

Index-based (re)insurance design in a dynamic setting

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Motivation

The convention insurance model

The index insurance model

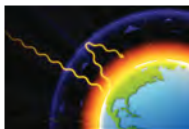
Numerical analysis

Conclusion

Climate change



(a) GHG concentrations



(b) Increasing temperature



(c) Sea level rising



(d) Melting glacier

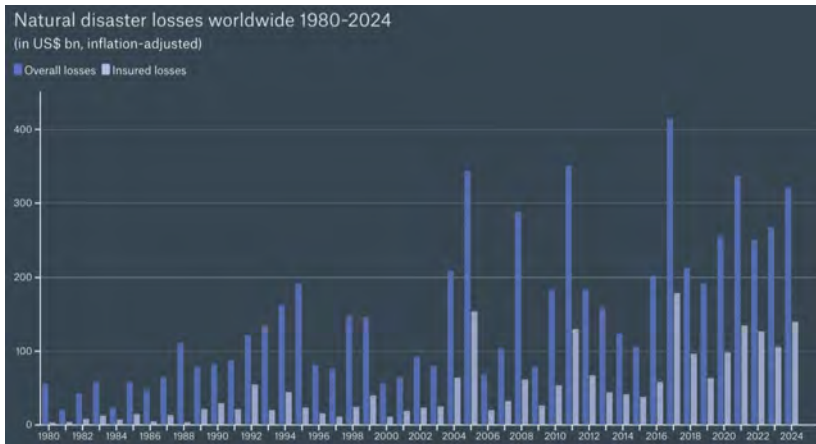


(e) Extreme events

Sources: MIT News; Google earth; BBC.

Natural disasters and protection gap

- Since 1980, natural disasters have cost US\$ 6.7 tn.
- Only around a third of these losses were insured



Two types of insurance

Based on how payout is determined,

- Conventional indemnity-based insurance (conventional insurance): insurance payments are directly calculated from actual losses incurred by insurance policyholders.
- Index-based insurance (index insurance or parametric insurance): payments are based on some pre-determined indices and parametric formulas.

Conventional insurance, as implied by its name, is more prevalent in the market.

Index insurance: benefits

However, conventional insurance sometimes fails to work satisfactorily, and index insurance inherently overcomes some major drawbacks of conventional insurance:

- Speed of liquidity: Payouts can arrive in days rather than after lengthy loss adjustment-crucial for working capital, claims handling, and disaster response.
- Capital/ratings optics: Faster cash reduces liquidity strain after a large event and can support solvency and ratings metrics
- Lower frictional costs: Lower loss adjustment expense and fewer coverage disputes.
- Non-damage business interruption: such as the supply chain disruption by Covid-19.

Index insurance: more benefits

Other benefits of index insurance make them potentially beneficial:

- Lower moral hazard: Because payout isn't tied to the buyer's individual loss handling or exposures, incentives to over report or delay repairs are reduced.
- Transparency: Objective triggers reduce information asymmetry and claims uncertainty
- Diversification & investor access: Standardized index triggers broaden the investor base and facilitate secondary trading, adding capacity to the market.
- Disaster risk financing & resilience: Rapid, rules based payouts help governments and communities fund relief and recovery when it matters most.

Literature: index insurance

Index insurance and index-based risk solutions in general, have been widely used to

- manage catastrophic risk (Biagini et al., 2008, Cummins et al., 2004, Gatzert et al., 2019),
- enhance climate change resilience (Broberg, 2020, Horton, 2018),
- securitize longevity/mortality risk (Cairns and El Boukfaoui, 2021, Denuit et al., 2007, Lin and Cox, 2005),
- pandemic risk (Huang et al., 2023),
- facilitate micro lending (Barnett et al., 2008, Skees et al., 2007),
- protect agricultural producers from weather risk (Brockett et al., 2005, Conradt et al., 2015, Mahul and Skees, 2007, Tan and Zhang, 2023, Zhu et al., 2018).

Literature: basis risk

Basis risk: the gap between the actual loss and the insurance payment (Clarke, 2016).

- It leads to communication and trust issues between insurance companies and policyholders (Jensen and Barrett, 2017, Norton et al., 2011).
- There is a rich literature investigating the design and pricing of index insurance to enhance mitigation effectiveness and encourage participation, including
 - improved modeling and pricing (Zhu et al., 2019a,b),
 - selection and construction of underlying indices (Porth et al., 2020),
 - design of payout mechanism under various risk objectives (Fan et al., 2023, Zhang et al., 2019).
- But there is yet any literature investigating the dynamic cases.

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The classical Cramér-Lundberg (CL) model

- Probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with filtration $\{\mathcal{F}_t\}_{t \geq 0}$
- Insurer's surplus $R_t = x + pt - \sum_{i=1}^{N_t} Z_i$
 - x is the initial reserve
 - N_t is a Poisson process with intensity $\lambda > 0$
 - $Z_1, Z_2, \dots$ are i.i.d losses
 - $p = (1 + \eta_0)\lambda\mathbb{E}[Z]$ is the premium rate, where $\eta_0 > 0$ is the insurer's safety loading.

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 - $p = (1 + \eta_0)\lambda\mathbb{E}[Z]$ is the premium rate, where $\eta_0 > 0$ is the insurer's safety loading.
- Insurer seeks a per-claim reinsurance policy with payment $l(Z_i)$.
 - $l(\cdot)$ satisfies the usual Lipchitz condition.
 - The premium is $p_1(l) = (1 + \eta_1)\lambda\mathbb{E}[l(Z)]$, where $\eta_1 > \eta_0$.

The ruin time

- The insurer's surplus process in the presence of reinsurance, R_t^I , becomes

$$R_t^I = x + (p - p_1(l))t - \sum_{i=1}^{N_t} (Z_i - l(Z_i))$$

The ruin time

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$$R_t^I = x + (p - p_1(I))t - \sum_{i=1}^{N_t} (Z_i - I(Z_i))$$

- Define the ruin time $\tau^I = \inf\{t > 0 | R_t^I \leq 0\}$
- The insurer aims to optimize:

$$V_1(x) = \inf_I \mathbb{P}(\tau^I < \infty | R_0^I = x) \quad (1)$$

Optimal solution for conventional insurance

The solution to (1) is well-studied in the literature (e.g., Hipp and Taksar, 2010, Meng and Zhang, 2010, Tan et al., 2020).

Proposition 1

The optimal reinsurance policy is static and is given by

$$I^*(z) = \left(z - \frac{\eta_1}{a^*}\right) \vee 0,$$

*and the minimal ruin probability is $V_1(x) = e^{-a^*x}$, where a^* satisfies $h_1(a^*) = 0$ and*

$$h_1(a) = \frac{a}{2} \mathbb{E} \left[\left(Z \wedge \frac{\eta_1}{a} \right)^2 \right] + \eta_1 \mathbb{E} \left[\left(Z - \frac{\eta_1}{a} \right) \vee 0 \right] - \eta_0 \mathbb{E}[Z].$$

Moreover, the optimal reinsurance policy and the minimal ruin probability are independent of λ .

Main observation for conventional insurance

From Proposition 1,

- Easy to show that $\frac{\eta_1}{a^*}$ is increasing with η_1 while a^* is decreasing with η_1 .
- As reinsurance becomes more expensive, the reinsurance demand decreases and the ruin probability increases.

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CL model with index insurance

- Now consider an index-based insurance payout $J(Y)$, where $Y_1, Y_2, \dots$ denote the sequence of i.i.d underlying index variables
- The dependence between claim size Z and index Y is given by joint pdf $f(y, z)$.

CL model with index insurance

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- The insurer's surplus process with index reinsurance, R_t^J , becomes

$$R_t^J = x + (p - p_2(J))t - \sum_{i=1}^{N_t} (Z_i - J(Y_i))$$

- premiums $p_2(J) = (1 + \eta_2)\lambda\mathbb{E}[J(Y)]$ with $\eta_2 \in (\eta_0, \eta_1)$

CL model with index insurance

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- premiums $p_2(J) = (1 + \eta_2)\lambda\mathbb{E}[J(Y)]$ with $\eta_2 \in (\eta_0, \eta_1)$
- We similarly consider the ruin probability minimization problem

$$V_2(x) = \inf_{\mathcal{J}} \mathbb{P}(\tau^{\mathcal{J}} < \infty | R_0^{\mathcal{J}} = x).$$

Case 1: Optimal solution for index reinsurance

Proposition 2

The optimal index reinsurance policy is static and is given by

$$J^*(y) = \left(\mathbb{E}[Z|Y=y] - \frac{\eta_2}{b^*} \right) \vee 0,$$

*and the minimal ruin probability is $V_2(x) = e^{-b^*x}$, where b^* is the unique positive solution to*

$h_2(b) = \frac{b}{2} \mathbb{E} \left[\left(Z - \left(\mathbb{E}[Z|Y] - \frac{\eta_2}{b} \right) \vee 0 \right)^2 \right] +$
 $\eta_2 \mathbb{E} \left[\left(\mathbb{E}[Z|Y] - \frac{\eta_2}{b} \right) \vee 0 \right] - \eta_0 \mathbb{E}[Z].$ *Moreover, the optimal reinsurance policy and the minimal ruin probability are independent of λ .*

Main observation for index reinsurance

Similar to the conventional reinsurance case, from Proposition 2,

- Easy to show that $\frac{\eta_2}{b^*}$ is increasing with η_2 while b^* is decreasing with η_2 .
- As reinsurance becomes more expensive, the reinsurance demand decreases and the ruin probability increases.
- It is consistent with the optimal solution in the one-period case (Zhang et al., 2019), if the utility function takes a quadratic form.
- The expectation form allows for convenient extensions to data-driven approaches, such as using neural networks (Chen et al., 2023, Zhang, 2024).
- The relative competitive between conventional and index is determined by a^* vs b^*

Case 2: Optimal solution for index-based derivative

Proposition 3

The optimal index-based derivative policy is static and is given by

$$J^{**}(y) = \mathbb{E}[Z|Y=y] - \frac{\eta_2}{b^{**}}$$

*and the minimal ruin probability is $V_2(x) = e^{-b^{**}x}$, where*

$$b^{**} = \frac{-(\eta_2 - \eta_0)\mathbb{E}[Z] + \sqrt{((\eta_2 - \eta_0)\mathbb{E}[Z])^2 + \eta_2^2 \sigma_\varepsilon^2}}{\sigma_\varepsilon^2} \text{ and}$$

$\sigma_\varepsilon^2 = \mathbb{E}[(Z - \mathbb{E}[Z|Y])^2] > 0$. *Moreover, the optimal reinsurance policy and the minimal ruin probability are independent of λ .*

Observation: It is also straightforward to verify that b^{**} is increasing with σ_ε^2 .

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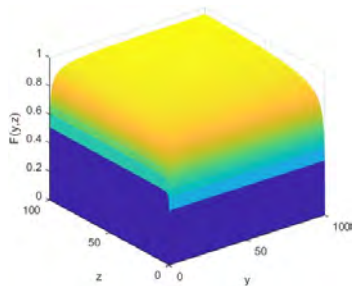
Conclusion

Data and assumptions

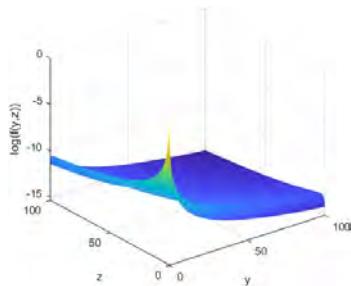
We assume an insurer that underwrites storm risk in Florida:

- The insurer has policyholders in a randomized set of counties, and therefore the total claim Z is the summation of losses in those counties.
- The underlying index Y is the industry loss, assumed to be the aggregate state-level loss.
- Joint pdf $f(y, z)$ are modelled by copula.
- Storm loss data are from January 1950 to April 2025 (adjusted for inflation), as provided by NOAA's National Weather Service (NWS).
- Assume that $\eta_0 < \eta_2 < \eta_1$.

Joint distribution

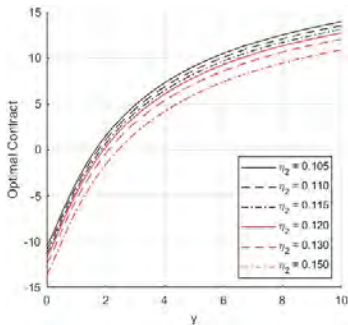


(f) CDF

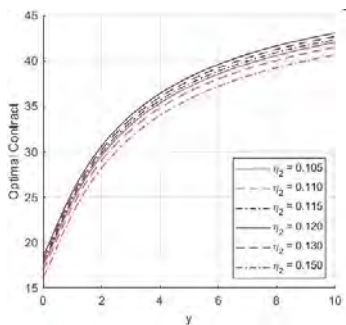


(g) PDF

Optimal payouts



(h) derivative



(i) reinsurance

Minimum ruin probability

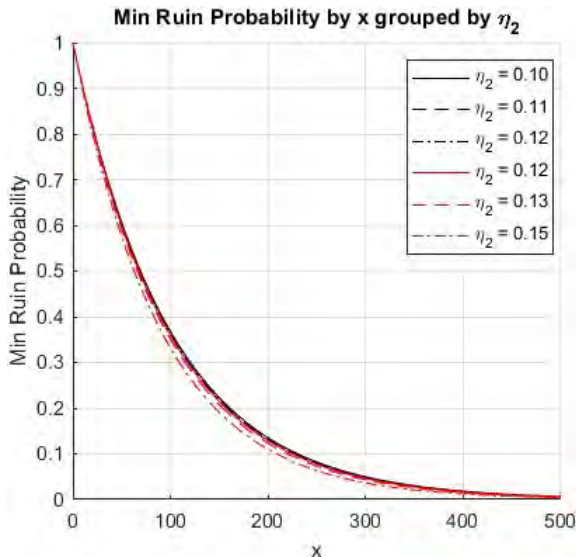


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Concluding remarks

- We study the optimal design of index-based risk solutions for insurers, including index-based reinsurance and financial derivatives, in a dynamic setting.
- Under the classical Cramér-Lundberg (CL) model, we derive closed-form solutions for the optimal payout structures which minimize insurers' ruin probability.
- We apply the framework to a storm loss reinsurance case, and numerically discuss the relative advantage of index-based over conventional risk solutions depends on the tradeoff between basis risk and cost efficiency.

Thank you

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