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Future Greeks Without Nested Stochastics

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Future Greeks Without Nested Stochastics

– A Neural Network Approach

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Transamerica Life Insurance Company

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Agenda

1. A brief introduction to neural network
2. Rediscover Black-Scholes with neural network
3. AAA scenarios and CTE optimization

A brief introduction to neural network

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History of neural network

- Neural Network research goes way back

- Only became particularly useful during last decade
    - Image recognition
    - Natural language processing

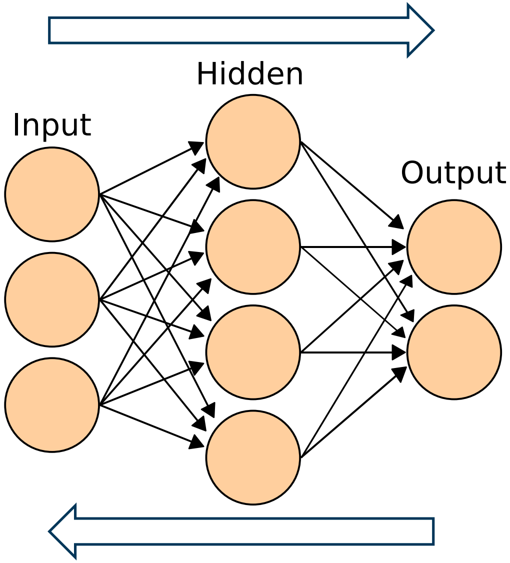
- Why now
    - Faster hardware: GPU, TPU, Neural Engine
    - Better software: improved network architecture, new activation functions, robust optimizer, modern software framework (tensorflow, pytorch etc.)
    - Bigger data to work with
    - Active community support, critical mass of interest

50’	perceptron
mid 70’	backpropagation
mid 90’	convolutional neural network
2009	ImageNet
2015	ResNet
2016	AlphaGo

How to train neural network

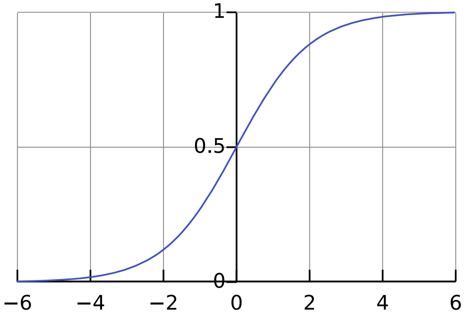
- Large labeled dataset
- Keep adjusting weights until desired outputs
- Gradient based weight adjustments

Forward pass - calculate loss



Backpropagation - adjust weights

Sigmoid activation function



Rediscover Black-Scholes with neural network

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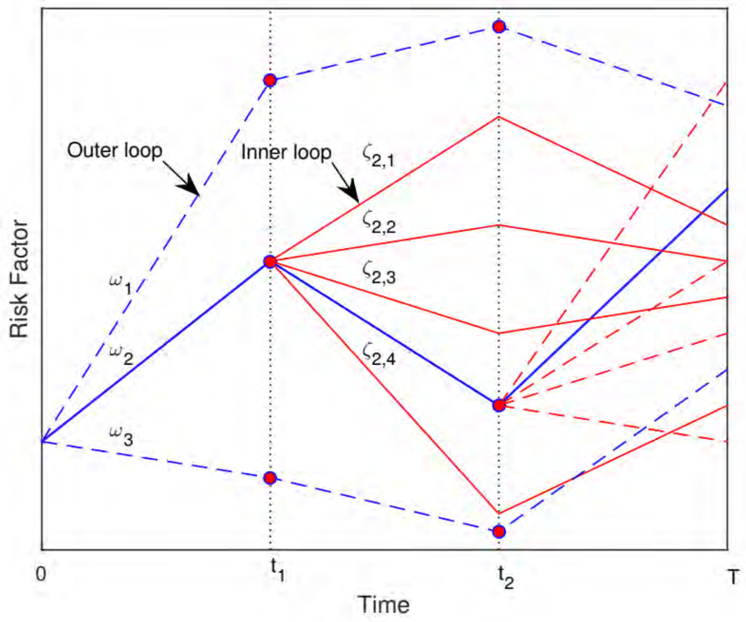
Future greeks

- Stochastic modeling is commonly used by actuaries
- Traditionally, nested stochastics is needed when calculating value/greeks for future node
- Nested stochastics with outer loop/inner loop setup is extremely computational intensive
- Least Square Monte Carlo. Reduce inner loops size by curve fitting.
- Some attempt to use neural network for the fitting



Train NN to produce Black-Scholes

- Question: How would you manage market risk from financial options if Black-Scholes has **NOT** been invented?
- Answer: Deep learning and AI

- An algorithm to train neural networks to discover the future greeks of financial options
- The inputs of the process are:
    1. One set of economic scenarios. Could be real world. Shocked scenarios are **NOT** needed
    2. Option cash flow associated with each scenario
    3. That's it. We do **NOT** need any prior knowledge of Black Scholes formula.
- The output of the process is a trained neural network, **with times and index levels as inputs, delta as output**.
- The training setup is original. We do not have a target output for neural network itself
- Instead, the training target is at batch level, where delta neural network is applied multiple times
    - We want the after hedge g/l the same among all scenarios (highest hedging effectiveness)!

Training setup, an example

- One year (252 days) at the money European call option
- 2% interest rate
- 4096 scenarios, daily time step, 0% drift and 16% volatility
    - **The drift rate is different from risk free rate**
- The delta network has
    - two inputs, time and index level
    - two hidden layers (16 nodes and 8 nodes, tanh activation)
    - One output, sigmoid activation, which will be trained as delta
- The training target is set up at batch level
    - 252 delta networks with shared weight to calculate delta at each time step
    - G/L of delta hedge is calculated
    - Then the after hedge cost of the options is calculated as the sum of hedge G/L and payout
    - The loss function is the variance of after hedge cost
    - **The average after hedge cost is the risk neutral price at time zero**

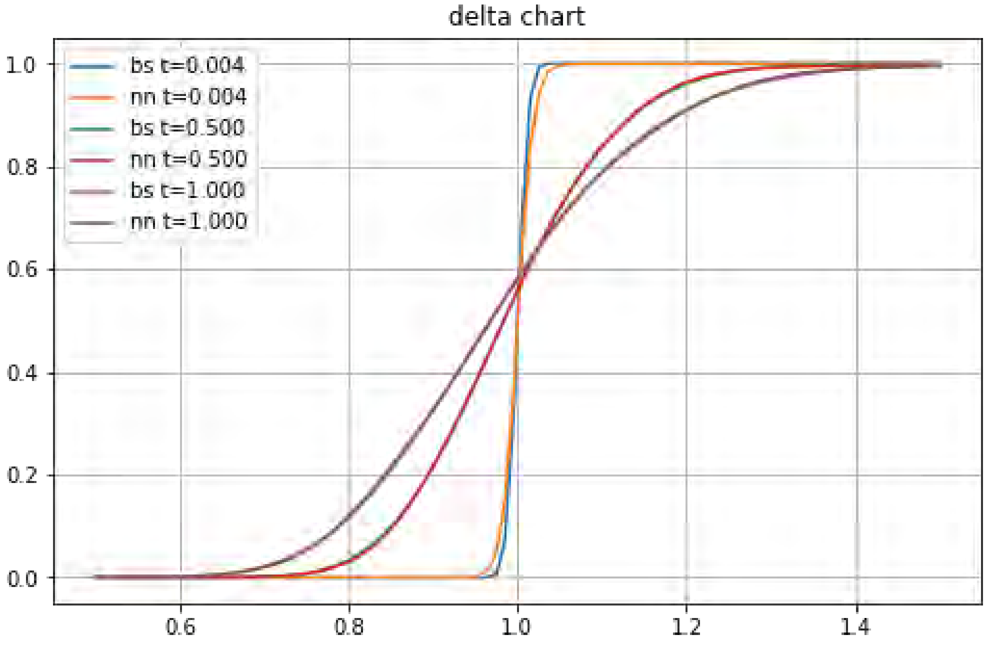
Jupyter notebook at https://colab.research.google.com/github/yufeng66/FutureGreeks/blob/master/SOA_talk_lognormal_scenario.ipynb

Training result (1)

- The program is developed with pytorch framework, using AdamW and LBFGS optimizer
- Only takes seconds to train on google Colab

- The neural network delta matches Black Scholes formula extremely well.
- The neural network delta also extrapolates well

- **The neural network independently rediscovered Black-Scholes formula!**



Training result (2)

	Before Hedge		Hedged with NN			Hedged with BS		
	Mean	Std	Mean	Std	HE	Mean	Std	HE
Training scenarios	6.258%	10.053%	7.353%	0.350%	96.518%	7.366%	0.366%	96.362%
Validation scenarios	6.237%	10.014%	7.353%	0.349%	96.514%	7.366%	0.363%	96.377%
risk neutral scenarios	7.329%	10.900%	7.350%	0.352%	96.767%	7.351%	0.365%	96.651%

- Hedge effectiveness with neural network is actually slightly better compared to Black-Scholes, even for out of sample scenarios
- The after hedge mean is very close to the Black-Scholes formula price of 7.352%

- We can now calculate future delta for a real world scenario set

AAA scenario and CTE training target

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AAA scenario

- 25 year put option, strike at 3 with initial index at 1
- AAA scenario for US Diversified Equity
- Still deterministic interest rate of 2.5%

- The delta network has
  - three inputs - time, index level and short volatility
  - two hidden layers (24 nodes and 12 nodes, tanh activation)
  - One output, sigmoid activation, which will be trained as delta

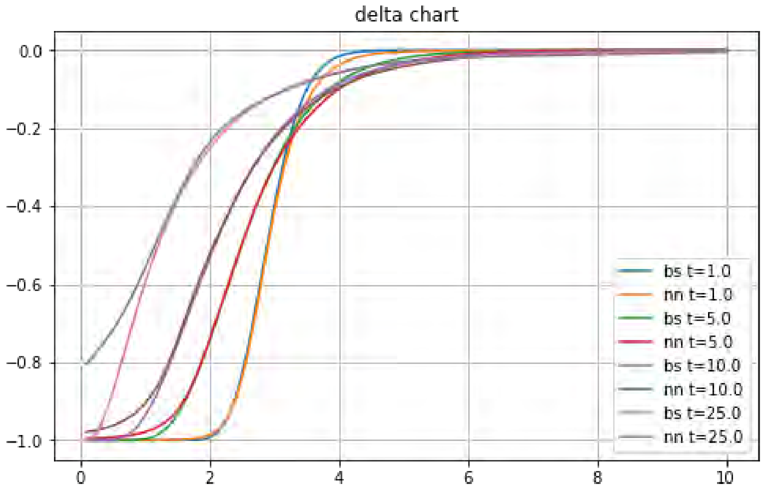
- Similar training setup
  - But need to approximate the daily rebalancing to get good hedge effectiveness

Jupyter notebook at https://colab.research.google.com/github/yufeng66/FutureGreeks/blob/master/AAA_scenario_25yr_put.ipynb

Training result, std target

	Mean	Stdev	HE	CTE70	CTE98
Before hedge	8.21%	22.80%		27.35%	109.81%
After hedge	74.69%	3.44%	84.92%	78.53%	85.65%

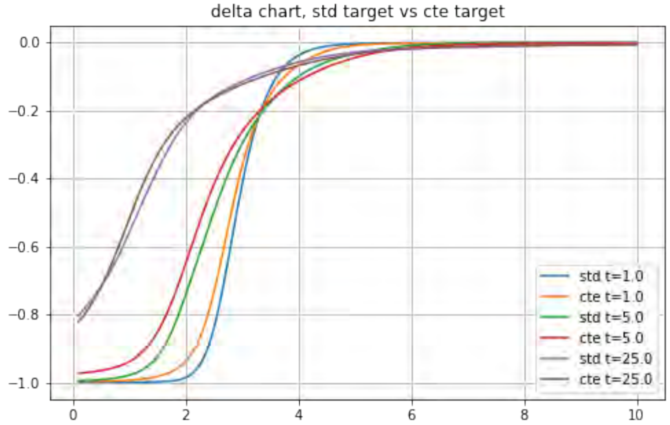
- Neural network delta curve is still similar to Black-Scholes delta
- Hedge Effectiveness is not as good as lognormal scenario case, most likely due to the randomness in volatility which is not hedged
- CTE98 decreased but CTE70 increased



CTE optimization

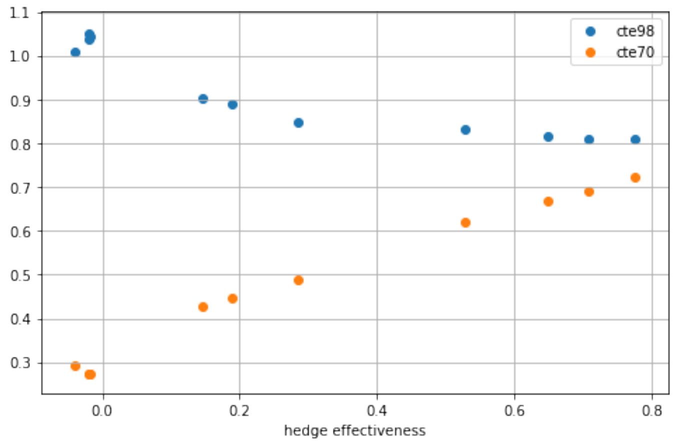
	Mean	Stdev	HE	CTE70	CTE98
Before hedge	8.21%	22.80%		27.35%	109.81%
Std hedge	74.69%	3.44%	84.92%	78.53%	85.65%
CTE hedge	68.20%	4.65%	79.63%	73.57%	80.65%

- CTE 98 can be used as optimization target
- The hedge size is smaller, or more positive delta due to high scenario drift
- Both CTE 70 and CTE98 decreased. But hedge effectiveness also decreased as a tradeoff



CTE98 vs CTE70 trade off

- We can also use a blended optimization target of CTE98 and CTE70
- When the target overweight on CTE70, we don’t need to hedge
- As target weight shift towards CTE98, hedge effectiveness increase, the CTE70 increase and the CTE98 decrease.
- We can value the option in a blended risk neutral world and real world by choosing different hedging strategy



Discussion and Conclusion

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Discussion

- We applied the technique to other options with more exotic features such as call spread, Asian, High Watermark and rainbow, etc. The same technique works with various degrees of success.
- The technique also works on a portfolio of options.
- We expected dynamic actuarial assumptions can be handled as well.
- For path dependent options, it is possible to structure the neural network to extract the path dependency automatically. But it is best to provide additional relevant inputs to the delta network.
- We will need to model interest rate hedge asset too, if stochastic interest rate is used.

- Pros
    - Efficient algorithm to produce future greeks with minimum requirement of data input.
    - Ability to explicitly dynamic hedge to statutory capital.

- Cons
    - Reproducibility. Need to watch out for optimizer stability
    - Black box natural. Not much intuition on individual neuron outputs.

Conclusion

- Given a lognormal scenario set, the algorithm can rediscover Black-Scholes delta curve independently
- The neural network can calculate risk neutral hedge position and price with real world scenarios
- The neural network also works well with the AAA scenario
- There could be further savings on CTE 98 if we optimize for it directly

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