



Mortality and Longevity



Aging and Retirement

# The Application of Affine Processes in Cohort Mortality Risk Models





# The Application of Affine Processes in Cohort Mortality Risk Models

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## Abstract

This paper assesses and compares multi-factor continuous time affine mortality models applied to age-cohort mortality curves that are well suited for theoretical and practical application in finance and insurance. Models based on Gaussian distributed mortality rates, as well as the Cox-Ingersoll-Ross (CIR) process allowing for Gamma distributed mortality rates, are compared, also quantifying the probability of negative rates in the Gaussian models. In particular, we introduce the Gaussian Arbitrage-Free Nelson-Siegel (AFNS) mortality model incorporating level, slope and curvature factors. The models have appealing features including efficient estimation and computation. We estimate models using age-cohort data to capture cohort effects more effectively and in order to explain the variability in cohort mortality curves in the continuous time framework. The models allow for Poisson variation in the model estimation using the Kalman filter. The affine mortality models facilitate the derivation of closed-form survivor curves allowing for efficient valuation of mortality-linked claims. The models can also incorporate factor dependence allowing for age-dependence in the mortality curves. Importantly we show that the Gaussian independent factor AFNS model performs very well in explaining and forecasting cohort mortality.

**Key words:** mortality models, continuous time, cohort curve, affine rates, Kalman filter

**JEL Classification Numbers:** G22, C13, C22, C52, J11

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