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12th Annual Survey of Emerging Risks

By Max Rudolph
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Council's Corner

By Frank Reynolds

The Joint Risk Management Section Council had a face-to-face meeting in connection with the 2019 Enterprise Risk Management Symposium this past May. The meeting concentrated on two ideas:

1. **How to increase participation by the younger members of the profession.** It was felt that the initiative to try to reach out to local actuarial clubs with assistance in finding speakers and in providing forums for networking at major meetings should be continued.

2. **Turning the sessions at the ERM Symposium into webcasts.** This would have the effect of making the sessions available to the general membership of the section and to members of other sections. Also, it would give the panel members more exposure for their ideas with little or no additional preparation. Finally, it could solve the funding problems the section faces without having to raise the section dues. It is hoped that we will be able to present a monthly webcast this fall and into the spring.

With the switch to electronic newsletters, the section will try to make better use of blast e-mails to publicize the work done by the section council and to provide better opportunities for the section's members to participate in the section's activities.

There is a good slate of candidates from all three sponsoring organizations for the fall elections. Please vote for the candidates of your choice.

For those who want to participate in the section's affairs but do not want to join the Joint Risk Management Section Council, I would urge you to register as a "friend of the council." These people participate in the discussions at all section council meetings, the same as elected council members, but cannot vote on resolutions and are not counted in determining if a quorum is present.

Finally, I would urge members to write articles and send in ideas as to how we can attract young and other new members. They are the lifeblood of our organization. ■

Frank Reynolds, FSA, FCIA, MAAA, is vice chairperson of the Joint Risk Management Section Council and has taught at the University of Waterloo for 35 years. He can be reached at *fgreynol@gmail.com*.

The CAS-branded foam cube contains a microphone. After asking a question to the panelist by talking into the cube, attendees at the 2019 ERM Symposium had fun throwing the cube across the room to the next person.

Editor's Note

By Florian Richard

They say that all good things eventually come to an end. This couldn't be truer than when it comes to our very own *Risk Management* newsletter, as we recently learned Cheryl Liu is leaving her role of newsletter editor after three years. I believe that I speak on behalf of most readers when I say that I am grateful for all the hard work that Cheryl has put into this newsletter to bring it to where it is today. As a reader of the newsletter during that span, I have thoroughly enjoyed the high standard of the editing as well as the variety of the articles featured. These are key reasons why I decided to volunteer earlier this year to help in any capacity. In fact, this first Editor's Note is a great opportunity for me to remind you that we always welcome new volunteers to help with the various aspects of managing a newsletter. If you are interested in volunteering for the newsletter, please contact David Schraub (*dschraub@soa.org*) or me.

The September issue of *Risk Management* inevitably draws inspiration from the annual Enterprise Risk Management Symposium that took place in Orlando on May 2 and 3. The ERM Symposium is a unique forum for industry professionals to come together and share their thoughts on a variety of enterprise risk management topics.

One topic that has now become a staple at the ERM Symposium is Max Rudolph's Annual Survey of Emerging Risks. The survey, which is sponsored by the Joint Risk Management Section, has now reached its 12th edition. The executive summary is included in this issue. I will avoid sharing any spoilers and I will simply mention that the order of the top emerging risks has changed this year and that new trends seem to be developing. Please note that the full report is now available on the Joint Risk Management Section pages of the Society of Actuaries (SOA) website.

The ERM Symposium is also an opportunity to encourage and celebrate research. This year, two prizes were given out for papers that promote "the practice of enterprise risk management by opening new perspectives and strengthening available insights, methods and tools."

The first prize, the Actuarial Foundation's ERM Research Excellence Award in Memory of Hubert Mueller for Best Overall Paper, was awarded to Kailan Shang for his paper on the estimation of wavelet-based equity VaR. An adapted version of his paper is featured as our second article in this issue. Estimating economic risk often means assuming that risk is time invariant. However, this is not necessarily true. This is where wavelet analysis can help address the time-horizon component of the risk analysis.

The second prize, the Joint CAS/CIA/SOA Risk Management Section Award for Practical Risk Management Applications, went to Dariush Akhtari for his paper on the valuation of economic surplus. After listing the deficiencies associated with the common practice of valuing market value of surplus by first valuing market value of liabilities, the article suggests a new way to directly calculate the market value of surplus that is "stable and reasonably immune to market noise." The paper has been adapted for inclusion in the newsletter.

"Introduction to the Research on Developing a Liability-Driven Investment (LDI) Benchmark Framework" is our fourth article of the September newsletter. This short article sets the stage for the full report and Excel tool that can be found on the SOA website.

Finally, our fifth article analyzes the methodologies used by industry professionals to value liability cash flows that extend beyond the maximum observable portion of the yield curve. The article looks into key assumptions, benefits, drawbacks and practical challenges associated with the various sample methods.

As usual, the newsletter concludes with a list of recent articles and papers that may be of interest to our members. These pieces can provide further information on a broad range of topics.

I would like to give a special thank-you to Cheryl Liu, David Schraub, Julia Anderson Bauer and Katherine Pickett for their help in pulling together this September issue.

Hope you enjoy the reading! ■

Florian Richard, FCAS, is in charge of risk management at AXA XL Reinsurance. He can be reached at *florian.richard@axaxl.com*.

Staff Corner

By David Schraub

The newsletter is one of our section members' most valued assets. This result has shown up in all the section surveys I have seen during my six-year tenure with the Society of Actuaries (SOA). This Staff Corner will shed some light on how newsletters are produced. Let's open up the hood and check it out.

Several groups of people take part in this initiative:

- **Article authors.** SOA members and nonmembers who volunteer to write articles.

- **Newsletter editors.** Volunteers who solicit and peer-review the articles and provide feedback to authors and SOA staff.

- **SOA section staff partner.** The liaison between the section, the volunteer newsletter editor and the newsletter staff. This person oversees reputation risk management and offers guidance as needed.

- **SOA staff editor.** An in-house editor who guides the newsletters from copyediting to publication. This person is the gatekeeper of the newsletter.

- **SOA graphic designer.** The person responsible for design and layout of the newsletters. The graphic designer also ensures the quality of graphics and tables.

In chronological order, the newsletter process looks like this:

1. Authors write articles. Generally, either the newsletter editor reaches out to potential authors with a request for an article on a specific topic, or an author reaches out to the newsletter editor and offers to write an article on a given topic. In some cases, authors are asked to republish an article that is already written.

2. Newsletter volunteer editors peer-review articles. They assess their fit within the newsletter regarding quality and topic and provide feedback on the content of each article.[1] For example, the topic of an article may be a better fit for a different section than originally intended. In that case, that article is forwarded to the other section's newsletter editor. After a few weeks of back-and-forth to firm up the content, the articles (along with author bios, head shots and figure and table source files) reach the staff partner. For a previously published article, the back-and-forth is replaced with a reach to the owner of the copyright for reprint permission.

3. The section staff partner reviews all the articles to assess whether there is any reputation risk regarding their content (e.g., self-advertising, lobbying or other pitfalls). This step sometimes takes place slightly later in the process.

4. The staff editor receives the finalized content and oversees copyediting for grammar and editorial style, as well as production of the newsletter. This is where the i's get dotted. The editor monitors the schedule, nudges volunteers as needed, and sends metadata[2] and copyright forms to the authors.

> Multiple pairs of eyes are key to the quality of the newsletter.

5. The staff editor and volunteer newsletter editor work together to address any challenges that go beyond punctuation. The newsletter editor answers the staff editor's questions directly or turns to the authors as needed. Common questions include, "Who should approach the coauthor to soften the tone of the conclusion, which is a bit too self-serving?" "Do we still have time for a last-minute announcement?" "Did anyone receive Jane Doe's article she promised us a while back?" "Should we keep that article for the next issue as it is not quite ready, and we have a lot of content already?" "Do we have head shots and authors' names correctly aligned?" This back-and-forth can take time, but multiple pairs of eyes are key to the quality of the newsletter.

6. The staff graphic designer makes the content look great. The newsletter editor and authors review the page proofs

for any typos and readability of the graphs, while the staff editor proofreads the full newsletter one more time. This is where loose ends are tied.

7. The staff editor sends the newsletter to the printer and/or digital vendor after green lights from all. Printing and shipping take place (as appropriate), the digital edition is created and, finally, the PDF version and links to the digital version are posted on the SOA website. This is the time to update the section's landing page with a link to the newsletter. For printed newsletters, readers at home receive their copies a few weeks later.

Toward step 5 of the current newsletter is when volunteer editors begin to gather articles for the next issue, whether it's the promise of an article or articles that are already in hand. Then the process begins all over again.

Want to join the fun? We are always looking for editors and authors to improve our content. ■

David Schraub, FSA, CERA, AQ, MAAA, is a staff actuary for the SOA. He can be contacted at *dschraub@soa.org*.

ENDNOTES

1 For some newsletters, the volunteer authors and volunteer editors are blended. For example, *Taxing Times* has a large group of newsletter editors who peer-review and cross-check every statement of every author (there are lawyers in the group).

2 Metadata includes topics, country of relevance, and keywords for each article. Topics and country of relevance are filters on the SOA website and help get readers to the content faster. Keywords are additional hints for search-engine optimization.

12th Annual Survey of Emerging Risks

By Max Rudolph

Editor's note: This article was originally published as an executive summary in conjunction with the full report of the 12th Annual Emerging Risks Survey. The full report is available on the Society of Actuaries website at www.soa.org/resources/research-reports/2019/12th-emerging-risks-survey/.

For the first time in the survey's history, climate change ranked as both the top current risk and the leading emerging risk—breaking cyber risk's four-year streak as number one—according to the 12th Annual Emerging Risks Survey from the Joint Risk Management Section (JRMS) of the Canadian Institute of Actuaries (CIA), the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA).
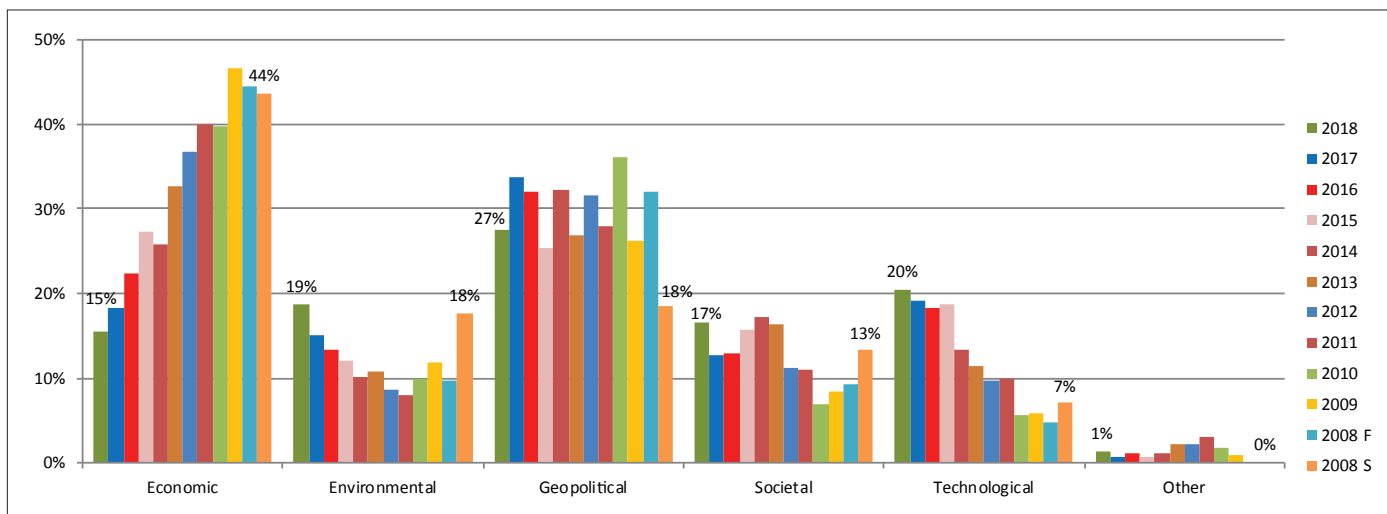
EMERGING RISK TRENDS

When survey respondents were asked to choose five emerging risks, the trend over the history of the survey shows economic risks reducing relative to environmental and technological,

with geopolitical risks experiencing spikes typically in even-numbered election years prior to the current U.S. administration (see Figure 1).

CLIMATE CHANGE TAKES THE TOP SPOTS

According to the survey fielded in November 2018, risk managers perceive climate change, cyber risk and financial volatility to be the three greatest current risks. While climate change edged out other risks this year, it is important to note that cyber risk—the previous top risk—is still a strong threat, ranking second among the current risks. The top emerging risks, as ranked by 267 risk managers from across the globe, follow a similar pattern, with climate change ranking first (22%), cyber risk ranking second (15%) and technology in third place (13%) (see Figure 2).

Figure 1
Emerging Risks by Category

Note: Up to five risks were chosen per survey response.

Figure 2
Top Current Risk, Year-Over-Year
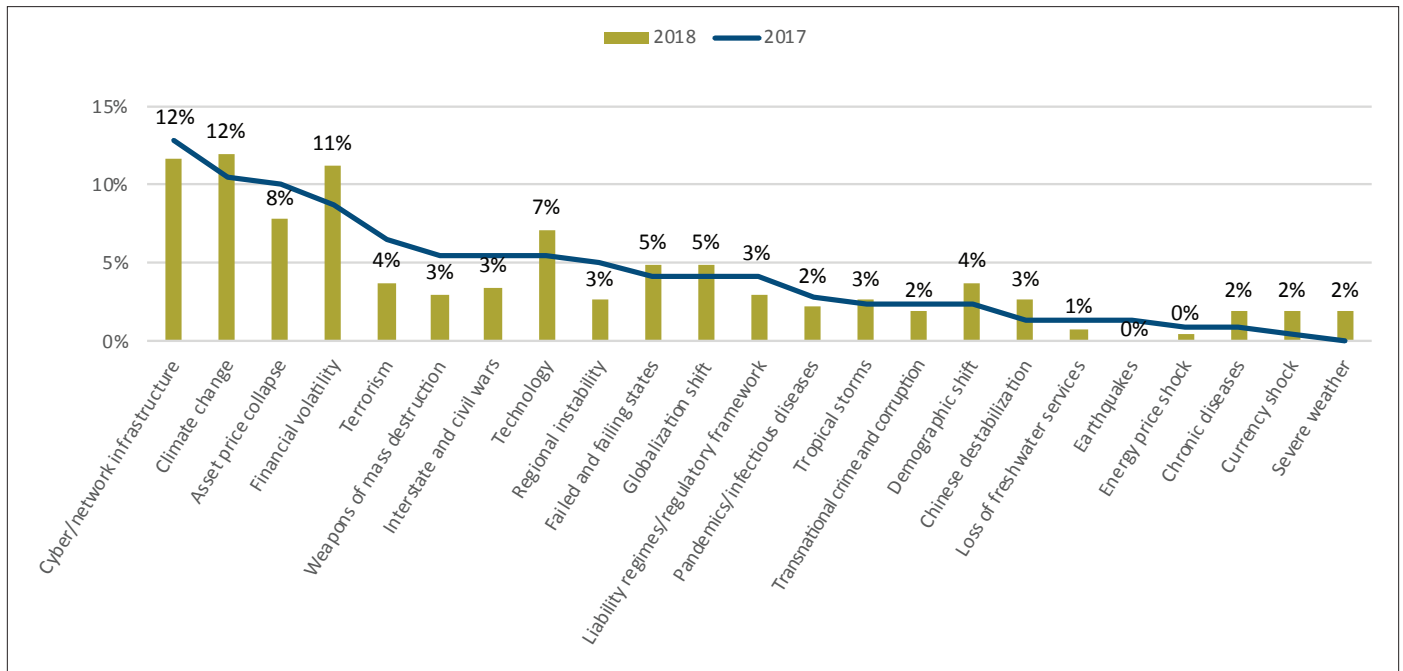
YEAR-OVER-YEAR COMPARISONS

- Climate change surpassed cyber risk as the top current risk, top emerging risk and top emerging risk combination. However, cyber risk remained first when respondents were asked to rank the top five emerging risks, at 56%.

- Geopolitical risk maintained the lead in 2018 among emerging risks by category, but the risk level decreased from 2017. This is unusual given that this category historically increases during even-numbered national election years. The change could be attributed to a decrease in rankings among the top five emerging risks across terrorism, regional instability, weapons of mass destruction and liability regimes.

- While global economic expectations remain relatively stable, they are down from the prior survey with a majority (67%) forecasting moderate or poor expectations for 2019.

- After dropping off the top five emerging risks list in 2017, financial volatility has reappeared, ranking number five.

Demographic shift has also appeared for the first time since 2013.

- Societal risk as an emerging risk category increased due to the combined impact of an influx of pandemics and infectious diseases, as well as the demographic shift risk.

NAMING CONVENTION

The survey respondents chose frequency and severity as their preferred description of how often an event could occur and how bad it could be. ∎

The 13th Annual Emerging Risks Survey will begin in early 2020.

Max Rudolph, FSA, CERA, MAAA, is the owner of Rudolph Financial Consulting, LLC. He can be reached at *max.rudolph@rudolph-financial.com*.

Wavelet-Based Equity VaR Estimation

By Kailan Shang

Editor's note: This article is adapted from the research paper Wavelet-Based Equity VaR Estimation, *part of the 2019 Enterprise Risk Management Symposium call for papers. It was awarded the Actuarial Foundation's ERM Research Excellence Award in Memory of Hubert Mueller for Best Overall Paper. The full report is available at* www.ermsymposium.org/wp-content/uploads/2019/05/Shang_Actuarial-Foundation-Prizewinner.pdf.

Economic risk is an important risk for insurers offering long-term products with guaranteed benefits. When estimating the magnitude of economic risk, historical data are usually used. However, an implicit assumption of this method is that the risk is time invariant. In reality, equity market volatility varies by time. It is caused by either economic cycles or economic structural changes. Figure 1 shows the annualized volatility using daily S&P 500 index return from 1990 to 2017. Assuming a time-invariant (constant) volatility, the annualized volatility is 17.7 percent. If calculating the annualized volatility on a yearly basis, the volatility could go above 40 percent, as evidenced during the 2008 financial crisis.

Another complication is the frequency of historical data to use. The annualized volatility calculated based on different frequencies varies a great deal. Table 1 shows the annualized volatility and empirical value at risk (VaR) of S&P 500 equity index return using daily, monthly and yearly data from 1990 to 2017. For simplicity, the calculation assumes that the volatility and VaR are time invariant and that the equity index follows a geometric Brownian motion. Here VaR measures the negative return value in the left tail. For example, a 99.5 percent VaR of 15 percent means that there is a 0.5 percent chance that the return will be less than –15 percent. It is the opposite of the negative return value in the left tail.

Historical equity index returns exhibit different risk levels by frequency. Annualized empirical VaR based on high-frequency data (daily and monthly) is higher than the VaR based on low-frequency data (quarterly and yearly). This phenomenon indicates the need to analyze the economic risk at different frequencies to get a holistic view.

Figure 1
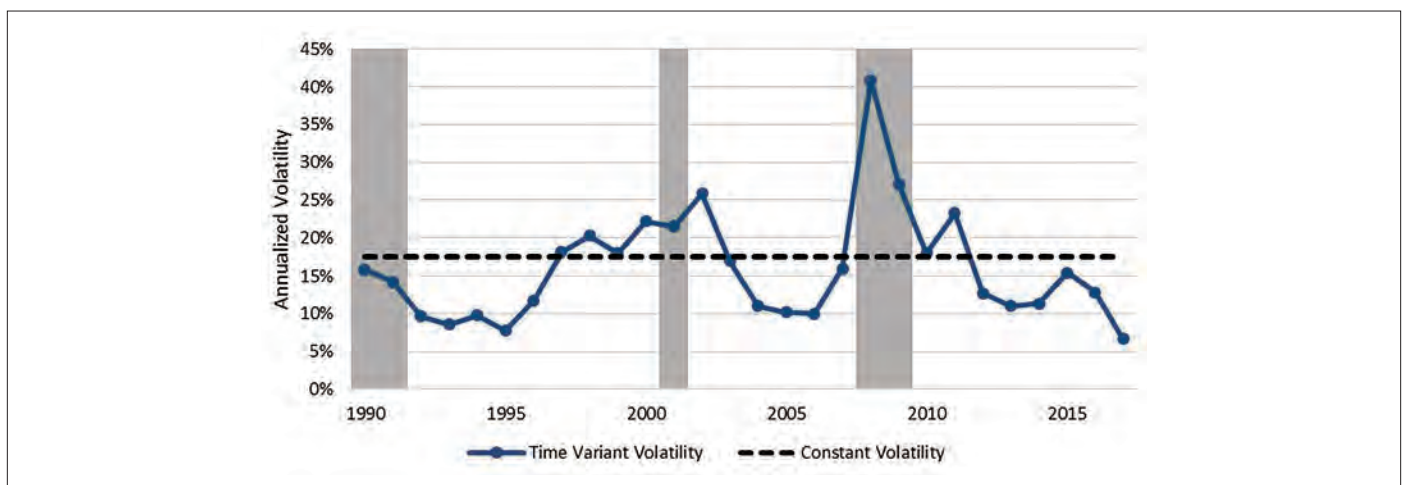S&P 500 Index Return Annualized Volatility (1990–2017)

Table 1
S&P 500 Index Return Volatility and VaR by Frequency

Frequency	Time-Invariant Volatility	Annualized Volatility[1]	99.5% Empirical VaR	Annualized Empirical VaR[2]
Daily	1.1%	17.5%	3.9%	69.3%
Monthly	4.2%	14.5%	19.3%	75.3%
Quarterly[3]	7.9%	15.5%	26.9%	64.2%
Yearly	17.7%	17.5%	43.5%	43.5%

[1] Annualized volatility = time-invariant volatility $\sqrt{n}$, where n equals 250/12/4/1 for daily/monthly/quarterly/yearly frequency.
[2] Annualized empirical VaR = (99.5% Empirical VaR – Mean return) $\sqrt{n}$ – Mean return n.
[3] Minimum value of quarterly and yearly return is used for 0.5% empirical VaR because the number of data points is less than 200.

TIME SERIES MODEL

Time series models, such as generalized autoregressive conditional heteroskedasticity (GARCH) and autoregressive moving average (ARMA), can be used to capture the time-variant feature of equity volatility. An ARMA-GARCH model is used to analyze historical S&P 500 index daily returns.

$$ARMA(p,q) \sim r_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j},$$

$$\varepsilon_t = z_t \sigma_t,$$

$$GARCH(p,q) \sim \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2,$$

where

r_t = S&P 500 index daily return. It is calculated as $log\left(S_t \big/ S_{t-1}\right)$.

z_t = i.i.d. with zero mean and unit variance.

The distribution of z_t that can more flexibly capture skewness and heavy tails should be chosen. In this example, z_t is assumed to follow to the skewed generalized error distribution (SGED). It has the following probability density function:

$$f_{SGED}(x;\mu,\sigma,\lambda,p) = \frac{pe^{-\left\{\frac{|x-\mu+m|}{v\sigma[1+\lambda sign(x-\mu+m)]}\right\}^p}}{2v\sigma\Gamma(1/p)},$$

where

μ = location parameter. It is zero for z_t,

σ = scale parameter. It is one for z_t,

λ = skewness parameter,

p = shape parameter,

$$m = \frac{2^{\frac{2}{p}} v\sigma\lambda\Gamma\left(0.5 + \frac{1}{p}\right)}{\sqrt{\pi}} \text{ if the mean of variable equals } \mu,$$

$$v = \sqrt{\frac{\pi\Gamma\left(\frac{1}{p}\right)}{\pi(1+3\lambda^2)\Gamma\left(\frac{3}{p}\right) - 16^{\frac{1}{p}}\lambda^2\Gamma\left(0.5 + \frac{1}{p}\right)^2\Gamma\left(\frac{1}{p}\right)}} \text{ if the}$$

volatility of variable x equals σ.

ARMA(3,3) and GARCH(2,2) with the SGED are used to analyze historical S&P 500 daily index returns from 1990 to 2017. The orders (p and q) are chosen based on Akaike information criterion (AIC).

Figure 2 shows the daily return and the conditional volatility σ_t based on the ARMA-GARCH model. The conditional volatility varies greatly, with the highest value observed during the 2008 financial crisis.

With the fitted model, future daily VaR can be predicted. Figure 3 shows the results based on 1,000 simulations for the 251 trading days from October 2017 to September 2018. Actual daily returns are compared with the projected ranges. While 10.4 percent of actual returns fall out of the middle 90 percent range (5th percentile to 95th percentile), 1.6 percent of actual returns fall out of the middle 99 percent range (0.5th percentile to 99.5th percentile). Although the SGED generates a better range prediction than the normal distribution, it still underestimates the probability of extreme returns for the projection period.

Figure 2
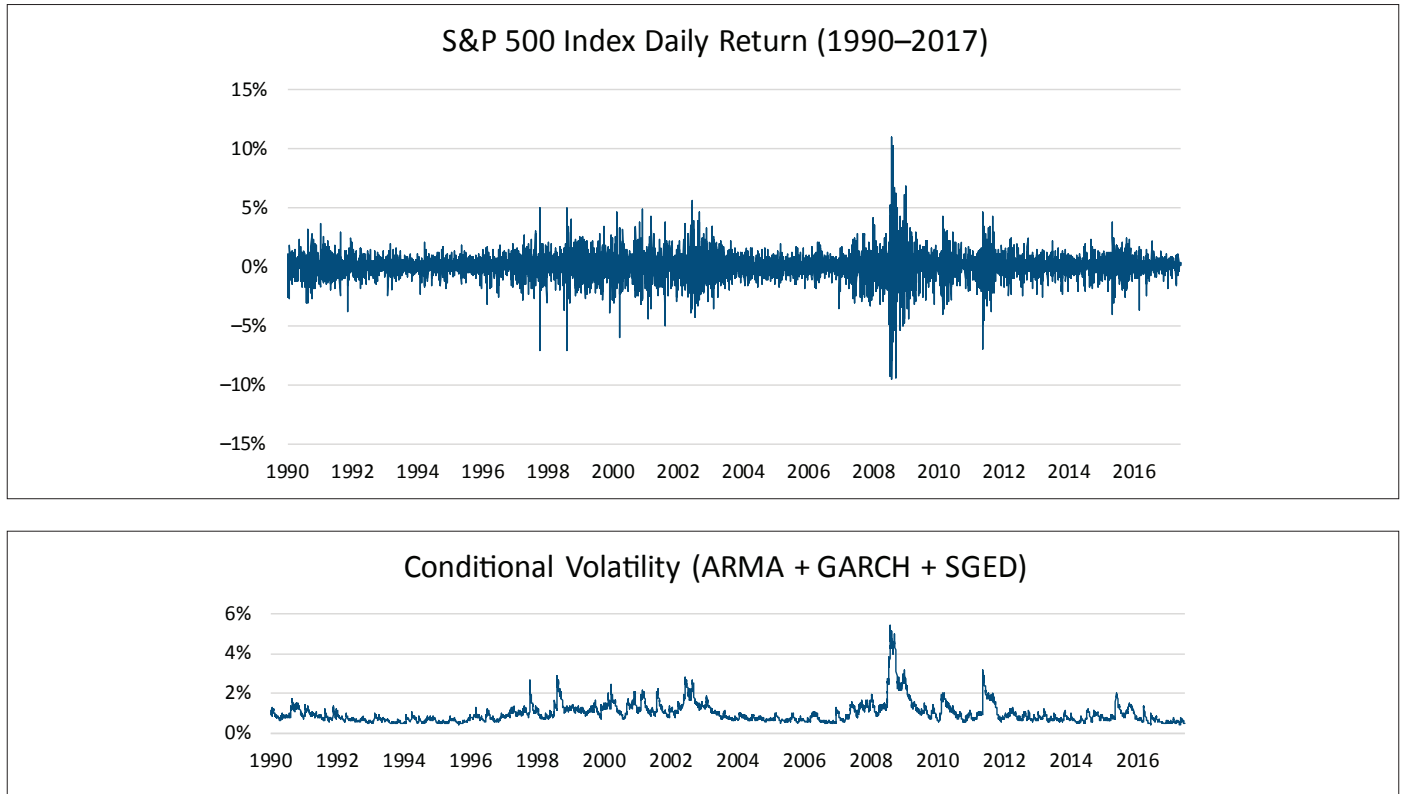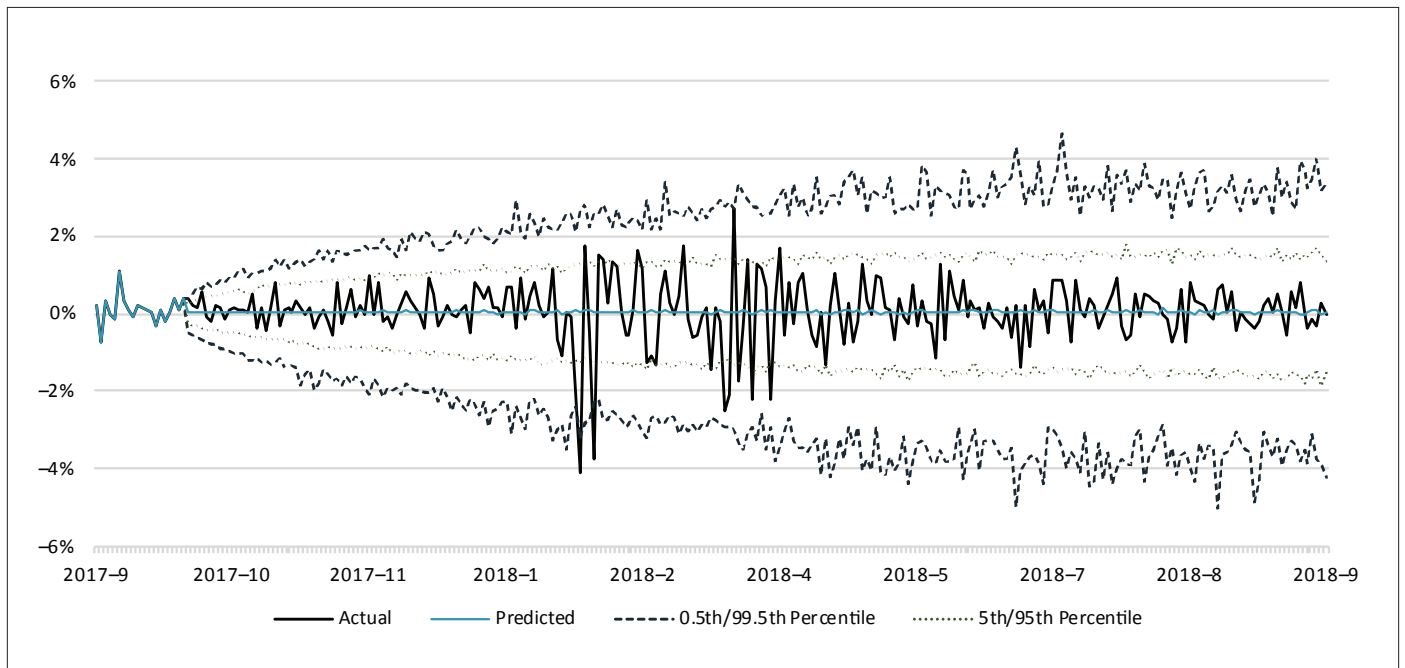S&P 500 Index Daily Return and Conditional Volatility

Figure 3
S&P 500 Index Daily Return Range Estimation

Instead of using closed-form formulas, annual VaR can be estimated based on simulated daily returns, as shown in Table 2. In this example, the SGED has a heavier left tail than the normal distribution.

Table 2
S&P 500 Index Return Annual VaR Estimation

	95% VaR	99.5% VaR
SGED	4.6%	24.2%
Normal distribution	5.0%	14.1%

WAVELET ANALYSIS

If the evolving of risk is driven by a few forces with different frequencies, a pure time series model may not be able to capture all the different patterns. When predicting the return and conditional volatility, the ARMA-GARCH model reflects only the direct impact of returns and volatilities in the past three days. The model cannot effectively capture the impacts for medium- and long-term patterns. People may argue that less frequent (such as annual) data can be used to estimate annual VaR. However, historical data may not be sufficient for a credible estimate, and valuable information in high-frequency data is lost.

Wavelet analysis can be used to analyze the historical data from two dimensions (time and frequency) at the same time. Wavelet analysis can be considered a combination of time series analysis and Fourier transform. Fourier transform analyzes the data purely from the frequency domain, assuming that patterns are time invariant. As shown in Figure 4, wavelet analysis keeps more time information for high-frequency data and less time information for low-frequency data.

Maximal overlap discrete wavelet transform (MODWT) is used to illustrate enhanced risk analysis based on wavelets. The MODWT is chosen over many other wavelets because its decomposition at different scales can easily be compared with original time series. The MODWT is also less sensitive than other wavelet transforms to the starting point of a time series. This is helpful to understand the patterns at different frequencies: short term, medium term or long term. Following the definition of Percival and Walden (2000), the MODWT of a time series X_t, $t = 1, 2, \ldots, N$ to the jth level works as the following:

Wavelet coefficient $\quad \tilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \; MOD \; N},$

Scale coefficient $\quad \tilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \; MOD \; N},$

where $\tilde{h}_{j,l}$ = wavelet filter constructed by convolving j filters composed of $\tilde{g}_l$ and $\tilde{h}_l$. It suffices the following conditions:

$$\sum_{l=0}^{L-1} \tilde{h}_l = 0 \qquad \sum_{l=0}^{L-1} \tilde{h}_l^2 = \frac{1}{2} \qquad \sum_{l=-\infty}^{\infty} \tilde{h}_l \tilde{h}_{l+2n} = 0 \text{ for all integers } n > 0,$$

$\tilde{g}_{j,l}$ = scale filter constructed by convolving j filters composed of $\tilde{g}_l$. It suffices the following conditions:

$$\sum_{l=0}^{L-1} \tilde{g}_l = 1 \qquad \sum_{l=0}^{L-1} \tilde{g}_l^2 = \frac{1}{2} \qquad \sum_{l=-\infty}^{\infty} \tilde{g}_l \tilde{g}_{l+2n} = 0 \text{ for all integers } n > 0,$$

$$\sum_{l=-\infty}^{\infty} \tilde{g}_l \tilde{h}_{l+2n} = 0 \text{ for all integers } n,$$

$L_j = (2^j - 1)(L - 1) + 1$. L is the width of the base level filter.

The maximum number of levels depends on the available data points. Table 3 lists the frequency of the first eight levels.
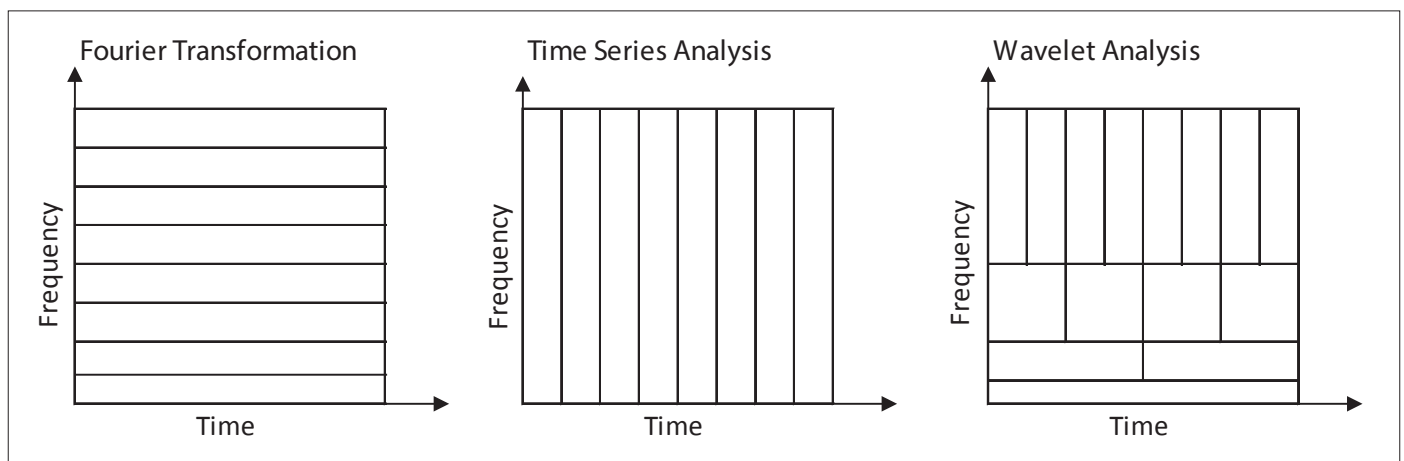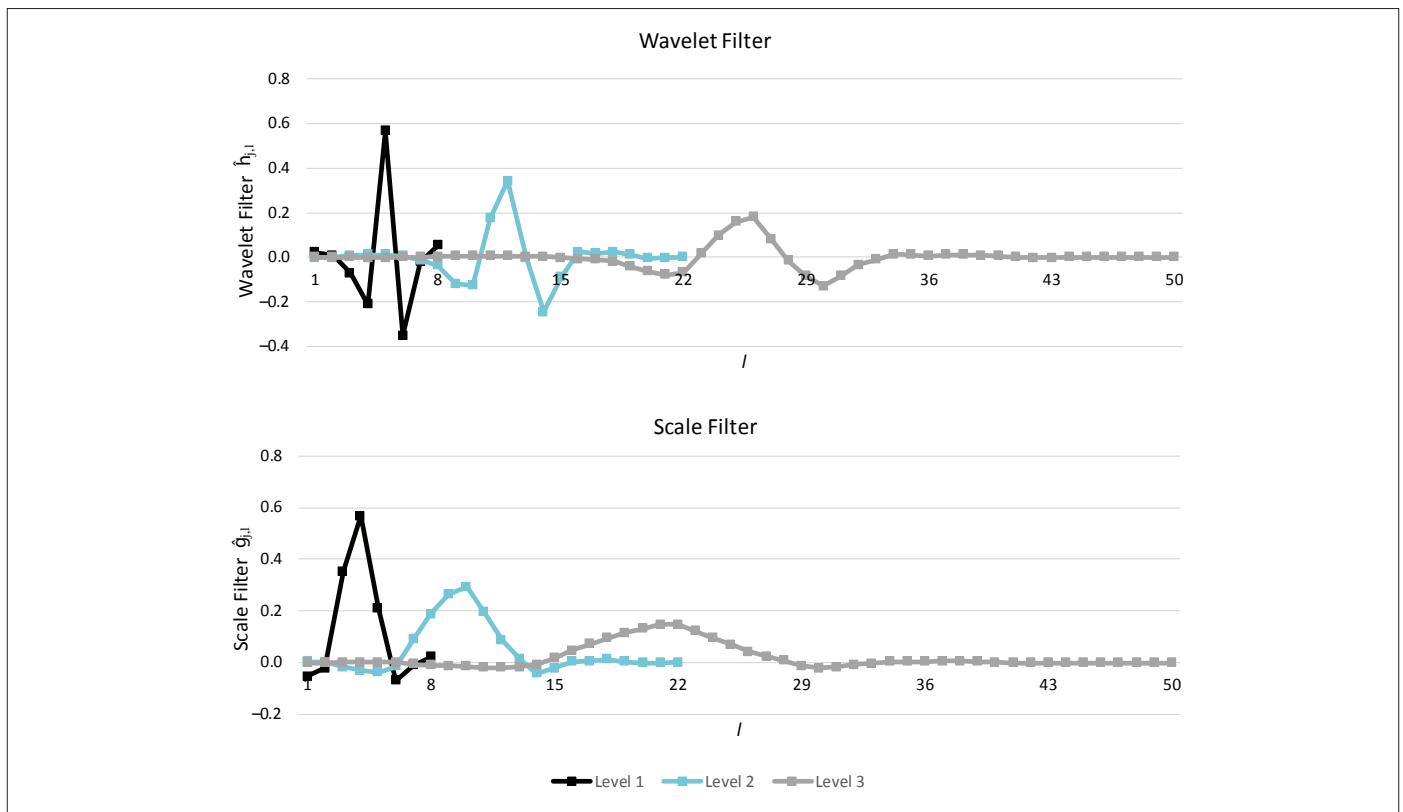
Figure 4
Wavelet Analysis Concept

Table 3
Frequency of Decomposition Levels

Level (*j*)	Frequency	Scale (1/Frequency)*
1	[1/4,1/2]	2–4 days
2	[1/8,1/4]	4–8 days
3	[1/16,1/8]	8–16 days
4	[1/32,1/16]	16–32 days
5	[1/64,1/32]	32–64 days
6	[1/128,1/64]	64–128 days
7	[1/256,1/128]	128–256 days
8	[1/512,1/256]	256–512 days

* The scale is measured in business days.

To analyze the equity risk, LA(8) (Daubechies least asymmetric filter with $L = 8$) is used to define $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$. Figure 5 shows the wavelet filters $\tilde{h}_{j,l}$ and scale filters $\tilde{g}_{j,l}$ for the first three levels. The wavelet dampens out with larger width as the level goes up. The same pattern applies when the level goes higher than level three.

The original time series (S&P 500 index daily return) is decomposed into eight levels. Figure 6 shows the wavelet coefficients ($\tilde{W}_{j,t}$) for all eight levels and the scale coefficients ($\tilde{V}_{j,t}$) for the eighth level. The wavelet coefficients are smoother at a higher level, representing longer-term volatility. The scale coefficients at the highest level represent the volatility that is not explained by wavelet coefficients.

TIME-INVARIANT RISK ANALYSIS

Wavelet analysis can be used to attribute the total volatility to different levels. The total variance can be calculated as the sum of the variances at each level:

$$\sigma_X^2 = \sum_{j=1}^{\mathcal{J}_M} \sigma_X^2(j),$$

where

σ_X^2 = total variance of the original time series,

$\sigma_X^2(j)$ = variance of the decomposition at level j,

$\mathcal{J}_M$ = number of levels used in wavelet analysis.

Figure 5
LA(8) Wavelet and Scale Filters for MODWT

Figure 6
MODWT Wavelet Coefficients and Scaling Coefficients
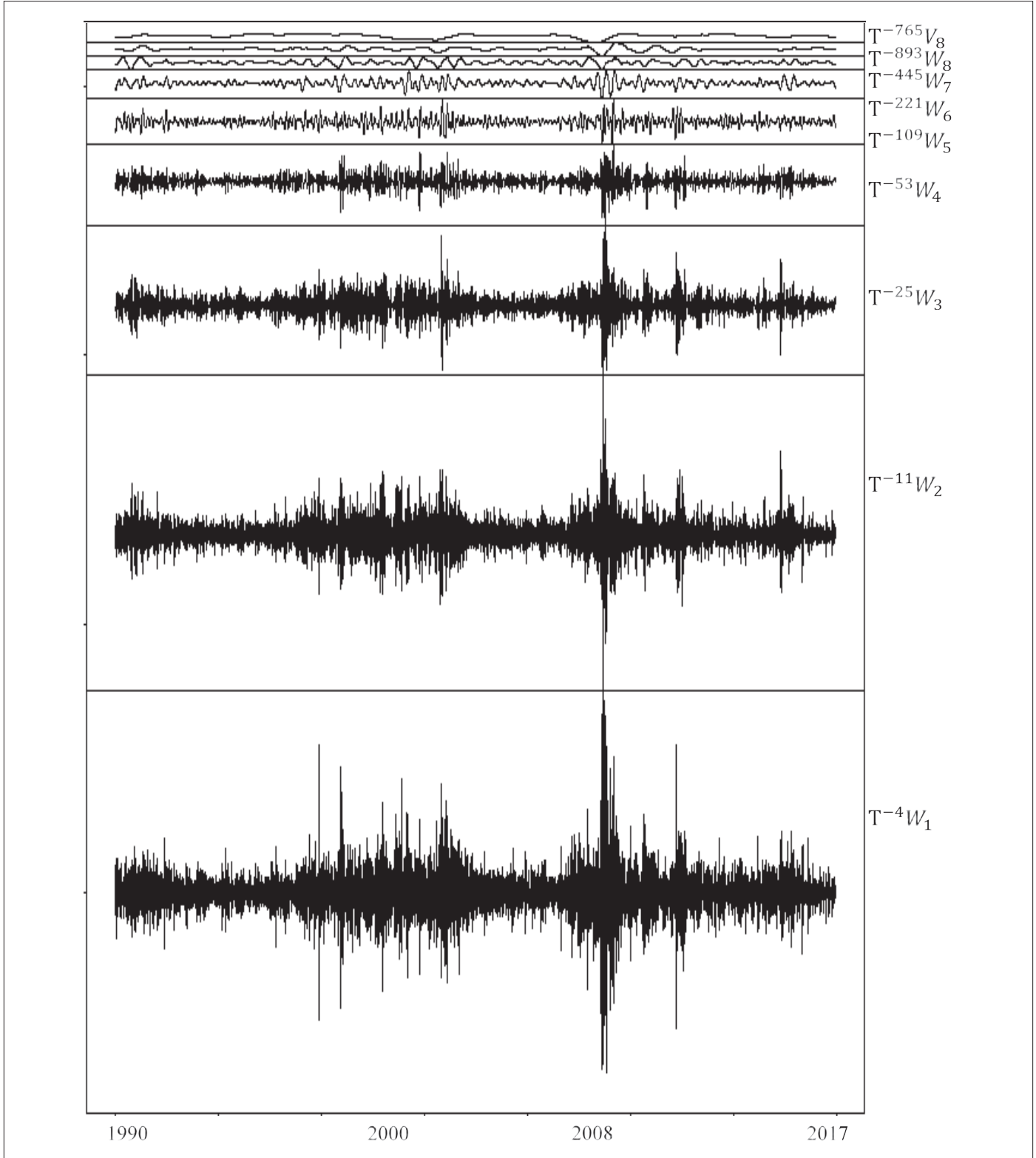
Note: T^{-i} means that the series of the coefficients is shifted by i positions backward so that all series are on the same timeline.

Also, $\sigma_X^2(j)$ has an unbiased estimator:

$$\hat{\sigma}_X^2(j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \tilde{W}_{j,t}^2,$$

where

$$M_j = N - L_j + 1.$$

Skewness and kurtosis of each level can be estimated as well:

$$\textbf{Skewness} \quad \hat{S}_X(j) = \frac{\frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \tilde{W}_{j,t}^3}{\hat{\sigma}_X^3(j)},$$

$$\textbf{Kurtosis} \quad \hat{K}_X(j) = \frac{\frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \tilde{W}_{j,t}^4}{\hat{\sigma}_X^4(j)}.$$

Table 4 lists the mean, variance, skewness and kurtosis for each decomposition level and the original time series. Low levels (high frequency/short term) contribute most of the variance of the original return series. Skewness and kurtosis are quite different among the eight levels, which indicates that the patterns at different frequencies are different, and it may be beneficial to model them separately.

The empirical VaR of the original time series can be approximated by aggregating the VaR at each decomposition level as follows:

$$VaR_{Agg} = \sqrt{\sum_{j=1}^{\bar{J}_M} VaR_j^2},$$

where

VaR_{Agg} = aggregated VaR,
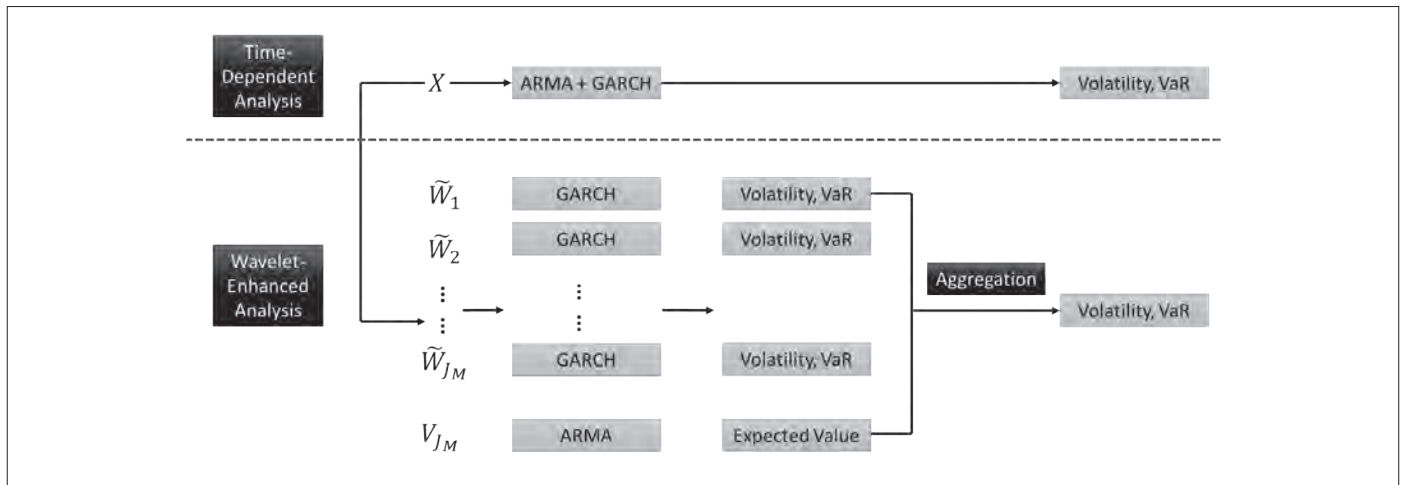
VaR_j = VaR at level j.

In this example, aggregated empirical VaR is 3.94 percent, compared to 3.93 percent calculated directly from the original time series. The non-normality of the original time series is preserved well by the wavelet coefficients in this example.

TIME-VARIANT RISK ANALYSIS

The wavelet analysis in the previous section assumes a constant volatility. Time-variant risk analysis can be enhanced with wavelet analysis as well to reflect different patterns at each wavelet decomposition level. This section builds on the ARMA-GARCH example to include analysis at each decomposition level. As shown in Figure 7, instead of modeling the original time series with one model, wavelet-enhanced time-dependent analysis studies wavelet coefficients at each level separately to understand the risk in different ranges of frequency. Wavelet coefficients are fitted into a GARCH model to get the volatility and VaR information. Scale coefficients at the highest level are fitted into ARMA and GARCH models to understand the trend of the time series. They are aggregated to get the predicted return, total volatility and VaR.

Following the simulation method used in the time series model to simulate future equity returns, wavelet coefficients can be simulated at each decomposition level. Conditional volatility and VaR can be projected for each level according to the

Table 4
Descriptive Statistics at Different Decomposition Levels

	Mean	Volatility	Variance Contribution	Skewness	Kurtosis	99.5% Empirical VaR	99.5% VaR (Normal)
Level 1	0.0000%	0.8%	53.5%	0.3	12.7	3.0%	2.1%
Level 2	0.0000%	0.6%	24.9%	0.2	11.3	2.0%	1.4%
Level 3	−0.0001%	0.4%	12.3%	0.1	7.6	1.2%	1.0%
Level 4	0.0000%	0.2%	5.0%	−0.1	6.3	0.9%	0.6%
Level 5	−0.0001%	0.2%	2.3%	0.1	5.5	0.5%	0.4%
Level 6	−0.0002%	0.1%	1.2%	0.03	5.2	0.4%	0.3%
Level 7	0.0001%	0.1%	0.4%	−0.2	3.7	0.2%	0.2%
Level 8	−0.0001%	0.1%	0.3%	−0.3	6.4	0.2%	0.2%
Original	0.0274%	1.1%	—	−0.2	11.9	3.93%	2.84%

Figure 7
Wavelet-Enhanced Time-Dependent Analysis Structure

calibrated GARCH model. They can be aggregated to predict the total VaR:

$$VaR_{Agg,T+l} = \sqrt{\sum_{j=1}^{J_M} VaR_{j,T+l}^2} - \mathbb{E}(r_{T+l}),$$

$$VaR_{j,T+l} = -\sigma_{j,T+l} SGED_j^{-1}(1-p),$$

where

$VaR_{Agg,T+l}$ = aggregated daily VaR at $T + l$, l periods ahead of T,

$VaR_{j,T+l}$ = daily VaR at $T + l$ at decomposition level j. The expected value of wavelet coefficients is zero and therefore is not included in the formula,

$\sigma_{j,T+l}$ = projected conditional volatility of level j wavelet coefficient at $T + l$,

$SGED_j^{-1}(1-p)$ = the $[100 \times (1 - p)]$th percentile of fitted SGED for level j wavelet coefficients.

Figure 8 shows the daily return range prediction based on 1,000 simulations for 250 trading days from the beginning of October 2017. Actual daily returns till September 2018 are compared with the projected ranges. While 10.2 percent of actual returns fall out of the middle 90 percent range (5th percentile to 95th percentile), 0.7 percent of actual returns fall out of the middle

99 percent range (0.5th percentile to 99.5th percentile). Compared to a pure time-dependent prediction, as in Figure 3, wavelet-enhanced prediction has a wider predicted range for extreme returns (0.5th percentile and 99.5th percentile).

> Time-variant risk analysis can be enhanced with wavelet analysis to reflect different patterns at each wavelet decomposition level.

For decision makers with a longer time horizon, annual VaR is a better measure than daily VaR for risk assessment. Multiresolution analysis (MRA) based on MODWT can be used to construct daily returns from transformed coefficients that preserve the autocorrelation of daily returns. Annual returns are then calculated based on simulated daily returns. Table 5 compares the annual VaR derived by different methods for the period from October 2017 to September 2018. Wavelet-enhanced time-dependent analysis provides a much higher annual VaR than a pure time-dependent analysis given a low volatility environment in September 2017. Wavelet analysis has a longer memory and helps preserve the long-term pattern much better than the time-dependent analysis in this example. Wavelet-enhanced time-dependent analysis also reflects current market conditions to predict the future risk in a given time horizon.

Figure 8
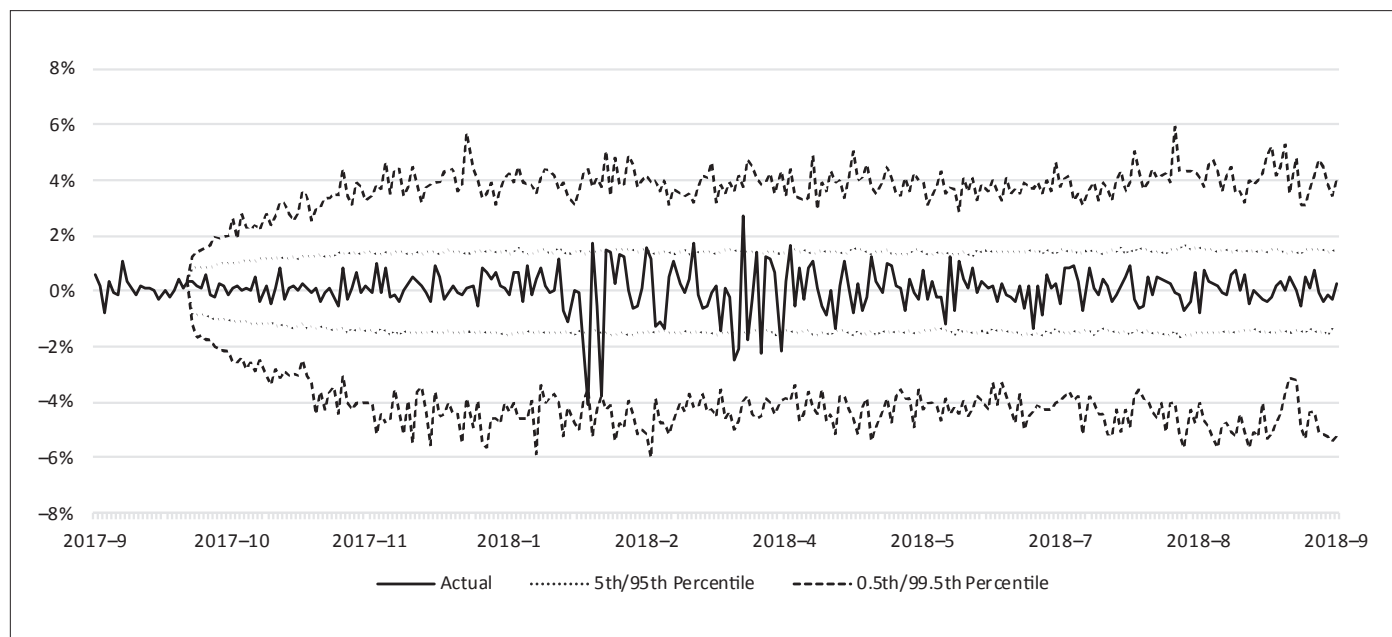Wavelet-Based S&P 500 Index Daily Return Range Estimation

Table 5
S&P 500 Index Return Annual VaR Estimation

	Projection Type	Model	95% VaR	99.5% VaR
Time-dependent analysis	Conditional	ARMA + GARCH	4.6%	24.2%
Wavelet-enhanced time-dependent analysis	Conditional	MODWT + MRA	17.6%	39.9%
Empirical analysis (Jan. 1990–Sept. 2017)	Unconditional	Statistical analysis	26.9%	43.5%

For VaR estimation at a high confidence level, wavelet-enhanced time-dependent analysis is the best option based on the back-testing results at different volatility levels. In addition, this type of analysis can adjust itself based on new information in a timely manner.

CONCLUSION

Unlike time series analysis, wavelet analysis can be used to systematically analyze historical time series data by time and frequency concurrently. Wavelet analysis provides a decomposition of the total risk and can tell whether short-, medium- or long-term risk is dominating. It can better capture different patterns at different frequency levels to improve risk estimation. Risk measures such as volatility and VaR can be calculated directly using wavelet models.

Wavelet analysis is especially useful when time horizon has a significant impact on risk analysis. It can help refine assumptions such as volatility, tail heaviness and correlation according to the time horizon of risk analysis. ■

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A Novel Approach to Valuing an Insurance Company's Economic Surplus

By Dariush Akhtari

Editor's note: This article is adapted from a research paper that was part of the 2019 Enterprise Risk Management Symposium call for papers. It was awarded the Joint CAS/CIA/SOA Risk Management Section Award for Practical Risk Management Applications. The full paper is available at www.ermsymposium.org/wp-content/uploads/2019/05/Akhtari_JRMS-Prizewinner.pdf.

A critical step in valuing a company for the purpose of financial reporting is the computation of the market value of economic surplus (MVES). This article provides a novel approach to the valuation of MVES that is stable and reasonably immune to "market noise."[1] Current approaches to the calculation of economic surplus of an insurance company generally define such surplus as the market value of the assets (MVA) supporting the liabilities less the market value of liabilities (MVL). While MVA is observable in the market, MVL is typically computed directly, without regard to the underlying supporting assets. Subtracting MVL from any value of assets that changes with market movements results in an unstable surplus. Consequently, a surplus that is not fully reflective of market sentiment has been rejected by many. This same approach is further used in the calculation of market-consistent embedded value (MCEV) when a balance sheet approach is used.[2]

Currently, most direct methods for computing MVL involve discounting at the risk-free rate plus a spread to account for the illiquidity of the liabilities. Some additional adjustments are typically made to account for (1) capital that must be held in case the future pattern of cash flows does not match the projected ones and (2) the fact that interest earned on funds in an insurance company is taxed inside the corporation before it is distributed to shareholders and becomes taxable income to the shareholder at distribution.

The most important driver of the direct method of calculating MVL is how the spread is calculated and how quickly this spread can react to market changes. Unfortunately, current proposed approaches rely on spreads that do not react quickly enough to market movements. This is because, unlike assets, there are no observable values for liabilities, resulting in a volatile economic surplus.

Since the economic value of surplus and MCEV are computed similarly, the terms "MCEV" and "economic surplus" (and MVES) will be used interchangeably in this article. MCEV is a great tool in evaluating company value and is widely used in Europe, yet its use in the United States has been curtailed because of its volatile nature.

GENERAL APPROACHES TO THE CALCULATION OF MVL

Approaches to the valuation of MVL can be generalized into two broad and distinct categories.

MVL Should Not Be Dependent on the Value of the Assets Backing Them

This concept stems from the belief that there is a unique value for every object independent of its owner. The proponents of this method come up with a unique value for an insurance liability, for which the market is neither liquid nor deep. To achieve that, they have concentrated on a unique discount curve that can be applied to the insurance cash flows to arrive at the liability's market value. A simple example highlights the flaw in this approach. Assume two identical term insurance contracts for the same face amount on the life of the same individual are held by two different insurance companies. Since the projected death claim by the two companies would most likely not be identical, discounting them using identical rates would not result in identical values.

MVL Should Reflect the Assets Backing Them

This concept reflects a number of extremely important elements of insurance markets and business models:

1. Due to the illiquidity of insurance cash flows, insurers could buy and hold an instrument to maturity, making them indifferent to the credit migration of these assets.

2. No liability is ever sold without the assets backing it.

3. Many insurance products' cash flows are dependent on the assets backing them (e.g., fixed annuity, universal life or variable annuity products).

4. MVL is used in the calculation of many asset and liability management (ALM) metrics, such as duration and convexity. Not reflecting the value in conjunction with the assets backing them will result in the mismanagement of the business. This important issue is explored further in the next section.

FLAWS IN THE CURRENT APPROACHES TO THE CALCULATION OF MVL

A simple example can be used to highlight a major flaw with valuing liabilities independent of the assets backing them. In this example, assume that the basket of assets backing the liabilities actually has cash flows that match those of the liabilities in every scenario. Now assume that a spread over the risk-free rate has been provided to calculate the value of the liabilities. Figure 1

shows three rates: the risk-free rate, the rate used to discount liabilities and the risk-adjusted rate of return of the supporting assets.[3] In this example, the average return on assets is about 84 basis points (bps) over the risk-free rate, and the average spread used for discounting liabilities is about 36 bps. This spread differential results in the value of assets being lower than the value of liabilities.

Table 1 reflects the ALM metrics and values based on the rates in Figure 1. Since the asset cash flows were identical to those of the liabilities in every scenario, one would expect a zero surplus from this combination of assets and liabilities. However, this approach does not produce a zero surplus either at the valuation rate or under any of the rate shocks. Only one deterministic rate has been used in this simple example to highlight the issue, but one could have used a set of stochastic runs and achieved a similar result.

Had a replicating portfolio technique been used to select the assets—and if the replicating portfolio technique had actually produced the exact basket of assets—it would be immune to this flaw because the value of the liabilities would be set equal to the value of the basket of assets. However, even this method has its limitations:

*   There is no guarantee that the method would produce the exact basket of assets. It is highly possible that two different baskets would be produced, depending on the starting universe of assets.
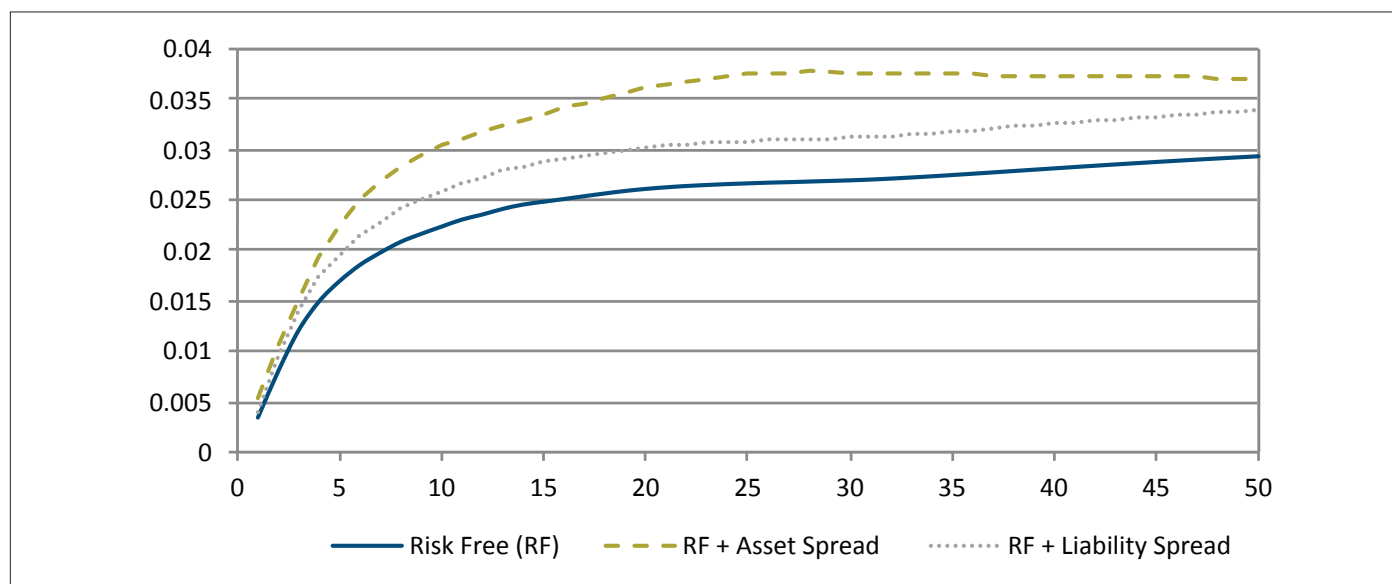
Figure 1
Discount Curves

Risk Free (RF) — — RF + Asset Spread .... RF + Liability Spread

Table 1
ALM Metrics—Current Method (Matching Asset and Liability Cash Flows), in Millions of Dollars

Metric	Base Curve	Rates Up 25 bps	Rates Up 300 bps	Rates Down 25 bps
MVA	714.9	692.9	507.6	737.9
Assets duration	12.6	12.4	10.3	12.8
Assets convexity	2.43	2.36	1.73	2.50
Assets DV01	(0.90)	(0.86)	(0.53)	(0.94)
MVL	765.2	740.7	535.3	791.0
Liabilities duration	13.2	12.9	10.8	13.4
Liabilities convexity	2.62	2.54	1.86	2.70
Liabilities DV01	(1.01)	(0.96)	(0.58)	(1.06)
Surplus	(50.4)	(47.8)	(27.8)	(53.1)
Surplus DV01	0.11	0.10	0.05	0.11

Abbreviations: bps, basis points; MVA, market value of assets; MVL, market value of liabilities.

- Policyholder behavior cannot be replicated with market instruments,[4] so it would be impossible to arrive at a basket of assets that replicates the liabilities.

- The replication techniques rely on linear regression that minimizes errors but does not necessarily match cash flows.

- The results are dependent on the scenarios that are run. Two companies using the same assumptions but different economic scenario generators (ESGs) could arrive at different baskets of assets.

- The recursive issue of products, the interdependency of liability cash flows and the assets backing them mean that this method cannot be applied to value nearly a third of the existing products in the insurance market.

Flaws in the Use of ESGs

ESGs are calibrated to reproduce the observed value of market instruments. Depending on the instruments used for the calibration of the parameters and the models, the scenarios in two different ESG models will differ. Such different scenarios are likely to generate liability cash flows that may be significantly different. In many cases, ALM metrics of liabilities are calculated using a set of scenarios based on some risk-neutral ESG. However, the ALM metrics of assets may not have been calculated using the same scenarios. When different ESGs are used to value assets and liabilities, revaluing liabilities using the same scenarios that are used to value assets may produce a significant change in the value of liabilities as well as the ALM metrics for those liabilities.

PROPOSED METHOD OF CALCULATING MVL

As indicated earlier, the main reason a value for MVL is important is to calculate the market value of surplus, which is obtained by subtracting MVL from the market value of assets. Formulaically, this means MVS = MVA – MVL. However, if the goal is to evaluate MVS, why not calculate MVS directly? MVL can then be obtained by subtracting MVS from MVA, avoiding the complexities associated with a direct computation of MVL, which involves discounting liability cash flows. In essence, the proposed approach in this article delivers a more stable market-based value of liabilities.

In the proposed methodology, terminology is borrowed from MCEV because of its acceptability in many parts of the world:[5]

ACF_t = Default-adjusted asset cash flow[6] at time t

LCF_t = Best-estimate liability cash flow at time t (inflow less outflow)

DR = Discount rate (time variant)

$Spread_t$ = Spread over risk-free rate (RF) = DR_t – RF_t

TVFOG = Time value of financial options and guarantees

CRNHR = Cost of residual non-hedgeable risks

FCRC = Frictional cost of required capital

NCF_t = Net cash flow at time t = ACF_t + LCF_t

S* = PV(@DR)(NCF_t)

Assume the assets backing the liabilities are set based on the amount needed to cover the statutory reserve amount. The excess of the value of assets over the assets needed to cover liabilities computed on a best-estimate set of assumptions without regard for solvency can be considered as surplus at the line-of-business level. In other words, this surplus is equivalent to the solvency margin as used in Solvency II or provisions for adverse deviation. This article further uses "company surplus" to refer to the sum of required capital (RC) and free surplus.

The first step in the proposed method is for the company to have a well-defined investment strategy that identifies asset classes, asset mix and asset quality that it intends to invest in to fulfill a year t expected cash flow.

In this approach, default-adjusted asset cash flows are projected using industry-accepted transition matrices.[7] Both asset and liability cash flows are projected using best-estimate assumptions for assets backing the liabilities under the same scenario. The goal here is to represent cash flows that are expected, as business is managed based on company-specific assumptions and management actions. Once both asset and liability cash flows are projected under the same scenario, their net cash flows for each period are produced. Net cash flow at time t will either be positive (surplus or asset) or negative (deficit or liability).

One approach with theoretical appeal involves using different risk discount rates to discount net asset cash flows and net liability cash flows. More specifically, a net asset cash flow one year from now represents an amount that can increase surplus at that time. To convert that amount immediately into cash, the company can borrow an amount today and pay the load in full with the cash flow one year from now. Hence, it makes sense that the discount rate used to convert a future positive net cash flow into cash would be the loan rate the company would be charged. This rate would be based on the company's credit rating. In contrast, a net liability cash flow should be completely funded by invested assets. Hence, the rate for discounting net liability cash flows should be based on the company's investment strategy, which includes the mix of assets and corresponding risk-adjusted rates of return. This combination of company credit rating and risk-adjusted return from the company's investment strategy defines the discount rate (DR).

Despite the theoretical appeal of using two different discount rates, there are some practical limitations. One could argue that should too much debt be used, the company's credit rating would deteriorate and the borrowing costs would increase. Further, a company's rating is also dependent on the type of invested assets, so the borrowing cost and risk-adjusted return on invested assets could converge. For these reasons, it is suggested that both positive and negative NCFs be discounted at the same risk-adjusted rate of return that will then define the DR. This means that all one needs are the rates used for discounting, as opposed to a spread over the risk-free rate. However, for reporting purposes, the spread could be calculated by subtracting the RF from the DR. It should be noted that spread is a curve, varying by period. To know the spread when many scenarios are run, one needs to convert both the DR and the RF to forward rates, thus defining "spread" as the spread over forward or short rates. This will allow the addition of a spread when discounting using short rates along each path or scenario.

For products with options and guarantees, a stochastic set of risk-neutral scenarios needs to be created. The cash flows of both assets and liabilities are projected using these scenarios. NCF_t in each scenario is calculated and discounted using the scenario's short rate plus the spread (calculated as spread over forward rate). By subtracting the average of the resulting S*s (derived from the set of stochastic scenarios) from S* (derived from a single deterministic scenario based on the prevailing RF at the valuation date), the TVFOG emerges. The average of S*s implicitly reflects TVFOG.

Since best-estimate assumptions are used in the calculation of NCFs, one needs to account for possible variance in the experience. The cost of capital approach could be used to account for this variance.[8] To make this approach consistent with MCEV, this article borrows the cost of residual non-hedgeable risks (CRNHR) calculation and uses it consistently.[9] The calculation of CRNHR should reflect the greater of statutory required capital and the value of capital derived using confidence levels for internal capital valuation/MCEV.

> The first step in the proposed method is for the company to have a well-defined investment strategy that identifies asset classes, asset mix and asset quality that it intends to invest in to fulfill a year t expected cash flow.

Generally, CRNHR refers to the capital charge for non-economic assumptions. However, since the generation of asset cash flows uses best-estimate default rates from transition matrices, one needs to account for the probability that actual defaults might be greater than projected. For this reason, this article uses the same cost of capital approach for capital charge on default as well. This means that CRNHR is extended to account for default risk beyond the best estimate.

Tax comes into the picture in two areas. One is tax on income generated from the release of the conservatism built into the held reserves, and the other is the tax on investment income earned on RC. In the proposed method, the liability cash flows include income taxes but not tax on investment income on assets supporting RC. Further, as investment income on RC is taxable, this article further borrows the FCRC from the American Academy of Actuaries (2011). It should be noted that if one assumes that the assets backing the liabilities include RC, the computed income tax would already include tax on investment income earned on RC, and FCRC would need to be adjusted to exclude such tax.

Extension of the risk-free rate beyond the observable values in the market is outside the scope of this article. However, the author is in favor of extending the risk-free rate using a mean reversion of forward rates over a long term, as such extension should contribute to a more stable surplus.

The market value of surplus then becomes MVS = S* – TVFOG – CRNHR – FCRC.[10] This valuation means that MVS has accounted for all capital and tax charges at a particular confidence level—MCEV asks for 99.5 percent.[11] Based on this approach, the calculated MVS plus the market value of a company's surplus would equal MCEV using the balance sheet approach.

For the valuation of liabilities with no assets, such as when valuing new business, the projected liability cash flows are discounted using the DR created from the assets to be invested to back the liability based on the investment strategy. Further, if the liability cash flows are dependent on the portfolio returns, the portfolio is assumed to earn the DR for the period.

RATIONALE FOR THE NEW PROPOSAL

It is important to remember that ALM risk is the risk created by a mismatch of asset and liability cash flows. As such, a method that incorporates these cash flow differentials is more advantageous for business management. This proposal offers two important benefits:

1. existing assets in the market, as opposed to synthetic assets that are generally used in the replicating portfolio methods, are used to cover the net cash flows; and

2. the company's investment strategy is incorporated in the choice of assets, which allows this method to be consistent with MCEV, ALM and how business is managed.

However, the most important benefit of this approach is the fact that it allows for more appropriate investment management using ALM metrics. This becomes clear through the use of an example. It was shown earlier that should assets backing liabilities have identical cash flows to the liabilities, the ALM metrics of these assets also should be identical to those of the liabilities. This will never be the case if the valuation of liabilities is independent of the assets backing them. For this example, the proposed approach would result in zero net cash flows at all durations, resulting in zero surplus in any scenario and the aggregate, ensuring that all asset ALM metrics also match those of liabilities.

In the example that follows, the same liability cash flows as before are used, while the asset cash flows are projected from the actual assets backing them. The discount curve used for discounting NCFs is assumed to be the same as that from the previous example, which could be achievable in the current market based on the documented investment strategy. It is assumed here that the cash flows of both assets and liabilities are not interest-rate sensitive, so only interest rates impact the ALM metrics. Figure 2 shows both the asset and liability cash flows, with liability cash flows depicted as inflow less outflow. As shown, in years 7 to 16, there are large excess net cash flows (LCF + ACF) to cover deficits in years 30 and over, where there are no asset cash flows.

Figure 2
Cash Flows
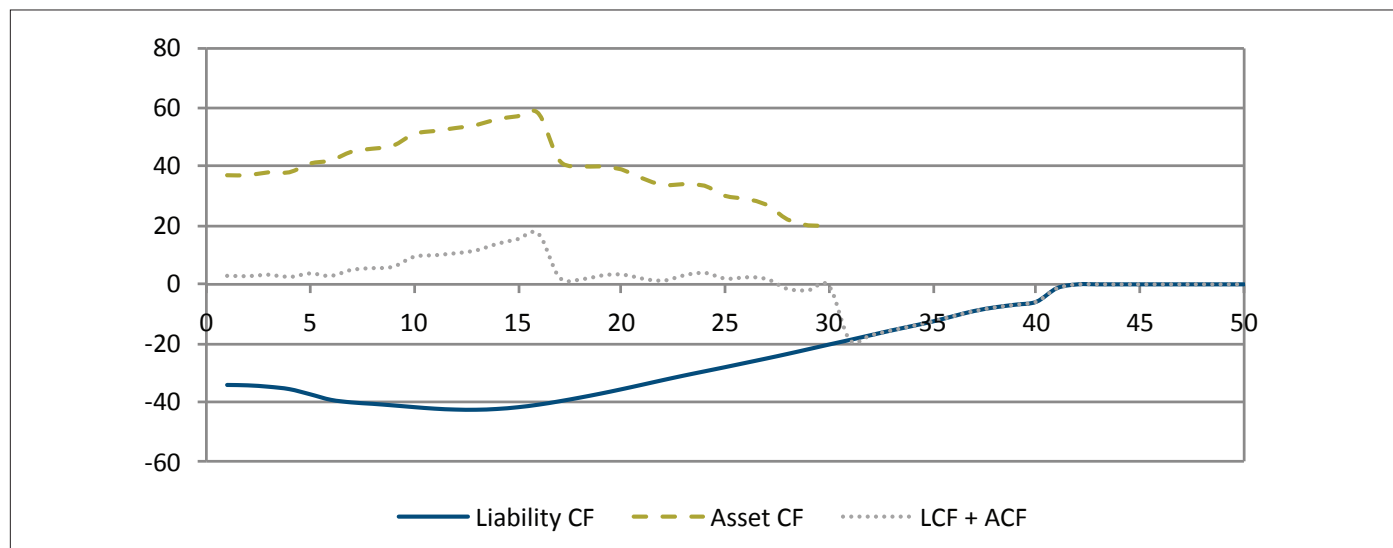
Abbreviations: ACF, asset cash flow; CF, cash flow; LCF, liability cash flow.

Table 2
Comparison of ALM Metrics Using Current and Proposed Methods, in Millions of Dollars

Metric	ALM Metrics Current Method (Discount L)				Metric	ALM Metrics Proposed Method (Discount NCF)			
	Base Curve	Rates Up 25 bps	Rates Up 300 bps	Rates Down 25 bps		Base Curve	Rates Up 25 bps	Rates Up 300 bps	Rates Down 25 bps
MVA	778.6	756.6	565.3	801.5	PV(NCF)	62.1	62.5	58.7	61.6
Assets duration	11.5	11.4	9.9	11.7	NCF duration	(2.7)	(1.6)	4.8	(4.0)
Assets convexity	1.95	1.90	1.49	1.99	NCF convexity	(4.71)	(4.16)	(0.97)	(5.33)
Assets DV01	(0.90)	(0.86)	(0.56)	(0.94)	NCF DV01	0.02	0.01	(0.03)	0.02
MVL	765.2	740.7	535.3	791.0	MVL	716.5	694.1	506.6	739.9
Liabilities duration	13.2	12.9	10.8	13.4	Liabilities duration	12.8	12.6	10.4	13.0
Liabilities convexity	2.62	2.54	1.86	2.70	Liabilities convexity	2.52	2.45	1.77	2.60
Liabilities DV01	(1.01)	(0.96)	(0.58)	(1.06)	Liabilities DV01	(0.92)	(0.87)	(0.53)	(0.96)
Surplus	13.4	15.9	30.0	10.5	Surplus	62.1	62.5	58.7	61.6
Surplus DV01	0.11	0.10	0.02	0.12	Surplus DV01	0.02	0.01	(0.03)	0.02

Abbreviations: bps, basis points; MVA, market value of assets; MVL, market value of liabilities; NCF, net cash flow.

Table 2 shows the result of the current method and the proposed method on surplus value and ALM metrics.

Examining Table 2, the current method suggests that there is a $13.4 million surplus, and should rates increase by 300 bps instantaneously, surplus will increase to $30 million (an increase of $16.6 million). The proposed method first indicates that this combination of assets and liabilities actually has $62.1 million of surplus, and a 300 bps rate increase reduces surplus by $3.4 million (versus *increasing* surplus by $16.6 million under the current method). Thus, a rate increase is detrimental to this combination of assets and liabilities, in contrast to what the current method suggests.

The current method first and foremost *underestimates* the value of this block of assets and liabilities. Further, it not only produces a larger surplus movement for interest rate shocks, but in this example, it also suggests that a large interest rate movement could be beneficial to the business, when it may actually be detrimental. In addition, note that under the proposed method a 25 bps rate change impacts surplus by only $0.4 million, which highlights the stability of this method over the current one.

Note that in the proposed method, MVL was not calculated directly but as the value of assets less value of surplus.

BENEFITS OF THE NEW PROPOSAL

The greatest benefit of the proposed approach is that the resulting MVS should be reasonably stable and far less susceptible to market noise than MVS obtained by current methods. Unless one uses the same ESG and discounting assumptions in the calculation of assets and liabilities, one will introduce volatility in the surplus value.

What if the Discount Rate is not Well Defined?

Because surplus is a fraction of the liability value, even with disagreement in the discount rates applied to the NCFs, the magnitude of disagreement is grossly mitigated. Generally, assets that back liabilities are set based on a statutory reserve that is slightly larger than the best-estimate liability—say, about 10 percent larger. Thus, when net cash flows are discounted, the discount rate is applied to this 10 percent as opposed to the entire liability. This means that should there be disagreement about the spread used for discounting, it impacts only 10 percent

of the value as opposed to the entire liability, resulting in more stable surplus and liability values.

In conclusion, this article has proposed a novel approach for calculating a stable economic surplus for an insurance business that allows better management of the business using more appropriate ALM metrics. It should achieve more acceptability by the industry, as it addresses many of the concerns with current approaches. ■

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ENDNOTES

1 "Market noise" is used in the context of insurance and signifies variation in the value of a company that is not reflective of overall market sentiment.

2 American Academy of Actuaries, 2011.

3 "Risk-adjusted rate of return" refers to a rate that can be used to discount an instrument's expected (or best-estimate) cash flows to reproduce the sum of the instrument's market value and its cost of capital. In this regard, "expected or best-estimate cash flows for a market instrument" refers to cash flows derived using management's best-estimate default assumptions. Risk adjustment will cover charges meant to reflect what a market participant would demand for accepting the risk of default (since defaults might actually turn out to be much higher than expected).

4 Koursaris, 2011; Hörig and Leitschkis, 2012.

5 American Academy of Actuaries, 2011.

6 Default-adjusted asset cash flows are created by assuming best-estimate default rates in the projection of asset cash flows. Best-estimate default rates are considered to be realistic, real-world assumptions as opposed to market-consistent or risk-neutral assumptions, which contain premiums that market participants demand for accepting the risk that defaults might be much higher than expected. As an example, if the best-estimate default rate of an asset over the year is 1 percent, and if the conditional (or promised) cash flow of that asset is $100 one year from now, the default-adjusted cash flow would be $99.

7 A number of industry-approved transition matrices (e.g., Moody's) provide best-estimate default assumptions for many market instruments in addition to the probability of the transition of assets from one rating to another.

8 For understanding the cost of capital approach, please refer to the American Academy of Actuaries (2011). In short, each assumption is shocked to a desired level of confidence for capital to be held, such as 99.5 percent. The resulting discounted liability cash flows less the best-estimate liability is considered capital needed for that assumption. This capital amount needs to be calculated for all future years (projected capital). A cost for this capital needs to be used—for example, 6 percent (this assumes that investors require a 10 percent return and the company earns 4 percent on that capital, netting a charge of 6 percent). In this example, a present value of 6 percent of projected capital is the cost of capital for that assumption. In essence, it is the cost to pay a potential risk buyer to take the risk.

9 American Academy of Actuaries, 2011.

10 If S* already includes TVFOG derived from a set of stochastic scenarios, only CRNHR and FCRC would be subtracted from S* to obtain MVS.

11 American Academy of Actuaries, 2011.

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Introduction to the Research on Developing a Liability-Driven Investment (LDI) Benchmark Framework

By Kailan Shang and Zakir Hossen

For defined benefit pension plans, liability-driven investment (LDI) strategies are becoming more popular as a way to reduce the risks associated with pension liability. The philosophy of LDI for pension funds is similar to asset liability management in the banking and insurance industry. It is a systematic approach to balancing pension liability hedging and pension asset growth.

A few challenges exist in LDI modeling and implementation for pension plans. In a low-interest-rate environment, alternative investments such as real estate, private equity, infrastructure and commodities are used to support the high expected asset return. These asset classes are less liquid or largely driven by specific factors in addition to the general market trend. The interdependency of asset subclasses in a pension asset portfolio also needs careful analysis. LDI cares about not only normal scenarios but also stress scenarios. The interdependency is usually stronger in stress scenarios. Because of the long-term nature of defined benefit pension plans, economic cyclical patterns also need to be embedded in economic scenarios. LDI analysis needs to be based on holistic, consistent and realistic scenarios.

To understand these challenges, an LDI benchmark model was developed in research sponsored by the Society of Actuaries (SOA) Retirement Section Research Committee and Committee on Finance Research. The model starts from an economic scenario generator, which includes fundamental economic factors and asset returns. The economic scenario generator provides a bridge between assets and liabilities in the LDI benchmark model. The exposure of assets and liabilities to common factors embedded in the scenarios can be assessed. With the LDI benchmark model, the financial outcome of LDI strategies can be predicted under different scenarios. This model allows users to test different LDI strategies for asset allocation purposes. The model is also helpful for measuring, optimizing and managing the risks arising from pension asset-liability mismatch. An Excel tool accompanying this report has also been developed to illustrate the LDI benchmark model with numerical examples.

Research materials, including a full research report and the Excel tool, can be found on the SOA website at *www.soa.org /resources/research-reports/2019/liability-driven-investment/*. ∎

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Methodologies for Valuing Cash Flows That Extend Beyond the Maximum Yield Curve

By Benjamin Leiser and Jack Kerbeshian

I n 2018, Risk & Regulatory Consulting conducted a study on methodologies used for yield curve extrapolation to value liability cash flows that extend beyond the maximum observable portion of the yield curve. The study was sponsored by the Society of Actuaries (SOA) Committee on Finance Research. We performed research on the methods available in theory and used in practice and also developed and provided questionnaires to a broad group of subject-matter experts with strong industry representation to comment on these methods, in order to provide a comprehensive view of the yield curve extrapolation methods. We developed the interview questionnaire based on an initial review of the literature and covered topics such as

- industry approaches for extrapolating the yield curve and the situations (specific products, specific applications) in which each is used;

- key assumptions and mechanics considered in the extrapolation of the yield curve;

- benefits and drawbacks of the various approaches; and

- practical challenges that arise from various methods.

We then supplemented the initial research with results of the survey and interviews, including any theoretical and practical issues noted with the methods. Both the research and the survey included details of how these methods are applied, as well as observations on the benefits, drawbacks and prevalence of their use. This article provides a summary of our study. The complete report, including a summary of panelists' views, can be found on the SOA's website (*www.soa.org/resources/research-reports/2019/yield-curve-report/*).

OVERVIEW

One of the most fundamental concepts in actuarial practice is the time value of money. For any work in which future cash flows are allowed for, such as reserving or pricing, it is natural to discount to present values so that an appropriate amount of money can be set aside today, allowing for future investment returns.

Risk-free yield curves are the building blocks for the valuation of future financial claims and long-term risk management work. Despite their fundamental importance, it turns out that measuring and estimating suitable risk-free interest rates present major challenges.

The liabilities of long-term financial institutions frequently extend beyond the term of available market instruments. To value these long-term claims and assess risk, practitioners must extrapolate yield curves to generate a set of "prices" for the assumed, inferred prices of discount bonds beyond the term of the longest available traded cash flow. A good yield curve estimation method must deliver extrapolated curves that are credible at a single point in time and where changes over time in extrapolated rates can be justified.

EXTRAPOLATION

Yield curve construction work requires completing two fundamental tasks: first, collating market data and fitting a continuous curve to the term of the longest available and reliable market instrument, and, second, extrapolating from the longest available and reliable market data toward some long-term assumption for forward interest rates.

Extrapolation also requires answering two questions about the path of forward interest rates beyond the longest market data point:

1. What is an appropriate assumption for the infinite-maturity, unconditional forward rate of interest?

2. What path is chosen between the longest (smoothed) market forward rate and this long-term rate? In particular, the analyst needs to determine the speed at which the extrapolated forward rate tends toward the long-term asymptote.

> A good yield curve estimation method must deliver extrapolated curves that are credible at a single point in time and where changes over time in extrapolated rates can be justified.

The initial goal when extrapolating the yield curve under many methods is to determine an ultimate long-term forward rate (UFR) to which the observable yield curve will converge. The components of the UFR are the following (a + b + c – d):

a. Expected future inflation.

b. Expected real short-term rate, which is the expected nominal short-term rate minus the expected future inflation.

c. Term premia, which are the additional returns an investor may expect as compensation for the longer-term investment and are represented by the difference between the forward rate and the expected future short-term interest rate. The term premium acts as compensation for holding long-term bonds, whose value will fluctuate in the face of interest rate uncertainty, exposing the holder to mark-to-market losses. Term premia have the following components:

   - Risk premia. Investors demand a premium for locking in long-term investments. This acts as compensation for holding long-term bonds, whose value will fluctuate in the face of interest rate uncertainty, exposing the holder to mark-to-market losses (not to be confused with credit or equity risk premia).

   - Term preference. Demand for long-term government securities from large institutional investors can drive down long-term forward rates because the long-term bonds offer a closer match to liabilities and are less risky investments to these investors.

d. Convexity effects. Fixed-income investments have positive convexity, which can cause longer-term bonds to trade at higher values (lower yields). Convexity adjustment arises because of the nonlinear (convex) relationship between interest rates and bond prices.

SAMPLE METHODS
After determining the UFR, the next step is to determine the appropriate methodology for extending or extrapolating the yield curve beyond the current investable universe. In this section we list several methodologies along with some detail on each method.

The Simple Extrapolation Method
The simple extrapolation method is simple to implement. It has two variations:

- **The simple monopole method.** This method assumes a constant single forward rate for all durations greater than 30.

- **The simple dipole method.** This variation uses the maximum observable (often 30-year) forward rate beyond that point.

The Flat Rate Extrapolation Method
The flat rate extrapolation method is similar to the simple extrapolation method. It assumes that the longest observable spot rate is extended infinitely throughout the non-observable portion of the yield curve. For any extrapolation, the long rate is guaranteed to exist and to be finite; however, it will not remain constant across periods. The usage of the observable yield curve is small, as the extrapolation relies entirely on the longest observable rate. The single factor driving the model is the longest observable rate, and while this is based on a tradable quantity, it could be limited when liquidity is low.

The Linear First-Order Extrapolation Method
The linear first-order extrapolation method assumes that a first-order linear relationship exists between forward rates beyond the longest observable spot rate. The two factors driving the model are gradient (slope of rates) and scale (level of rates). If the two factors are determined exclusively from the observable yield curve, then they will be hedgeable. This method assumes that the forward rates beyond M years follow a first-order linear progression of the form

$$f_+(\tau) = a + b \times \tau, \tau > M,$$

where

- a and b are the parameters of the extrapolation, estimated via least squares and

- τ represents the term of the forward rate and

- M represents the term of the longest observable (and tradable) spot rate.

Other First-Order Extrapolation Methods

Two more first-order extrapolation methods bear discussing:

- **The power spot rate extrapolation method**. This model assumes that forward rates beyond the longest observable spot rate follow a power relation. This method assumes that the forward rates beyond M years follow a power progression of the form

$$f_t(\tau) = a \times \tau^b, \tau > M.$$

- **The exponential spot rate extrapolation method.** This method assumes that forward rates beyond the longest observable spot rate follow an exponential relation. It assumes that the forward rates beyond M years follow an exponential progression of the form

$$f_t(\tau) = a \times e^{bxt}, \tau > M.$$

The Nelson-Siegel-Svensson Extrapolation Methods

The Nelson-Siegel-Svensson extrapolation methods place lower reliance on the 30-year spot rate for extrapolation purposes, and as a result, the hedging portfolios derived using these methods tend to be highly spread across the range of tradable and observable interest rates.

For the Nelson-Siegel method, the spot curve is expressed as a linear combination of three component functions with different shapes: a flat curve, a sloped curve and a humped curve. The Svensson method is an extension of the Nelson-Siegel model that adds an additional humped curve and allows a more diverse set of yield curves to be modeled.

The forward rate curve estimation is

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} e^{-\tau/\lambda} + \beta_{3,t}(\tau/\lambda)e^{-\tau/\lambda},$$

where

- τ represents the term of the forward rate,

- $\beta_{1,t}$, $\beta_{2,t}$, $\beta_{3,t}$ represent time-dependent stochastic variables and

- λ is a shape parameter.

The Svensson approach proposes an extension of the Nelson-Siegel model by adding another hump-shaped element, as shown in Figure 1.

Figure 1
Comparing Nelson-Siegel Curve with Nelson-Siegel-Svensson Curve
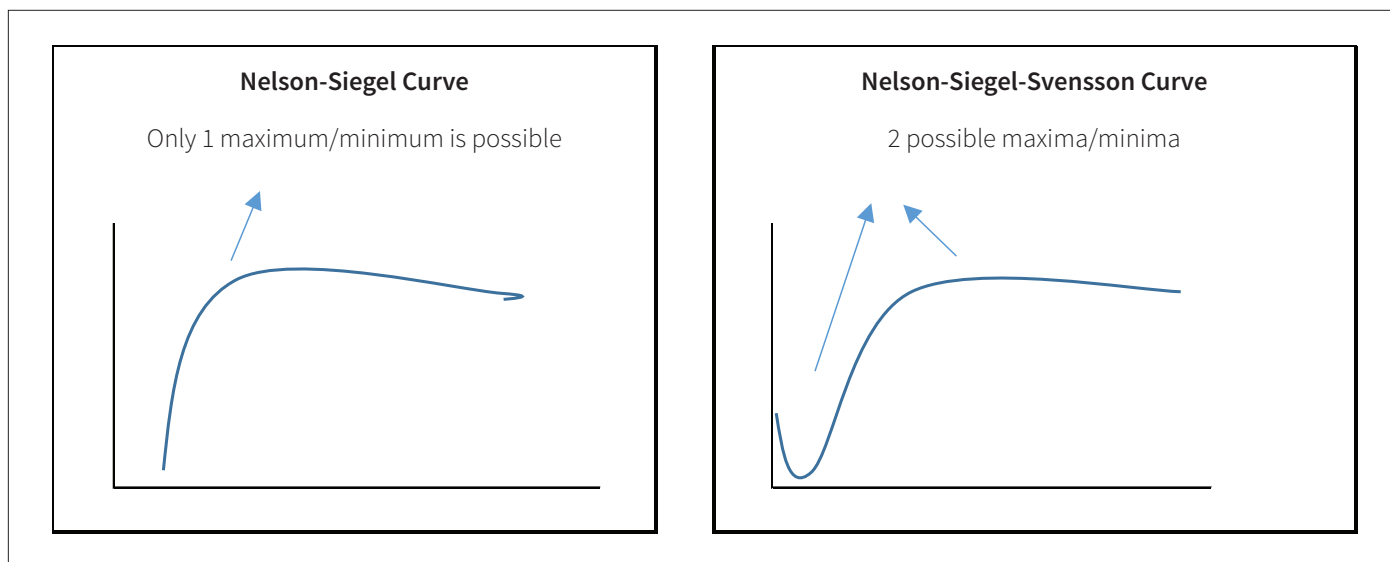
The Smith-Wilson Extrapolation Method

The Smith-Wilson extrapolation method is a class of models in which the long forward rate is a fixed input parameter and does not vary over time as bond prices change. It allows the long-term forward rates to converge toward the chosen "infinite" rate and provides a strong basis for hedging the long-term interest rate risk.

The input parameters are

- the UFR, and

- α, the speed of convergence to the UFR.

Smith-Wilson assumes that the discount factor, $P(\iota)$, at time t is determined by

$$P_t(\iota) = e^{-f\infty\iota} + \Sigma\ \xi_{i,t}K_i(\iota)\ \text{(sum from } i = 1 \text{ to I)},$$

$$K_i(\iota) = \Sigma c_{ij}W(\iota,u_j)\ \text{(sum from } j = 1 \text{ to } \mathcal{J}_i\text{)},$$

$$W(\iota,u) = e^{-f\infty(\iota+u)}[\alpha\min\{\iota,u\} - e^{\alpha\max\{\iota,u\}}\sinh(\alpha\min\{\iota,u\})],$$

where

- c_{ij} represents the jth cash flow on the ith bond used to calibrate the price function, and u_j represents the term of the respective cash flow;

- ξ represents a series of time-varying parameters used to fit the actual yield curve;

- K represents a set of kernel functions for each input observable bond price; and

- W is a symmetric function known as Wilson's function.

The Cubic Spline Extrapolation Method

The cubic spline extrapolation method extends the cubic spline used to fit the market data to the unconditional horizon. It is a class of models in which the long forward rate is a fixed input parameter and does not vary over time as bond prices change. It allows the long-term forward rates to converge toward the chosen "infinite" rate.

COMPARING THE NELSON-SIEGEL, SVENSSON AND SMITH-WILSON METHODS

In Table 1, we compare some of the more often utilized "complex" methods.

The Nelson-Siegel and Smith-Wilson methods are quite different in the way they are formulated. For extrapolation, the Smith-Wilson method relies on the last known observation (at the last liquid point [LLP]) and on the defined UFR, and the curve is created based on a weighted average of both for the period of convergence. For its part, the Nelson-Siegel method uses all the observed data to fit a curve and then uses

Table 1
Comparison of Frequently Used "Complex" Methods

Model	Pros	Cons
Nelson-Siegel	The three components give the model enough flexibility to capture monotonic, humped and S-type curves often typically observed in yield curve data	Highly nonlinear, which has been reported to cause estimation problems
	Parameters are easy to estimate and have simple, intuitive explanations	Cannot handle *all* yield curve shapes
	Widely used by central banks and practitioners	Assumes forward rates are always positive and the discount factor approaches zero as maturity increases
Svensson	Can more easily fit term structures with more than one local maximum or minimum, thereby allowing for a broader and more complicated range of yield curves	No significant improvement of the estimates when compared with the Nelson-Siegel model Highly nonlinear, which can make the estimate of the model difficult Overparameterization of the model can cause convergence problems
Smith-Wilson	Can be applied to raw market data	Requires expert judgment for the choice of alpha (the speed of convergence to the ultimate forward rate)
	Provides a perfect fit to liquid market data	$P(t)$, the discount factor, may become negative

the factor loadings, or the component coefficients, to extrapolate the remainder of the curve beyond the LLP. The Svensson method is an extension of the Nelson-Siegel in which a second medium-term "hump" factor with a separate decay parameter is added.

SAMPLE OF PANELIST VIEWS

Expert industry panelists were given questionnaires to comment on methods used to extrapolate the yield curve both in practice and in theory. We include here some of the responses they provided that helped to supplement our research with respect to the various methods that are commonly used for yield curve extrapolation.

These days many tend to use Smith-Wilson where mandated and cubic spline Nelson-Siegel (CSNS) in other situations. The key benefit of the CSNS method is that, when appropriately parameterized, it allows curves to be produced in a highly automated way while reliably meeting quality criteria:

- good quality of fit to market data;

- smooth transition between market data and extrapolation phase; and

- appropriate convergence to UFR.

A potential limitation is that the parameterization requires some care in the setup, but this is achievable with appropriate attention and expertise.

Some practical challenges that have arisen from the various applied approaches include the following:

- Many approaches seem to be very complex while still requiring a large amount of judgment and discretion.

- Any method not based on setting the future forward curve can lead to unusual and unrealistic patterns of forward rates.

- Generally, parametric methods lack the flexibility to accurately fit market data and extrapolation behavior; particularly for liability valuation, this is a critical requirement.

- Flat extrapolations are potentially oversimplified (particularly in markets where liabilities are longer than the longest dated traded instrument) and fail to account for many practitioners' preference for a UFR.

- A 50-year discount rate curve (or 200 quarters) makes the curve a little unwieldy for valuations where the

long-term rate is less relevant. Some actuaries prefer a simpler approach for their valuations.

- Thinking through the last liquid point can be a challenge.

- Getting agreement from stakeholders is always a challenge.

> When appropriately parameterized, the CSNS method allows curves to be produced in a highly automated way while reliably meeting quality criteria.

Any extrapolation method involves significant risk, and because the potential riskiness, accuracy and bias of various methods is hard to evaluate, it may be appropriate to choose to put more resources into evaluating the risk of any proposed rate structure than into attempting to make "better" forecasts.

- For the UFR, including components such as the expected inflation and expected real short-term rate, the rate is usually a combination of judgment and officially published requirements. Generally, a simple extrapolation of the current long spot and forward rates is used. Using each of the components, these are estimated from pooled (across multiple currencies) historical data. Consulting with other long-term rates (such as the government's intergenerational reports) as well as historical data on long-dated bonds is helpful to check for reasonableness. The UFR accounts for expectations of long-term real interest rate and inflation. Term premia and convexity adjustment are not included in the determination of the UFR.

- For the duration of the UFR, one panelist stated, lacking anything definitive, using 20 years is a reasonable approach. Another stated they use 30 years and then grade over another 30 years. Some comments were more general such as, "as needed for product pricing application and as long as needed for projection." Others go as long as 50 years, which seems plausible when looking at countries with longer-dated bonds (U.K./Canada/U.S.). The duration is driven by the last liquid point for market data and a reasonable convergence period.

- The speed of convergence to the UFR is based on judgment and historical data, and it could be defined by a simple method. One panelist stated the convergence is linear from around 15 years through to year 50.

- The shape and smoothness of the transition from the observed rates to the extrapolated rate generated by the algorithm might be defined by a simple method. An important consideration is the smoothness of the transition. Some prefer a smoother transition, while others indicate the transition should jump from discontinuity to smooth. Linear interpolation is a popular transition despite being slightly nonintuitive compared to a decay curve.

- The mechanics or processes used to fit the curve include least squares with some judgment, automated processes, implemented as a solver optimization in Excel, linear programming and an interpolation method called monotone convex, which ensures that the continuous forward rates are positive.

Many of the panelists seem to agree that there isn't any "right" answer and, therefore, ease of explanation, simplicity and consistency with markets with long observable rates can be more important than theoretical "purity."

CONCLUSION

It is important to recognize that these extrapolation methods are models, and both the models and the assumptions going into the models need to be strongly vetted by the user to ensure applicability of the model and the appropriateness of the assumptions for the purpose for which it is used. One must determine if the assumptions and model result in an average or extreme view. A company wanting to be more conservative may model with more extreme down assumptions, whereas another that wants stability may use more average assumptions.

There is a wide range of modeling methods, from simple linear models to more complex spline models. A good extrapolation model strikes the right balance—practicality on the one hand, with the ability to capture the most important attributes and most critical features of history on the other.

Based on the research and the survey of industry experts, it appears that many of the methods used in practice are similar to those discussed and analyzed in the theoretical literature. The assumptions that feed into an extrapolation method may have as great, or at times even a greater impact than the technical methodology. However, the choice of the method itself does have an impact on the results. Using the current year forward rate extrapolated out into the future will have a much different result from a method that grades over time.

A large number of experts seem to be using the simpler models, favoring simplicity over complex models. Models that are too simple, however, can miss the true risks and may not appropriately capture tail events. At the other extreme, a good model does not "overfit" the data, reducing the ability to produce simulations beyond the historical data itself.

A good extrapolation model will produce results that are relevant to historical facts. A common tendency is to overweight the importance of the recent past. The danger in placing too much focus on recent risks is that one can forget that, over long periods, the economy can move rates to new and different places. With a longer-term horizon, one must avoid the temptation to influence a view.

Through our research and the survey as described in this article, we took a closer look at a few of the more popular methods, each having their own pros and cons. We were not surprised to find that in selecting an extrapolation method, users must determine the best fit for their particular needs. Many of the panelists stated that their method is simple and adequate. They believe that other methods involve more complex math without much evidence that they are any more theoretically justifiable. As one panelist stated, they endeavor to use the simplest model possible, but no simpler than what is necessary to be consistent with the market and economic principles. ■

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