# QFI – Quantitative Finance Exam

Spring 2023/Fall 2023

**Important Exam Information:**

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| Exam Registration | Candidates may register online or with an application. |
| Order Study Notes | Study notes are part of the required syllabus and are not available electronically but may be purchased through the online store. |
| Syllabus Readings | Readings listed in this syllabus may include study notes, online readings and textbooks. Candidates are responsible for all readings in their entirety, including sections such as Appendices, unless it is stated otherwise in the syllabus. |
| Introductory Study Note | The Introductory Study Note has a complete listing of all study notes as well as errata and other important information. |
| Case Study | A case study will not be used for this examination. |
| Past Exams | Past Exams from 2000-present are available on SOA website. |
| Updates | Candidates should be sure to check the Updates page on the exam home page periodically for additional corrections or notices to the current syllabus. |
| Formula Package | A Formula Package will be provided with the exam. Please see the Introductory Study Note for more information. |
| Table | A Cumulative normal distribution table will be provided with the exam. |

## 1. **Topic: Stochastic Calculus**

**Learning Objectives**

The candidate will understand the foundations of quantitative finance.

**Learning Outcomes**

The Candidate will be able to:

a) Understand and apply concepts of probability and statistics important in mathematical finance

b) Understand the importance of the no-arbitrage condition in asset pricing

c) Understand the Ito integral and stochastic differential equations

d) Understand and apply Ito's Lemma

e) Understand the Black-Scholes-Merton PDE (partial differential equation)

f) Understand and apply Jensen's Inequality

g) Understand the distinction between complete and incomplete markets

h) Define and apply the concepts of martingale, market price of risk, and measures in single and multiple state variable contexts

i) Demonstrate understanding of the differences and implications of real-world versus risk-neutral probability measures, and when the use of each is appropriate

j) Understand and apply Girsanov's theorem in changing measures

**Resources**

- *An Introduction to the Mathematics of Financial Derivatives,* Hirsa, Ali and Neftci, Salih N., 3rd Edition 2nd Printing, 2014
    - Chapter 1 (background)
    - Chapters 2-3
    - Chapters 4-5 (background)
    - Chapters 6-15 (excluding section 8.2.4)

  ***Note**:* Candidates should verify that their copy of the Hirsa & Neftci textbook is the 3rd edition, second printing by checking that page iv, *Notices* section, begins with "Knowledge and best practice in this field…". If the *Notices* section begins "No responsibility is assumed…" it is the first printing and should be replaced.

- *Problems and Solutions in Mathematical Finance: Stochastic Calculus*, Chin, Eric, Nel, Dian and Olafsson, Sverrir, 2014

  ***Note:*** Candidates should study the following problems in Chin, Nel, and Ólafsson alongside the matching chapters of *An Introduction to the Mathematics of Financial Derivatives,* Hirsa and Neftci to reinforce the standard techniques used in stochastic calculus. Please note that formulas from Chin, Nel, and Ólafsson are not included in the formula package.

| Chin et al. Chapter | Pages | Item | Corresponding Hirsa & Neftci Chapter sections |
|---|---|---|---|
| 1 | 1 to 3 | Definitions 1.1 to 1.7<br>(Note that statement (b) of Definition 1.7 involves integration using a measure-theoretic approach. An equally valid statement can be made using a Riemann-Stieltjes integral for continuous distributions or a sum for discrete distributions.) | 5.2 |
| | | | |
| | 18 to 19 | Q7 | 15.4.1 |
| | 43 to 44 | Q4 | 14.2.2 |
| | 43 to 44 | Q5 | Pg. 245 paragraph containing equations from (14.91) - (14.93) |
| 2 | 52 | Definitions 2.1, 2.2 | 8.2.1 |
| | 52 | Theorems 2.3 and 2.4 | 5.8 |
| | 52 | Definitions 2.5 and 2.6 | 2.2.9, 6.2 |
| | 55 to 57 | Q1 to Q3 | 6.4.1 |
| | 57 to 68 | Q4 to Q13, except Q11 | 8.2.1 |
| | | | |
| | 71 | Q1 | 6.6.1 |
| | 72 | Q2 | 6.6.2 |
| | 72 to 74 | Q2-Q5 | 6.6.3 |
| 3 | 96 to 97 | Theorem 3.2 | 10.4 |
| | 97 to 98 | Theorem 3.3 | 21.4 |
| | 99 to 100 | Definition 3.6 | 10.4 |
| | 115 | Q13 | First part is different approach to part of question #12 in pg. 64 |
| | 123 to 147 | Q1 to Q19 | 10.5 |
| | 155 to 158 | Q1 to Q3 | 10.7 |
| | 175 to 178 | Q10 | 10.7 |
| 4 | 186 to 187 | Definitions 4.1(a) - (f) | H&N Introduces self-financing portfolios in 6.11.3 and goes into detail in 12.3 |
| | 189 | Theorem 4.6 | 14.3 |
| | 192 to 194 | Q1 | 6.11 and exercise 1 in pg. 109 |
| | 192 to 194 | Q2 | 6.11 and exercise 4 in pg. 110 |
| | 194 to 197 | Q1 to Q3 | 14.3 |
| | 221 to 242 | Q1 to Q17 | 15 |
| 5 | 262 to 264 | Q9 to Q11 | 5.5.5, 8.2.2 |
| | 281 to 285 | Q1, Q2 Jump diffusion | Chapter 11: section on pure jump framework |

*** means there is no direct correspondence

## 2. **Topic: Interest Rate Models and Hedging**

**Learning Objectives**

The candidate will understand:

- The quantitative tools and techniques for modeling the term structure of interest rates.
- The standard yield curve models.
- The tools and techniques for managing interest rate risk.

**Learning Outcomes**

The Candidate will be able to:

a) Understand and apply the concepts of risk-neutral measure, forward measure, normalization, and the market price of risk, in the pricing of interest rate derivatives

b) Understand and apply various one-factor interest rate models and various simulation techniques including Euler-Maruyama discretization and transition density methods

c) Calibrate a model to observed prices of traded securities

d) Describe the practical issues related to calibration, including yield curve fitting

e) Demonstrate understanding of option pricing theory and techniques for interest rate derivatives

f) Apply the models to price common interest sensitive instruments including: callable bonds, bond options, caps, floors and swaptions

g) Understand and apply the techniques of interest rate risk hedging

h) Understand and apply the Heath-Jarrow-Morton approach including the LIBOR Market model

i) Understand and apply multifactor interest rate models

**Resources**

- *An Introduction to the Mathematics of Financial Derivatives,* Hirsa, Ali and Neftci, Salih N., 3rd Edition 2nd Printing, 2014
  - Ch. 16-20
- *Fixed Income Securities: Valuation, Risk, and Risk Management*, Veronesi, Pietro, 2010
  - Ch. 14
  - Ch. 15 (excluding Appendix)
  - Ch. 16 (including Case Study, excluding Appendix)
  - Ch. 18 (excluding Appendix)
  - Ch. 19 (excluding Appendix)
  - Ch. 20
  - Ch. 21 (excluding Appendix)
  - Ch. 22.1-22.4
- QFIQ-116-17: Low Yield Curves and Absolute/Normal Volatilities

- QFIQ-130-21: Interest Rate Models – Theory and Practice, Second Edition, Brigo, Damiano and Mercurio, Fabio, 2006, sections 4.2.1, 4.2.2, and 4.2.5 only
- QFIQ-136-23: Calibrating Interest Rate Models
- Understanding the Connection between Real-World and Risk-Neutral Scenario Generators
    - Introduction
    - Section 1
    - Section 2
    - Section 3
    - Appendix D

3. **Topic: Equity Option Pricing and Hedging**

**Learning Objectives**

The candidate will understand:

- How to apply the standard models for pricing financial derivatives.
- The implications for option pricing when markets do not satisfy the common assumptions used in option pricing theory.
- How to evaluate risk exposures and the issues in hedging them.

**Learning Outcomes**

The Candidate will be able to:

a) Demonstrate an understanding of option pricing techniques and theory for equity derivatives

b) Identify limitations of the Black-Scholes-Merton pricing formula

c) Demonstrate an understanding of the different approaches to hedging – static and dynamic

d) Demonstrate an understanding of how to delta hedge, and the interplay between hedging assumptions and hedging outcomes

e) Analyze the Greeks of common option strategies

f) Appreciate how hedge strategies may go awry

g) Describe and explain some approaches for relaxing the assumptions used in the Black-Scholes-Merton formula

h) Compare and contrast the various kinds of volatility, e.g., actual, realized, implied and forward, etc.

i) Define and explain the concept of volatility smile and some arguments for its existence

j) Compare and contrast “floating” and “sticky” smiles

k) Describe and contrast several approaches for modeling smiles, including stochastic volatility, local-volatility, jump-diffusions, variance-gamma, and mixture models

l) Explain various issues and approaches for fitting a volatility surface

**Resources**

- *The Volatility Smile*, Derman, Emanuel and Miller, Michael B., 2016
  - Ch. 1, 2 (background)
  - Ch. 3-11
  - Ch. 14
  - Ch. 17-19
- QFIQ-115-17: Which Free Lunch Would You Like Today, Sir?: Delta Hedging, Volatility Arbitrage and Optimal Portfolios
- QFIQ-120-19: Chapters 6 and 7 of *Pricing and Hedging Financial Derivatives*, Marroni, Leonardo and Perdomo, Irene, 2014

## 4. Topic: Applications

**Learning Objectives**

The candidate will learn how to apply the techniques of quantitative finance to applied business contexts.

**Learning Outcomes**

The Candidate will be able to:

a) Identify and evaluate embedded options in liabilities, e.g., indexed annuity, structured product based variable annuity, and variable annuity guarantee riders including GMxB, etc.

b) Demonstrate an understanding of embedded guarantee risk including market, insurance, policyholder behavior, and basis risk

c) Demonstrate an understanding of dynamic and static hedging for embedded guarantees, including:

   i. Risks that can be hedged, including equity, interest rate, volatility, and cross Greeks

   ii. Risks that can only be partially hedged or cannot be hedged including policyholder behavior, mortality and lapse, basis risk, counterparty exposure, foreign bonds and equities, correlation, and operational failures

d) Demonstrate an understanding of target volatility funds and their effect on fund performance, guarantee cost, and risk control

e) Demonstrate an understanding of how differences between modeled and actual outcomes for guarantees affect financial results over time

**Resources**

- QFIQ-124-20: Variable Annuity Volatility Management: An Era of Risk-Control
- QFIQ-128-20: Mitigating Interest Rate Risk in Variable Annuities: An Analysis of Hedging Effectiveness under Model Risk
- QFIQ-132-21: Investment Instruments with Volatility Target Mechanism, Albeverio, Steblovskaya, and Wallbaum, 2013
- QFIQ-134-22: An Introduction to Computational Risk Management of Equity-Linked Insurance, Feng, 2018 (sections 1.2-1.3, 4.7,4.8 (background), 6.2-6.3)
- QFIQ-135-22: Structured Product Based Variable Annuities, Deng, Dulaney, Husson, McCann (sections 2 & 3)