SOCIETY OF ACTUARIES

Advanced Long-Term Actuarial Mathematics Exam

Exam ALTAM

Date: October 21, 2025

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.
2. **Question 1 is to be answered in the Excel workbook. For this question only the work in the Excel workbook will be graded.**
3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations, for referencing tables, or for statistical functions, but any work in the Excel booklet will not be graded.
4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Excel Answer Instructions

1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Excel questions will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. When you finish, save your Excel workbook with a filename in the format xxxxx_ALTAM where xxxxx is your candidate number. Your name must not appear in the filename.
6. Record your candidate number in the indicated cell in the Excel Question worksheet.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.
2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.
3. Write in pen on the lined side of the answer sheet.
4. The answer should be confined to the question as set.
5. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet.
6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support may not receive full or partial credit.
7. When you finish, hand in <u>all</u> your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

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****BEGINNING OF EXAMINATION****
****ADVANCED LONG-TERM ACTUARIAL MATHEMATICS****

Provide the response for Question 1 in the Excel Question worksheet

1.

(*10 points*) An insurer issues a 10-year Equity-Linked product with a Guaranteed Minimum Maturity Benefit (GMMB) and a Guaranteed Minimum Death Benefit (GMDB).

You are given the following additional information:

(i) The policyholder is age 70 at issue.
(ii) The initial premium is P = 10,000.
(iii) The death benefit is payable at the end of the year of death.
(iv) Mortality follows the Standard Ultimate Survival Model.
(v) There are no lapses or transaction costs.
(vi) Management charges of 2% of the fund value are deducted at the start of each year.
(vii) The premium is invested in a stock index fund with volatility $\sigma = 25\%$.
(viii) The risk free rate is $r = 0.05$ compounded continuously.

(a) (*2 points*) The GMMB is 100% of the initial premium. Calculate the value at issue of the GMMB.

The GMDB is equal to the initial premium accumulated to the end of the year of death at an annual effective interest rate of $i_g = 5\%$.

(b) (*1 point*) Your colleague says that you should use American option pricing for the GMDB, rather than European option pricing, because there are multiple possible payment dates. Explain briefly the difference between an American option and the GMDB.

1. Continued

(c) (*4 points*) The hedge cost at issue for the GMDB payable at t, assuming the policyholder dies in the t-th year is denoted $v(0,t)$. Complete Table 1 below to calculate $v(0,t)$ for $t = 1,2,\ldots,10$. You should find that $v(0,1) = 1080.99$.

Table 1: GMDB Hedge Calculations

Year of death *t*	Guarantee on death in year t	$d_1(0)$	$d_2(0)$	$v(0,t)$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

(d) (*2 points*) Calculate the value at issue of the GMDB.

(e) (*1 point*) Explain briefly why the GMMB value at issue is greater than the GMDB value at issue, even though the GMMB guarantee is smaller.

2.

(*7 points*) An individual age 40 purchases a Type A Universal Life insurance policy with a Face Amount of 100,000.

The policy offers a no-lapse guarantee that specifies that from time 20, the policy will not lapse before age 90, provided that the policyholder pays a minimum premium of 1000 each year for the first 20 years.

You are given the following information:

(i) Cost of Insurance rates are set at 120% of the Standard Ultimate Survival Model (SUSM).
(ii) The interest rate for discounting the cost of insurance charge is $i_q = 0.02$.
(iii) The credited rate for year 1 is 4%.
(iv) Expense charges are 50 per policy per year plus 3% of premiums.
(v) There are no corridor factors.
(vi) There is no additional charge for the no-lapse guarantee.

(a) (*1 point*) Briefly describe the difference between Type A and Type B Universal Life products.

(b) (*2 points*) The policyholder pays a premium of 1500 at the start of the first year. Calculate the account value at the end of the first year, AV_1.

You are also given:

(i) After 20 years, the policy qualifies for the no-lapse guarantee.
(ii) The valuation basis for the no-lapse guarantee is the SUSM with $i = 5\%$.
(iii) The account value at the end of year 20 is $AV_{20} = 33{,}590$.
(iv) The account value at the end of year 40 is $AV_{40} = 26{,}403$.

(c) (*2 points*)

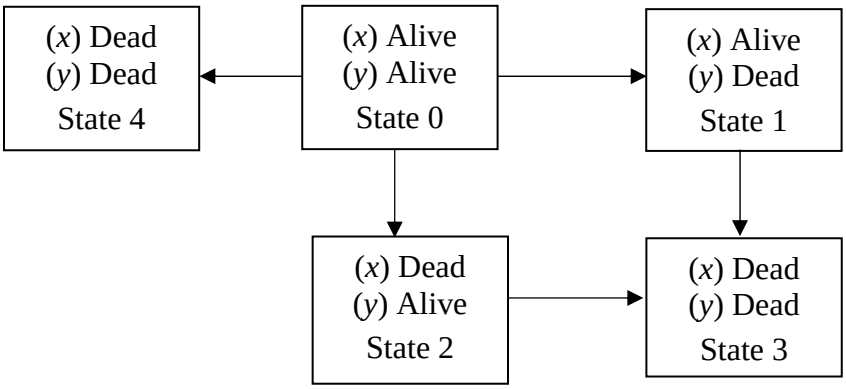
(i) Calculate the reserve for the no-lapse guarantee at the end of year 20.

(ii) Calculate the reserve for the no-lapse guarantee at the end of year 40.

(d) (*2 points*) Give two advantages and two disadvantages from the perspective of the insurer of offering a no-lapse guarantee on a Universal Life insurance product.

3.

(*11 points*) An insurance company uses the following model to price joint life insurance products.



Let μ_z^s denote the force of mortality for a life age z under the Standard Ultimate Survival Model (SUSM).

The transition intensities of the joint life model are:

$$\mu_{x:y}^{01} = \mu_y^s - \lambda; \quad \mu_{x:y}^{02} = \mu_x^s - \lambda; \quad \mu_{x:y}^{04} = \lambda$$
$$\mu_{x:y}^{13} = \mu_x^s; \quad \mu_{x:y}^{23} = \mu_y^s$$

(a) (*4 points*)

(i) Show that the individual force of mortality for (x) is the same whether (y) is alive or dead, and hence that the marginal distribution of T_x is the same as the distribution under the SUSM.

(ii) Your colleague states that because the individual force of mortality for (x) is the same whether (y) is alive or dead, and similarly for (y), then T_x and T_y must be independent. Explain in words why he is wrong.

(iii) Another colleague states that because $\mu_x^{02} < \mu_x^{13}$ this model incorporates a "broken heart" effect. State with reasons whether she is correct.

3. Continued

(b) (*2 points*) Let ${}_t p_z^s$ denote the *t*-year survival probability for a life age *z* under the SUSM.

Show that ${}_t p_{x:y}^{00} = {}_t p_x^s \, {}_t p_y^s \, e^{\lambda t}$.

A couple, $x = 50$ and $y = 55$, purchases a special whole life reversionary annuity. The annuity is valued using the model above.

You are given the following information:

(i) An annuity of 50,000 per year is payable continuously, starting immediately on the first death, and continuing until the second death.
(ii) Premiums are paid continuously at a rate of *P* per year while both (*x*) and (*y*) are alive.
(iii) If both lives die simultaneously, a death benefit of the total premium paid is paid immediately on death. That is, the death benefit payable at time *t* if both lives die simultaneously at that time is *tP*.
(iv) You should use Woolhouse's two term formula to evaluate continuous annuity functions.
(v) $i = 0.05$
(vi) $\lambda = 0.0005$
(vii) $\bar{a}_{50:55}^{00} = 14.7443$

(c) (*4 points*) Let $\left(\overline{Ia}\right)_{x:y}^{00}$ denote the EPV of a continuously increasing joint life annuity, with a rate of payment of *t* per year at time *t*, payable continuously while (*x*) and (*y*) both survive.

(i) Show that the EPV of the common shock death benefit is $\lambda P\left(\overline{Ia}\right)_{50:55}^{00}$.

(ii) Show that the EPV of the annuity benefit is 129,800 to the nearest 100. You should calculate the value to the nearest 0.1.

(iii) You are given that $\left(\overline{Ia}\right)_{50:55}^{00} = 167.13$. Calculate the annual net premium for the policy.

(d) (*1 point*) Write down Thiele's differential equation for the net premium policy value at time $t > 0$, assuming both lives are alive.

4.

(*10 points*) An insurer is analyzing its mortality experience, using the Alive-Dead model, assuming the force of mortality μ_x is constant within the age range [*x*, *x*+1). There are *n* independent lives in the group.

Let t_j denote the waiting time in $[x, x+1)$ for the *j*-th life in the group, and let $\delta_j = 1$ if the *j*-th life died in $[x, x+1)$, $\delta_j = 0$ otherwise.

You are given that the contribution of the *j*-th life to the likelihood function for μ_x may be written as $f_j(t_j, \delta_j) = e^{-t_j \mu_x} (\mu_x)^{\delta_j}$.

(a) (*5 points*)

(i) Briefly describe the circumstances under which $t_j < 1$ and $\delta_j = 0$ for the *j*-th life.

(ii) Write down the equation for log-likelihood function for μ_x in terms of t_j and δ_j, $j = 1, 2, \cdots, n$.

(iii) Show that the maximum likelihood estimator for μ_x is $\hat{\mu}_x = \frac{d_x}{E_x^c}$, where E_x^c and d_x are functions you should specify.

(iv) Show that $\text{Var}[\hat{\mu}_x] \approx \frac{d_x}{(E_x^c)^2}$.

The following data have been collected for age 65 for two different policy lines. Assume all lives are independent.

	Line A		**Line B**	
x	E_x^c	d_x	E_x^c	d_x
65	150,150	1,550	18,070	132

(b) (*1 point*)

(i) Calculate $\hat{\mu}_{65}$ for each line.

(ii) Estimate the standard deviation of $\hat{\mu}_{65}$ for each line.

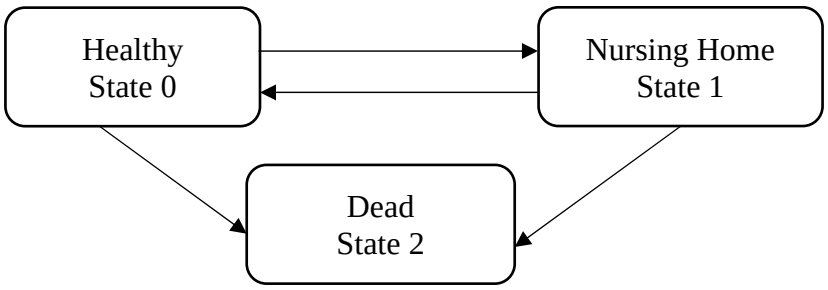
4. Continued

(c) (*2 points*) Show that the age 65 forces of mortality for the two lines are significantly different from each other, at a 95% confidence level.

(d) (*2 points*) Write down three possible reasons why the mortality experience of the two lines might be significantly different from each other.

5.

(*12 points*) NED Insurance Company uses the following multi-state model to price and reserve term life insurance policies offering an additional benefit for lives in Nursing Home care.



Let P_x denote the one-year probability transition matrix for a person age *x*.
You are given:

$$P_{80} = \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.40 & 0.40 & 0.20 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } P_{81} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.30 & 0.40 & 0.30 \\ 0 & 0 & 1 \end{bmatrix}$$

You are also given that all transitions occur before the year end.

A Healthy life age 80 purchases a 2-year policy with the following benefits:

- A death benefit of 100,000 payable at the end of the year of death.
- A benefit of 50,000 payable at the end of each year if the policyholder is in a Nursing Home at that time.
- Level gross annual premiums are payable conditional on the policyholder being in State 0 at the payment date.

You are also given:

(i) $v = 0.95$

(ii) The expense to issue the policy is 500. This would be considered a pre-contract cost in a profit testing exercise.

(iii) The maintenance expense for the policy is 300 at the beginning of each year including the first.

5. Continued

(a) (*3 points*)

(i) Calculate the 2-year transition matrix for a life age 80.

(ii) Show that the expected present value of benefits for this policy is 33,000 to the nearest 1000. You should calculate the value to the nearest 1.

(b) (*1 point*) The gross annual premium for this policy is calculated using the equivalence principle. Show that the gross annual premium is 19,400 to the nearest 100. You should calculate the value to the nearest 1.

The gross premium policy value at time *t* for a policy in state *j* is denoted ${}_tV^{(j)}$.

(c) (*2 points*)

(i) Show that ${}_1V^{(0)}$ is -100 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Show that ${}_1V^{(1)}$ is 48,000 to the nearest 1000. You should calculate the value to the nearest 1.

NED conducts a profit test of the contract using the following assumptions:

(i) The earned interest is 12%.
(ii) The hurdle rate is 10%.
(iii) Reserves are the gross premium policy values.

All other assumptions are the same as the premium basis above.

The expected profit at time *t* per policy in state *j* at time $t-1$ is denoted by $Pr_t{}^{(j)}$.

(d) (*5 points*)

(i) Show that $Pr_1{}^{(0)}$ is 1800 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Calculate $Pr_2{}^{(0)}$ and $Pr_2{}^{(1)}$.

(iii) Calculate the profit signature for this policy.

(iv) Calculate the profit margin for this policy.

5. Continued

(e) (*1 point*) Explain why the profit margin for this contract will increase if NED increases its reserves at time 1.

6.

(*10 points*) Lin is a member of a final average salary pension plan. On December 31, 2024, Lin is exactly 50 years old, with 20 years of pensionable service. Lin's salary in 2024 was 100,000.

You are given the following information for a plan funding valuation on January 1, 2025:

(i) The accrual rate is 1.5% per year of service.
(ii) Final average salary is defined as the average salary in the two years before retirement.
(iii) The benefit on retirement is a pension payable monthly in advance for life. There are no spousal benefits.
(iv) Annual salaries are assumed to increase by 3% each year on January 1. Salaries are paid in level installments at the end of each month within each calendar year of employment.
(v) Pension payments are assumed to increase monthly at an annual effective rate of 2%.
(vi) The pension plan uses the projected unit credit method to determine pension reserves and funding.
(vii) You are given the following valuation assumptions:
- All lives remain in service until retirement (that is, there are no deaths or withdrawals before retirement).
- $i = 0.06$
- At an interest rate of $i^* = \frac{1.06}{1.02} - 1$, $\ddot{a}^{(12)}_{60|i^*} = 16.2437$ and $\ddot{a}^{(12)}_{65|i^*} = 14.5260$

Lin is considering two retirement options.

Option A: Retire at age 60 with an actuarial reduction of 0.3% per month up to age 65.
Option B: Retire at age 65 with no actuarial reduction.

(a) (*2 points*)

(i) Calculate the amount of the first month's pension payment under Option A.

(ii) Calculate the amount of the first month's pension payment under Option B.

(b) (*1 point*) Describe briefly the circumstances that would justify the valuation assumption that there are no pre-retirement exits.

6. Continued

(c) (3 *points*)

(i) Calculate the actuarial liability assuming Lin will choose Option A.

(ii) Calculate the actuarial liability assuming Lin will choose Option B.

(iii) State with reasons whether the actuarial reduction factor appears to be appropriate.

(d) (*2 points*) You are also given:

- Lin pays contributions of 8% of her salary at the end of each month. The balance of the cost of her benefits is paid by her employer.
- Employer contributions are deposited into the pension plan each month, at the same time as the employee contributions.

(i) Calculate the EPV at the valuation date of Lin's pension contributions in 2025.

(ii) Calculate the employer's net normal contribution rate for Lin's retirement benefits in 2025, after taking Lin's contributions into consideration, assuming Lin will choose Option B.

(e) (*2 points*)

(i) The plan actuary currently assumes that all plan members choose Option B. Critique this assumption.

(ii) The plan actuary currently assumes that the mortality after age 65 of retirees who chose option A is the same as the mortality of those who chose option B. Critique this assumption.

****END OF EXAMINATION****