

Equity-Based Insurance Guarantees Conference

Nov. 11-12, 2019

Chicago, IL

Dynamic vs Static Replication

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Dynamic vs Static Replication

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November 12, 2019 (1330 - 1430 hours)

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Overview of Options and Replicating Strategies



The World of Options

- Exposures can go far beyond vanilla calls/puts
 - Exercise types: European, American, Bermudan, Asian, ...
 - Underlying: single name, basket, chooser, rainbow, spread, variance/correlation, ...
 - Payoff profile: binary, power, range, ...
 - Boundary conditions: barrier, knock-in/out, ...
 - Exotic terms: cancelable, extendable, amortizing, ...
 - Exotic strikes: lookback, cliquet, ...
- Frequently insurance liabilities (particularly life) are long dated and impacted by policyholder behavior

Dynamic vs Static Replication

- Delta hedge alone?
- Include market options to lower trade rebalancing?
- Not a yes/no choice, more of a continuum
- Even (relatively) model independent static replication of variance swaps requires continuous delta hedging

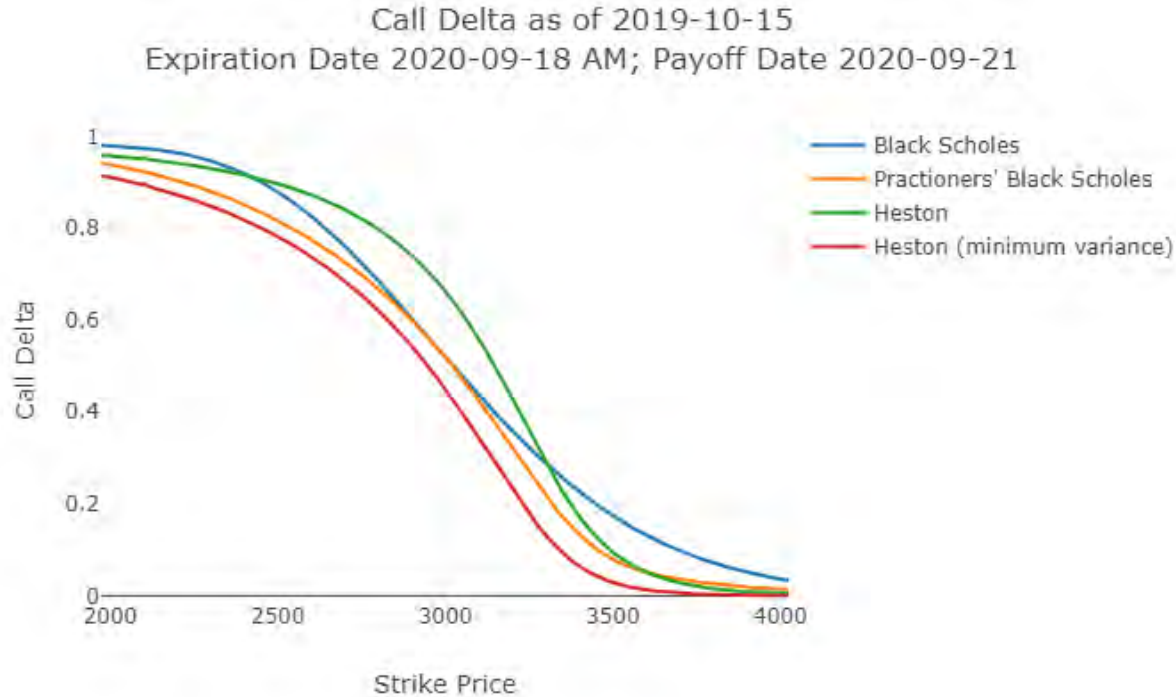
Hedge Effectiveness

- Accuracy is constrained
- Limited market instruments (e.g. long dated)
- Liquidity, transaction costs, slippage
- Model/data dependency
- Policyholder behavior

Costs of Trade Rebalancing

- Execution commission: roughly \$1.00/contract
- Exchange commission: CBOE charges up to \$0.80/contract (\$0.35-\$0.55/contract on SPX)
- Clearing commission: roughly \$1.00/contract
- Clearing fees: OCC charges up to \$0.055/contract
- Options Regulatory Fee: \$0.0388/contract
- Bid/ask spread: at least a few basis points

Hedge Ratio Model Dependency



Opportunities

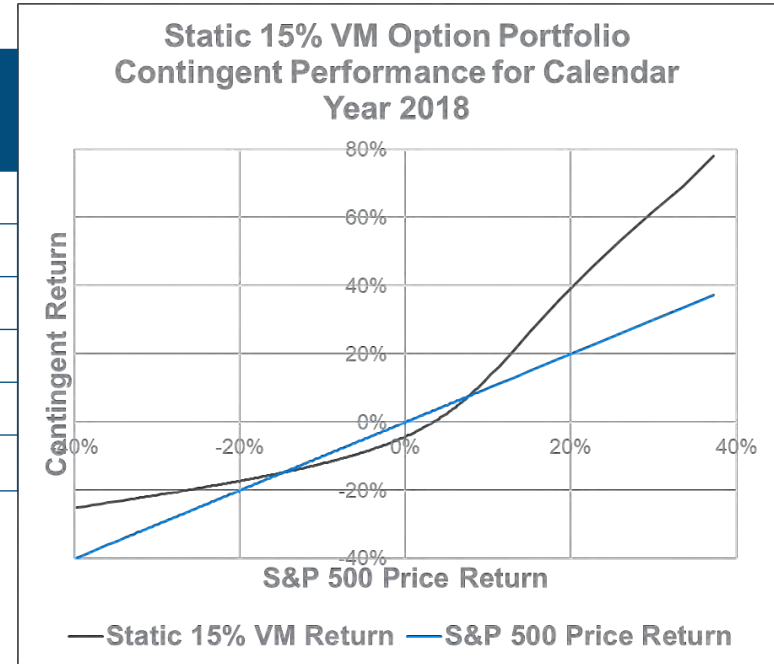
- Dynamic replication can harvest implied volatility premium/term structure
- Can earn spread via cash management
- Optimized ALM, statistical hedge replication
- Maintain adaptability/flexibility/control

Example: “Static” 1-Year S&P 500 Volatility Management

Calendar Year 2018, Target Volatility = 15%

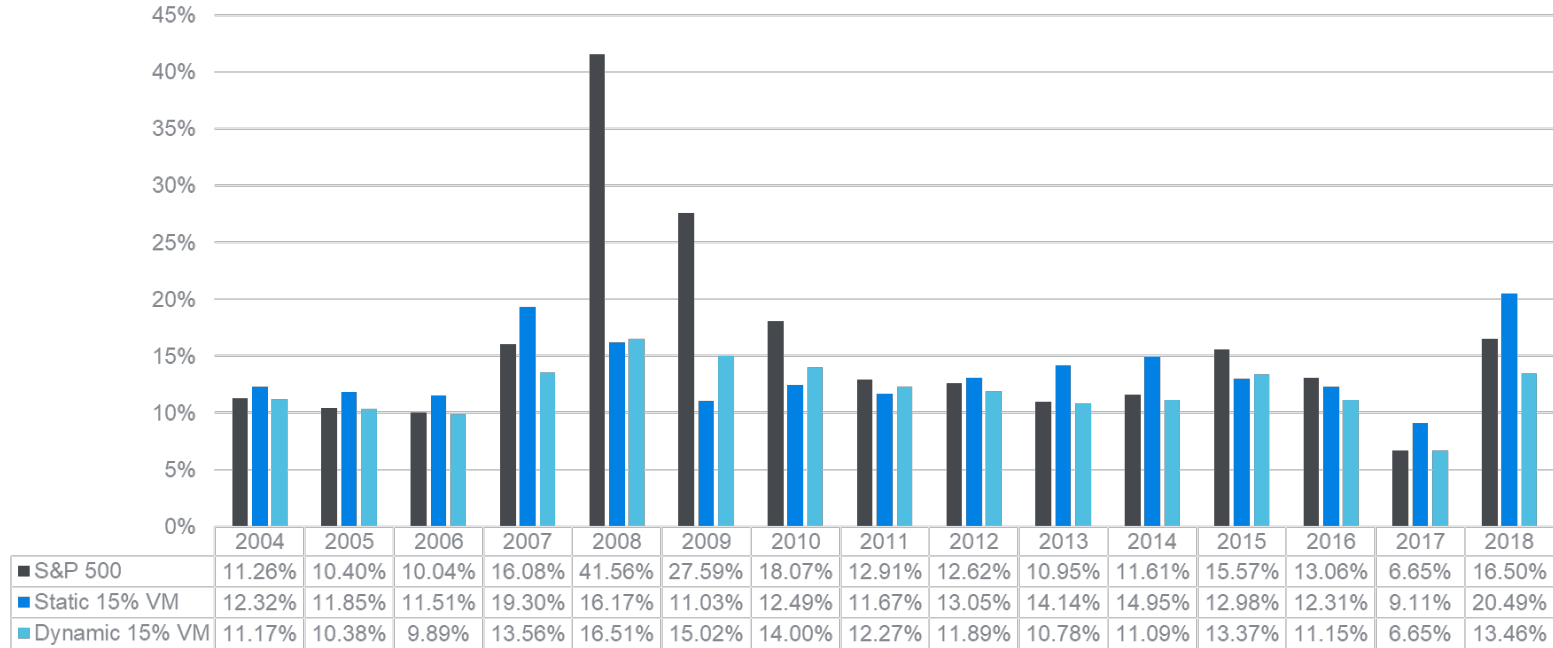
S&P 500 Price Return 1/2/2018-12/31/2018	Static 15% VM Option Portfolio	Dynamic 15% VM Account (no leverage)
< -14.73%	Outperform	?
-14.73% - 0%	Underperform	?
0%	-4.23%	?
0% - +7.69%	Underperform	?
> +7.69%	Outperform	?
Realized: -7.01%	-10.21%	-5.87%

- S&P 500 price return between -14.73% - +7.69
(S&P 500 total return including dividends = -5.18%)
- Outcome: “static” 15% VM underperforms



RESULTS BASED ON SIMULATED OR HYPOTHETICAL PERFORMANCE RESULTS HAVE CERTAIN INHERENT LIMITATIONS. UNLIKE THE RESULTS SHOWN IN AN ACTUAL PERFORMANCE RECORD, THESE RESULTS DO NOT REPRESENT ACTUAL TRADING. ALSO, BECAUSE THESE TRADES HAVE NOT ACTUALLY BEEN EXECUTED, THESE RESULTS MAY HAVE UNDER-OR OVER-COMPENSATED FOR THE IMPACT, IF ANY, OF CERTAIN MARKET FACTORS, SUCH AS LACK OF LIQUIDITY. SIMULATED OR HYPOTHETICAL TRADING PROGRAMS IN GENERAL ARE ALSO SUBJECT TO THE FACT THAT THEY ARE DESIGNED WITH THE BENEFIT OF HINDSIGHT. NO REPRESENTATION IS BEING MADE THAT ANY ACCOUNT WILL OR IS LIKELY TO ACHIEVE PROFITS OR LOSSES SIMILAR TO THESE BEING SHOWN. MILLIMAN DOES NOT MANAGED THE UNDERLYING FUND.

“Static” VM Realized Volatility

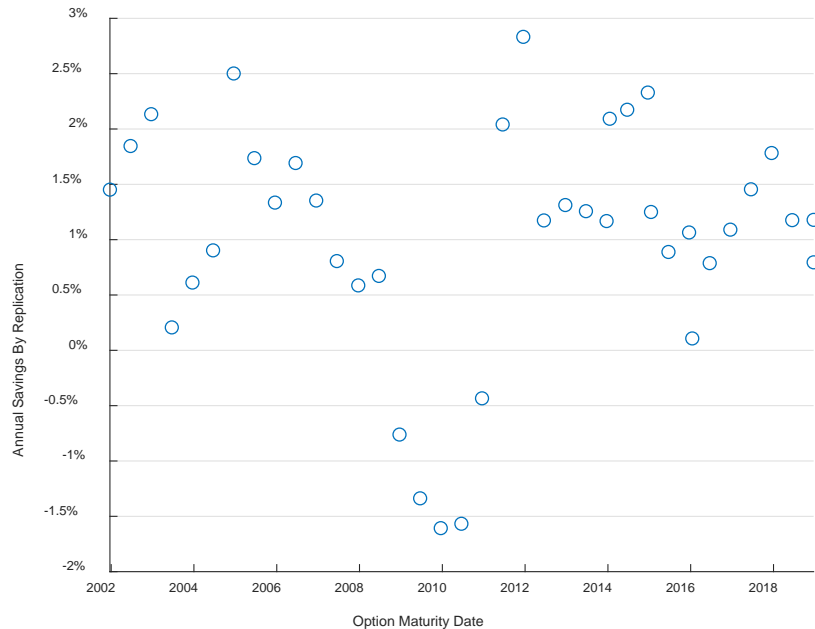


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Dynamic vs Static Hedging



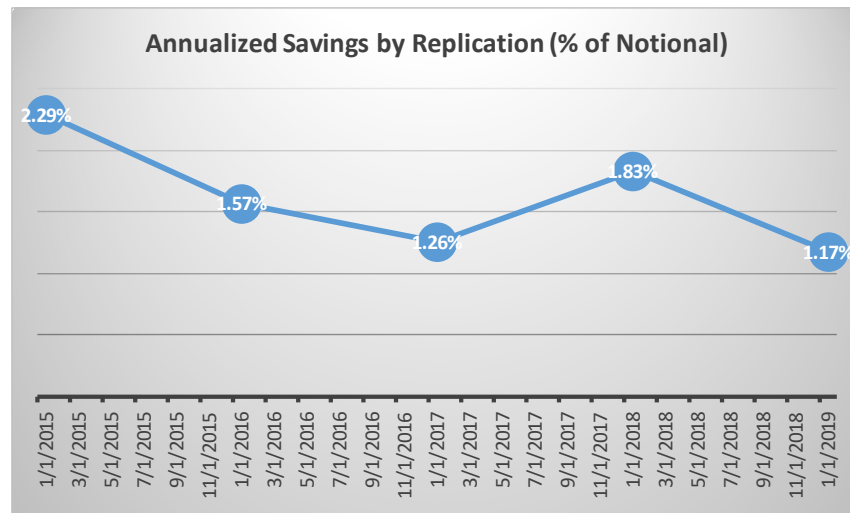
Annualized Savings from CBOE Options Replication



- In this example, we replicated listed CBOE call options with futures contracts on the S&P 500.
- Savings are annualized and reflect the difference between the implied volatility of the call option versus the realized volatility of the replicated position.
- The assumed volatility used to calculate option delta for replication was based on the MGI value for the remaining time-to-maturity of the option.
- Longest option tenors were three years, but we modeled multiple tenors here.
- All savings figures were annualized for comparability.

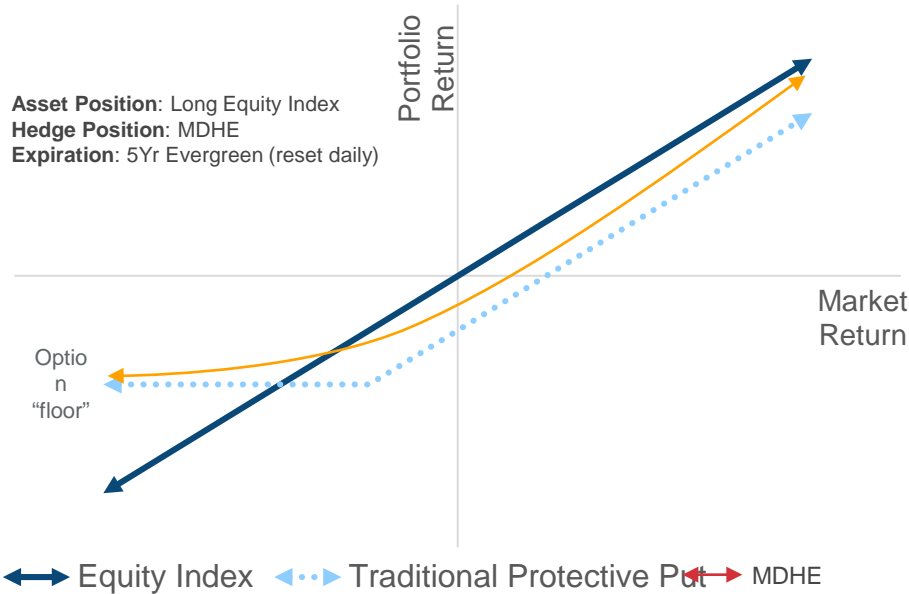
Replicating 6Y Option – Stylized Example

- Used OTC Dealer Implied Data
- Only Data From 2009 – Present is Available
- 5 Options Replicated (annually, starting at 1/9/2009, with 6-year maturity)
- Volatility Used for Delta Target for Replication Taken From MGI
- Bond Portfolio is ZCB with Maturity Matching Time to Expiry of Synthetic Option



Average Annualized Savings Via Replication: 1.62%

Milliman Dynamic Hedged Equity Strategy (MDHE)



For illustrative purposes only, does not represent the performance of any actual investment or portfolio, and should not be viewed as a recommendation to buy/sell.

MDHE has been a component of the Milliman Managed Risk Strategy since MMRS was inception.

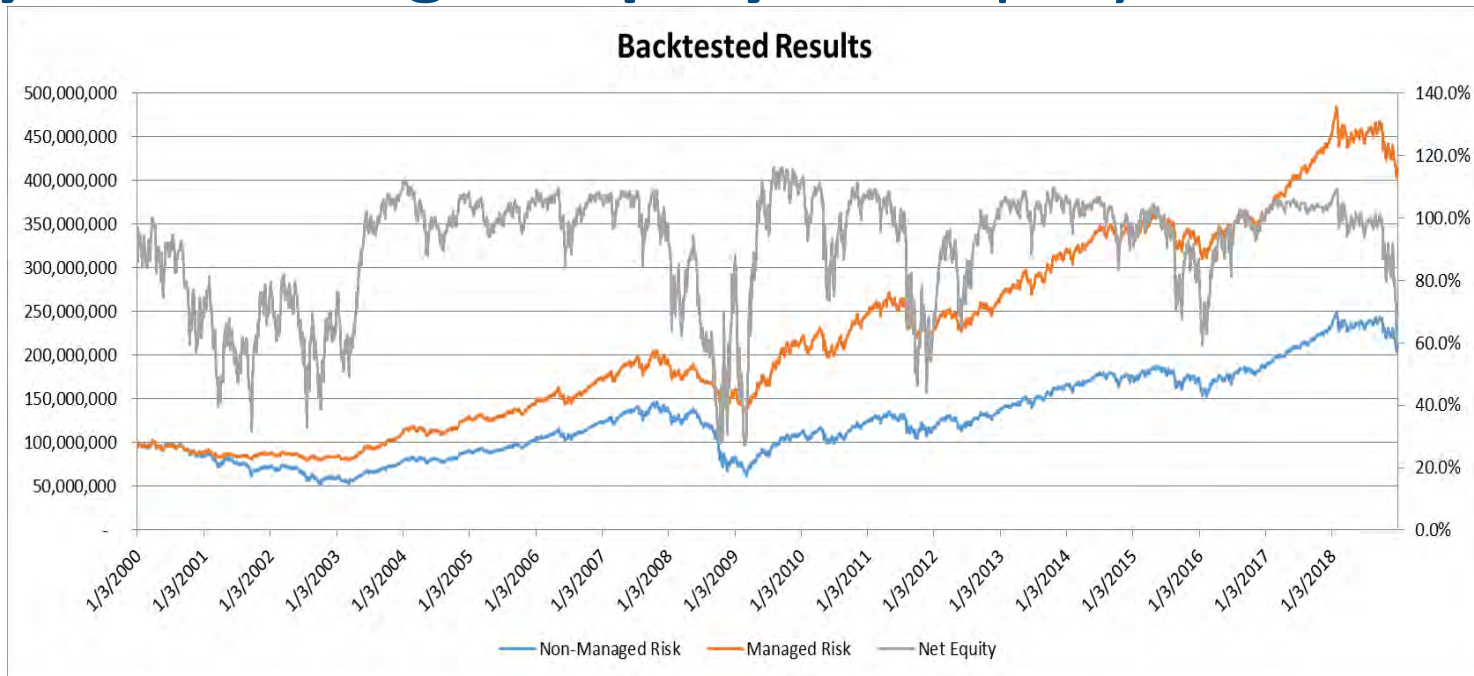
- MDHE is a long-dated constant maturity put replication that uses a delta adjustment to the equity exposure
 - Similar to a protective put, or synthetic long call
 - Implemented with futures contracts
- Seeks to reduce the downside exposure of the portfolio during significant and sustained market declines by:
 - Capturing gains after favorable returns on the portfolio's underlying holdings
 - Harvesting gains from the portfolio's offsetting positions after severe market downturns
- Can augment the strategy with options which may reduce capital in shock scenarios and increase downside protection

Milliman Dynamic Hedged Equity Strategy (MDHE)

- The addition of the options is designed to further enhance the tail risk protection provided by the capital protection strategy
- Long S&P 500 put options are incorporated into Milliman Dynamic Hedged Equity (MDHE) strategy in a delta-neutral manner by utilizing long equity futures to maintain consistent net equity exposure with the core MDHE strategy
- The amount of equity options to be held by the fund is established by evaluating a 5% equity shock scenario on a daily basis and the resulting projected trading requirements that are generated by the shock in order to restore the fund to its volatility and target equity range.
- The options strategy is designed to hold options that will provide 80% of these projected equity trading requirements
- The put options purchased are ~6 month (180-day) to expiration and are sold when there is ~35 days to expiration

Growth of \$100mm

Dynamic Hedged Equity: All Equity Portfolio

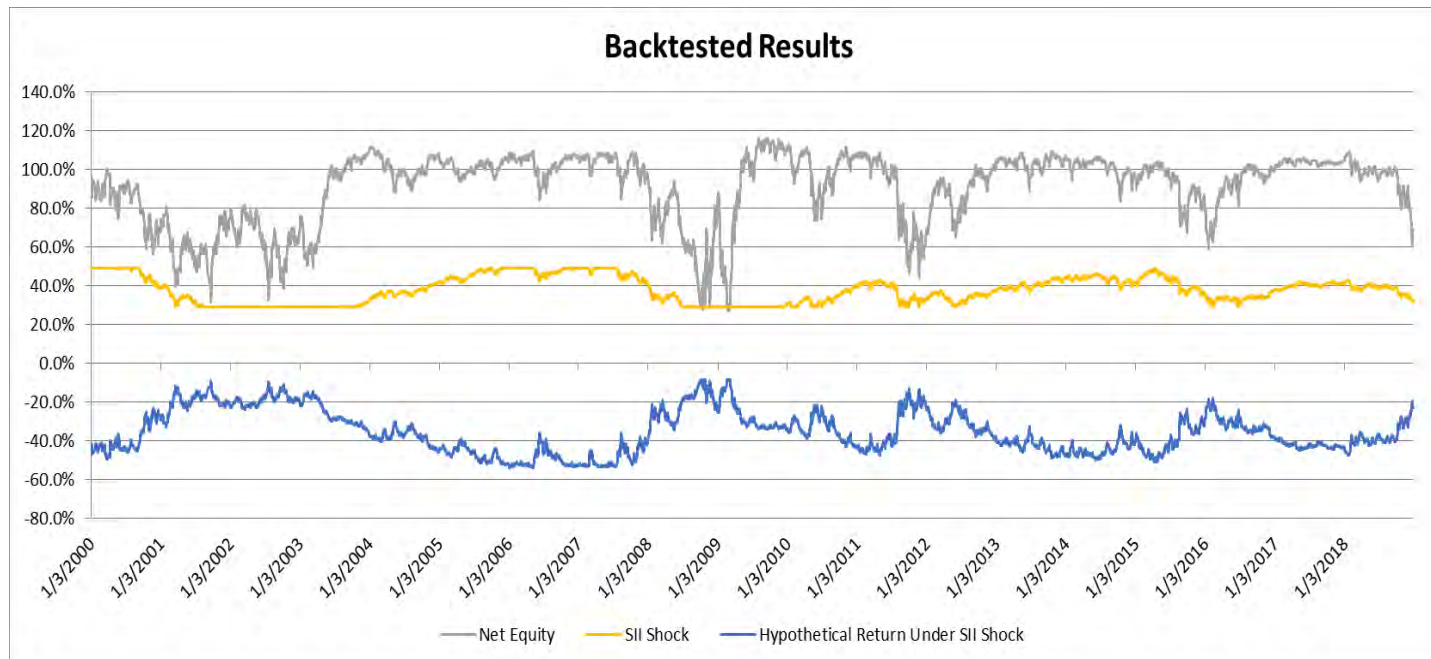


SEE SLIDE 23 FOR ADDITIONAL INFORMATION

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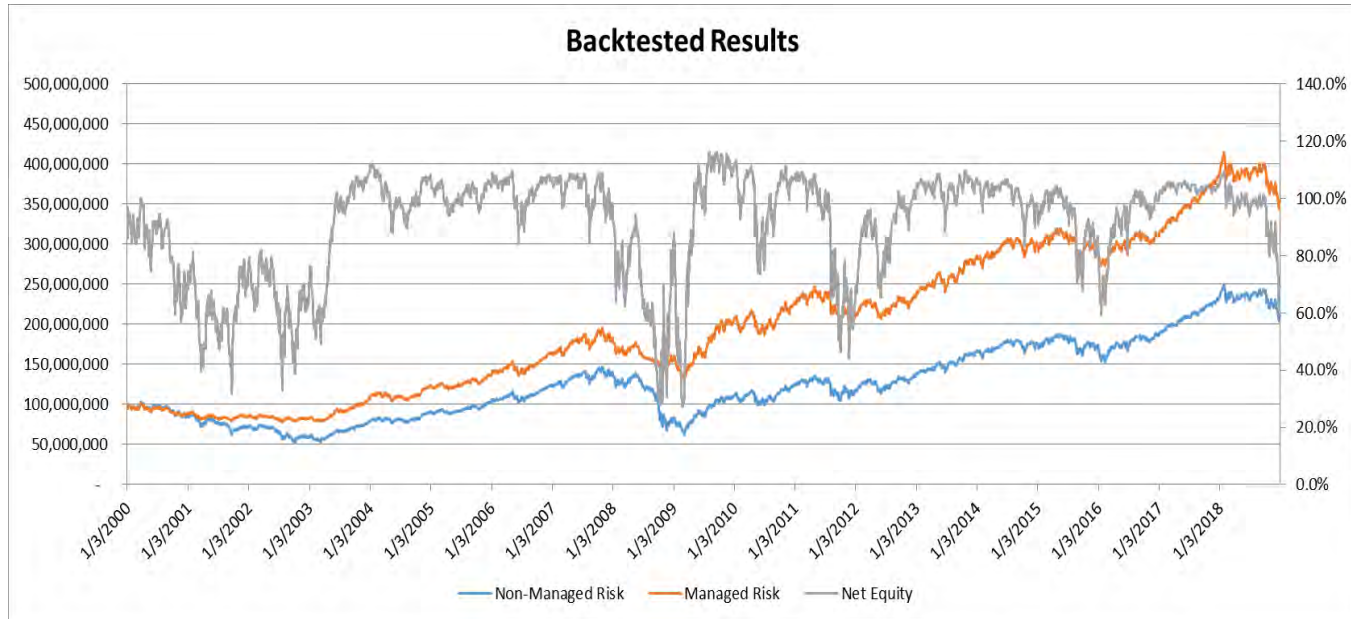


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Growth of \$100mm

Dynamic Hedged Equity + Options: All Equity Portfolio

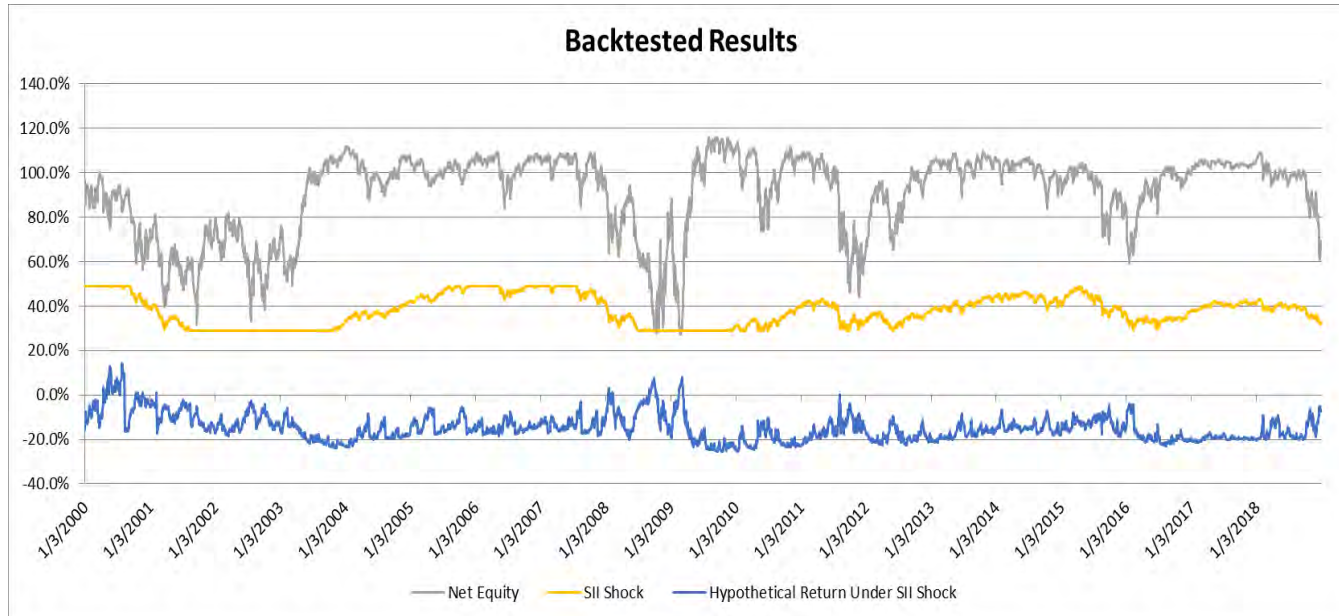


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Growth of \$100mm

Dynamic Hedged Equity + Options: All Equity Portfolio



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Backtested Performance Analysis: All Equity Portfolio

Analytics and Standardized Performance (2000-2018)			
Full Period	Non-Mgd	MDHE	MDHE + Options
Return	4.03%	7.77%	6.90%
Volatility	15.96%	11.79%	11.36%
Drawdown	-58.00%	-34.53%	-33.10%
Max Volatility	73.40%	30.17%	24.68%
Return/Risk	0.25	0.66	0.61

AVG SII CHARGE	37.95%	35.22%	14.70%
RETURN/CAPITAL	0.11	0.22	0.47

Time Period	Ann. Returns			Volatility		
	Non-Mgd	MDHE	MDHE + Options	Non-Mgd	MDHE	MDHE + Options
1 YR	-8.95%	-7.88%	-7.91%	12.52%	11.58%	10.97%
3 YR	7.20%	7.63%	6.83%	10.96%	9.72%	9.32%
5 YR	4.85%	5.19%	4.44%	11.04%	9.77%	9.40%

Time Period	Max Drawdown			Net Equity (Avg.)		
	Non-Mgd	MDHE	MDHE + Options	Non-Mgd	MDHE	MDHE + Options
1 YR	-18.56%	-16.99%	-17.09%	-	96.26%	96.26%
3 YR	-18.56%	-16.99%	-17.09%	-	97.09%	97.09%
5 YR	-18.88%	-16.99%	-17.09%	-	97.08%	97.08%

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Backtested Performance Analysis: All Equity Portfolio

Year	Annual Returns			Annual Volatility			Annual Max Drawdowns			Net Equity (Avg.)	
	Non-Mgd	MDHE	MDHE + Options	Non-Mgd	MDHE	MDHE + Options	Non-Mgd	MDHE	MDHE + Options	MDHE	MDHE + Options
2000	-14.0%	-10.8%	-11.8%	15.6%	12.8%	12.6%	-18.8%	-14.9%	-15.7%	83.7%	83.7%
2001	-15.8%	-1.7%	-2.9%	17.3%	9.5%	9.4%	-30.7%	-12.4%	-12.8%	60.9%	60.9%
2002	-18.9%	-4.1%	-3.5%	20.3%	9.8%	9.6%	-30.2%	-11.3%	-10.9%	64.0%	64.0%
2003	34.7%	33.3%	31.3%	13.6%	10.1%	10.0%	-14.0%	-6.7%	-7.1%	88.1%	88.1%
2004	15.8%	16.1%	14.5%	9.8%	10.0%	9.9%	-8.0%	-9.1%	-9.4%	101.4%	101.4%
2005	11.4%	10.2%	9.3%	7.9%	7.9%	7.9%	-6.3%	-6.2%	-6.3%	101.3%	101.3%
2006	21.6%	21.4%	21.0%	10.3%	10.3%	10.1%	-12.2%	-12.3%	-11.8%	103.0%	103.0%
2007	12.3%	11.8%	11.4%	13.3%	13.2%	12.6%	-11.3%	-10.9%	-10.1%	103.3%	103.3%
2008	-41.8%	-19.0%	-14.4%	33.2%	15.8%	14.7%	-51.5%	-27.5%	-22.3%	65.8%	65.8%
2009	35.5%	36.1%	29.4%	23.6%	18.5%	18.0%	-26.4%	-16.5%	-18.5%	93.0%	93.0%
2010	13.3%	15.4%	11.9%	16.4%	15.1%	14.1%	-15.5%	-14.5%	-13.9%	100.0%	100.0%
2011	-6.8%	-7.7%	-7.3%	21.1%	15.4%	14.3%	-22.9%	-19.3%	-18.0%	87.0%	87.0%
2012	16.8%	14.6%	11.5%	12.7%	10.4%	10.2%	-12.8%	-10.1%	-10.2%	88.7%	88.7%
2013	23.5%	22.7%	21.9%	9.9%	10.2%	10.0%	-8.5%	-9.6%	-9.4%	104.0%	104.0%
2014	4.8%	5.1%	4.8%	8.9%	8.6%	8.4%	-9.2%	-8.2%	-8.0%	101.2%	101.2%
2015	-1.8%	-1.7%	-2.7%	13.0%	11.0%	10.5%	-14.7%	-12.7%	-12.4%	93.0%	93.0%
2016	8.5%	8.0%	6.6%	13.0%	10.7%	10.3%	-11.3%	-6.4%	-6.3%	91.2%	91.2%
2017	24.7%	25.3%	24.2%	5.7%	5.9%	5.7%	-1.9%	-1.9%	-1.9%	103.9%	103.9%
2018	-9.0%	-7.9%	-7.9%	12.5%	11.6%	11.0%	-18.6%	-17.0%	-17.1%	96.3%	96.3%

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Additional information

- **Net Equity:** represents the net effective equity exposure of the Managed Risk Portfolio, including the baseline exposure from the underlying holdings, and the effect of the hedging strategy, which dials up or down the equity exposure in response to risk signals
- **Dynamic Hedged Equity: All Equity Portfolio (slides 17, 18, 21, 22)**
 - **Non-Managed Risk:** represents the hypothetical historical performance of a 100% MSCI ACWI Index portfolio, assuming that they did not employ an active risk management strategy
 - **Managed Risk:** represents the hypothetical historical performance of the non-managed risk investment portfolio, assuming they employed the Milliman Dynamic Hedge Equity (MDHE) Strategy over the entire time period
- **Dynamic Hedged Equity + Options: All Equity Portfolio (slides 19-20)**
 - **Non-Managed Risk:** represents the hypothetical historical performance of a 100% MSCI ACWI Index portfolio, assuming that they did not employ an active risk management strategy
 - **Managed Risk:** represents the hypothetical historical performance of the non-managed risk investment portfolio, assuming they employed the Milliman Dynamic Hedge Equity + Options (MDHE + Options) Strategy over the entire time period

Deep Reinforcement Learning Application



Overview of Deep Reinforcement Learning

What is Reinforcement Learning?

- Training an intelligent agent by allowing it to interact with a given environment and learning from trial and error within that environment

Why in Portfolio Management?

- Rather less mathematical constraints / assumptions for modeling – model-free
- Proven to work well to capture complex / non-linear patterns
- Neural net based deep learning can alleviate curse of dimensionality – enabling large scale portfolio management

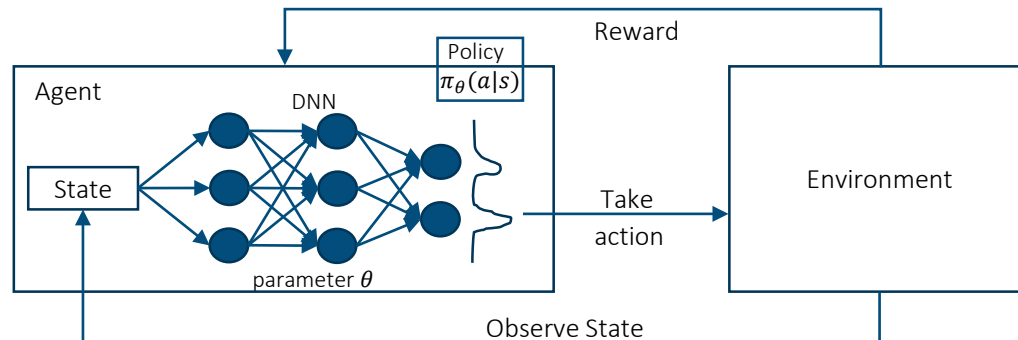
Examples of Different Reinforcement Agents

- Deep Q-Network (DQN): Generic Q-network with deep learning overlay
- Policy Search Based
 - ✓ Policy Gradient (PG)
 - ✓ Generic Actor-Critic
 - ✓ Proximal Policy Optimization (PPO): Surrogate objective function

Deep Reinforcement Learning – Deep Hedging (1)

Objective of Deep Hedging

- Given market signal information up to time t , liabilities, and a number of hedging instruments in a pre-defined asset universe, it is trying to determine the most optimized holdings across different hedging instruments while meeting some of the key constraints (i.e. liquidity limit, trading costs, etc) within their risk appetite (i.e. convex risk measure such as CTE)
- And, this is done through the Deep Reinforcement Learning (DRL) by providing “proper” rewards within the environment for certain actions (how to allocate holdings across different assets to hedge liabilities) that the agent takes



Deep Reinforcement Learning – Deep Hedging (2)

Overall Set-up

- Discrete time and market with friction
- $\{I_0, \dots, I_k\}$ is a set of market signals up to time k (t_k) that forms the filtration up to t_k
- Z is a F_T measurable random variable indicating liabilities (or, contingent claims)
- δ_k^i is i^{th} asset holdings at time t_k
- H_k is a set of constraints that δ_k is subject to at t_k
- $(\delta * S)_T$ is defined as $\sum \delta_k * (S_{k+1} - S_k)$
- $C_T(\delta)$ is defined as $\sum C_k(\delta_{k+1} - \delta_k)$, where C_k can take fixed, proportional, and rather complicated cross-asset cost functional forms
- P_0 is defined as cash injection or extraction
- ρ is a convex risk measure meeting the following three properties:
 - Monotone decreasing: if $x_1 \geq x_2$, then $\rho(x_1) \leq \rho(x_2)$; in words, this means more favorable positions require less cash injection
 - Convex: $\rho(\alpha x_1 + (1 - \alpha)x_2) \leq \alpha \rho(x_1) + (1 - \alpha)\rho(x_2)$; in words, diversification works
 - Cash-invariant: $\rho(x + c) = \rho(x) - c$, where $c \in \mathbb{R}$; in words, adding cash to a position reduces the need for more by that amount

Deep Reinforcement Learning – Deep Hedging (3)

Overall Set-up Cont.

- The problem we are trying to solve here becomes solving the following convex objective function:
- $\pi(-Z) := \inf_{\delta \in H} \rho(-Z + P_0 + (\delta * S)_T - C_T(\delta))$
- , where **inf** indicates the greatest lower bound
- In the neural net sense, what we are after using that objective function here is getting the most optimized holdings across different hedging instruments while meeting some of the key constraints (i.e. liquidity limit, trading costs, etc.) within certain risk appetite (i.e. convex risk measure such as CTE)
- $\delta_k^\theta := f^\theta(I_k, \delta_{k-1}^\theta)$
- , where θ indicates a set of parameters for the trained neural net
- , where f is a composite functional form of neural nets (i. e. $f(g(k(h(x))))$)
- , where δ_{k-1}^θ indicates the recurrent nature of the neural nets
- (a. k. a. past information from the previous time steps cascade forward to form time dependencies)
- And, we do this rebalancing exercise on a daily basis
 - ✓ The key structure of training an artificial agent is achieved through the DRL environment

Deep Reinforcement Learning – Equity Allocation Example (1)

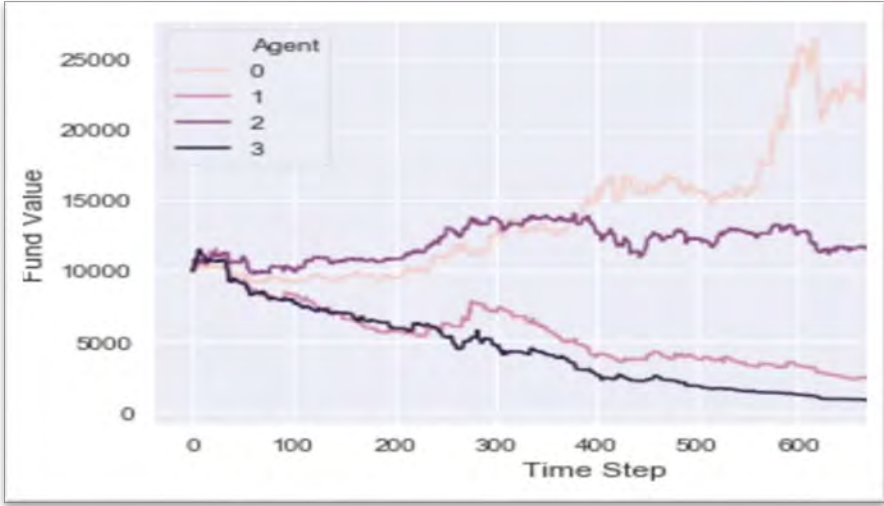
Overall Environment Set-up and Assumptions

- Portfolio: M risk assets (in my case, plain equities) + 1 risk-free asset (cash); in this exercise, M == 5
- State: state space indicates the “market condition” at a specific point in time, such as closing prices, mid prices, volume, PE ratio, PB ratio, etc. – can stem from one of those “signal processed” data by Natural Language Processing (NLP)
- In this study, somehow we use closing and high prices and we claimed that the combination of these two produced rather “better” results
- Fixed window for a time-series training == 10
- Action: action space is defined here as the “proper” or “desirable” allocating weights. Obviously $\sum_{i=1}^{M+1} a_{i,t} = 1$
- Reallocation is assumed to be done once a day in this exercise
- Reward: fluctuation of wealth minus transaction cost. Just for the experiment purpose, the transaction cost was assumed to be 0.25%
- Train the agent in a way that it maximizes the reward (i.e. Profit or Sharpe ratio in this example)

Deep Reinforcement Learning – Equity Allocation Example (2)

Sample Example Testing Results

- Agent 0: Deep Reinforced Agent
- Agent 1, 2, and 3: Other Sub-optimal Allocation Strategies



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