Quantitative Finance

Exam INV 201

Date: Monday, November 17, 2025

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 7 questions numbered 1 through 7 with a total of 50 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

3. Each question part or subpart should be answered either in the Excel document or the paper provided as directed. Graders will only look at the work as indicated.

4. In the Excel document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 (and ^ used to indicate a superscript).

5. Prior to uploading your Word and Excel files, each file should be saved and renamed with your unique candidate number in the filename. To maintain anonymity, please refrain from using your name and instead use your candidate number.

6. The Excel file that contain your answers must be uploaded before the five-minute upload period expires.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, hand in all your written-answer sheets to the Prometric center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

8770 W. Bryn Mawr Avenue, Suite 1000
Chicago, IL 60631

The responses for all parts of this question are required on the paper provided to you.

1.

(*5 points*) You are a corporate actuary at ABC. You are overseeing the negotiation with XYZ in a fixed-for-floating rate swap transaction.

- ABC has a credit rating of AAA while XYZ has a credit rating of BBB.
- Both ABC and XYZ wish to borrow $100 million for 10 years.
- ABC wants to borrow at a floating rate while XYZ wants to borrow at a fixed rate.

The two companies have been offered the following per annum rates by external lenders:

	Corporate Fixed Rate	Floating Rate
ABC	5.00%	Floating - 0.1%
XYZ	6.10%	Floating + 0.6%

Alternatively, ABC and XYZ can enter the swap below:

- ABC will pay XYZ the floating rate each quarter
- XYZ will pay ABC a fixed rate of 5.3% each quarter
- To fund the swap, ABC will take out a fixed rate loan from an outside lender payable at the end of year 10. XYZ will take out a 3-month loan, refinanced every 3-months at floating rate + 0.6%.

(a) (*2 points*) Show that this fixed-for-floating rate swap is advantageous for both ABC and XYZ, compared to their own external lending opportunities.

(b) (*1 point*) Explain the risks to ABC if XYZ credit rating is lowered to CCC.

(c) (*1 point*) Propose two strategies for ABC to manage the risks in part (b).

1. **Continued**

The swap between ABC and XYZ was initiated in 2021 using LIBOR as the reference floating rate. Since then, LIBOR has been discontinued and the new floating reference rate is the SOFR rate.

(d) (*1 point*) List three differences between LIBOR and SOFR.

The responses for all parts of this question are required on the paper provided to you.

2.

(*8 points*) Consider a single-period Arrow-Debreu economy with three states of the world and two tradable assets: A zero-coupon bond and a non-dividend paying stock. The bond pays 1 unit of currency at maturity regardless of the state, while the stock has state-dependent payoffs. The initial price vector and payoff matrix are given by:

$$S_0 = \begin{bmatrix} 0.95 \\ 100 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 1 & 1 \\ 120 & 96 & 64 \end{bmatrix},$$

where the rows correspond to the two assets (the bond and the stock, respectively) and the three columns represent the three possible states.

(a) (*2 points*) Assess whether this model is arbitrage-free.

Consider a European call option on this stock with strike price $K = 90$ and the same maturity as the bond.

(b) (*2 points*) Verify that the call option's payoff is not attainable.

Now assume that the call option from part (b) is added as a third tradable asset, and its initial price is 21.

(c) (*2 points*) Verify that there is an arbitrage opportunity by means of a non-strictly positive state vector.

Your coworker claims that the Arrow-Debreu security $\mathcal{E}^2$ can be used to construct an arbitrage strategy.

(d) (*2 points*) Justify your coworker's claim with an explicit calculation.

The responses for all parts of this question are required on the paper provided to you.

3.

(*8 points*) In the real world, the price of a stock follows the following stochastic model

$$dS_t = 0.15\, S_t\, dt + 0.2\, S_t\, dB_t$$

where S_t is the stock price at time t and $(B_t)_{t \geq 0}$ is a standard Brownian motion. The risk-free continuously compounded rate is 10%.

(a) (*1 point*)

(i) (*0.5 points*) Explain what can be measured by the market price of risk of a stock.

(ii) (*0.5 points*) Calculate the market price of risk for this stock.

(b) (*2.5 points*) Calculate $Var(S_1 | S_{0.5} = 100)$.

(c) (*1.5 points*) Derive the standard Brownian motion under the risk-neutral measure.

Assume the spot stock price is $S_0 = 100$ and consider a one-year European option on the stock with the following payoff:

$$f_1 = \begin{cases} 1 & \text{if } S_1 < 80 \text{ or } S_1 > 120 \\ 0 & \text{if } S_1 \in [80, 120] \end{cases}$$

(d) (*3 points*) Calculate the price of the option at time 0 under the risk-neutral measure.

The responses for all parts of this question are required on the Excel provided to you.

4.

(*7 points*) Your company is planning to sell options on coupon bonds and is evaluating two interest rate models for pricing these options.

You are provided with the following data:

- 25 Treasury STRIPS on Jan 2, 2025 with maturities varying from 0.3 years to 10 years.
- Overnight rates from Jan 2, 2021 to Jan 2, 2025.
- Overnight rate on Jan 2, 2025 is 3%.
- 10 swaption prices on Jan 2, 2025 with various expiry dates and tenor combinations.

(a) (*2 points*) Explain how you would use the above data for calibrating

- The Vasicek model
- The Hull-White model.

The response for this part is to be provided in the Excel spreadsheet.

A consultant has calibrated the models using the provided data, resulting in the following parameters:

- The Vasicek model with parameters $\bar{r}^* = 0.0634, \gamma^* = 0.4653$ and $\sigma = 0.0221$.
- The Hull-White model with parameters $\gamma^* = 0.19$ and $\sigma = 0.0196$.

You are also given the market price per 100 of one-year zero-coupon bonds and five-year zero-coupon bonds are 97.625 and 79.2813 respectively.

(b) (*2 points*) Calculate the prices of one-year and five-year zero-coupon bonds with the Vasicek model.

The response for this part is to be provided in the Excel spreadsheet.

The market price of a one-year European call option (expiring in one year) on a 5-year zero-coupon bond, with a strike price of 80% of the face value, is 0.021.
(At option maturity, the underlying will be a 4-year zero-coupon bond.)

(c) (*1 point*) Calculate the option price using the Vasicek model.

The response for this part is to be provided in the Excel spreadsheet.

4. Continued

(d) (*1 point*) Calculate the option price with the Hull-White model.

The response for this part is to be provided in the Excel spreadsheet.

(e) (*1 point*) Recommend which model you would choose to price and hedge European options on coupon bonds.

The response for this part is to be provided in the Excel spreadsheet.

The responses for some parts of this question are required on the paper provided to you.

5.

(*5 points*) You are managing a well-diversified portfolio that mirrors the performance of S&P 500. For simplification, the following assumptions are made:

- The portfolio value is $500 million USD.
- The price of S&P 500 is 5,000.
- The risk-free interest rate is 4% per annum, continuously compounding.
- The dividend yield on both the portfolio and the S&P 500 is 0%.
- The volatility of the S&P 500 index is 20% per annum.
- You can buy/sell S&P 500 underlying stocks to replicate the index movement.

You seek downside protection against a decline of more than 10% in the value of the portfolio over the next year.

(a) (*1 point*) Calculate the cost of purchasing European put options to achieve the downside protection against a decline of more than 10%.

The response for this part is to be provided in the Excel spreadsheet.

(b) (*1 point*) Describe an alternative strategy using traded European call options to achieve the same protection.

(c) (*1 point*) Calculate the portion of the portfolio to be sold initially and invested in risk-free securities to achieve the same protection.

(d) (*1 point*) Calculate the initial position in one-year S&P 500 futures to provide the same protection.

Seeking another strategy to provide protection, you will consider using a forward contract on the S&P 500 index.

(e) (*1 point*) Explain whether a forward contract on the S&P 500 index has the same delta as the corresponding futures contract.

The responses for all parts of this question are required on the paper provided to you.

6.

(*7 points*) Suppose you want to statically and approximately replicate a 6-month cash-or-nothing European call option on a stock that pays a fixed amount of $1 if the stock ends up above the strike price or pays nothing otherwise.

(a) (*1 point*) Recommend a strategy using plain-vanilla European call options to statically and approximately replicate the cash-or-nothing European call option with the strike price $K_1 = 50$.

Now consider a 6-month up-and-out European call option on the same stock with a barrier of $60.

You are given a similar 6-month cash-or-nothing European call option to that in part (a) with a different strike $K_2 = 60$. Assume both of these cash-or-nothing European calls are available.

(b) (*2 points*) Recommend a strategy using the 6-month cash-or-nothing call options and plain-vanilla European call options to statically and approximately replicate the up-and-out call option.

(c) (*2 points*)

(i) (*1 point*) Describe two advantages of static replication of an exotic option over dynamic delta hedging.

(ii) (*1 point*) Describe two limitations of static replication of an exotic option with respect to dynamic delta hedging.

In the context of Derivatives Mishaps described in Hull's *Options, Futures and Other Derivatives*, you are concerned about conflicts and risks associated with the traders tasked with static replication.

(d) (*2 points*)

(i) (*1 point*) Explain how the recognition of "inception profit" could lead to an increase in the marking-to-model risk.

(ii) (*1 point*) Recommend two mitigation strategies to reduce the potential conflict of interest.

The responses for some parts of this question are required on the paper provided to you.

7.

(*10 points*) Your company is looking to enter the equity indexed annuity (EIA) market. The product design is as follows:

Credit a proportion of index return, i.e. participation rate α, subject to the minimum guaranteed interest of g per year, compounded annually.

You have considered crediting interest with continuous compounding as well but think customers would be more receptive to annualized guarantee rates and participation rates as a multiplicative constant applied to the underlying index.

The underlying index, $S(t)$, follows a geometric Brownian motion under the risk-neutral measure $\mathbb{Q}$ with volatility σ and initial value $S(0) = 1$. The underlying pays no dividends.

Denote the value of the EIA by $V(\alpha, g, T)$, where T is the remaining guarantee period.

(a) (*1 point*) Verify that $V(\alpha, g, T) = e^{-rT}\mathbb{E}\left[\max\left((1-\alpha) + \alpha e^{\left(r-\frac{1}{2}\sigma^2\right)T+\sigma W_T}, (1+g)^T\right)\right]$.

Market volatility has raised awareness of hedging strategy programs within the company.

A colleague of yours has suggested this product can easily be hedged with vanilla European options.

(b) (*1.5 points*) Construct a hedging strategy that validates your colleague's claim and could be implemented at $t = 0$.

7. **Continued**

Let $C(S,K)$ denote the price of a vanilla European call option with underlying asset S, strike $K = \frac{(1+g)^T-(1-\alpha)}{\alpha}$ and expiry T.

Observe that the value of EIA can be written as:

$$V(\alpha,g,T) = (1+g)^T e^{-rT} + \alpha C(S,K).$$

(c) (*2.5 points*)

(i) (*1.5 points*) Verify that:

$$\frac{\partial V}{\partial \alpha} = C(S,K) - \frac{\partial C}{\partial K} * (K-1)$$

(ii) (*1 point*) Verify that:

$$\frac{\partial V}{\partial \alpha} = \frac{\partial C}{\partial S} + \frac{\partial C}{\partial K}.$$

Hint: It is useful to recall that $\frac{\partial \mathrm{C}}{\partial \mathrm{K}} = -e^{-rT}N(d_2)$

You are given that the volatility for the selected index σ is 20% and the continuously compounded risk-free rate r is 6%.

Your colleague calculated the value of a 5-year EIA using a guaranteed rate of 3.0% with a 75% participation rate to be 1.0265.

You would like to adjust the participation rate to maintain the break-even point, i.e., $V(\alpha, 3.0\%, 5) = S(0)$.

(d) (*1.5 points*) Derive the approximate participation rate that aligns with the break-even point for a 3.0% guarantee.

The response for this part is to be provided in the Excel spreadsheet.

7. Continued

Another actuary on your team has also suggested the company may want to enter the RILA market instead of EIA. Their impression is the RILA market is growing at a faster pace and may offer a greater customer base.

(e) (*1 point*) Compare RILAs and EIAs.

You've proposed a RILA design based on initial market research of competitor offerings, where participation rate varies by performance: is 90% on upside returns, but only 50% on losses. In addition, you'd like to incorporate a cap of 18%.

Assume the reference stock has a current value of 100.

(f) (*2.5 points*)

(i) (*1 point*) Sketch the payoff structure of proposed design, clearly labeling how the characteristics are reflected.

(ii) (*1 point*) Develop a hedge strategy for this structure using vanilla European options on the underlying stock.

(iii) (*0.5 points*) Verify that your strategy is appropriate.

****END OF EXAMINATION****