Equity-Based Insurance Guarantees Conference

Nov. 11-12, 2019

Chicago, IL

From Theory to Practice: FIA Index Crediting Hedging

Alex Hookway

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From Theory to Practice: FIA Index Crediting Hedging

ALEX HOOKWAY

AVP, Quantitative Risk Management, Nationwide Financial

November 11, 2019 (1045 – 1215 hours)

SOCIETY OF ACTUARIES®

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The United States antitrust laws aim to protect consumers by preserving the free economy and prohibiting anti-competitive business practices; they promote competition. There are both state and federal antitrust laws, although state antitrust laws closely follow federal law. The Sherman Act, is the primary U.S. antitrust law pertaining to association activities. The Sherman Act prohibits every contract, combination or conspiracy that places an unreasonable restraint on trade. There are, however, some activities that are illegal under all circumstances, such as price fixing, market allocation and collusive bidding.

There is no safe harbor under the antitrust law for professional association activities. Therefore, association meeting participants should refrain from discussing any activity that could potentially be construed as having an anti-competitive effect. Discussions relating to product or service pricing, market allocations, membership restrictions, product standardization or other conditions on trade could arguably be perceived as a restraint on trade and may expose the SOA and its members to antitrust enforcement procedures.

While participating in all SOA in person meetings, webinars, teleconferences or side discussions, you should avoid discussing competitively sensitive information with competitors and follow these guidelines:

- -Do not discuss prices for services or products or anything else that might affect prices
- -Do not discuss what you or other entities plan to do in a particular geographic or product markets or with particular customers.
- -Do not speak on behalf of the SOA or any of its committees unless specifically authorized to do so.
- -Do leave a meeting where any anticompetitive pricing or market allocation discussion occurs.
- -Do alert SOA staff and/or legal counsel to any concerning discussions
- -Do consult with legal counsel before raising any matter or making a statement that may involve competitively sensitive information.

Adherence to these guidelines involves not only avoidance of antitrust violations, but avoidance of behavior which might be so construed. These guidelines only provide an overview of prohibited activities. SOA legal counsel reviews meeting agenda and materials as deemed appropriate and any discussion that departs from the formal agenda should be scrutinized carefully. Antitrust compliance is everyone's responsibility; however, please seek legal counsel if you have any questions or concerns.

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What I thought the challenges of hedging would be

$$\frac{\partial u}{\partial t}(x,t) + \mu(x,t)\frac{\partial u}{\partial x}(x,t) + \tfrac{1}{2}\sigma^2(x,t)\frac{\partial^2 u}{\partial x^2}(x,t) - V(x,t)u(x,t) + f(x,t) = 0$$

$$u(x,t) = E^Q\left[\int_t^T e^{-\int_t^\tau V(X_\tau,\tau)\,d\tau} f(X_r,r)dr + e^{-\int_t^T V(X_\tau,\tau)\,d\tau}\psi(X_T)\middle| X_t = x\right]$$

$$\text{Vera} = \frac{\partial \rho}{\partial \sigma} = \frac{\partial^2 V}{\partial \sigma\,\partial r}$$

$$dX = \mu(X,t)\,dt + \sigma(X,t)\,dW^Q$$

$$\text{Zomma} = \frac{\partial \Gamma}{\partial \sigma} = \frac{\partial \text{vanna}}{\partial S} = \frac{\partial^3 V}{\partial S^2\,\partial \sigma}$$

$$\text{Charm} = -\frac{\partial \Delta}{\partial \tau} = -\frac{\partial \Theta}{\partial S} = -\frac{\partial^2 V}{\partial \tau\,\partial S}$$

$$P(S,T) = A(S,T)\exp(-B(S,T)r(S))$$

$$dr(t) = [\theta(t) - \alpha(t)r(t)]\,dt + \sigma(t)\,dW(t)$$

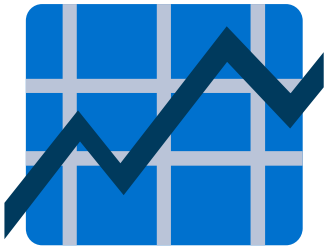
$$B(S,T) = \frac{1-\exp(-\alpha(T-S))}{\alpha}$$

$$A(S,T) = \frac{P(0,T)}{P(0,S)}\exp\left(-B(S,T)\frac{\partial \log(P(0,S))}{\partial S} - \frac{\sigma^2(\exp(-\alpha T) - \exp(-\alpha S))^2(\exp(2\alpha S)-1)}{4\alpha^3}\right)$$

But some of our hardest problems are more mundane.

Source of prop equations: Wikipedia Hull-White Model, Feynman–Kac formula, Greeks (finance)

Proprietary Indices Overview

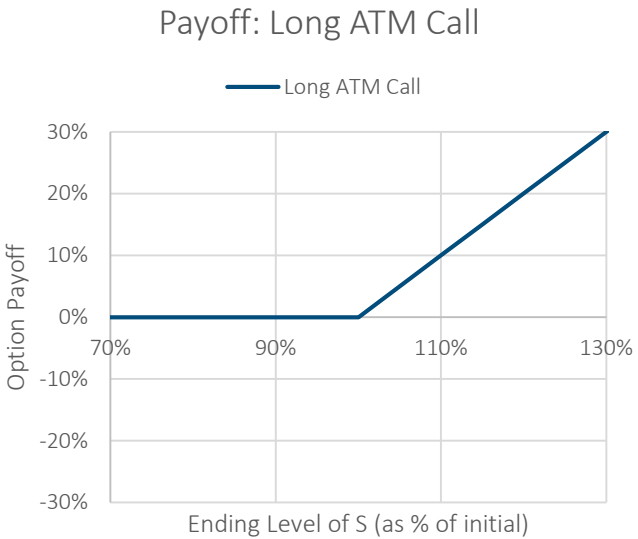


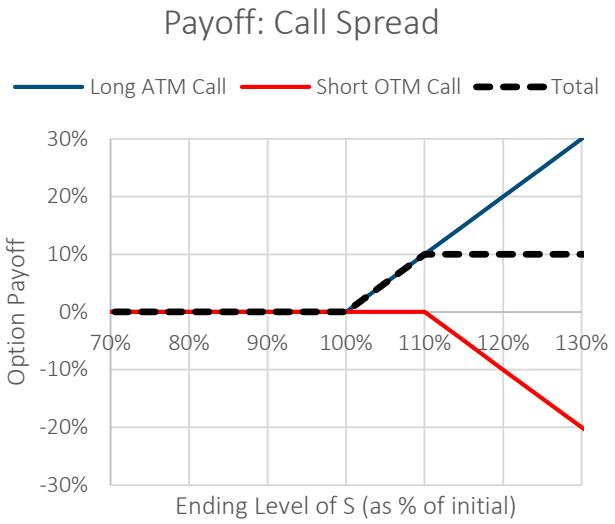
- Proprietary indices are often complex
  - Many underlyings, potentially in international markets
  - Volatility control and other complexities
- Hedge the market exposure by:
  1. Purchasing static options from a counterparty and/or
  2. Dynamic hedging of underlying exposures

Focus of this presentation will be #1.

This will be easy…

- I can hedge this perfectly, just match the payoffs!

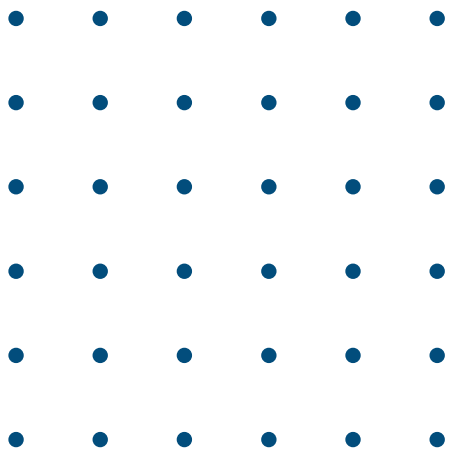




- Automatically match all of the greeks, even the weird ones.

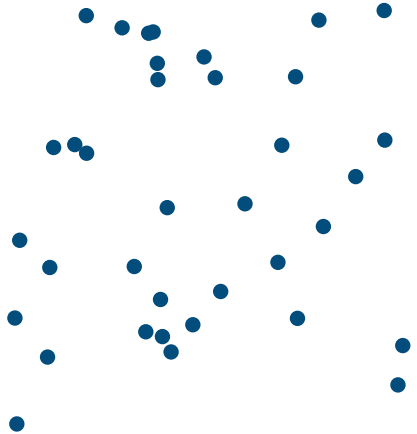
The Idealized Pricing World

- Issue a bunch of business all at once (or maybe at a regular time interval if you are fancy)
- Buy a hedge asset for each policy
- On modeled lapses/deaths, sell back part of the hedge asset
- (Let hedge team figure out the pesky details)



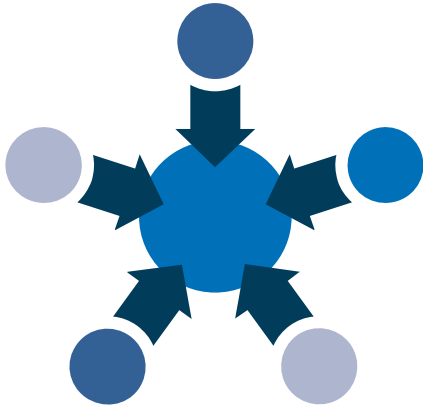
Messy Reality

- Policies are issued every day with crediting term starting immediately
- Not feasible to buy an option for every policy
- Unmanageable to sell back a portion of every option as experience emerges
- (We'll figure out an approach…)

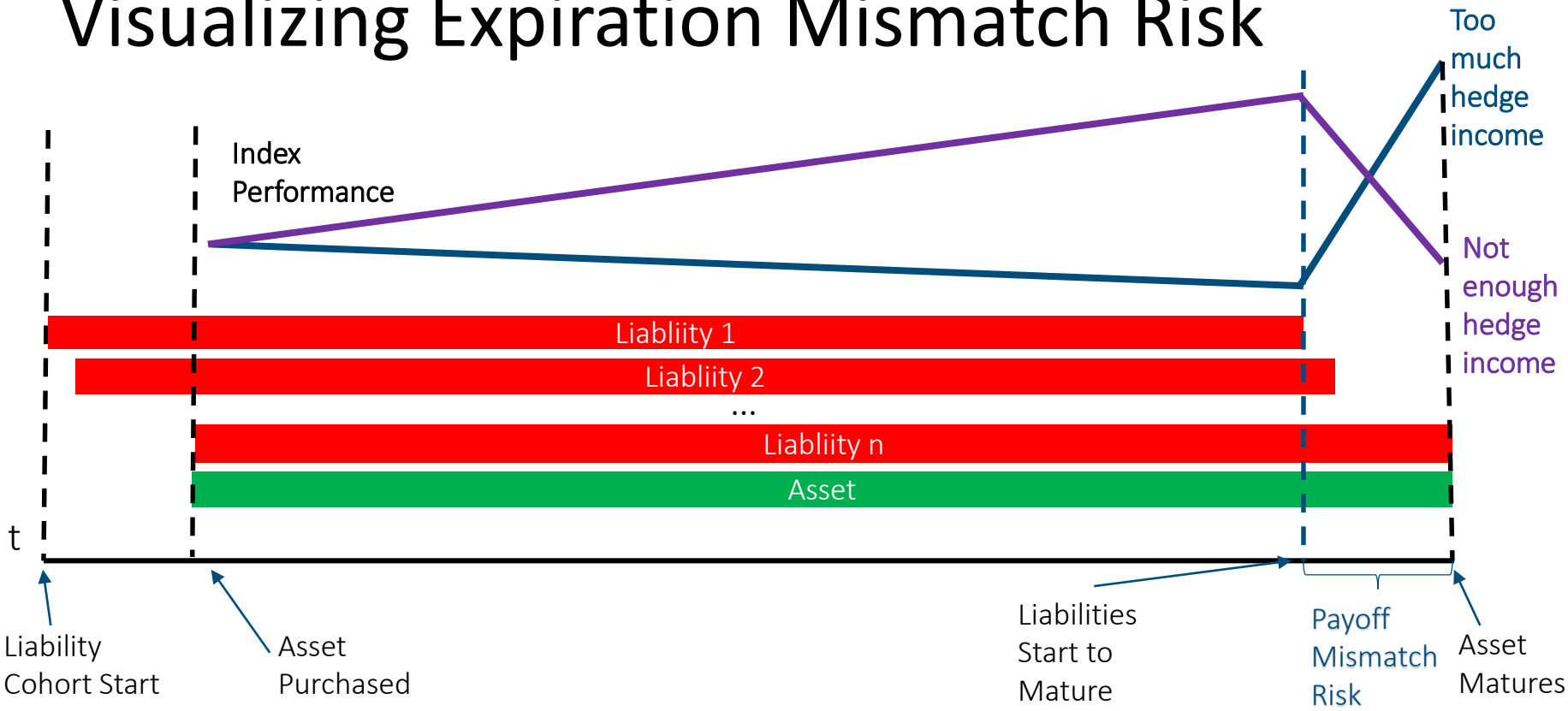


Buying for a Cohort of Policies

- Can’t buy an asset for every policy, so what can we do?
- Group policies by similar characteristics
  - What characteristics?
  - How to choose asset characteristics?
    - Optimize?
      - To what objective?
      - Varying what?



Visualizing Expiration Mismatch Risk



FIA crediting isn't a European Call

- Policyholders can lapse and perhaps receive a partial credit
- Complexity of crediting
- Unique product features (e.g. policyholders may be able to elect to "lock-in" index levels as of a given date)

But hedge instruments are usually vanilla European calls

Unanticipated Complications

Source: https://www.kantei.go.jp/jp/98_abe/actions/201904/30taiirei.html

Actuarial Assumptions for Asset Purchases

- Can manage assumption experience by:
  - Assuming some rates of decrements (mortality/lapse) in purchasing the options
  - Periodically buying/selling back to rebalance based on experience
  - Rebalancing using another hedge instrument
  - (or some combination)
- During the surrender charge period, impact is relatively small
  - After the surrender charge, potentially massive!

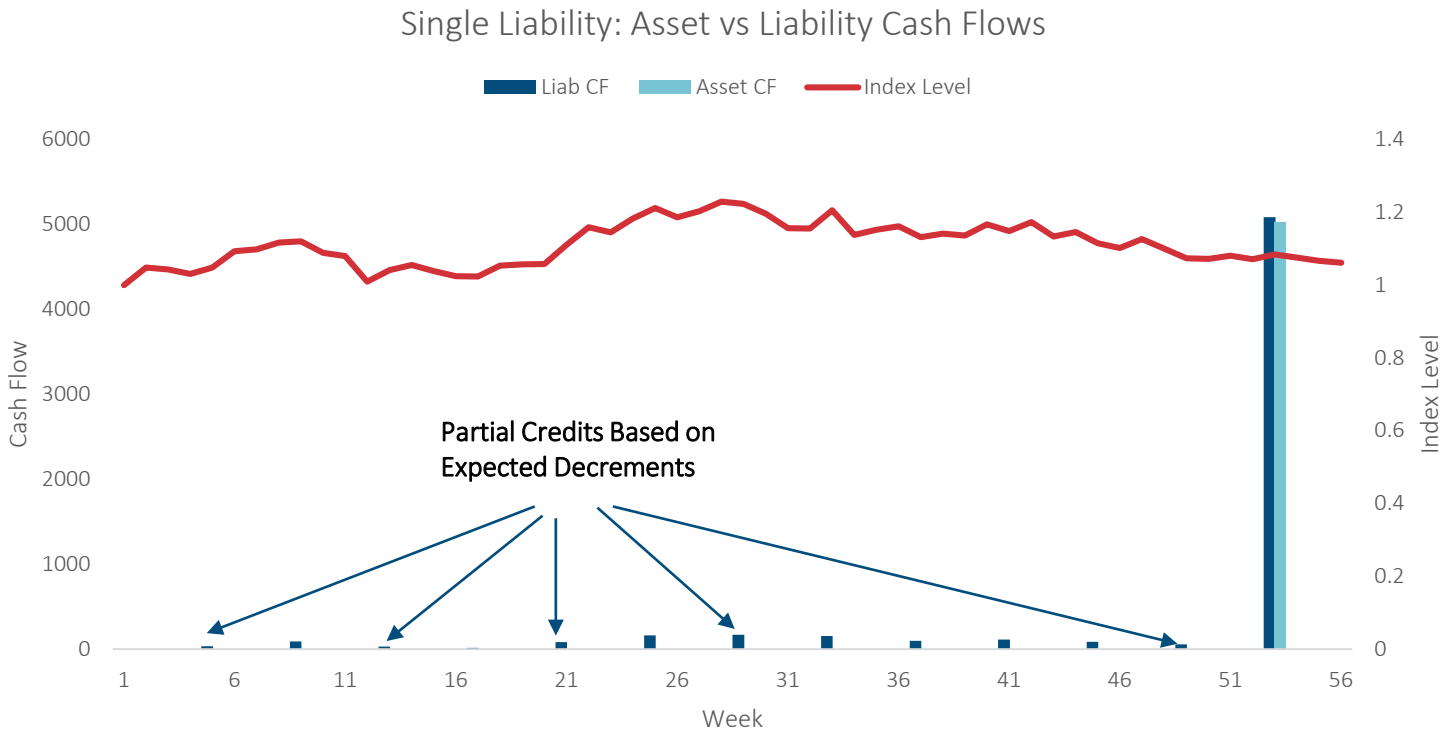
Choosing and Optimizing Cohorts

- Tradeoff: Minimize potential mismatch while limiting number of purchases to a manageable level.
- Path dependent payoff for liability
- Simulation is your friend
  - Playing your strategy though many realistic scenarios can reveal weaknesses
  - Simulation results can also drive optimization

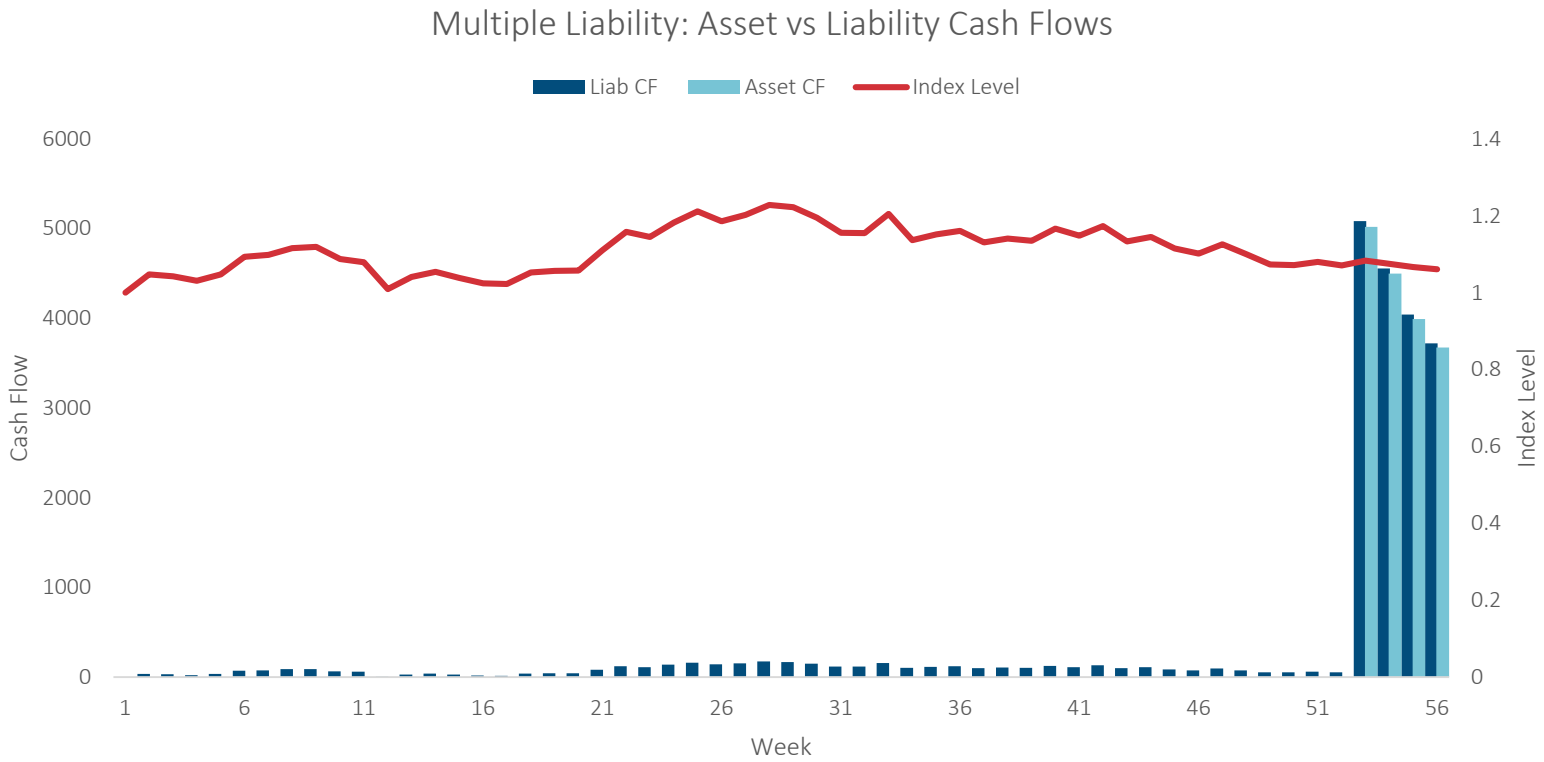
“Perfect” Hedging

- An asset for every day of issues isn’t enough
  - Need a tiny asset for each expected decrement also to be perfectly immunized
  - AND I need to predict the rate of decrements perfectly
  - Perfect hedging isn’t feasible, so what level of mismatch can we accept?

Cash Flow Comparison: Single Day of Issues



Cash Flow Comparison: Multiple Days of Issues

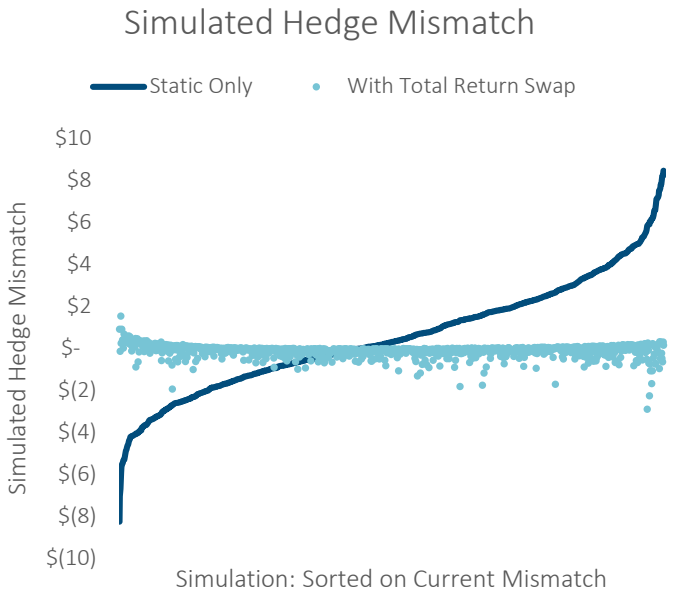


Results of Simulation

- Tradeoff: Minimize potential mismatch while limiting number of purchases to a manageable level.
- Path dependent payoff for liability
- Minimizing expected hedge mismatch via simulation
  - Rule of thumb: Buy an asset for each issue date/index combination if the volume is “big enough”
  - Choose asset notional and strike based on simulated payoffs

Total Return Swaps

- Another powerful tool to manage exposure: Total Return Swaps on the index
  - OTC instruments that provide “Delta-1” exposure to the index
  - Allows for rebalancing “around the edges” for emerging experience while most exposure is covered by call options



Emerging Issues

- End of the surrender charge period
  - Lapse rates are tame during surrender charge period, but (likely) large and (definitely) uncertain after
- Developing capability to manage portion of risk internally

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