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Empirical Results on the Rotation of the Age Pattern of Mortality Decline

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Abstract Rotation of the age pattern of mortality decline refers to two phenomena supposedly occurring simultaneously: decelerating mortality decreases at younger ages and accelerating improvements in elderly populations. Several researchers have documented these processes in the literature, especially in highly developed countries.

After a concise summary of the most relevant sources, a simple, largely data-driven methodology with few assumptions is used to empirically examine the rotation phenomenon in historical mortality datasets of the G7 countries[1], using United Nations data from the period between 1950 and 2015 for both genders.

In line with earlier findings about European Union member states, my results indicate that the presence of rotation is far from universal, even in highly developed countries. There is strong evidence of rotation in both male and female populations only in the case of Japan, and no evidence of rotation whatsoever in US data. Therefore, it is necessary to exercise appropriate caution before applying forecasting procedures such as the variant of the popular Lee–Carter model including rotation.

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[1] The Group of Seven consists of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States of America.

1 Introduction

Several mortality researchers have noted a historical pattern of diminishing mortality decline at relatively younger ages, accompanied by accelerating improvements at more advanced ages (Christensen et al. [2009]). Li–Lee–Gerland [2013] call this phenomenon the "rotation" of the age pattern of mortality decline, which is captured by a counterclockwise rotation in Figure 1.

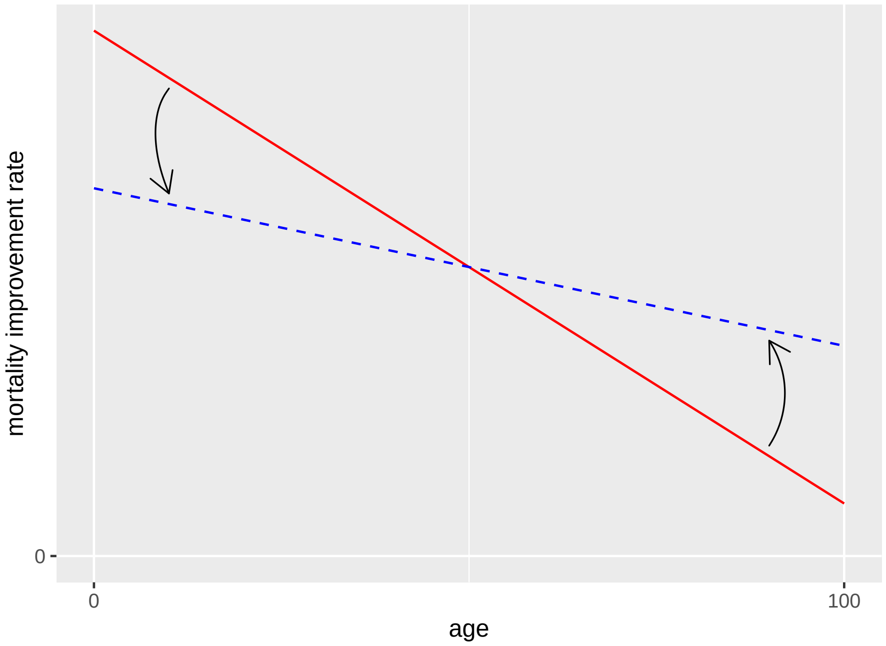

Fig. 1 Rotation of the age pattern of mortality decline (stylized illustration, source: Vékás [2019])

A somewhat simplistic explanation of the rotation is that longevity increases used to be driven by rapidly declining infant and childhood mortality rates (e.g., due to widespread vaccination programs and improved child nutrition) – and to some extent, by improvements in middle-aged mortality –, where spectacular advances are less and less possible, but on the other hand, better medications, nutrition and lifestyle choices for the elderly and costly medical procedures to extend life at higher ages are increasingly available.[2] It should be noted that the investigation of the causes of the rotation falls outside the scope this paper.

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[2] Li–Lee–Gerland [2013] argue that the rotation is more prevalent in developed countries characterized by low mortality, which is consistent with this explanation. Elderly mortality itself is far from homogeneous, and this general description may hold for some age groups and countries and not for others.

The practical significance of the topic lies in the fact that ignoring rotation in long-term mortality forecasts leads to the systematic underestimation of the old-aged population, which exacerbates longevity risk. These underestimation errors have a cumulative nature and may be suprisingly severe in the long run (see e.g. Vékás [2018]). This may lead to serious financial consequences for life and health insurers as well as pension schemes.

Mortality forecasting techniques play a key role in demography, life insurance and pensions. Due to the immense and ever-growing literature on these methods (see e.g. Booth–Tickle [2008] and Pitacco et al. [2009] for comprehensive reviews), an exhaustive overview is not attempted here, but instead, this paper will only focus on sources related to the rotation phenomenon.

The famous paper of Lee–Carter [1992] has probably been the most important breakthrough in the history of mortality forecasting. The authors model the logarithm of the central mortality rate at age x and calendar year t as

$$\log m_{xt} = a_x + b_x k_t + \varepsilon_{xt}, \tag{1}$$

where a_x represents the mean of the observed logarithmic central mortality rates for a given age, the time series k_t captures the evolution of the overall level of mortality across time, and b_x denotes the speed of mortality decline for every age.

As the parameters b_x do not depend on time, and the time series k_t is overwhelmingly assumed to follow a linear pattern (Tuljapurkar et al. [2000]), age-specific mortality declines at a constant speed in the Lee–Carter model, and the rate of improvement only depends on the age of the individual in question. The latter implicit assumption of the model has attracted intense scrutiny by the scientific community (see e.g. Kannisto et al. [1994], Horiuchi–Wilmoth [1995], Lee–Miller [2001], Carter–Prskawetz [2001], Rau et al. [2008] and Christensen et al. [2009]).

Several approaches have been developed to address this inflexibility of the classic Lee–Carter framework. Notably, Li–Lee–Gerland [2013] have incorporated the rotation into the original procedure[3], where instead of Equation (1), they model the logarithms of central mortality rates as

$$\log m_{xt} = a_x + B(x,t) k_t + \varepsilon_{xt}. \tag{2}$$

The parameters $B(x,t)$ in Equation (2) capture the rotation phenomenon by converging smoothly across time from their initial levels corresponding to b_x in Equation (1) to their assumed ultimate levels, as life expectancy at birth advances from an initial threshold to an upper ceiling (the authors propose 80 and 102 years, respectively) in the original model described by Equation (1). It is important to note that the authors recommend their model for low-mortality countries and very long forecasting horizons, and knowledge of the estimated parameters of the original Lee–Carter model is sufficient to fit the rotated model to data. Ševčíková et al. [2016] and Dion et al. [2015] recently incorporated this technique into population projections for the United Nations Population Division and Statistics Canada, respectively.

Another solution is to capture the rotation by modeling the evolution of age-specific

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[3] Li–Gerland [2011] present an earlier, not fully developed version of this approach.

mortality improvement rates instead of mortality rates, as proposed by Haberman–Renshaw [2012] and Mitchell et al. [2013], among others. Bohk-Ewald–Rau [2017] follow this line in a Bayesian framework capable of combining mortality trends of different countries. These approaches are data-driven, as opposed to Li–Lee–Gerland [2013], who impose a somewhat arbitrary process on age-specific mortality improvement rates, as they are of the opinion that empirical evidence of the rotation is too subtle to govern forecasts.

Yet another alternative is the approach of Booth et al. [2002] and Hyndman–Ullah [2007], who recommend using more than one interaction of age- and time-dependent parameters in Equation (1) in order to capture the non-constant evolution of age-specific mortality improvement rates, which produces so-called multi-factor mortality forecasting models. Bongaarts [2005] proposes a shifting logistic model to describe the transition in the age pattern of mortality decline. Li–Lee [2005], Cairns et al. [2011], Russolillio et al. [2011] and Hyndman et al. [2013] model mortality rates of several populations in a coherent framework. In a multi-population setting, age-specific rates of mortality improvement are not necessarily constant due to interactions among different populations. Further recent developments in this field are described by de Beer–Janssen [2016] and Li–Li [2017].

Based on data from 28 European Union member states and the period between 1950 and 2015, Vékás [2019] concludes that the rotation only took place in a few member states, with only 11 of them displaying statistically significant evidence for rotation at the 5% level in case of both genders, while apparently no rotation at all (or even on the contrary, an anti-rotation) in many others. Additionally, the rotation was more prevalent in female than male populations. Contrary to Li–Lee–Gerland [2013], Vékás [2019] argues that the presence and strength of the rotation phenomenon appear to be largely unrelated to life expectancies at birth in the European Union as a whole: positive and negative cases appear among both low- and high-mortality countries, and the strength of the assocation between these two variables is apparently statistically negligible. On the other hand, there is significant evidence for a positive correlation between degrees of rotation and life expectancies at birth among member states that used to belong to the Eastern Bloc during the Cold War.

2 Data and methods

2.1 Demographic data

The statistical analysis presented in this paper was performed in R (R Development Core Team [2008]) using mortality rates, life expectancies at birth and population counts of the Group of Seven, which consists of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States of America These indicators are available for both genders, all G7 countries, 22 age groups (0, 1-4, 5-9, 10-14, ..., 95-99 and 100 years and older) and 13 calendar periods (1950-1955, 1955-1960, ...2010-2015).[4] The grouping of ages and calendar years smooths the data (akin to moving averages) so that they contain less undesirable random fluctuations. All data are the courtesy of the UN World Population Prospects 2017 ([United Nations [2018]]).

Mortality improvement rates pertaining to age group $x \in \{x_1, x_2, \ldots, x_{22}\}$, calendar period $t \in \{1, 2, \ldots, 12\}$, country $c \in \{c_1, c_2, \ldots, c_{28}\}$ and gender $g \in \{M, W\}$, denoted by r_{xt}^{cg} and computed as

$$r_{xt}^{cg} = -\log\left(\frac{m_{x,t+1}^{cg}}{m_{xt}^{cg}}\right)$$

will be used throughout this paper instead of the corresponding mortality rates m_{xt}^{cg}. Based on these quantities, acceleration rates β_x^{cg} may be computed for every age group $x \in \{x_1, x_2, \ldots, x_{22}\}$ and country $c \in \{c_1, c_2, \ldots, c_7\}$ as well as both genders $g \in \{M, W\}$. Long-term mean acceleration is measured by the slope of the linear trend of mortality improvement rates (Vékás [2019]):

$$\beta_x^{cg} = \frac{\sum_{t=1}^{12}(r_{xt}^{cg} - \bar{r}_x^{cg})(t - \bar{t})}{\sum_{t=1}^{12}(t - \bar{t})^2}. \tag{3}$$

β_x^{cg} in Equation (3) may be interpreted as the mean growth of the mortality improvement rate for age group x, country c and gender g over a 5-year period assuming a linear trend. To determine the degree to which rotation has taken place (if at all) for a given country and gender, it has to be examined whether the acceleration of mortality decline has been more pronounced at advanced ages than in the earlier and middle stages of life (possibly characterized by deceleration). Following Vékás [2019], Spearman's ρ (Pinto da Costa [2015]), weighted by mean population counts over the period 1990-2015, is used for this purpose:

$$\rho^{cg} = \frac{\sum_{i=1}^{22} P_{x_i}^{cg}(\mathrm{rank}(\beta_{x_i}^{cg}) - \mu^{cg})(i - \nu^{cg})}{\sqrt{\sum_{i=1}^{22} P_{x_i}^{cg}(\mathrm{rank}(\beta_{x_i}^{cg}) - \mu^{cg})^2}\sqrt{\sum_{i=1}^{22} P_{x_i}^{cg}(i - \nu^{cg})^2}} \tag{4}$$
$$((c,g) \in \{c_1, c_2, \ldots, c_7\} \times \{M, W\}),$$

where

$$\mu^{cg} = \frac{\sum_{i=1}^{22} P_{x_i}^{cg}\mathrm{rank}(\beta_{x_i}^{cg})}{\sum_{i=1}^{22} P_{x_i}^{cg}} \quad \text{and} \quad \nu^{cg} = \frac{\sum_{i=1}^{22} P_{x_i}^{cg} i}{\sum_{i=1}^{22} P_{x_i}^{cg}}.$$

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[4] Every period spans 5 years and starts and ends on July 1 of the respective years.

Additionally, the one-sided z-test (Pinto da Costa [2015]) with

$$H_0 : \rho^{cg} \leq 0, \quad H_1 : \rho^{cg} > 0 \quad ((c,g) \in \{c_1, c_2, \ldots, c_7\} \times \{M, W\}). \tag{5}$$

is used to test whether degrees of rotation are significantly different from zero.

3 Conclusions

Figures 2 and 3 display degrees of rotation in male and female populations of the G7 countries, based on Equation (4), as well as the critical values at the 5% and 1% significance levels of the hypothesis test defined by Equation (5). Table 1 in the Appendix contains the exact numeric values of ρ^{cg} as well as the p-values of the above test by country and gender.

Evidence for rotation is significant at the 5% level in male populations of Canada, Germany, Italy and Japan, as well as in female populations of Italy, Japan and the United Kingdom. This suggests that rotation of the age pattern of mortality decline was far from universal in the G7 countries between 1950 and 2015, similarly to the

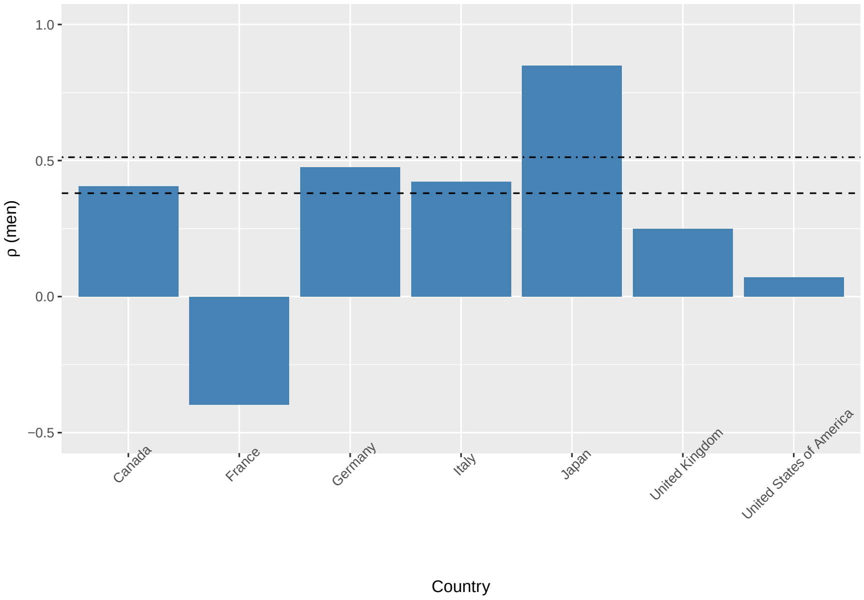

Fig. 2 Degrees of rotation (measured by Spearman's ρ) by country for male populations. The dashed and dotted-dashed lines denote the one-sided critical values at the 5% and 1% significance levels, respectively.

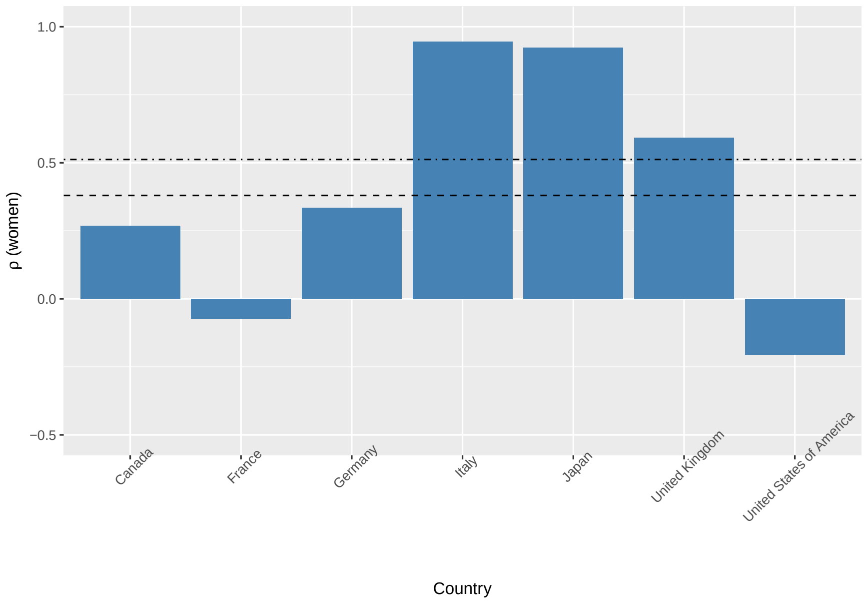

Fig. 3 Degrees of rotation (measured by Spearman's ρ) by country for female populations. The dashed and dotted-dashed lines denote the one-sided critical values at the 5% and 1% significance levels, respectively.

findings of Vékás [2019] about European Union member states.[5]

Apparently, no statistically significant rotation took place among either males or females in France and the United States of America. Results are even more mixed and provide less evidence of the rotation if a 1% significance level or the Bonferroni adjustment are applied. Only Japan has strong evidence of rotation for both genders at the 1% significance level.

As the rotation phenomenon may jeopardize the reliability of mortality forecasts for pension schemes as well as life and health insurers, which may lead to severe financial consequences (Vékás [2018]), it is essential to be aware of the possibility of its presence and apply appropriate forecasting procedures that take it into consideration, whenever necessary.

As the immensely popular Lee–Carter [1992] mortality forecasting model ignores rotation, in some cases, it is advisable to use the particularly promising Li–Lee–Gerland [2013] variant of the original method, but if and only if there is enough evidence for rotation in the data series. The methodology and results presented in this paper may facilitate the choice of the appropriate forecasting technique in actuarial practice.

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[5] A stricter testing framework might take into account that $2 \cdot 7 = 14$ null hypotheses are being tested simultaneously. Hence applying the popular Bonferroni adjustment for controlling the familywise error rate (see Frane [2015] for a critical discussion), the p-values below $0.05/14 \approx 0.0036$ imply statistical significance at the 5% level.

4 Appendix

	Men			**Women**		
Country	ρ	p-**value**		ρ	p-**value**	
Canada	0.405	0.039	*	0.268	0.13	
France	-0.397	0.958		-0.072	0.617	
Germany	0.475	0.017	*	0.334	0.076	
Italy	0.422	0.032	*	0.946	< 0.001	***
Japan	0.85	< 0.001	***	0.924	< 0.001	***
United Kingdom	0.249	0.148		0.592	0.003	**
United States of America	0.071	0.385		-0.206	0.805	

Table 1 Degrees of rotation ρ^{cg} by country and gender and one-sided p-values (.: $0.05 < p < 0.1$, *: $0.01 < p < 0.05$, **: $0.001 < p < 0.01$, ***: $p < 0.001$)

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