests trend adjustments to premium.

Solution:

(a) Recommend a semi-annual premium trend to account for changes in the proportion of policyholders with the 15% discount. Justify your recommendation.

Calendar Half-Year	Proportion of Policyholders with 15% Discount	Average Discount Factor	Semi-Annual Change
2020-1	14.50%	0.978	
2020-2	15.16%	0.977	–0.10%
2021-1	15.80%	0.976	–0.10%
2021-2	16.45%	0.975	–0.10%
2022-1	18.79%	0.972	–0.36%
2022-2	19.85%	0.970	–0.16%
2023-1	20.86%	0.969	–0.16%
2023-2	21.95%	0.967	–0.17%
2024-1	23.01%	0.965	–0.16%
2024-2	24.02%	0.964	–0.16%

Selected semi-annual trend: –0.16%

Justification: average of all semi-annual periods after 2022-1 as there appears to have been a change since then.

14. Continued

(b) Calculate the premium trend factor to use for 2022, using written premiums for the trending analysis and the trend recommended in part (a).

Rates to be effective:	Sep. 1, 2025
Average written date in CY2022	Jul. 1, 2022
Average written date in future rating period*	Mar. 1, 2026
Trending Period (months)	44
Trending Period (years)	3.667
Annualized trend = $(1 + -0.16\%)^2 - 1$	–0.324%
Premium trend factor = $(1 + -0.324\%)^{3.667}$ =	0.9882

** Note: Halfway through the period the rates are effective, as it's written and not earned.*