SOCIETY OF ACTUARIES

Advanced Short-Term Actuarial Mathematics Exam

Exam ASTAM

Date: October 22, 2025

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.

2. **Question 1 is to be answered in the Excel workbook. For this question only the work in the Excel workbook will be graded.**

3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations or for statistical functions, but any work in the Excel booklet will not be graded.

4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Excel Answer Instructions

1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Question 1 will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. When you finish, save your Excel workbook with a filename in the format xxxxx_ASTAM where xxxxx is your candidate number. Your name must not appear in the filename.
6. Record your candidate number in the indicated cell in the Excel Question worksheet.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.

2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.

3. Write in pen on the lined side of the answer sheet.

4. The answer should be confined to the question as set.

5. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet.

6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support may not receive full or partial credit.

7. When you finish, hand in all your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

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****BEGINNING OF EXAMINATION****
*****ADVANCED SHORT-TERM ACTUARIAL MATHEMATICS*****

Provide the response for Question 1 in the Excel Question worksheet

1.

(*12 points*) Table 1 gives the aggregate claims (in 000s) for property claims under travel insurance policies. The policies were written in bulk through r=5 different tour companies over an n=8-year period.

Table 1								
Aggregate claims (in 000's) by company/year, Y_{ij}								
Tour Company i	Year j							
	1	2	3	4	5	6	7	8
1	615.32	726.24	587.88	847.74	660.05	344.59	305.93	460.34
2	345.50	386.99	615.22	625.12	510.00	269.89	306.32	444.64
3	256.57	287.73	312.75	444.19	337.47	212.68	253.59	390.12
4	191.13	225.07	225.04	245.24	267.69	145.71	169.79	260.33
5	128.85	119.92	167.10	273.09	287.92	161.85	164.44	176.13

(a) (*4 points*) You will use the data in Table 1 to calculate the credibility premium in year 9, for each tour company using the Bühlmann model with empirical Bayes parameter estimation.

(i) Calculate the empirical Bayes estimates of v, a, Z_1, and μ. You should find that the estimate of a is 23,850 to the nearest 10.

(ii) Calculate the year 9 premiums in 000's for each tour company.

(b) (*2 points*) Explain briefly why the Bühlmann-Straub model with empirical Bayes parameter estimation might be more appropriate than the Bühlmann model for these data.

(c) (*1 point*) Below are graphs showing the aggregate claims per insured traveller (in Figure 1), and per insured traveller-day (in Figure 2). State with reasons which risk volume measure you would recommend.

1. Continued

(d) (*4 points*) The insurer decides to use the number of traveller-days for the risk volume measure. The claims per insured traveller-day for each year and for each company are given in Table 2 below. The numbers of insured traveller-days for each year and for each company are given in Table 3 below.

(i) Calculate the following empirical Bayes estimates of v, a, Z_1, and μ for the Bühlmann-Straub model. You should find that the estimate of a is 4 to the nearest integer.

(ii) Calculate the credibility premium in year 9 for each tour company, 000's, using the Bühlmann-Straub model, assuming the number of traveller-days is the same for each company in year 9 as in year 8.

(e) (*1 point*) Explain briefly why the Year 9 premium for Company 1 is smaller using the Bühlmann-Straub model than with the Bühlmann model.

1. Continued

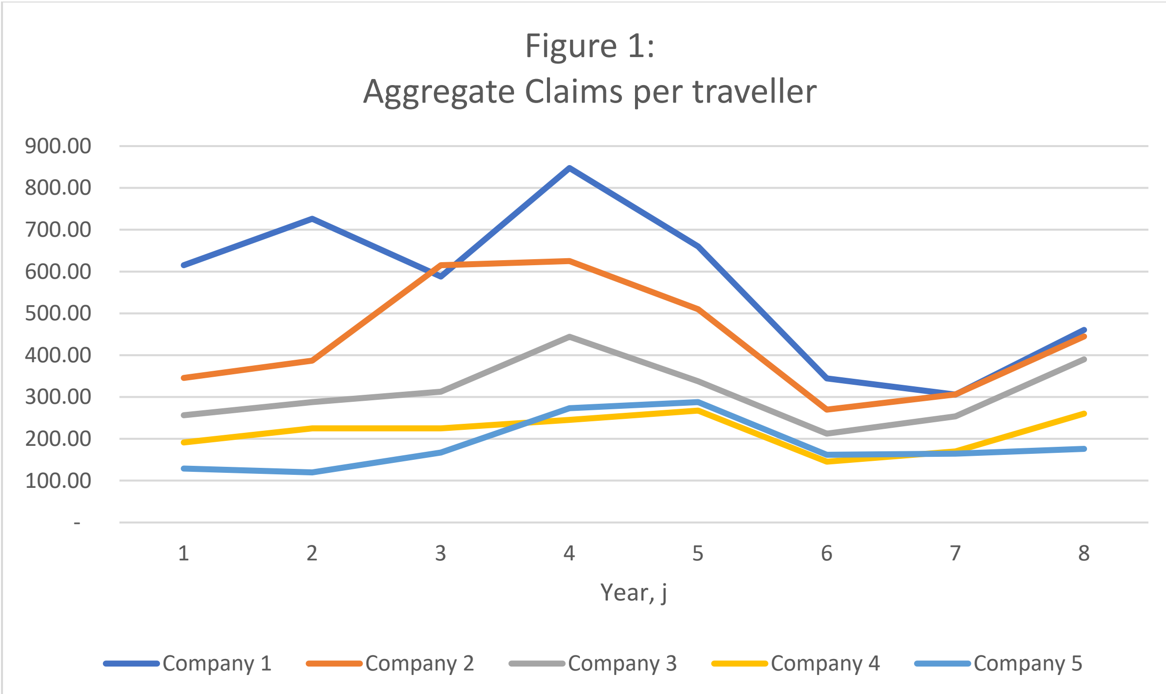

Figure 1:
Aggregate Claims per traveller
900.00
800.00
700.00
600.00
500.00
400.00
300.00
200.00
100.00
-
1
2
3
4
5
6
7
8
Year, j
Company 1
Company 2
Company 3
Company 4
Company 5

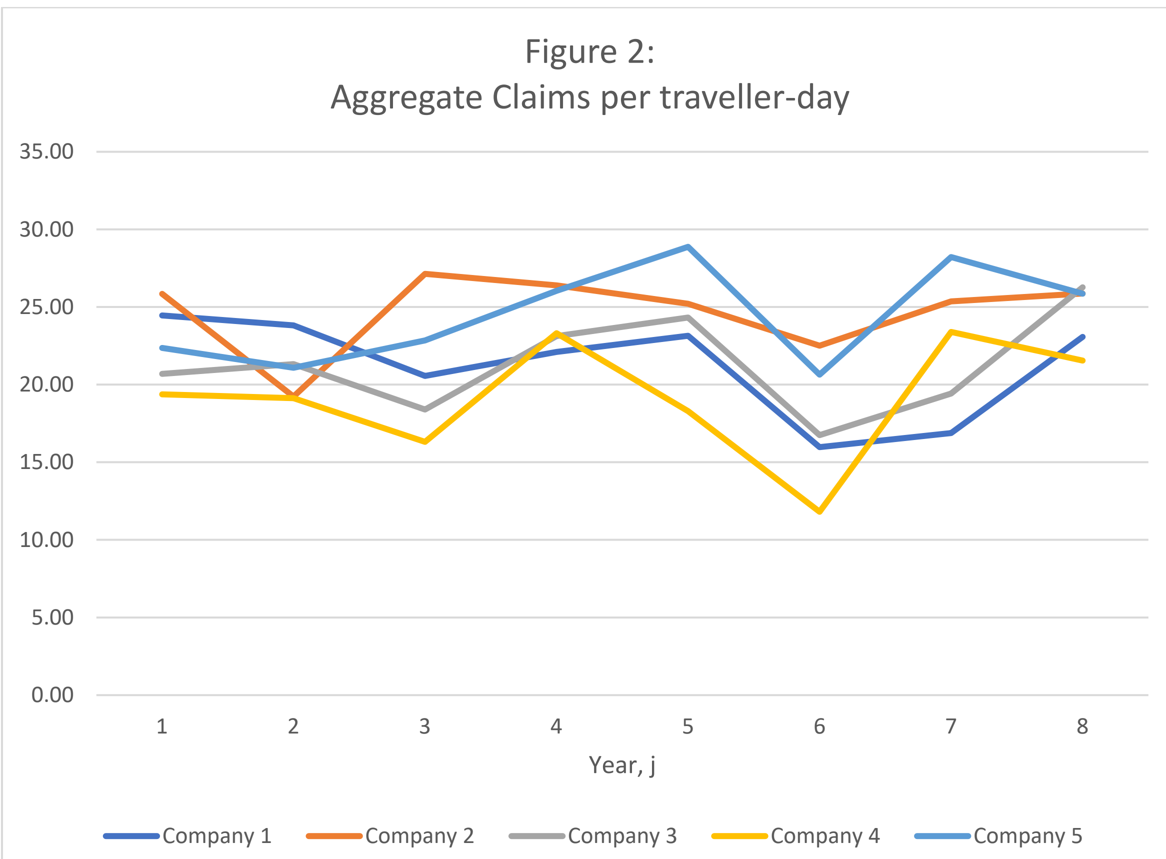

Figure 2:
Aggregate Claims per traveller-day
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
1
2
3
4
5
6
7
8
Year, j
Company 1
Company 2
Company 3
Company 4
Company 5

1. **Continued**

Table 2 Aggregate claims per traveller-day of insurance cover by company/year, X_{ij}								
Tour Company i	Year j							
	1	2	3	4	5	6	7	8
1	24.46	23.82	20.56	22.11	23.14	15.98	16.88	23.07
2	25.85	19.24	27.14	26.40	25.20	22.50	25.36	25.87
3	20.69	21.32	18.39	23.13	24.32	16.75	19.43	26.28
4	19.37	19.13	16.31	23.31	18.29	11.81	23.39	21.55
5	22.36	21.09	22.84	26.05	28.88	20.65	28.22	25.86

Table 3 Number of traveller-days of insurance cover by company/year, m_{ij}								
Tour Company i	Year j							
	1	2	3	4	5	6	7	8
1	25155	30489	28597	38349	28519	21562	18122	19951
2	13368	20116	22667	23681	20236	11994	12080	17187
3	12400	13493	17003	19208	13876	12696	13054	14846
4	9868	11768	13795	10522	14637	12334	7258	12080
5	5763	5687	7315	10485	9970	7839	5828	6812

2.

(*9 points*) You are analyzing the distribution of the number of claims during a one-year period for a portfolio of 2000 property insurance policies. The number of claims for each policy are assumed to be independent and identically distributed. You are given the following observed data:

Number of Claims	Number of Policies
0	1325
1	516
2	120
3	39
4 or more	0

You first fit the data using a negative binomial distribution with $r = 2$, and unknown β.

You are given that the maximum likelihood estimate of β is $\hat{\beta} = 0.21825$.

(a) (*2.5 points*) Show that the maximum value of the log-likelihood function under this model is -1747 to the nearest 1. You should calculate the value to the nearest 0.01.

(b) (*2.5 points*) You conduct a chi-square goodness of fit test with 5 cells at the 5% significance level.

(i) Show that the test statistic for this test is 14 to the nearest 1. You should calculate the value to the nearest 0.1.

(ii) Determine the critical value for this test.

(iii) State the conclusion of this test.

You next consider an alternative negative binomial model with both r and β estimated by maximum likelihood. You are given the following information.

- The maximum likelihood estimates are $\hat{r} = 3.814$, $\hat{\beta} = 0.114$.
- The value of the log-likelihood function with these parameters is -1744.43.
- The test statistic for a chi-square goodness of fit test with 5 cells is 12.982.

2. **Continued**

(c) (*3 points*) State with reasons which of the two models is preferred based on each of the following criteria:

(i) The Akaike Information Criterion (AIC).

(ii) The Schwartz Bayes Criterion (SBC/BIC).

(iii) The likelihood ratio test at the 5% significance level.

(iv) The chi-square goodness of fit test.

(d) (*1 point*) Your colleague suggests using a zero-modified negative binomial instead of either of the models used above. Explain why this suggestion is unlikely to significantly improve the chi-square goodness of fit of the negative binomial distribution for these data.

3.

(*9 points*) The loss severity random variable X has a Uniform distribution on the interval $[a, b]$. We denote this by $X \sim \text{Unif}(a,b)$.

You are given that the survival function of X is $S_X(x) = \dfrac{b-x}{b-a}, \quad a < x < b$.

Let $Y = X - d \mid X > d$ denote the excess loss random variable where $0 \le a \le d < b$.

(a) (*2 points*) Show that $Y \sim \text{Unif}(0, b-d)$.

Now let $a = 0, b = 10,$ and $d = 2$.

(b) (*3 points*) Let H_ξ denote the Generalized Extreme Value distribution with parameter ξ. You are given that Y is in the MDA of H_ξ with $\xi = -1$. Derive the normalizing sequences, c_n and d_n, for the limiting extreme value distribution of Y.

(c) (*2 points*) Let $Y_1, Y_2, \ldots, Y_n$ be independent and identically distributed random variables such that $Y_j \sim \text{Unif}(0,8)$ for $j = 1, 2, \cdots, n$, and let $M_n = \max\{Y_1, Y_2, \cdots, Y_n\}$.

(i) Calculate $\Pr[M_{10} \le 7]$ directly.

(ii) Estimate $\Pr[M_{10} \le 7]$ using the limiting extreme value distribution.

(d) (*2 points*)

(i) State with reasons whether Y is in the MDA of the Fréchet, Gumbel, or Weibull Extreme Value (EV) distribution.

(ii) Describe one key difference between the Weibull EV distribution and the Gumbel distribution.

4.

(*11 points*) For a health insurance policy, the number of losses in a year is distributed as a Poisson distribution with an expected number of losses equal to 1.

The ground-up loss severity distribution is given in the following table:

Amount	Probability
100	0.6
500	0.3
1000	0.1

Number of losses and loss severity are independent.

(a) (*3 points*)

(i) Calculate the Loss Elimination Ratio (LER) with an ordinary deductible of 60.

(ii) Calculate the insurer's expected payment per loss with an ordinary deductible of 60.

(iii) Calculate the ordinary deductible that would be necessary to achieve a LER of 40%.

(b) (*2 points*)

(i) Describe the difference between an ordinary deductible and a franchise deductible.

(ii) Describe a type of insurance which might typically use a franchise deductible.

The insurer decides to impose an ordinary deductible on each claim that is the greater of 20% of the loss or 100. Let *S* denote the aggregate claims paid after the deductible for each policy.

(c) (*4 points*)

(i) Show that $\Pr[S=0]$ is 0.67 to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(ii) Using the recursive method, calculate $\Pr[S<1000]$. (Hint: Work in units of 400.)

4. **Continued**

(d) (*2 points*) The insurer purchases stop-loss insurance with an aggregate deductible of 1000, based on claims paid by the insurer. Calculate the net premium for this coverage.

5.

(*10 points*) A casualty insurance company sells a product that pays a single fixed payment of 200 if the insured is hospitalized.

The following are the cumulative claim payments, in 000's, from the third quarter of 2023 through to the end of 2024.

You can assume that there are no claim payments after Development Quarter 5.

Cumulative claim payments by quarter-year (in 000's)						
Accident Quarter	Development Quarter					
	0	1	2	3	4	5
Q3-2023	70	190	236	276	310	316
Q4-2023	80	220	280	330	360	
Q1-2024	80	230	300	360		
Q2-2024	90	260	340			
Q3-2024	100	290				
Q4-2024	100					

(a) (*4 points*)

(i) Show that the projected cumulative claims incurred in 2024, using the Chain Ladder method, total 1840 to the nearest 10. You should calculate your answer to the nearest 0.1.

(ii) Explain why the inflation-adjusted chain ladder would not be appropriate in this case.

(iii) Calculate the projected claims payments in Quarter 2 of 2025 in respect of claims arising in 2024.

(b) (*1 point*) Explain what is meant by the term "calendar year effect" in relation to estimating outstanding claims using the chain ladder method.

5. Continued

The insurer uses the 'Large (L) – Small (S)' test of development factors to test for calendar year effects by quarter-year. The L-S triangle for the development factors is given below. Median values are indicated by *.

Accident Quarter	Development Quarter 0	1	2	3	4
Q3-2023	S	S	S	L	*
Q4-2023	S	S	*	S	
Q1-2024	*	L	L		
Q2-2024	L	L			
Q3-2024	L				

Let $k=1$ denote the quarter year Q4-2023, $k=2$ denote the quarter year Q1-2024, etc.

(c) (*4 points*)

(i) Write down the null hypothesis for the test.

(ii) Write down the values of n_k, m_k, and Z_k for $k=1,2,3,4$.

(iii) Calculate $E\left[Z_4\right]$ under the null hypothesis.

(iv) You are given that under the null hypothesis, $E\left[Z\right]=3.0$ and $\text{Var}\left[Z\right]=1.125$. Calculate the *p*-value of the test.

(v) State the conclusion of the test.

(d) (*1 point*) Describe one reason why there might be a calendar year effect in a claims development triangle.

6.

(*9 points*) Claims from an insurance portfolio have a compound distribution. The primary distribution is negative binomial, with parameters $r=5$ and $\beta=3$. The secondary distribution is gamma with parameters $\alpha=0.035$ and $\theta=36,000$.

(a) (*2 points*)

(i) Show that the mean of the aggregate distribution is 19,000 to the nearest 1,000. You should calculate the value to the nearest 10.

(ii) Show that the standard deviation of the aggregate distribution is 28,000 to the nearest 1,000. You should calculate the value to the nearest 10.

(b) (*2 points*) The insurer first estimates the aggregate distribution using a Normal approximation.

(i) Calculate the 95% Value at Risk (VaR) of the aggregate claims using the Normal approximation.

(ii) Calculate the 95% Expected Shortfall of the aggregate claims using the Normal approximation.

(c) (*3 points*) The insurer then estimates the aggregate distribution using a lognormal approximation, with parameters μ^* and σ^* fitted by matching moments.

(i) Show that $\sigma^*=1.1$ to the nearest 0.1. You should calculate the value to the nearest 0.001.

(ii) Calculate the 95% Value at Risk (VaR) of the aggregate claims using the lognormal approximation.

(iii) Calculate the 95% Expected Shortfall of the aggregate claims using the lognormal approximation.

(d) (*2 points*)

(i) State with reasons which of these two approximations is expected to give a more accurate approximation of the 95% Expected Shortfall in this case.

(ii) Explain briefly how a more accurate estimate of the Expected Shortfall could be determined based on a discretized severity (secondary) distribution.

****END OF EXAMINATION****