Aging and Retirement

A Multifaceted Analysis of Dynamic Pension Plan Designs

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A Multifaceted Analysis of Dynamic Pension Plan Designs

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Executive Summary

The evolution of collective defined contribution (CDC) plans has gained momentum in countries such as the Netherlands, the UK, and Canada. These plans blend defined benefit (DB) and defined contribution (DC) elements, offering more predictable retirement income, while managing funding risks through risk-sharing among participants. This paper examines the roles of various designs of decumulation-only CDC plans, referred to as Dynamic Pension (DP) plans, in providing sustainable retirement income for retirees.

We use two key measures to evaluate DP plans: the repayment ratio (RR) and the annual pension payment ratio (APPR). The RR measures the extent to which participants recapture their contributions through pension payments, while the APPR assesses the annual pension payments relative to target pensions. Our analysis reveals the impacts of mortality and investment risks on pension outcomes within DP plans.

A critical focus of this paper is the exploration of optimal investment strategies under the Type 3 (Target Benefit Funded Ratio Linked, described later in this report) benefit adjustment mechanism. We apply bivariate spline interpolation to identify combinations of equity allocation and adjustment parameters that balance pension payment volatility with intergenerational wealth transfers. This quantitative approach offers plan administrators actionable strategies to manage risk and increase fairness across generations.

Additionally, the paper explores crucial design elements for DP plans, such as valuation interest rates, benefit-smoothing mechanisms, and pension annuity forms. Our findings highlight the necessity of a comprehensive design strategy for DP plans, incorporating regular reviews and adjustments to ensure fairness, stability, and participant satisfaction. By addressing the unique challenges of open versus closed group plans and optimizing design elements, DP plans can better achieve their goals of sustainable and equitable pension outcomes.

Section 1 Introduction

The development of collective defined contribution (CDC) plans, also known as target benefit plans, has been gaining momentum worldwide, with notable implementations in the Netherlands, the UK, and Canada. In the Netherlands, CDC plans have long been a cornerstone of their pension system, balancing defined benefit and defined contribution elements. Recent legislative changes have introduced further reforms to enhance the sustainability and fairness of these plans (Hoekert & Eikelboom, 2020). A new type of DC contract with collective investments will be an option available to pension funds, where contribution rates are fixed and not determined based on an explicit targeted level of benefits, and there will be no funding buffer. In the UK, the introduction of CDC schemes has been encouraged as a viable alternative to traditional pension models, promising a more predictable retirement income for participants. Canada has also embraced target benefit plans, particularly in the multi-employer sector, where they are seen as a means to share risks more equitably among members.

Parallel to these developments, Australia's superannuation funds are maturing, with a significant number of retirees entering the decumulation phase (Brinsmead & Edibam, 2023). This shift necessitates a focus on ensuring that retirement savings are managed efficiently to provide sustainable income streams for an aging population. The Australian model, characterized by its mandatory contribution system, offers valuable insights into managing large-scale pension funds transitioning to decumulation.

A recent paper by the author, titled "*Balancing Act: Exploring Intergenerational Risks in Target Benefit Plans,*" (Ma, 2024) delved into the intergenerational risk-sharing mechanisms within CDC plans, primarily focusing on the accumulation phase and associated investment risks. This paper serves as a continuation, concentrating on decumulation-only CDC plans, which are designed to provide sustainable retirement income through pooling and sharing of investment and mortality risks among a large number of retirees.

In the context of decumulation-only CDC plans, this paper identifies three commonly used mechanisms for adjusting pension payments to ensure that the gains and losses—primarily arising from deviations in mortality and investment experience from expectations—are equitably distributed among members of the retiree pool:

- **Type 1: Adjustment to Maintain Current Funding Level**
  Pension payments are adjusted annually to maintain a predetermined funding ratio, typically set at 1.0, thereby aligning the plan's assets and liabilities on an ongoing basis.
- **Type 2: Future Pension Escalation Adjustment**
  This approach modifies the expected escalation rate of future pension payments to preserve financial balance, particularly useful in smoothing adjustments over time.
- **Type 3: Target Benefit-Funded Ratio-Linked Adjustment**
  Annual pension payments are recalibrated to achieve a funded ratio relative to participants' target benefits, enabling a more direct link between funding status and target benefit levels.

These mechanisms play a central role in managing the financial health of the plan by adjusting payouts in response to changing economic conditions and mortality trends. They facilitate the collective sharing of risks and rewards inherent in retirement systems and embody the principles of flexibility and sustainability. The resulting benefit streams exhibit the characteristics of "Dynamic Pension" arrangements as discussed in *Affordable Lifetime Pension Income for a Better Tomorrow* (MacDonald, Sanders, Strachan, & Frazer, 2021).

For the purposes of this paper, we refer to decumulation-only CDC arrangements employing any of the above mechanisms as **Dynamic Pension (DP)** plans. This terminology underscores their adaptability in responding to evolving demographic and financial realities while promoting equitable outcomes among participants.

To analyze the investment and mortality risks (considering only mortality shifts within a participant cohort rather than individual longevity risks[1]) in DP plans, this paper applies two key measures: the repayment ratio (RR) and the annual pension payment ratio (APPR). The RR measures the extent to which participants recapture their contributions through pension payments, while the APPR assesses the level of annual pension payments relative to the participants' target pensions. These measures provide valuable insights into the effectiveness of DP plan designs in achieving their goals.

Building on these analytical measures, Section 11 of this paper introduces a comprehensive evaluation of investment strategies under the Type 3 benefit adjustment mechanism. By applying a combination of equity allocation (π) and adjustment parameters (θ), the section explores how these variables influence pension payment volatility and intergenerational wealth transfers. Using bivariate spline interpolation, optimal combinations of these parameters are identified, targeting specific levels of pension volatility. The insights gained from this analysis are critical for plan administrators seeking to balance pension outcomes and equity among generations.

The paper is organized as follows: It begins with an introduction to the advantages of risk pooling and sharing, followed by an examination of common designs of DP plans, including participant characteristics and types of benefit adjustment mechanisms. A framework for benefit adjustments is then presented, detailing the calculation of pension adjustment factors and evaluation measures. The impact of mortality and investment risks on pension outcomes is analyzed, focusing on intra-cohort wealth transfers, mortality risk, and investment risk. The paper also discusses the specific challenges and strategies for closed DP plans with non-homogeneous participants and open DP plans. Subsequent sections cover the design elements of DP plans, such as valuation interest rates, benefit smoothing, and pension annuity forms, and explore the optimal investment strategies under the Type 3 benefit adjustment. The paper concludes with implications for DP plan design and a summary of findings.

[1] This paper focuses on the impacts of systematic deviations from expected mortality, specifically those arising from misstatement of initial mortality assumptions. Idiosyncratic longevity risk is not modeled, nor are unanticipated mortality improvements incorporated. This simplification is intentional, as the primary objective is to examine how aggregate plan-level mortality and investment experience affect pension outcomes and wealth transfers.

Section 2 Advantages of Risk Pooling and Sharing

Upon retirement, participants in individual defined contribution (IDC) plans can either use their accumulated savings to purchase a lifetime annuity or systematically draw down their savings. Alternatively, they can participate in dynamic pension plans, which enable risk pooling among a large number of retirees. DP plans offer several advantages, including the mitigation of longevity risk, management of sophisticated investment strategies, and reduction of the impact of volatile investment returns. These benefits collectively contribute to provide retirees with a more stable and enhanced income.

2.1 LONGEVITY RISK POOLING

One of the primary advantages of DP plans is the pooling of mortality or longevity risk. Because the pooled funds in DP plans are used to provide lifetime income to a large number of participants, the plan can balance the needs of those who live longer against those who do not, ensuring that funds are available for the duration of each participant's retirement. This risk pooling mechanism significantly reduces the likelihood that any single retiree will outlive their assets. In contrast, systematic withdrawals place the responsibility on individual retirees to manage their own retirement savings, leading to a significantly higher risk of asset depletion. This collective approach reduces the pressure on individual retirees to make their savings last and provides a sense of security that their income will not run out.

2.2 INVESTMENT RISK POOLING

While IDC retirees can invest in pooled investment funds to achieve some level of risk mitigation, DP plans offer a more integrated approach. These plans pool contributions from a large number of retirees and manage investments collectively. This pooling of resources allows for the collective management of investment risks and, through intergenerational risk-sharing, smooths out the effects of market volatility over time. As a result, retirees receive a more consistent income under unpredictable market conditions.

The collective investment strategy in DP plans can lead to enhanced returns through economies of scale and sophisticated investment management. This approach helps to provide retirees with higher income compared to the outcomes achievable by individually managed IDC accounts, even those invested in pooled funds.

2.3 ADDRESSING INDIVIDUAL CHALLENGES IN RETIREMENT PLANNING

In addition to the advantages mentioned above, DP plans address several problems faced by individuals planning for retirement. Retirees relying on systematic withdrawals often adopt a conservative approach to avoid depleting their savings, resulting in a lower standard of living than necessary. Furthermore, they tend to keep their assets in short-term and highly liquid investments, which typically yield lower rates of return. DP plans, by commingling assets and pooling risks, eliminate these inefficiencies. The collective approach ensures that retirees can enjoy a more secure and potentially higher income without the need for overly cautious withdrawal strategies or suboptimal asset allocation.

In summary, DP plans offer significant advantages over IDC plans during the decumulation phase by pooling and sharing risks. This collective approach mitigates the risk of outliving one’s assets and reduces the impact of volatile investment returns. The result is a more stable and predictable retirement income for participants.

Section 3 Common Designs of Dynamic Pension Plans

DP plans are financial instruments designed to provide retirees with a steady income stream throughout their retirement years. These plans aggregate contributions from numerous participants and invest them to generate returns that fund periodic payouts. Various designs of DP plans cater to different participant characteristics and employ distinct benefit adjustment mechanisms to maintain the plan's financial health. Here, the common designs of DP plans based on participant characteristics and types of benefit adjustment mechanisms are explored.

3.1 PARTICIPANT CHARACTERISTICS

The design of DP plans can vary significantly based on the characteristics of the participants. One common design involves a **closed homogeneous group**. In this setup, all retirees joined the plan at the same age and elect the same form of pension (with or without guarantees or survivor benefits) when the plan was established. The uniformity in age and pension choices simplifies actuarial calculations and risk assessments, making it easier to project future liabilities and adjust benefits accordingly. The primary advantage of this design is its simplicity in administration and predictable risk management. However, it offers limited flexibility, as all participants must conform to the same age and pension options, which may not cater to diverse retiree needs.

Another design features a **closed non-homogeneous group**, where retirees join the plan at different ages and/or elect different forms of pension. This approach introduces more complexity in managing the pool but provides greater flexibility for participants. The advantage of this design is that it accommodates a wider range of retiree circumstances, allowing participants to choose their retirement age and pension type. However, the increased complexity in actuarial calculations and risk management can present challenges in maintaining equitable adjustments across diverse participant profiles.

The third common design is an **open group**, where retirees continuously join the plan at different times. This ongoing influx of new participants helps to mitigate longevity risk and supports the sustainability of the pool. The open group design ensures continuous replenishment, maintaining funding levels and reducing the impact of mortality on the plan's sustainability. It also offers enhanced adaptability to changing demographics and economic conditions. However, managing an ever-changing participant base can be complex, and ensuring fair benefit adjustments across different cohorts of retirees is challenging.

3.2 TYPES OF BENEFIT ADJUSTMENT MECHANISMS

Benefit adjustment mechanisms are fundamental to maintaining the financial health and sustainability of DP plans. As introduced in Section 1, three general types of adjustment mechanisms are commonly used—each reflecting a different approach to balancing plan assets and liabilities over time. While the basic concepts were outlined earlier, this section expands on their operational implications and comparative features within the context of DP plan design.

Type 1 mechanisms[2] focus on maintaining a stable funding level by adjusting annual pension payments. While this promotes short-term solvency and simplicity in application, it tends to concentrate the impact of investment fluctuations on current retirees, with limited intergenerational risk-sharing. Smoothing techniques, such as multi-year averaging, are sometimes layered onto this method to dampen short-term volatility, but they may delay necessary adjustments, potentially leading to funding imbalances.

[2] This approach is employed by some DP plans in the United States and Canada, such as The Wisconsin Retirement System, The College Retirement Equities Fund (CREF), and the UBC Faculty Pension Plan.

Type 2 mechanisms adjust the future rate of pension escalation based on the plan's evolving financial status. This approach, seen in various international CDC models (e.g., in the UK (Barajaz-Paz & Donnelly, 2023)), facilitates a longer-term alignment between asset growth and benefit progression. It introduces a forward-looking element to adjustment design but may result in uncertainty for retirees regarding their future income trajectory—particularly in closed plans without new entrants.

Type 3 mechanisms directly link annual pension payments to the funded ratio derived from participants' target benefits. By allowing both investment and mortality risks to be pooled and shared across cohorts, this approach inherently supports intergenerational risk-sharing. It offers greater resilience to economic shocks by softening the impact on older retirees through partial offsetting from younger cohorts' contributions or benefit adjustments.

Adjustment factors for these mechanisms can be determined either on a cohort-by-cohort basis or on a group basis. When adjustments are made on a **cohort-by-cohort** basis, they are tailored to specific groups of retirees, taking into account the unique characteristics and financial status of each cohort. This method provides more precise and equitable adjustments, reflecting the specific risks and funding needs of each cohort. However, it increases administrative complexity and can create perceived inequities if different cohorts receive significantly different adjustments.

Alternatively, adjustments can be applied uniformly across all participants in the plan, regardless of cohort, referred to as a **group basis**. This method simplifies administration and communication and ensures uniform treatment of all participants, promoting a sense of equity. However, it may not accurately reflect the unique risks and funding needs of different cohorts and can lead to cross-subsidization, where some participants indirectly support others.

In summary, the design of DP plans involves careful consideration of participant characteristics and the choice of benefit adjustment mechanisms. Closed homogeneous groups offer simplicity, closed non-homogeneous groups provide flexibility, and open groups ensure sustainability. The choice among different types of adjustment mechanisms, along with the determination of adjustment factors on a cohort or group basis, further influences the stability and predictability of retirement income.

In the following sections, we explore the impact of mortality and investment risks within DP plans, on wealth transfers, pension stability, and financial equity on intra-cohort, inter-cohort, and intergenerational bases.

Section 4 A Framework for Benefit Adjustments

This section describes a general framework for benefit adjustments under a DP plan, focusing on the Type 1 adjustment mechanism. This plan consists of a group of retired participants of the same age, each contributing the same amount and receiving the same form and amount of pension at the plan's inception. A valuation of the plan's assets and liabilities is performed annually. If observed asset returns or mortality rates deviate from the expected values, pension benefits are adjusted to maintain the plan's funding level.

Details for different types of benefit adjustments under DP plans, consisting of closed non-homogeneous groups or open groups of participants, are provided in Appendix A.

4.1 PENSION ADJUSTMENT FACTOR CALCULATION

The pension adjustment factor is calculated based on various parameters, including the participant's age, mortality experience, and the investment return of the pension fund. This factor ensures that the funded ratio of the plan aligns with expected values over time.

Let C be the contribution paid by a participant aged x, P_0 the initial annual pension, and $\ddot{a}_x$ the annual life annuity due factor at age x based on a discount rate i and mortality table. The relationship between these variables can be expressed as:

$$C = P_0 \cdot \ddot{a}_x$$

The valuation basis, comprising the discount rate (often referred to as a hurdle rate in DP plans) and mortality rates, is assumed to remain unchanged throughout the life of the plan.

Let F_t and L_t denote the fund balance and liability at time t. At time 0, $F_0 = L_0 = 0$.

Let G_t denote the group of surviving participants at time t. The expected value of the fund and liability at time 1 are:

$${}^eF_1 = \left[F_0 + \sum_{G_0}(C - P_0)\right](1 + i)$$

$${}^eL_1 = \sum_{G_0} P_0 \cdot p_x \, \ddot{a}_{x+1}$$

Here, p_x is the participants' expected survival probability in the first year and is calculated as $1 - q_x$, where q_x is the assumed probability of death in that year.

Given the initial conditions, the expected funded ratio at time 1 is equal to 1[3]:

$${}^eFR_1 = \frac{{}^eF_1}{{}^eL_1} = 1$$

[3] If the assumptions underlying P_0 differ from those used in the valuation basis, the expected funded ratio could deviate from 1.

The actual fund value and liability based on P_0 are:

$$F_1 = \left[F_0 + \sum_{G_0}(C - P_0)\right](1 + r_0)$$

$$\widetilde{L_1} = \sum_{G_0} P_0 \cdot p'_x \ddot{a}_{x+1} = \sum_{G_1} P_0 \cdot \ddot{a}_{x+1}$$

where r_0 is the annual rate of fund return and p'_x is the participants' actual survival probability in the first year. The group G_1 is comprised of all surviving participants at time 1. The total number of participants in this group is equal to $\sum_{G_0} p'_x$.

The funded ratio of the plan at time 1 is:

$$FR_1 = \frac{F_1}{\widetilde{L_1}}$$

To equate the plan's funded ratio to the expected funded ratio at time 1, the pension payment P_0 upon which $\widetilde{L_1}$ is based must be adjusted by a factor h_1 such that:

$$\frac{F_1}{h_1 \widetilde{L_1}} = \frac{{}^eF_1}{{}^eL_1}$$

The adjustment factor h_1 can be decomposed into two components: (1) the investment adjustment factor h_1^I, and (2) the liability adjustment factor h_1^L:

$$h_1^I = \frac{F_1}{{}^eF_1} = \frac{1 + r_0}{1 + i}$$

$$h_1^L = \frac{{}^eL_1}{\widetilde{L_1}} = \frac{p_x}{p'_x}$$

The adjusted pension payable at time 1 is then equal to:

$$P_1 = h_1 P_0$$

The expected fund and liability values at time 2 are:

$${}^eF_2 = (F_1 - \sum_{G_1} P_1)(1 + i)$$

$${}^eL_2 = \sum_{G_1} P_1 \cdot p_{x+1}\, \ddot{a}_{x+2}$$

where p_{x+1} is the participants' survival probability in the second year.

The actual fund and liability values based on P_1 are:

$$F_2 = \left(F_1 - \sum_{G_1} P_1\right)(1 + r_1)$$

$$\widetilde{L_2} = \sum_{G_1} P_1 \cdot p'_{x+1}\ddot{a}_{x+2}$$

where r_1 is the annual rate of fund return and p'_{x+1} is the participants' actual survival probability in the second year.

To match the funded ratio at time 2, the pension payment P_1 must be adjusted by a factor h_2:

$$h_2 = \frac{F_2}{{}^{e}F_2} \cdot \frac{{}^{e}L_2}{\widetilde{L_2}}$$

$$h_2 = h_2^I \cdot h_2^L$$

where h_2^I is the investment adjustment factor and h_2^L is the liability adjustment factor defined as follows:

$$h_2^I = \frac{1 + r_1}{1 + i}$$

$$h_2^L = \frac{p_{x+1}}{p'_{x+1}}$$

The adjusted pension payment at time 2 is:

$$P_2 = h_2 P_1 = h_1 h_2 P_0$$

In general, the adjustment factor at time t can be expressed as:

$$h_t = h_t^I \cdot h_t^L$$

where:

$$h_t^I = \frac{1 + r_{t-1}}{1 + i}$$

$$h_t^L = \frac{p_{x+t-1}}{p'_{x+t-1}}$$

The adjusted pension payable at time t is:

$$P_t = h_t P_{t-1} = \left(\prod_{s=1}^{t} h_s\right) P_0 = \left[\prod_{s=0}^{t-1} \left(\frac{1 + r_s}{1 + i}\right)\left(\frac{p_{x+s}}{p_{x+s}{}'}\right)\right] \cdot P_0 = \left[\prod_{s=0}^{t-1} \left(\frac{1 + r_s}{1 + i}\right)\right]\left(\frac{{}_t p_x}{{}_t p'_x}\right) \cdot P_0 \qquad (1)$$

where ${}_t p_x$ and ${}_t p'_x$ are the expected and actual surviving probabilities from age x to age $x + t$, respectively.

4.2 EVALUATION MEASURES

The primary goal of this paper is to assess how different designs of DP plans affect wealth transfers, pension stability, and financial equity on either an intra-beneficiary or intergenerational basis. To perform these assessments, the authors introduce the following two measures: the annual pension payment ratio (APPR) and the repayment ratio (RR).

Annual Pension Payment Ratio (APPR)

The annual pension payment ratio at time t is defined as:

$$APPR_t = \frac{P_t}{P_0} = \left[\prod_{s=0}^{t-1}\left(\frac{1+r_s}{1+i}\right)\right]\left(\frac{{}_tp_x}{{}_tp'_x}\right) \quad (2)$$

This ratio compares the actual pension (P_t) to the expected pension (P_0) payable at time t[4].

The average APPR over the life of all participants in the group is given by:

$$\overline{APPR} = \left(\sum_{t=0}^{120-x} {}_tp'_x \cdot APPR_t\right) / \left(\sum_{t=0}^{120-x} {}_tp'_x\right) \quad (3)$$

In this formula, 120 represents the terminal age by which the participant is expected to die with certainty. This measure provides a summary view of how the actual pensions compare to the expected pensions over the lifetime of the participants.

Repayment Ratio (RR)

The Repayment Ratio for a participant who survives to time t represents the present value of pensions received up to that time divided by the initial contribution made. It reflects the extent to which the participant's initial contribution has been recaptured through the pension payments received.

For a participant who survives to time t, the present value at plan inception of pensions received up to that time is:

$$PV = P_0 + \frac{P_1}{1+r_0} + \cdots + \frac{P_t}{\prod_0^{t-1}(1+r_s)}$$

4 If the pension payable under the plan is an indexed life annuity with a target indexing rate k, the expected pension payable at time t is $P_0(1+k)^t$. Consequently, the annual pension payment ratio at time t is defined as:

$$APPR_t = P_t/P_0(1+k)^t.$$

All annuity factors illustrated in this section should incorporate this target indexing rate. This adjustment ensures that the APPR appropriately reflects the impact of indexing on the pension payouts over time.

The repayment ratio at time t is defined as this present value divided by the initial contribution *C*:

$$RR_t = \frac{PV}{C} \tag{4}$$

The repayment ratio for all participants in the group is calculated as follows:

$$RR = \sum_{t=0}^{120-x} {}_t p'_x \cdot q'_{x+t} \cdot RR_t \tag{5}$$

where q'_{x+t} is the actual probability of death in the year of age $x + t$. Under the benefit adjustment procedure described above, the participants' contributions are expected to be fully utilized to provide their pension benefits. In other words, the repayment ratio is expected to equal one. This can also be proven mathematically; see Appendix B.

Note that, by definition, the repayment ratio is calculated by discounting at actual, rather than assumed, investment returns and that the repayment ratio for all participants in the group is calculated using actual, rather than assumed, mortality. Thus, deviations from the assumed investment return or mortality will not, by themselves, cause RR to deviate from 1.0[5].

A Special Case

Assuming mortality experience is as expected, the pension payable at time t can be expressed as:

$$P_t = \left[\prod_{s=0}^{t-1} \left(\frac{1 + r_s}{1 + i}\right)\right] \cdot P_0$$

The present value of pensions received up to time t can then be written as:

$$P_0 \left(1 + \frac{1}{1+i} + \cdots + \frac{1}{(1+i)^t}\right) = P_0 \cdot \frac{1 - \left(\frac{1}{1+i}\right)^{t+1}}{i} (1 + i)$$

The repayment ratio at time t, being the present value divided by the initial contribution *C*, is:

$$RR_t = \frac{P_0}{C} \cdot \frac{1 - \left(\frac{1}{1+i}\right)^{t+1}}{i} (1 + i) = \frac{1}{\ddot{a}_x} \cdot \left[\frac{1 - \left(\frac{1}{1+i}\right)^{t+1}}{i}\right] (1 + i)$$

Note that RR_t is an increasing function of t and is independent of the actual returns experienced by the fund. The first time RR_t reaches 1 or above can be solved by this equation:

$$\frac{1}{\ddot{a}_x} \cdot \frac{1 - \left(\frac{1}{1+i}\right)^{t+1}}{i} (1 + i) = 1$$

[5] This may not be true in the case of a closed non-homogeneous group or open group DP plan. The RR for a participant cohort may differ from 1.0 if there are deviations from the mortality assumption underlying the valuation basis. See Sections 6 and 7.

Rearranging,

$$\left(\frac{1}{1+i}\right)^{t+1} = 1 - \frac{i\ddot{a}_x}{1+i}$$

$$-(t+1)\ln(1+i) = \ln\left(1 - \frac{i\ddot{a}_x}{1+i}\right)$$

$$t = -\frac{\ln\left(1 - \frac{i\ddot{a}_x}{1+i}\right)}{\ln(1+i)} - 1 \qquad (6)$$

Finally, t is rounded up to the next integer.

The age at which the repayment ratio (RR_{t}) reaches 1.0—i.e., when the present value of pension payments received equals the participant's initial contribution—is often referred to as the break-even age. While this metric can provide analytical insight into intra-cohort wealth transfers, it may not serve as an effective tool for guiding individual decision-making. As noted by the (National Institute on Ageing, 2024), framing retirement choices around break-even ages can trigger loss aversion and lead individuals to undervalue the insurance provided by lifetime income. Communicating plan features in terms of income stability and longevity protection may be more effective.

Section 5 Impact of Mortality and Investment Risks on Pension Outcomes in DP Plans

DP plans offer a unique approach to providing retirement income by pooling longevity and investment risks among participants. The primary risks that influence pension outcomes are mortality risk and investment risk. Mortality risk pertains to the uncertainty regarding the lifespan of participants, which affects the duration and total amount of pension payouts. Investment risk encompasses the variability in returns from fund investments but also includes timing of return risk. Both risks play a crucial role in determining the financial stability and adequacy of pensions provided by DP plans. Understanding how these risks impact pension outcomes is essential for ensuring that retirees receive sustainable and predictable income throughout their retirement years. This section delves into the dynamics of mortality and investment risks within DP plans, illustrating their effects on pension outcomes through a detailed analysis of a closed homogeneous group of participants.

For analysis purposes, we consider a simple DP plan consisting of 1,000 male retirees, each aged 65 and contributing $500,000 upon joining the plan at plan inception. The plan provides a lifetime annuity without survivor benefits or targeted benefit increases. We assume mortality follows the "Pri-2012 Male Mortality Table - Amount" with future improvement rates from "MP-2021," as published by the Society of Actuaries (Society of Actuaries, 2019) (Society of Actuaries, 2021). The interest rate is set at 6% for both pricing and valuation purposes. On this basis, the annual pension expected to be paid is approximately $42,600. The plan begins its operation on January 1, 2024, which is considered time 0. The fund invests in a balanced portfolio with 50% in equities and 50% in long bonds. The expected rate of return net of expenses is 6% per annum.

Unless otherwise stated, we will apply the above mortality and interest assumptions for numerical illustrations throughout this paper.

5.1 INTRA-COHORT WEALTH TRANSFERS

This section examines the effects of wealth transfers within DP plans. Under a DP plan, participants who die early receive pensions that are less than the contributions they made to the plan. This results in a transfer of wealth from those who die young to those who live longer lives, ensuring that the funds remain sufficient to provide ongoing benefits to long-lived participants. Previous studies have extensively analyzed these wealth transfer dynamics (Weil & Fisher, 1974).

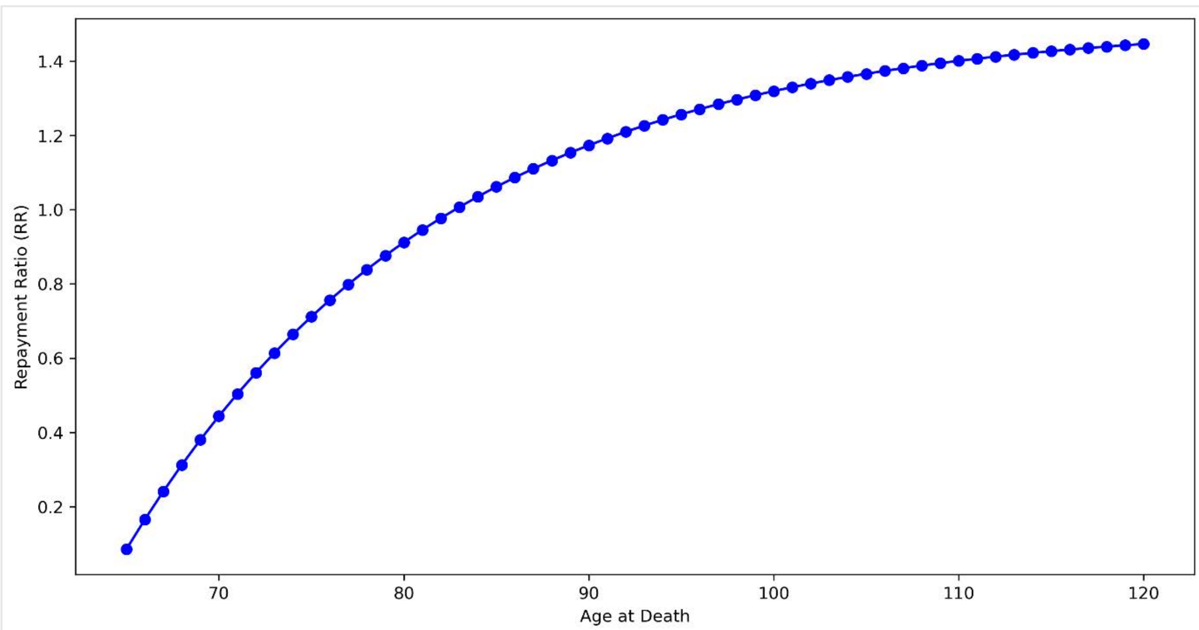
To assess how wealth is transferred among participants, the authors use the measure called the repayment ratio (RR), as described in Section 4. The RR quantifies the relationship between the total pension benefits received by a participant and their initial contributions. An RR less than 1 indicates that a participant receives benefits that are lower in value than their initial contribution, whereas an RR greater than 1 indicates that a participant receives more than their initial contribution. This measure is essential for understanding the redistribution of wealth within the DP plan, highlighting how the contributions of deceased participants support the longevity of others.

This redistribution of retirement wealth is fundamental to the sustainability of DP plans. By applying parts of participants' contributions who die early to support those who live longer, the plan maintains its financial ability to provide lifetime benefits. This process is crucial for the long-term viability of DP plans, as it mitigates the impact of longevity risk and ensures that the collective contributions are utilized efficiently to benefit all participants over time.

Let's assume that the mortality assumption underlying the valuation of the sample plan materializes. Figure 5.1 below plots the distribution of RRs for participants dying at different ages. The RR increases from 0.085 for participants who die at age 65 (i.e., in the year of age 65), indicating that these participants only recapture 8.5% of their contributions through pension payments. It rises to 1.007 for those who die at age 83 and further increases to 1.407 for participants dying at the terminal age of 120. This means that those who die at the terminal age receive 1.4 times the contributions they made. Age 83 is significant as it is the first age when the RR exceeds 1, meaning that participants begin to receive more than their initial contributions. This age can be derived directly from Equation (6) given in Section 4.

Note that repayment ratios below 1.0 indicate that participants receive less than their original contribution in discounted terms—primarily due to early death. These forfeitures are not lost in aggregate but are reallocated within the pool, forming the basis of longevity credits received by longer-lived members.

Figure 5.1
DISTRIBUTION OF REPAYMENT RATIOS



For participants who die before age 83, their expected loss is 11.13% of their initial contribution. Conversely, for those who die after age 82, their expected gain is also 11.13%. This expected loss or gain is calculated using the following formula:

$$Expected\ Loss\ (Gain) = \sum \frac{(Contribution - PV(Benefits\ Received))}{Contribution},$$

where the sum is made over participants who die before age 83 or those who die after age 82 and then divided by the number of participants in the respective group.

This formula considers the contributions made by the participants and the benefits they receive over their lifetime. Participants who die earlier than expected subsidize the benefits of those who live longer.

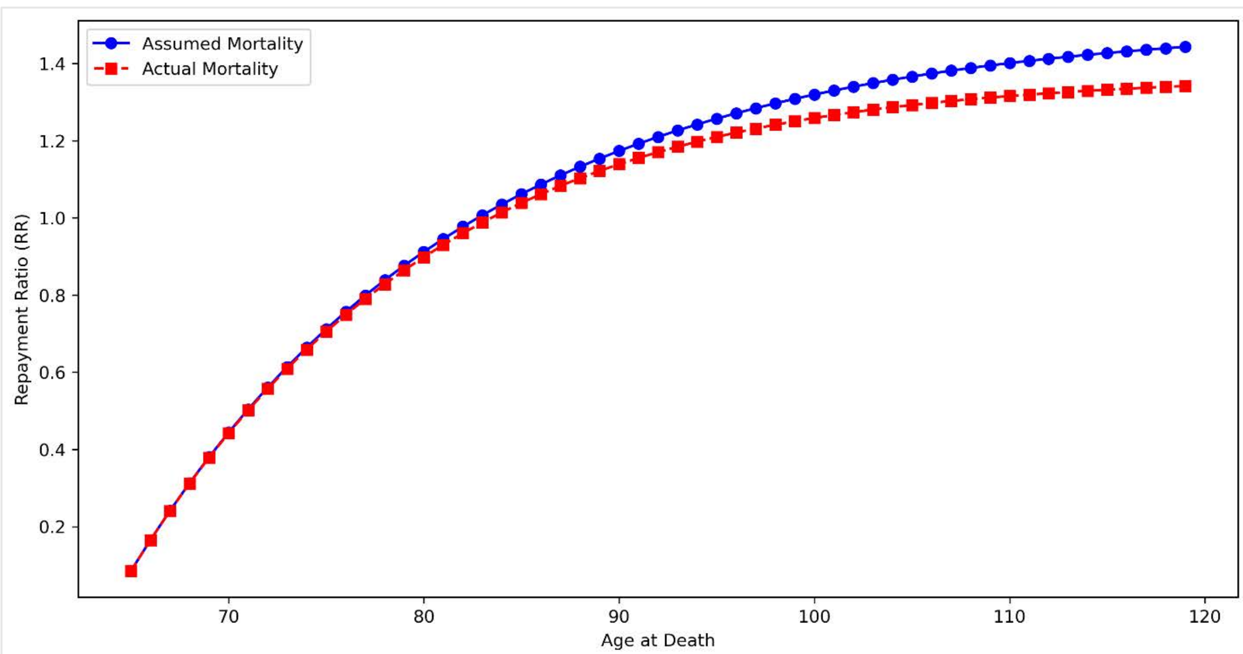
5.2 MORTALITY RISK

This section explores the impact on pension outcomes under the sample DP plan due to shifts in mortality from the assumptions made in the plan's valuation, focusing on the risk that retirees live longer than expected. When retirees live longer than expected, the plan may face increased pressure to meet the target pension benefit, potentially leading to lower-than-expected income for participants. Additionally, this mortality risk could change the distribution of wealth among participants in the plan.

We assume that the actual mortality experienced by participants in the plan is lower than initially assumed. Specifically, while the assumed mortality provides a life expectancy of 21.2 years, the actual mortality suggests a life expectancy of 22.2 years—one year longer[6]. Figure 5.2 plots the distribution of RRs for participants dying at different ages, compared with the distribution presented in Section 5.1. Under the actual mortality, the distribution of RRs is consistently lower than under the assumed mortality. It starts with an RR of 0.085 for participants who die at age 65, increasing to 1.014 for those who die at age 84, and further increasing to 1.344 at age 120.

For participants who die before age 84, their expected loss is 9.91% of their initial contribution, whereas for those who die after age 83, their expected gain is 9.91%. This is lower than the 11.13% expected loss or gain under the assumed mortality. Thus, lower-than-expected mortality reduces the extent of intra-cohort wealth transfers.

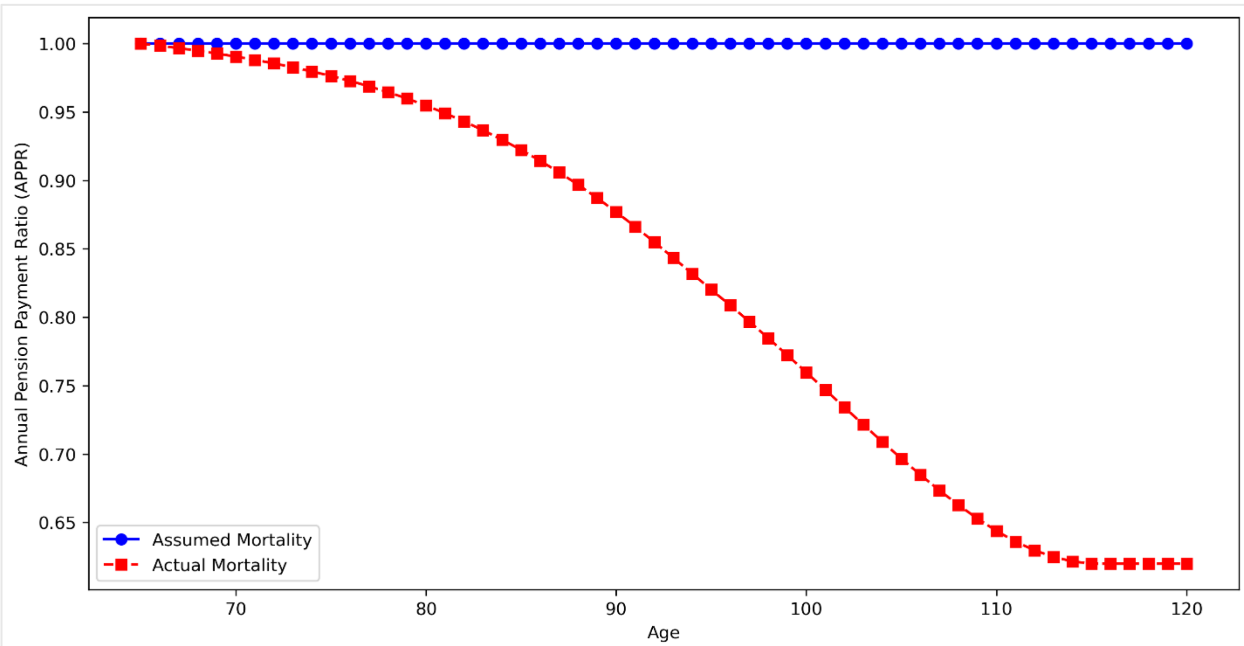
Figure 5.2
DISTRIBUTION OF REPAYMENT RATIOS



[6] For reference, the additional life expectancy under the mortality risk scenario is roughly equivalent to the life expectancy difference between a 50% male/50% female population and a 100% female population or between the total dataset table and the white-collar table from Pri-2012.

Figure 5.3 shows the distribution of annual pension payment ratios (APPRs) throughout the participants' lifespans, assuming the fund earns the expected annual return of 6%. The APPRs fall below the expected level of 1.0, declining progressively from 1 at age 65 to below 0.88 after age 90. The average APPR for participants is 0.953, which is about 5% lower than the target pension payment. This discrepancy highlights the importance of accurate mortality projections in the valuation of DP plans. Overly optimistic mortality assumptions can lead to unrealistic pension targets, potentially undermining the plan's design objectives and its ability to provide sustainable benefits to participants.

Figure 5.3
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS



5.3 INVESTMENT RISK

This section explores the impact of investment risk on pension outcomes under a DP plan. Investment risk can significantly affect the stability and adequacy of retirement income. To illustrate its effects, the authors analyze the sample DP plan described earlier, assuming that participant mortality aligns with the valuation assumptions. By simulating a range of investment scenarios, the authors examine how variations in return volatility and the timing of investment shocks influence pension outcomes.

Timing of Investment Shocks

The authors consider scenarios where the fund experiences a return that is higher or lower than expected at different points in the retirement period (e.g., early, middle, or late), while all other assumptions remain unchanged. Although this analysis is related to the concept of sequence-of-returns risk, it does not hold the long-term average return constant across scenarios. Instead of modeling return paths with identical time-weighted averages, the authors introduce isolated return shocks and assess their impact on annual pension payments. This approach is intended as a form of stress testing specific market events rather than a formal sequence-of-returns framework.

First, the authors assume the fund experiences a single-year loss of 10% relative to the expected 6% return (i.e., a return of -4%) at different points in time after the plan's inception: specifically, in the 5th, 10th, 15th, 20th, 25th, and 30th years. Figure 5.4 shows the distribution of APPRs over the lifetime of participants by the year of loss. The APPR drops from 1.00 to 0.91 in the year of loss and remains at that lower level subsequently. Consequently, the average APPRs for the group of participants vary from 0.927 when the loss occurs in the 5th year to 0.997 when the loss occurs in the 30th year. The earlier the year of loss, the greater the impact on the participants' overall income level.

Figure 5.4
DISTRIBUTION OF APPRS BY YEAR OF LOSS

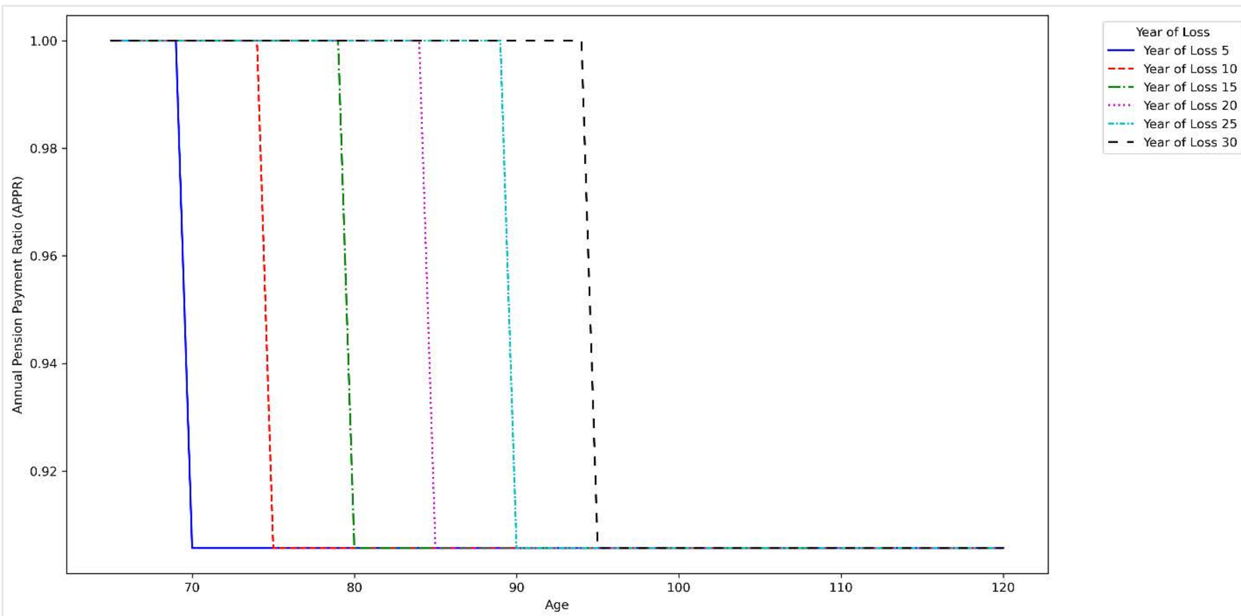
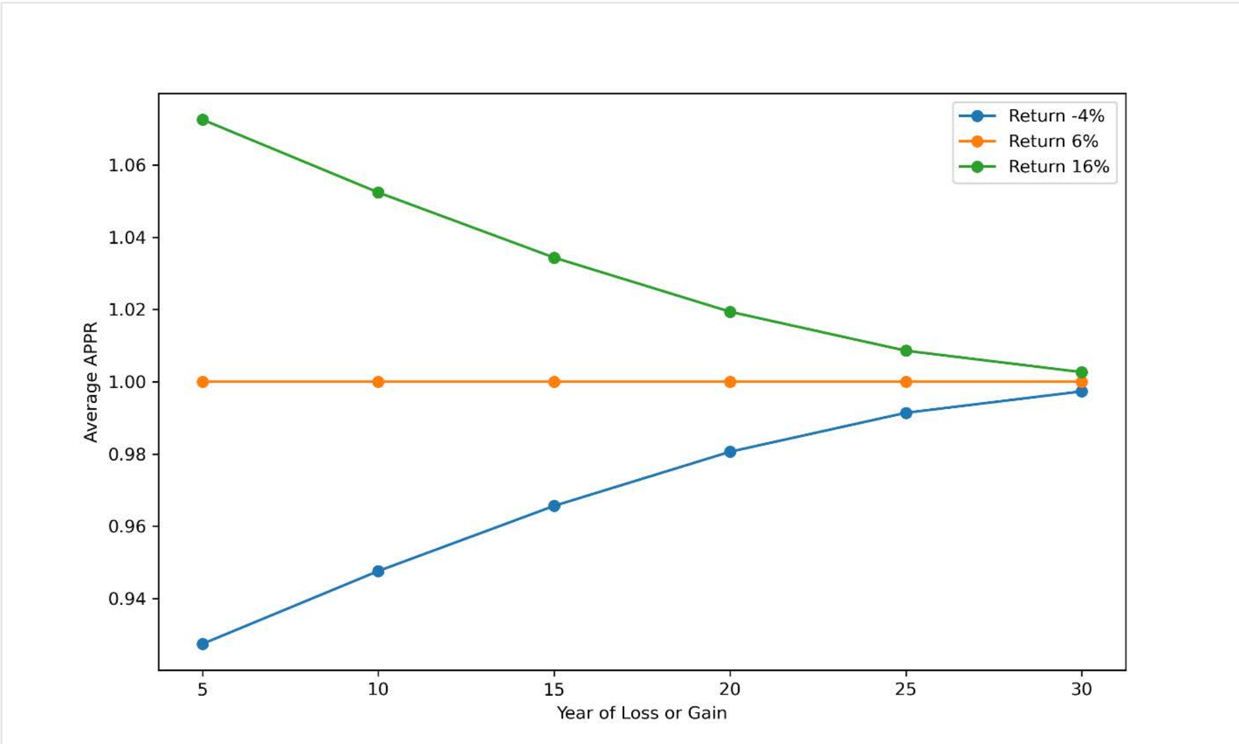


Figure 5.5 plots the *average* APPRs for three different investment returns: -4%, 6%, and 16%, occurring at different points during the participants' retirement years. These returns correspond to a loss of 10%, no loss or gain, and a gain of 10%, respectively, relative to the expected 6% return. This graph demonstrates the impact of timing of investment return risk on the retirement income levels of participants as a group, underscoring the need for attention to this risk in the design of DP plans, although the authors do not see any way to design a DP plan to significantly mitigate the timing of return risk without material intergenerational wealth transfers (see discussions in later sections).

Figure 5.5
TIMING OF INVESTMENT RETURN RISK



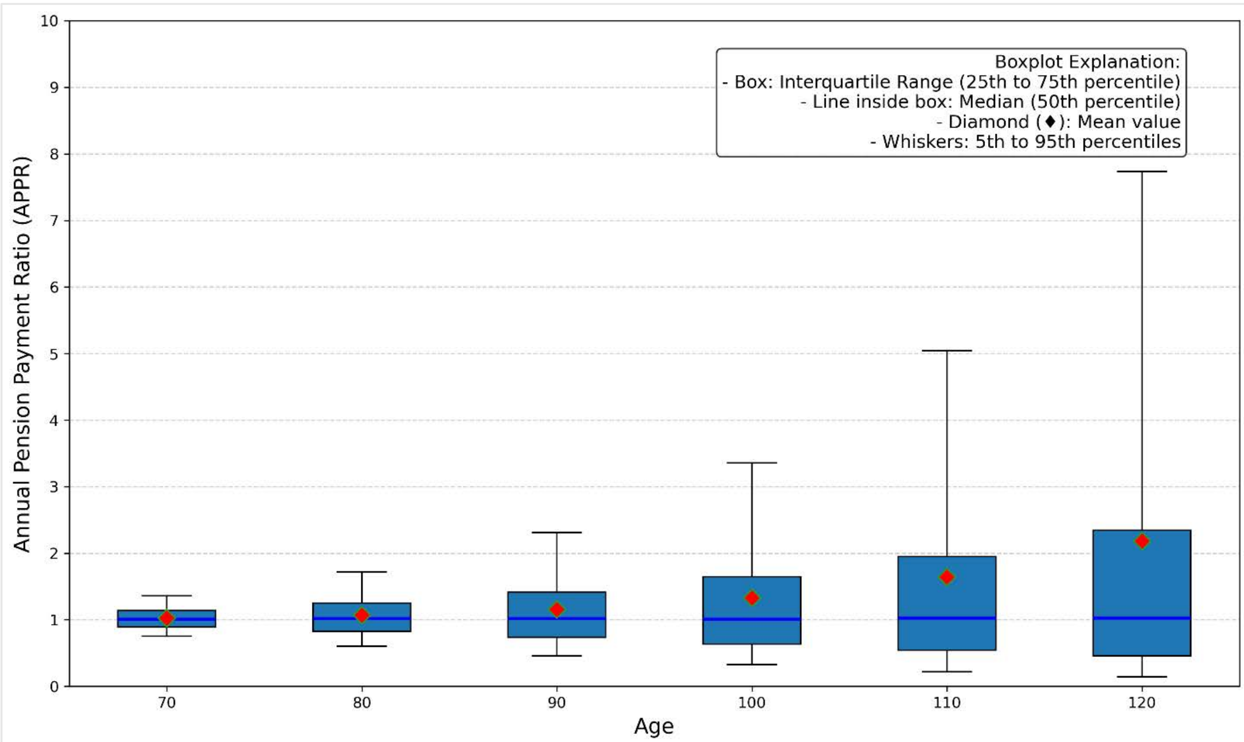
Return Volatility

To demonstrate the impact of volatile returns on the pension outcomes of DP plans, the authors use the economic scenario generator outlined in (Ma, 2023) to construct 1,000 investment return scenarios. Each scenario encompasses annual time series data for long-term bond yields and equity returns over a 55-year period. The parameters for the bond yield model—including the long-run mean, rate of reversion, and standard deviation—along with other factors such as the equity risk premium, standard deviation in the equity price model, diversification return, and expense loading, are consistent with the specifications provided in Ma's study[7]. Employing a balanced investment strategy, the authors evenly allocated the funds between equities and bonds, projecting an expected 6% annual return based on these models. All stochastic demonstrations in this paper will be based on these generated return scenarios.

[7] The long-run mean, rate of reversion, and standard deviation for the bond yield model are set as 0.04, 0.0194 and 0.0076, respectively. The equity risk premium and the standard deviation for the equity price model are specified as 0.04 and 0.15, respectively. Furthermore, we assume a diversification return of 0.35% and an expense loading of 0.52%.

For the sample DP plan, the authors assume the mortality experienced by participants matches that assumed in the plan's valuation and use the simulated return scenarios to project the pension payments. Figure 5.6 shows the range of APPRs at the 5th, 25th, 50th, 75th, and 95th percentiles for selected ages of participants. As illustrated, there is considerable uncertainty in the levels of APPRs; the older the age, the more uncertain the APPR level becomes.

Figure 5.6
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS BY AGE



The average APPR for the group of participants ranges from 0.747 (5th percentile) to 1.490 (95th percentile) with a median of 1.021. There is a 46% probability that the average APPR will fall below 1, and a 27% probability it will fall below 0.9. These findings suggest that there is considerable risk to the income levels for participants when the plan adopts an investment strategy with significant exposure to equity assets.

It is important to note, however, that participants' repayment ratios (RRs) under the sample DP plan are independent of the returns of the fund. In other words, investment risk does not impact intra-cohort wealth transfers, which are solely dependent on participants' mortality.

Mitigating Investment Risks

In analyzing the impact of investment risk compared to mortality risk, the authors refer to the mortality risk scenario in subsection 5.2. Under that scenario, the APPRs displayed in Figure 5.3 are between the 25th and 50th percentile shown in Figure 5.6, resulting in an average APPR of 0.953 for the group of participants in the plan. This indicates that, on average, participants receive only 95.3% of the expected pension payment under the assumed mortality conditions. In contrast, when considering investment risk, there is a 37.7% probability that the average APPR could fall below this level. This comparison highlights that, while mortality risk is significant, investment risk poses an even greater threat to the stability of pension outcomes.

One way to mitigate investment risks is to pursue a more conservative investment strategy for the fund. This typically involves allocating a higher proportion of the fund to fixed-income securities and other low-risk assets. While this approach can reduce exposure to severe market downturns and stabilize returns, it comes at a cost. Lower-risk investments generally offer lower expected returns, which can translate into lower pension income for participants. Therefore, there is a trade-off between reducing risk and achieving higher pension income.

Section 6 Closed DP Plans with Non-homogeneous Participants

In this section, the authors extend the analysis to a DP plan where retirees join at different ages, creating a non-homogeneous participant base. The purposes are:

1. To explore the impact of mortality risk on inter-cohort wealth transfers within the plan.
2. To explore the impact of investment risk on the pension outcomes of respective cohorts, assuming actual mortality matches the assumptions used in the plan's valuation.

Impact of Mortality Risk on Inter-Cohort Wealth Transfers

When participants join a DP plan at different ages, mortality risk introduces a layer of complexity to the redistribution of wealth among cohorts. The longevity of each cohort can vary significantly, affecting the stability of the pension fund and the fairness of benefit distribution. For instance, if participants of a particular age cohort experience longer-than-expected life expectancy, they might receive benefits for a longer period, potentially at the expense of participants in other cohorts. This scenario could lead to inter-cohort wealth transfers where the contributions of participants in other age cohorts subsidize the benefits of those in the longer-living cohort.

To illustrate the effect of inter-cohort wealth transfers, the authors consider a sample DP plan consisting of 500 male retirees joining at age 65 and 500 male retirees joining at age 70, each contributing $500,000 for a lifetime pension. Two benefit adjustment mechanisms are applied: cohort-based as described in Section 4 and group-based Type 1 adjustment as detailed in Appendix A. The authors employ the same pricing and valuation assumptions as those in Section 5. The initial annual pensions provided to these two cohorts are $42,600 and $48,000, respectively.

The authors assume the fund earns the expected 6% return throughout the life of the plan. The pension outcomes are simulated under the following two scenarios, comparing cohort-based and group-based adjustments:

- **Scenario A:** The mortality experienced by the Age 65 cohort is lower than assumed, resulting in a 1-year increase in life expectancy (from 21.2 years to 22.2 years), while the mortality experienced by the Age 70 cohort is as assumed.
- **Scenario B:** The mortality experienced by the Age 65 cohort is as assumed, while the mortality experienced by the Age 70 cohort is lower than assumed, resulting in a 1.2-year increase in life expectancy (from 17.1 years to 18.3 years).

Table 6.1 below summarizes the results on RRs and average APPRs for the respective age cohorts.

Table 6.1
SUMMARY OF PENSION OUTCOMES

		Age 65 Cohort		Age 70 Cohort	
	Benefit Adjustments	RR	Average APPR	RR	Average APPR
Scenario A	Cohort-based	1.000	0.953	1.000	1.000
	Group-based	1.012	0.970	0.988	0.981
Scenario B	Cohort-based	1.000	1.000	1.000	0.933
	Group-based	0.976	0.962	1.024	0.969

Under a cohort-based adjustment, each cohort is treated independently, and benefits are adjusted based solely on the experience of that specific cohort. This approach results in an RR of 1, indicating that there are no cross-subsidies between the age cohorts. In Scenario A, while the RR for the Age 65 cohort remains 1, the average APPR is less than 1 (i.e., 0.953) since participants live longer than expected. Similarly, in Scenario B, the average APPR for the Age 70 cohort is 0.933.

Under a group-based adjustment, a single adjustment factor reflecting the combined experience of both cohorts is applied annually. As a result, the RR for each group deviates from 1, indicating inter-cohort wealth transfers. For example, in Scenario A, the RR for the Age 65 cohort is higher than 1, while the RR for the Age 70 cohort is lower than 1, as the mortality loss attributable to the former cohort is shared by the latter cohort. The average APPR for the Age 65 cohort is somewhat higher than under the cohort-based adjustment, while the average APPR for the Age 70 cohort is lower. The opposite effects are observed in Scenario B: the RR for the Age 65 cohort is lower, while the RR for the Age 70 cohort is higher.

Where there are deviations in both cohorts' mortality from the assumptions used in the plan valuations, the inter-cohort wealth transfers will vary based on the relative weights of mortality loss or gain from each cohort. These weights depend on several factors, including the number of participants in each cohort, their ages, pension levels, and the actual mortality experience. For example, if one cohort has a larger number of participants or higher average pension levels, the impact of their mortality deviations on the overall plan will be more significant. Conversely, smaller or lower-pension cohorts will have a lesser impact. Additionally, the actual mortality experience—whether it deviates positively or negatively from the assumptions—will influence the direction and magnitude of wealth transfers. Understanding these dynamics is crucial for plan administrators to ensure equitable treatment of all cohorts and maintain the plan's financial stability.

Impact of Investment Risk on Pension Outcomes

Investment risk remains a significant factor in determining the pension outcomes under a DP plan with non-homogeneous participants. Each cohort's pension benefits are subject to the performance of the plan's investments, which can vary from year to year.

To illustrate the impact of investment risk, the authors assume that the actual mortality experienced by participants matches the plan's valuation assumptions. The authors then apply the investment return scenarios as indicated in subsection 5.3 to project the pension payments for different cohorts. Variations are observed in the APPRs for the two cohorts. Notably, the APPR distribution for the cohort entering the plan at age 65 is identical to that observed in Figure 5.6.

Under the sample DP plan, if actual mortality is as assumed in the plan valuation, the RR of each cohort is equal to 1. This means that investment return variations do not result in inter-cohort wealth transfers, even under a group-based adjustment. However, the distribution of APPRs can vary widely due to investment performance, indicating the importance of managing investment risk to ensure stable pension outcomes for all cohorts.

Section 7 Open DP Plans

This section extends the analysis from Sections 5 and 6 to open DP plans, which continuously incorporate new retirees after plan inception. The purposes of this analysis are:

1. To examine how the continuous influx of new participants affects the distribution of pension benefits across generations.
2. To explore the impact of mortality and investment risks on intergenerational equity and the overall pension outcomes for all participants.

Impact of Mortality Risk on Intergenerational Equity

Mortality risk in an open DP plan introduces complexities in maintaining intergenerational equity. The plan must balance the longevity expectations of different cohorts. If a generation experiences significantly different mortality rates than assumed, it could affect the benefits received by all participants, including those who join the plan subsequently.

To analyze the impact of mortality risk, we consider a sample DP plan consisting of 100 retirees aged 65 at plan inception, with the same number of 65-year-old retirees joining the plan on each subsequent anniversary. Each participant contributes an initial amount of $500,000 and receives an annual lifetime pension based on the fund's performance and the mortality experience among participants. The authors employ the same pricing and valuation assumptions as those described in Section 5.

The authors assume that participants of the initial generation experience lower-than-expected mortality, resulting in a life expectancy one year longer than anticipated. Future generations, however, match the mortality assumptions used in the valuation. Furthermore, it is assumed the fund earns the expected annual return of 6%.

Table 7.1 compares the pension outcomes between the closed group plan described in Section 5 and the initial generation of the open group plan. The RR for the initial generation is above 1.0, indicating that the participants in this generation receive more than their initial contributions, but this is at the expense of subsequent generations, as Table 7.2 demonstrates. Unlike the closed group plan, the annual pension payments for the open group plan participants are more stable, staying close to 1.0, and their average APPR is higher than that of the closed group plan participants (0.994 versus 0.953[8]).

Table 7.1
APPRS IN CLOSED GROUP PLAN AND OPEN GROUP PLAN

		APPR (by age)							
	RR	65	66	….	90	…..	119	120	Average APPR
Closed group plan	1.000	1.000	0.998	….	0.877	….	0.620	0.620	0.953
Open group plan	1.024	1.000	0.998	….	0.991	….	0.991	0.991	0.994

[8] The Average APPR for the closed group in Table 7.1 can be compared to the value of 0.953 in subsection 5.2, as both reflect average pension outcomes under the same underlying assumptions.

Table 7.2 shows the RR and average APPR by generation in the open group plan. Although the mortality rates for generations subsequent to the initial generation align with the assumptions, their RR and average APPR are both less than 1.0. This indicates that the higher pension payments received by the initial generation, compared to closed group plan participants, are subsidized by subsequent generations, illustrating an intergenerational wealth transfer that benefits the initial generation.

Table 7.2
PENSION OUTCOMES IN OPEN GROUP PLAN

Generation #	1	2	3	….	9	10	11	….
RR	1.024	0.997	0.997	….	0.998	0.998	0.998	….
Average APPR	0.994	0.996	0.996	….	0.997	0.998	0.998	….

This analysis provides insights into how mortality risk affects pension outcomes and intergenerational equity in open DP plans, highlighting the need for careful management of longevity expectations to ensure fair benefit distribution across all generations.

Impact of Investment Risk on Pension Outcomes

Investment risk remains a critical factor in open DP plans. The inherent volatility of investment returns can significantly impact the stability and predictability of pension payments, making it essential to understand and manage this risk effectively.

The authors use the investment return scenarios indicated in subsection 5.3 to project the pension payments under the sample plan for both existing and new cohorts over time. Under the Type 1 benefit adjustment mechanism described in Appendix A, pension payments for all participants are directly adjusted according to fund performance, ensuring no cross-subsidies across generations. This results in an RR of 1.0 for all generations, indicating that each cohort's contributions and benefits are balanced. However, while this mechanism avoids intergenerational wealth transfers, it does not shield participants from the uncertainties of investment returns. The APPRs for participants can vary widely throughout their lifetime, like those illustrated in Figure 5.6.

Section 8 Design Element: Valuation Interest Rates

This section and the next two sections explore different design elements of DP plans, focusing on valuation interest rates, benefit smoothing approaches, and pension annuity forms. The analysis begins on valuation interest rates by assessing how the choice of different rates would impact pension outcomes in DP plans, considering both closed and open group plans.

Valuation interest rates, often referred to as hurdle rates, play a crucial role in determining the distribution of pension benefits among participants in DP plans. These rates are essential in calculating the present value of future pension payments, thereby affecting both initial benefits and subsequent adjustments over time.

In the context of funding defined benefit pension plans, it is a common practice in Canada to adopt a conservative valuation basis for prudent management. This approach typically involves incorporating margins for adverse deviations into the discount rate assumption, as described in actuarial standards such as those outlined in the Final Standards - Part 3000 by the Canadian Institute of Actuaries (Canadian Institute of Actuaries, 2022). These margins serve as a financial buffer against unexpected downturns, increasing the likelihood that the pension plan remains solvent and capable of fulfilling its obligations even under adverse conditions.

However, this conservative approach, while suitable for defined benefit plans where the plan sponsor bears the investment and other risks, may not be appropriate for DP plans where the risks are borne solely by the participants. In DP plans, the valuation interest rate has a direct impact on the pension benefits received by participants. Setting the rate too conservatively can result in lower initial pension benefits, as the present value of future payments is overestimated. Conversely, an overly optimistic rate can lead to higher initial benefits but may cause a decline in income over time if the expected returns do not materialize.

The study by (Weil & Fisher, 1974) examines the intra-beneficiary wealth transfer effect in DP plans caused by the use of conservative valuation interest rates. An undesirable retirement income pattern emerges: those who live shorter lives are worse off than those who live longer. We illustrate this effect using our sample closed DP plan, assuming that there is no deviation in the participants' mortality from the assumptions made.

Figure 8.1 shows the distributions of APPRs under both 5% and 6% interest rates, where the fund is assumed to earn an annual return of 6%. Under the 5% assumption, the initial pension amount is $39,200, which is lower than that under the 6% assumption. The APPR increases by 0.95% per year from 1.0 at age 65 to 1.684 at age 120, compared to the APPR of 1.0 throughout under the 6% assumption. The use of a conservative valuation interest rate results in an increasing pattern of pension payments. This is not in line with the objective of the sample plan, which aims to provide a level pension.

Figure 8.1
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS

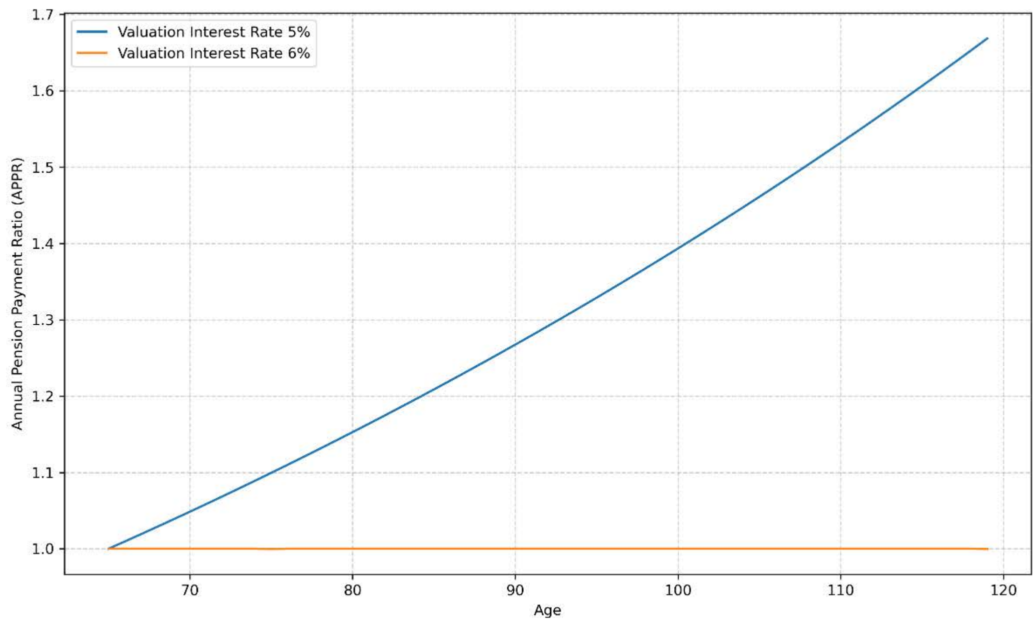
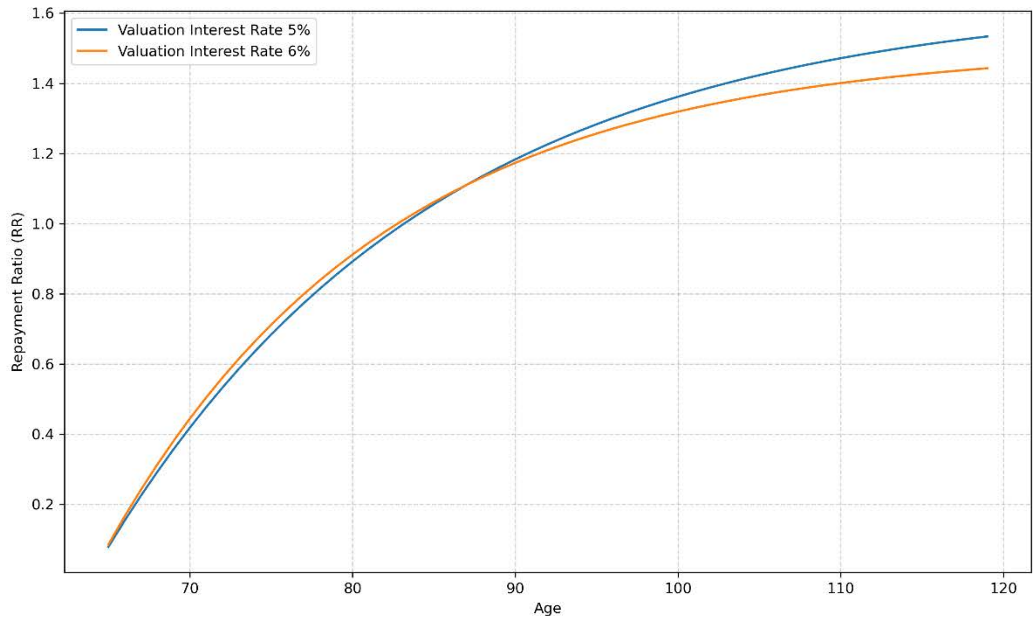


Figure 8.2 shows the distributions of RRs for different ages at death. The RR under the 5% assumption is lower than under the 6% assumption up to age 87 and higher beyond that age. This demonstrates that participants who live shorter lives are disadvantaged due to the use of a conservative valuation assumption. Note, however, that the RR for the participants as a whole is equal to 1.0 under either the 5% or 6% assumption. All contributions made by participants are applied to provide their lifetime benefits.

Figure 8.2
DISTRIBUTION OF REPAYMENT RATIOS



The authors perform similar simulations for the sample open DP plan in Section 7 and observe that the RRs for all generations are not affected by the choice of valuation interest rate under the Type 1 benefit adjustment mechanism described in Appendix A.

To avoid undesirable intra-cohort wealth transfers, the valuation interest rate should reflect the expected long-term returns of the pension fund. If the plan aims to provide an escalation of pension benefits to compensate for the effect of inflation, an appropriate indexing rate should be incorporated into the valuation assumptions. This increases the likelihood that the distribution of pension benefits remains fair and equitable across all participants, maintaining the plan's long-term sustainability and integrity.

Section 9 Design Element: Benefit Smoothing

As discussed in subsection 5.3 and Section 7, there are considerable uncertainties in pension levels over time, especially when fund investments have substantial exposure to equity markets. Applying smoothing mechanisms can help mitigate the effects of short-term market fluctuations, leading to more stable retirement income.

In this section, the authors explore the impacts on DP plans using two smoothing approaches:

1. **Staggered Adjustments** as discussed in (Begin & Sanders, 2023)
2. **Type 2 – Future Pension Escalation Adjustment**, referred to as CDC-type adjustment hereafter, as explained in subsection 3.2 and Appendix A.

For the staggered adjustment approach, the authors calculate a benefit adjustment factor as the geometric average of the last five adjustment factors including the current one. Using the notation from Section 4, the actual adjustments applied to the time t benefits can be written as:

$$\widetilde{h_t} = \left(\prod_{s=0}^{4} h_{t-s}\right)^{1/5}$$

This staggered adjustment method aims to smooth out the impact of volatile market conditions by averaging the adjustment factors over a set period. The authors use a 5-year period as an illustration[9].

9.1 CLOSED GROUP PLANS

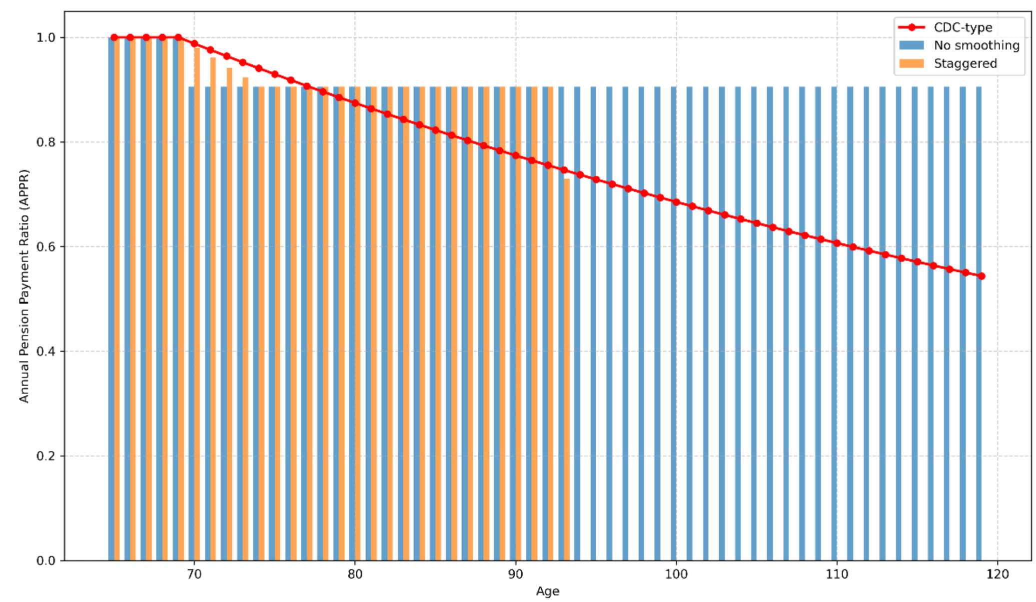
First, the authors examine the impact on the sample closed group plan described in Section 5. It is assumed the fund experiences a negative 4% return, a loss of 10% relative to the expected 6% return, in the 5^{th} year from plan inception. Figure 9.1 illustrates the distribution of APPRs under both adjustment approaches compared to the case with no smoothing adjustments.

With no smoothing, the APPR drops from 1.0 to 0.906 at age 70 and remains at that level until age 120. Under the staggered adjustment approach, the APPR decreases from 1.0 to 0.906 over five years starting at age 70, maintains that level until age 92, and then drops to 0.537 at age 93, reaching zero thereafter due to fund depletion at age 94. Conversely, under the CDC-type adjustment, the APPR declines from 1.0 to 0.988 at age 70 and continues to decrease progressively to 0.537 by age 120. The RR for the participant group equals 1.0 in all cases, indicating that participants' contributions are fully applied to their benefits under the indicated investment scenario.

[9] While this "staggered adjustment" can mitigate year-to-year income volatility, it does not guarantee that assets and liabilities remain perfectly balanced at each intermediate valuation.

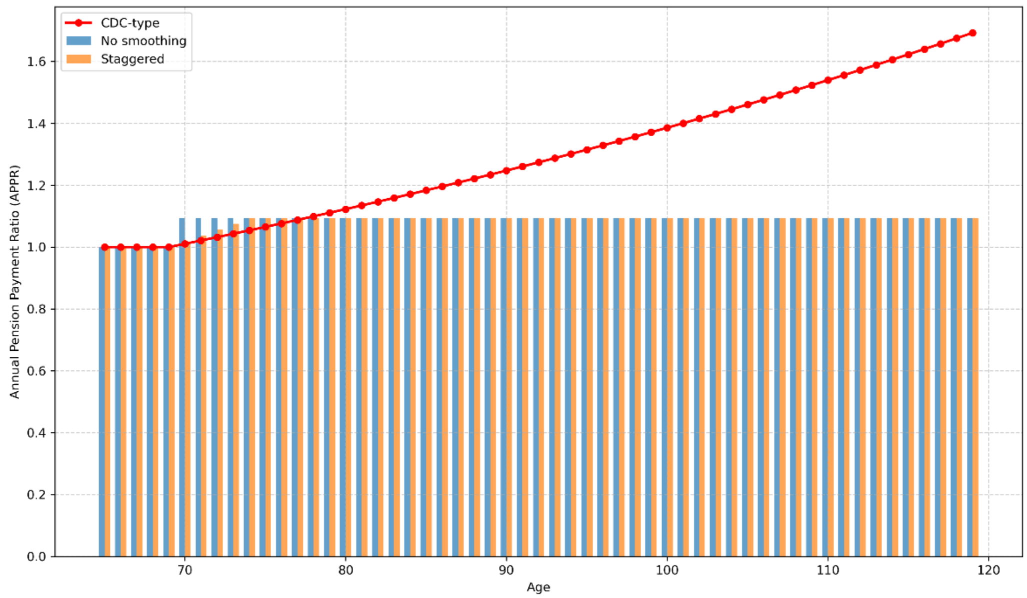
Figure 9.1

DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (-4% RETURN IN YEAR 5)



Next, the authors assume the fund earns a 16% return (i.e., 10% higher than the expected 6% return) in year 5. Figure 9.2 shows the pattern of APPRs under the three benefit adjustment approaches. In this scenario, the RR for the staggered adjustment approach is 0.99, while it remains 1.0 for the other approaches. This suggests that the fund has not been fully utilized to pay benefits under the staggered adjustment approach.

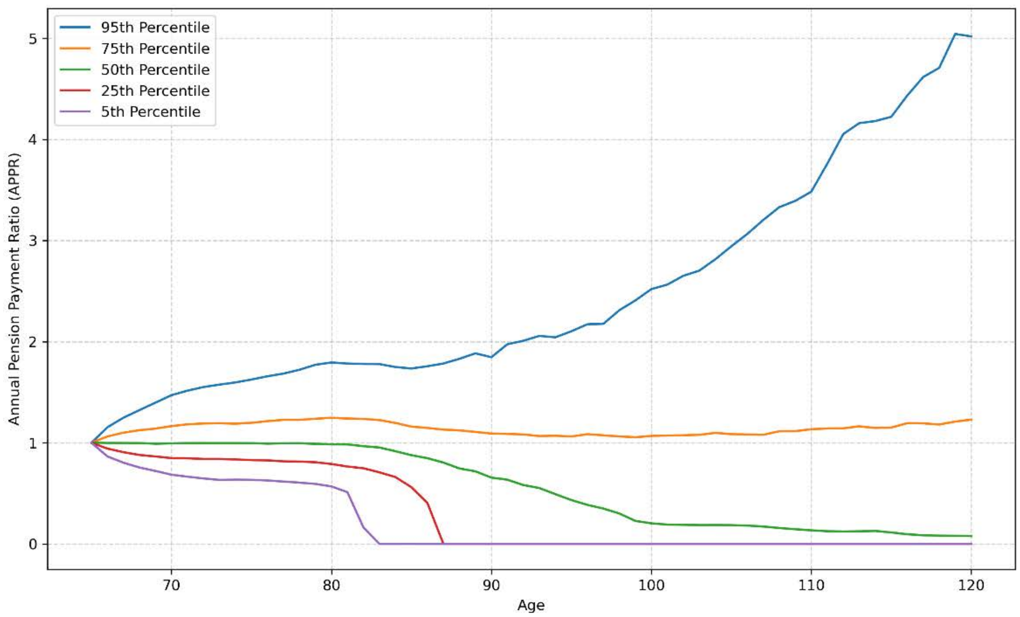
Figure 9.2
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (16% RETURN IN YEAR 5)



While smoothing can enhance benefit stability, the staggered adjustment approach runs the risk of asset depletion under unfavorable investment conditions and does not fully utilize the contributions to provide benefits under favorable conditions. Therefore, it is not a desirable adjustment method. Figure 9.3 provides a stochastic illustration of the APPR distribution under this smoothing approach.

Figure 9.3

DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (STAGGERED ADJUSTMENTS)



For the CDC-type adjustment approach, while year-over-year changes in pension payments during the early stages of participants' lives could be reduced, it may increase uncertainties in pension levels during the later stages of participants' lives.

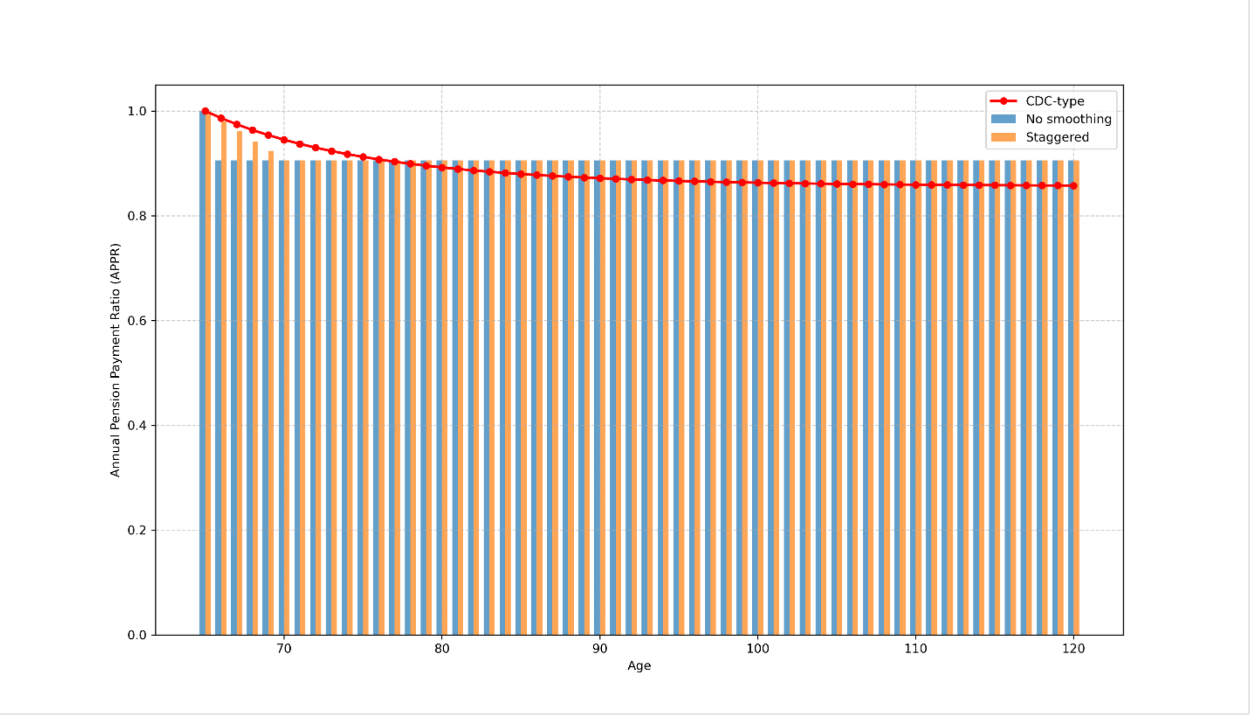
Based on the above analysis, the authors contend that there are doubts about the efficacy of applying a smoothing approach in closed group plans.

9.2 OPEN GROUP PLANS

The authors now examine how smoothing approaches impact the pension outcomes of open DP plans. Assume the sample DP plan described in Section 7 has been operating for 54 years. At the beginning of the 55th year, the participants include surviving members from Generations #1 to #55, aged 65 to 119. Based on past mortality experience, the total number of surviving participants at this point in time is 2,285, with 100 new retirees aged 65 joining the plan each year thereafter. The fund has been earning the expected annual return of 6%.

Assume the fund experiences a negative 4% return in year 55. Figure 9.4 shows the distribution of APPRs for Generation #55 under staggered and CDC-type adjustments compared to the case with no smoothing adjustments.

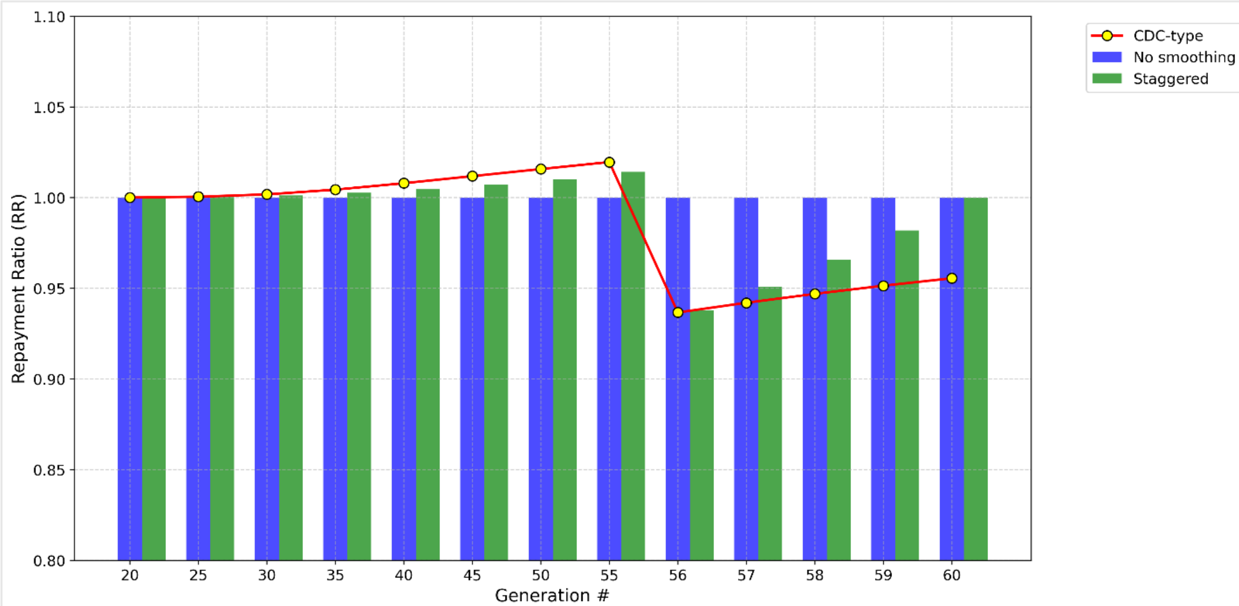
Figure 9.4
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (GENERATION #55)



With no smoothing, the APPR drops from 1.0 to 0.906 at age 66 and remains at that level until age 120. Under the staggered adjustment approach, the APPR decreases gradually from 1.0 at age 65 to 0.906 at age 70 and remains at that level thereafter. Unlike closed group plans, the APPR does not drop to zero later in participants' lives due to the continuing influx of new participants. However, if new retirees stopped joining the plan, the APPR could drop to zero when the fund's assets were depleted. Under the CDC-type adjustment, the APPR declines gradually from 1.0 at age 65 to approximately 0.86 in the later stages of participants' lives, demonstrating a greater smoothing impact in open group plans than in closed group plans. The average APPRs for Generation #55 under the three smoothing approaches are 0.910, 0.917, and 0.913, respectively.

In open group plans, intergenerational wealth transfers occur when applying a smoothing adjustment mechanism. Figure 9.5 shows the RRs for seven older generations (i.e., Generations #20, #25, ..., #50) and five immediate new generations after the 55th year.

Figure 9.5
DISTRIBUTION OF REPAYMENT RATIOS BY GENERATION



With no smoothing, the RRs are equal to 1.0 for all generations, indicating no cross-subsidies between generations. Under staggered adjustments, the investment loss is not fully recovered by benefit reductions applied to existing generations. Consequently, the value of benefits received by existing generations exceeds their initial contributions, with younger generations gaining a greater benefit advantage. In contrast, the four new generations following Generation #55, i.e., Generations #56 to #59, have an RR less than 1.0, as their benefits are subject to reduction due to the delayed adjustments.

Note that the benefit adjustment factors under the staggered adjustment approach are determined based on the investment gains or losses of the fund relative to the valuation interest rates employed in valuations. They do not vary according to the funding position of the plan (i.e., the ratio of assets to liabilities). Application of such benefit adjustments results in a decline in the funded ratio of the plan subsequent to the year of loss (YOL), as illustrated in Table 9.1 below.

Table 9.1
CHANGES IN THE FUNDED STATUS OF THE PLAN

Time of Valuation	Beginning of							
	Year of loss (YOL)	YOL+1	YOL+2	YOL+3	YOL+4	YOL+5	YOL+6	YOL+7
Funded ratio	1.000	0.906	0.920	0.935	0.951	0.968	0.987	0.984

The investment loss in Year 55 is borne partly by existing participants (Generations #1 to #55) and partly by the immediate four generations (Generations #56 to #59) with the balance absorbed by the fund, leading to a funding shortfall. This raises questions about the sustainability of the plan, as there might not be adequate assets to provide participants with lifetime benefits. In contrast, the plan is always maintained in a fully funded position through the reductions in participants' benefits under no smoothing or CDC-type adjustments.

Under the CDC-type adjustment, the RRs for existing generations are higher than 1.0, while the RRs for future generations are lower than 1.0. The closer the generations are to the time of the investment loss, the greater the deviation of RR from 1.0. This phenomenon results from retirement wealth transfers across generations.

9.3 SCENARIO TESTING WITH STOCHASTIC TRIALS

In the preceding section, the authors illustrated how benefit adjustment mechanisms influence pension outcomes in open group plans following a single investment shock. In this section, the authors conduct scenario-based testing using a small number of representative return paths drawn from a broader stochastic framework. These scenarios, selected from a larger set of simulated investment return trials, allow the authors to illustrate the benefit smoothing and intergenerational wealth transfer effects of each adjustment mechanism under more realistic—but still stylized—market conditions.

For our analysis, we specifically chose three sample investment return scenarios from the 1,000 generated scenarios indicated in subsection 5.3[10]:

- **Consistent markets**: Reflecting the median value (6.6%)
- **Unfavorable markets**: Representing the 25th percentile (5.4%)
- **Favorable markets**: Representing the 75th percentile (7.9%)

We simulated the pension outcomes of our sample plan starting from the beginning of the 55th year over the ensuing 55 years. Thus, participants of Generation #55, aged 65 at the start of simulation, are exposed to these market scenarios for the entire 55 years.

The annual benefit adjustment rates applied to the sample plan under the three investment scenarios are visually depicted in Appendix C for reference.

9.3.1 CONSISTENT MARKETS

Figure 9.6 illustrates the distribution of APPRs for Generation #55 under the consistent market scenario. The average APPRs, weighted by participants' survival probabilities, under the three benefit adjustment approaches—no smoothing, staggered, and CDC-type—are 1.078, 1.069, and 1.067, respectively, which deviate modestly from 1.0. However, the plan with no smoothing adjustments exhibits a wider spectrum of pension payments.

To assess benefit stability over time, the authors calculated log changes in APPRs[11] for the three adjustment approaches. The statistical results showed that the CDC-type adjustment had a greater stabilizing effect on pension payments compared to the staggered adjustment (Table 9.2). This is likely because it adjusted the pension payments by spreading investment gains or losses over a longer period, specifically the average remaining lifespan of participants, including future participants. Additionally, this adjustment approach is backend-loaded.

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[10] The investment scenarios are ranked according to their arithmetic average returns, weighted by the survival probabilities of the participants in Generations #55, over a 55-year period.

[11] The log change in the APPR is the logarithm of the ratio between two consecutive APPR values. Letting $APPR_x$ denote the APPR at age x, the log change between two consecutive values, $APPR_x$ and $APPR_{x-1}$, is calculated as:

$$Log\ Change\ =\ ln(\frac{APPR_x}{APPR_{x-1}})$$

The log change represents the relative change between two consecutive observations. A positive log difference indicates an increase, while a negative log difference indicates a decrease. A log difference of 0 indicates no change.

Figure 9.6
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (GENERATION #55)

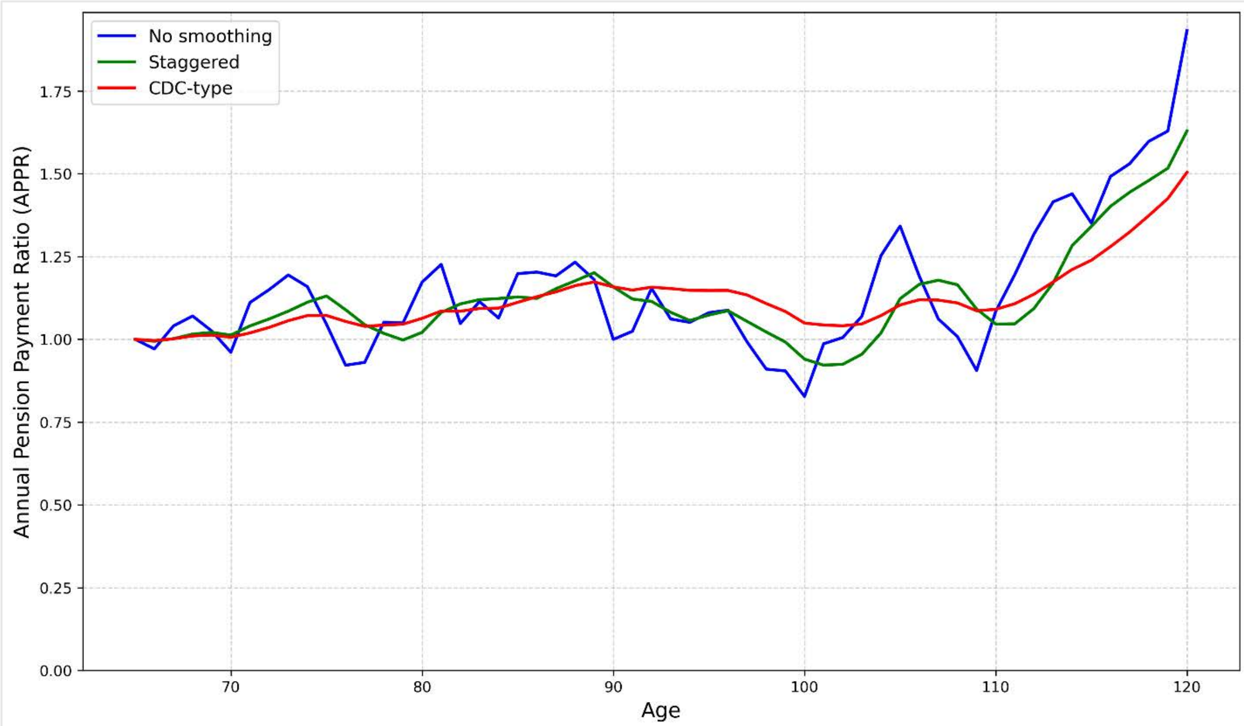


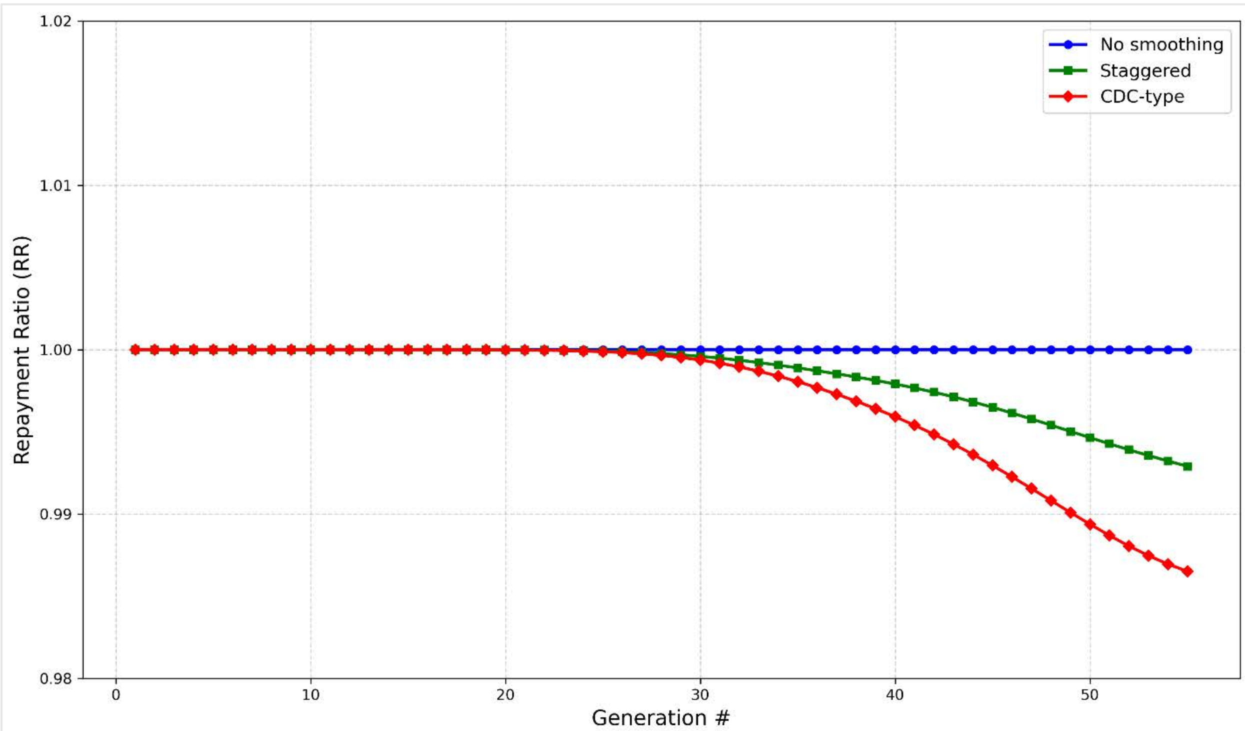
Table 9.2
SUMMARY STATISTICS OF APPRS

	APPR			Log Change in APPR		
	No smoothing	Staggered	CDC-type	No smoothing	Staggered	CDC-type
Mean (unweighted)	1.147	1.117	1.119	1.2%	0.9%	0.7%
Standard deviation	0.206	0.145	0.102	8.8%	3.5%	1.7%
Minimum	0.828	0.922	0.996	-16.6%	-6.5%	-3.3%
Maximum	1.932	1.629	1.504	18.0%	9.7%	5.4%

The RRs for Generation #55 under the three adjustment approaches are 1.000, 0.993, and 0.987, respectively. This indicates a small transfer of wealth to future participants when benefit smoothing adjustments are applied in this specific investment scenario. It is important to note that the RR for no smoothing is always equal to 1.0, as participants in the generation, as a group, receive a return of their contributions with investment earnings.

Figure 9.7 shows the RRs for existing participants at the beginning of the 55th year. Under the CDC-type adjustment, the area between the red line and the blue line represents the extent of retirement wealth being transferred from the current plan participants, i.e., Generations #1 to #55, to future generations.

Figure 9.7
DISTRIBUTION OF REPAYMENT RATIOS BY GENERATION



9.3.2 UNFAVORABLE MARKETS

Figure 9.8 illustrates the distribution of APPRs for Generation #55 under the unfavorable market scenario. The APPR falls progressively over the lifetime of participants. The average APPRs under the three smoothing approaches are 0.840, 0.861, and 0.877, respectively. With staggered or CDC-type adjustments, participants receive, on average, higher benefits than in the no smoothing case. This suggests that a portion of investment losses relative to the expected 6% return is transferred from this generation to future generations, lowering the benefits for new participants.

Specifically, the RR for Generation #55 under the CDC-type adjustment is 1.047, indicating that participants in this generation receive benefits 4.7% higher in value than their initial contributions. The RR for Generation #55 under the staggered adjustment is comparatively lower at 1.027. Figure 9.9 shows the distribution of RRs for existing participants at the beginning of the 55th year.

The log changes in APPRs highlight the higher volatility in pension payments under the no smoothing case relative to the cases where smoothing adjustments are applied (Table 9.3).

Figure 9.8
DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (GENERATION #55)

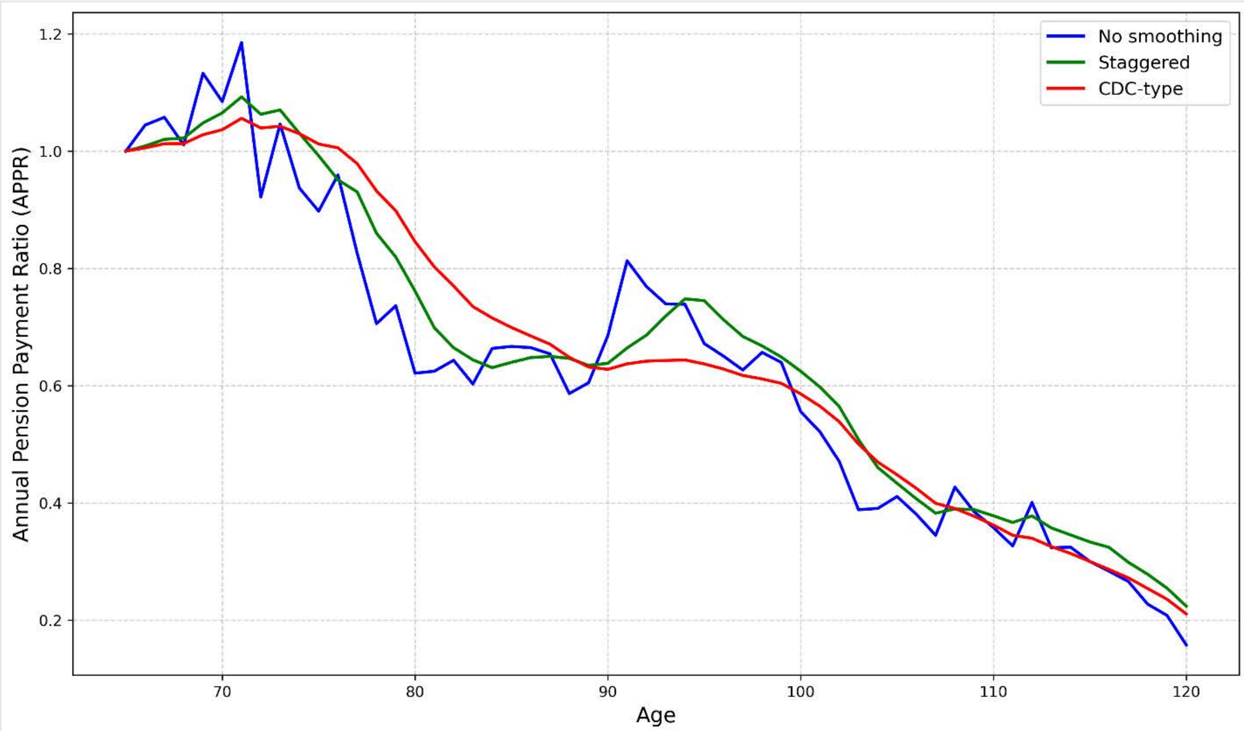
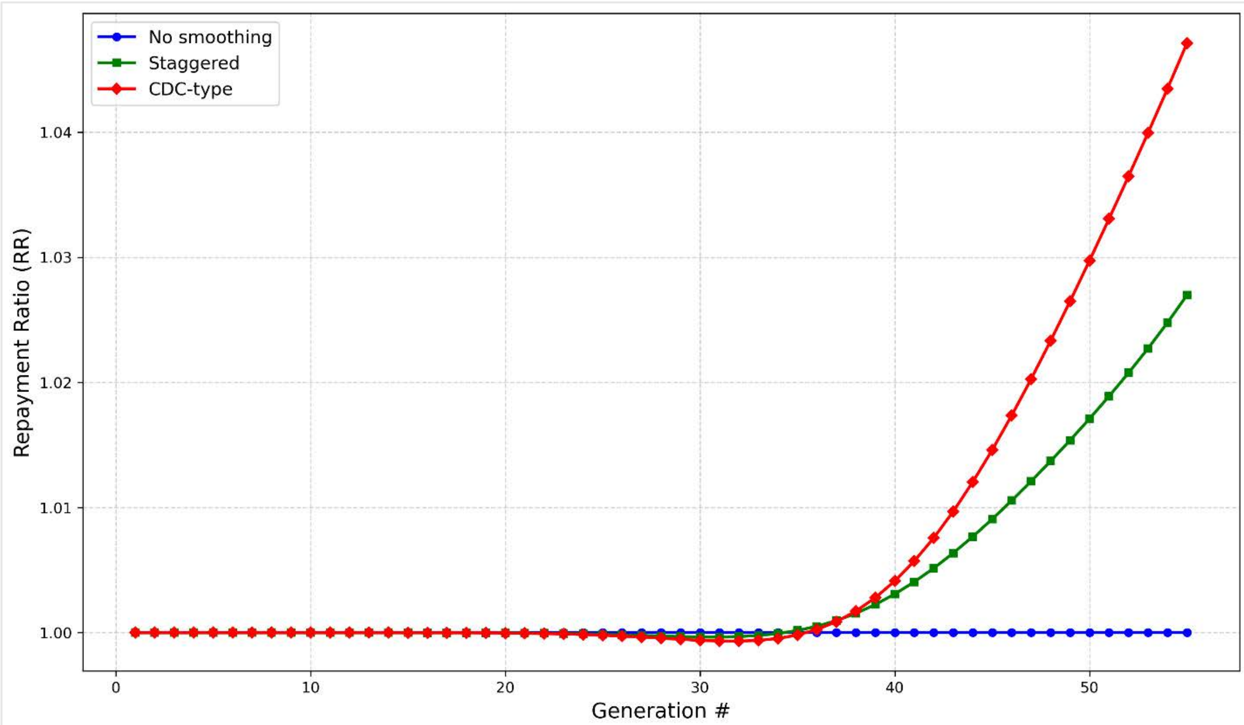


Table 9.3
SUMMARY STATISTICS OF APPRS

	APPR			Log Change in APPR		
	No smoothing	Staggered	CDC-type	No smoothing	Staggered	CDC-type
Mean	0.631	0.657	0.652	-3.4%	-2.7%	-2.8%
Standard deviation	0.265	0.254	0.263	10.6%	4.1%	2.7%
Minimum	0.158	0.224	0.210	-27.7%	-12.9%	-11.3%
Maximum	1.185	1.093	1.056	21.4%	4.6%	1.9%

Figure 9.9
DISTRIBUTION OF REPAYMENT RATIOS BY GENERATION



9.3.3 FAVORABLE MARKETS

Under the favorable market scenario (Figure 9.10), the average APPRs for the three smoothing approaches are 1.363, 1.326, and 1.331, respectively. With staggered or CDC-type adjustments, participants receive, on average, lower benefits compared to the no smoothing case. This suggests that a portion of the investment gains relative to the expected 6% return is transferred from the current generation to future generations, benefiting future participants.

Specifically, the RR for Generation #55 under the CDC-type adjustment is 0.949, indicating that participants in this generation receive benefits 5.1% lower in value than their initial contributions. The RR for Generation #55 under the staggered adjustment is slightly higher at 0.966. Figure 9.11 shows the distribution of RRs for existing participants at the beginning of the 55th year.

The log changes in APPRs indicate higher volatility and wider fluctuations in pension payments under the no smoothing case compared to the cases where smoothing adjustments are applied (Table 9.4).

Figure 9.10

DISTRIBUTION OF ANNUAL PENSION PAYMENT RATIOS (GENERATION #55)

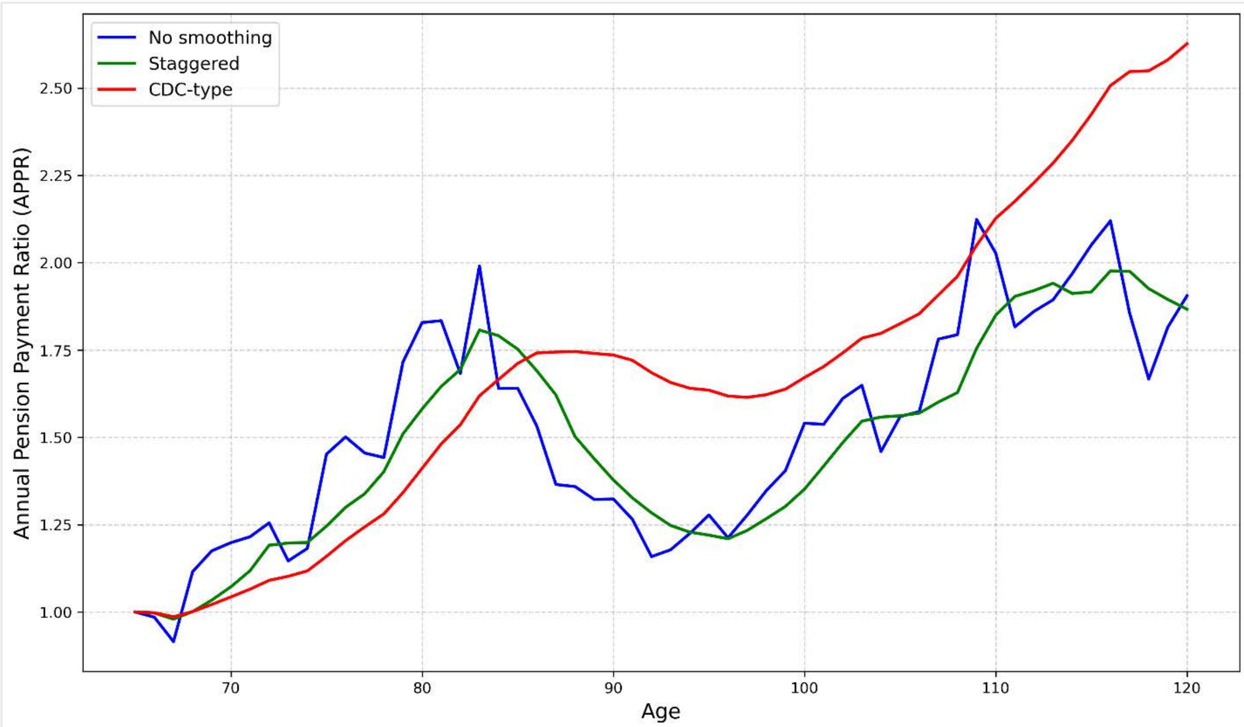
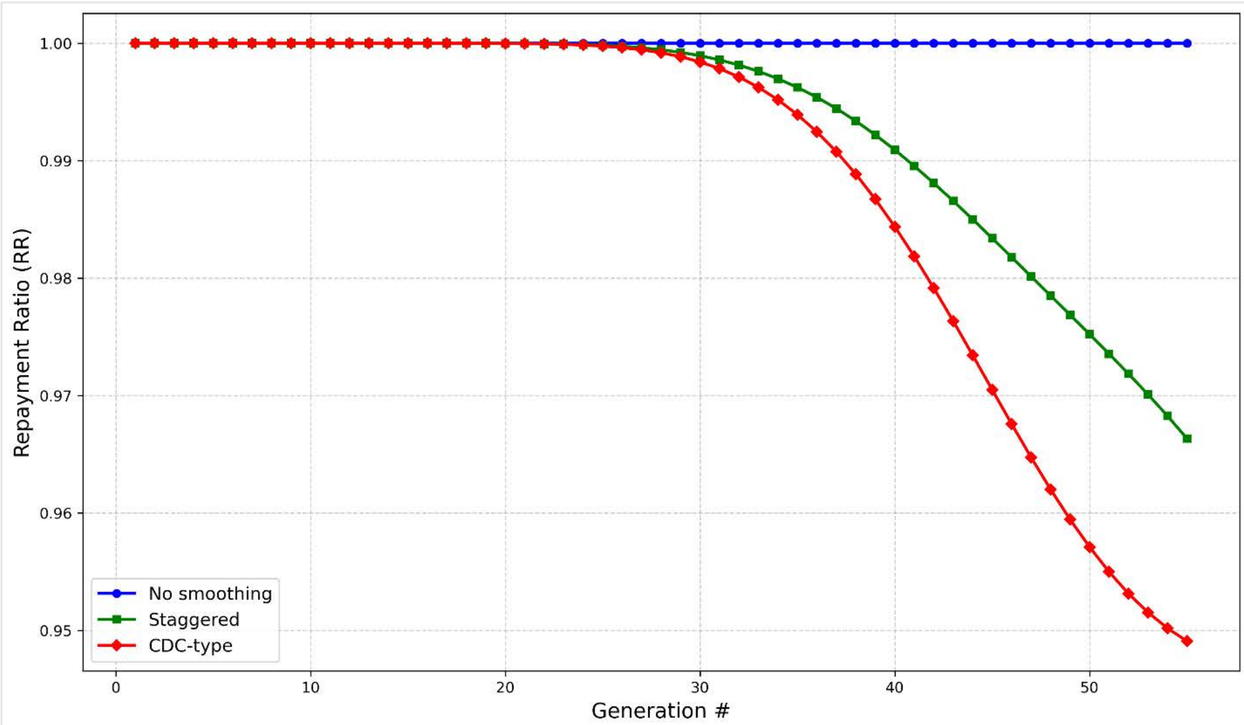


Table 9.4

SUMMARY STATISTICS OF APPRS

	APPR			Log Change in APPR		
	No smoothing	Staggered	CDC-type	No smoothing	Staggered	CDC-type
Mean	1.522	1.489	1.684	1.2%	1.1%	1.8%
Standard deviation	0.313	0.300	0.459	8.4%	3.4%	1.8%
Minimum	0.915	0.980	0.986	-19.4%	-7.6%	-2.1%
Maximum	2.124	1.976	2.627	20.5%	7.5%	5.2%

Figure 9.11
DISTRIBUTION OF REPAYMENT RATIOS BY GENERATION



In summary, the CDC-type adjustment enhances stability in pension payments by reducing variations in APPRs, thus promoting a more stable distribution of benefits across generations. While the staggered adjustment also provides some smoothing benefits, they are less pronounced. The stability of pension payments over time relies significantly on intergenerational wealth transfers, particularly evident in non-neutral market conditions.

Section 10 Design Element: Pension Annuity Forms

The choice of pension annuity form significantly affects the distribution of retirement wealth among participants. Preferences for either lifetime income or the ability to bequeath funds to beneficiaries shape the overall design of the DP plan.

In an article by (Peterson, Shemtob, & Gold, 2024), the authors highlight these differing preferences. For instance, some participants may prefer a life-only annuity for maximum lifetime income, while others might opt for a 20-year certain life annuity or a life annuity with survivor benefits to ensure their beneficiaries receive some level of benefit.

As discussed in subsection 5.1, a life-only annuity results in longer-living participants benefiting from the contributions of those who die earlier, creating a natural wealth transfer. Conversely, a 20-year certain life annuity reduces this transfer, as benefits are guaranteed for a minimum period regardless of the participant's lifespan. This design choice can address concerns about equity and fairness, ensuring that participants who die earlier do not feel their contributions are wasted.

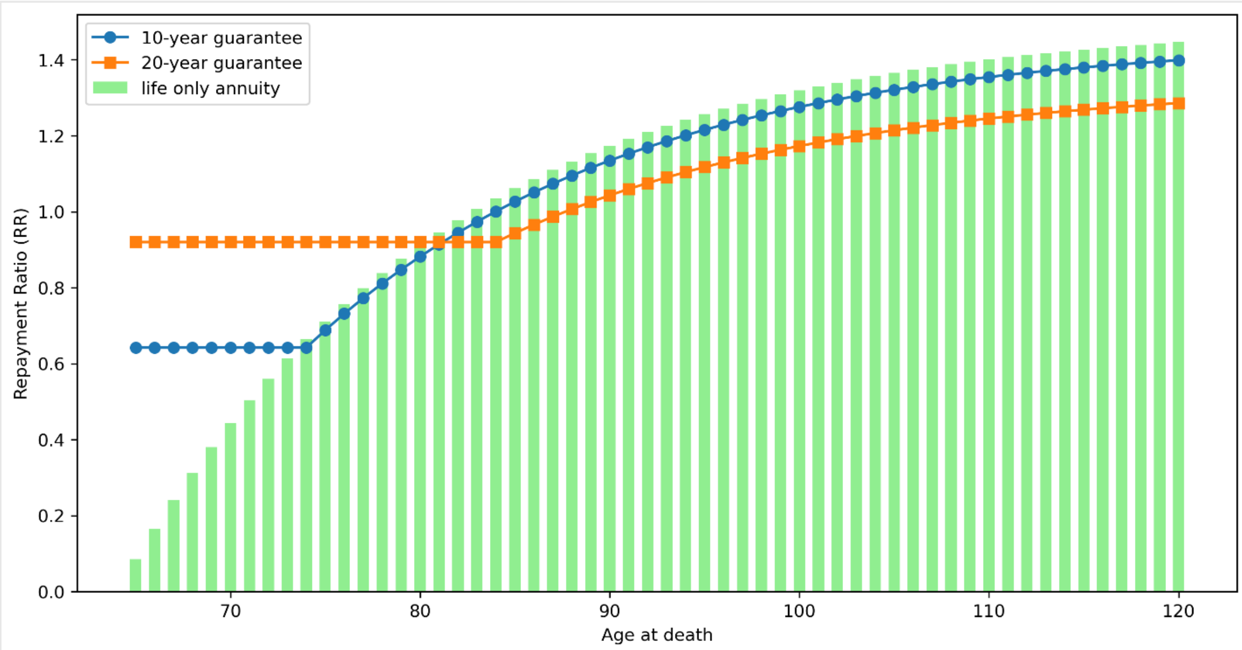
Based on the sample DP plan described in Section 5, we compare the pension outcomes under three annuity forms:

- Life-only annuity
- Life annuity with a 10-year guarantee
- Life annuity with a 20-year guarantee

The initial annual pensions for the three annuity types are $42,600, $41,200, and $37,800, respectively. Figure 10.1 shows the distribution of RRs under the three annuity types. For instance, under the annuity with a 20-year guarantee, the RRs for participants who die during the guarantee period stand at 0.92, whereas the RRs under the life-only annuity range from 0.085 to 1.035. The first age at which the RR exceeds 1.0 is 88, compared to 83 under the life-only annuity. The expected loss in retirement wealth for participants who die before age 88 is 3.9% of their initial contributions, significantly lower than the 11.1% expected loss under the life-only form for participants who die before age 83. This illustrates that a life annuity with a guarantee can reduce the extent of wealth transfers from participants who die young to those who live longer lives.

Figure 10.1

DISTRIBUTION OF REPAYMENT RATIOS



Section 11 Type 3 Benefit Adjustment and Optimal Investment Strategies

In this section, the authors analyze the Type 3 adjustment mechanism, known as the Target Benefit Funded Ratio Linked Adjustment, as described in subsection 3.2. This mechanism is unique in its handling of the redistribution of investment gains and losses across generations, providing a flexible approach to managing intergenerational risk. Additionally, the authors propose an approach to finding optimal investment strategies for plans with Type 3 benefit adjustment.

The Type 1 adjustment mechanism allows for mortality risk pooling across generations but does not account for investment risk diversification. This lack of investment risk-sharing exposes individuals within the same generation to elevated risk, particularly if their retirement coincides with unfavorable economic conditions. A generation retiring during an economic downturn may experience significantly reduced retirement income, with no mechanism for redistributing investment losses across other cohorts.

On the other hand, the Type 3 adjustment mechanism fills this gap by continuously redistributing investment gains or losses across the remaining lifetimes of all participants. Negative or positive investment shocks are not solely borne by the current generation but spread across both current and future retirees. This intergenerational risk-sharing feature leads to a more balanced distribution of financial outcomes, mitigating the risk of adverse economic conditions disproportionately affecting any one generation.

11.1 TYPE 1 VERSUS TYPE 3 ADJUSTMENTS

Type 1 adjustment calculates the liability based on the pension currently payable, while Type 3 bases the liability on the target pension, which is the initial pension set when a retiree joins the plan. In closed group scenarios, both Type 1 and Type 3 lead to the same projected retirement income as there is no intergenerational risk-sharing. However, in open group scenarios—where new participants continually join the plan—the differences between these two mechanisms become more apparent.

This can be illustrated with a simple example.

Consider the plan scenario described in the first paragraph of subsection 9.2. Suppose the fund experiences a -4% return in year 55, representing a 10% loss relative to the assumed valuation interest rate of 6%. Let the beginning of year 55 be denoted as time 0.

At time 1, the Type 1 benefit adjustment factor h_1 is calculated using Equation (8) from Appendix A:

$$h_1 = \frac{0.96}{1.06} = 0.906$$

Similarly, the Type 3 adjustment factor at time 1, derived from the target benefit funded ratio as described in Equation (11) of Appendix A, also equals 0.906.

Thus, the pension paid to each surviving retiree at time 1, under either Type 1 or Type 3, is:

$$P_1 = 0.906 \cdot P_0$$

Now, assume the fund earns the expected 6% return in subsequent years.

- **Type 1 Adjustment:** The pension payable in future years is determined by applying the adjustment factor to the previous year's pension. Since the fund earns exactly the expected return, the adjustment factors stabilize at 1.0, and the pension remains at $0.906 \cdot P_0$ for all future years.
- **Type 3 Adjustment:** In contrast, the pension is adjusted by applying the benefit adjustment factors based on the target pension (see Appendix A). For any time $t > 1$, the adjustment factor is determined by the target benefit funded ratio:

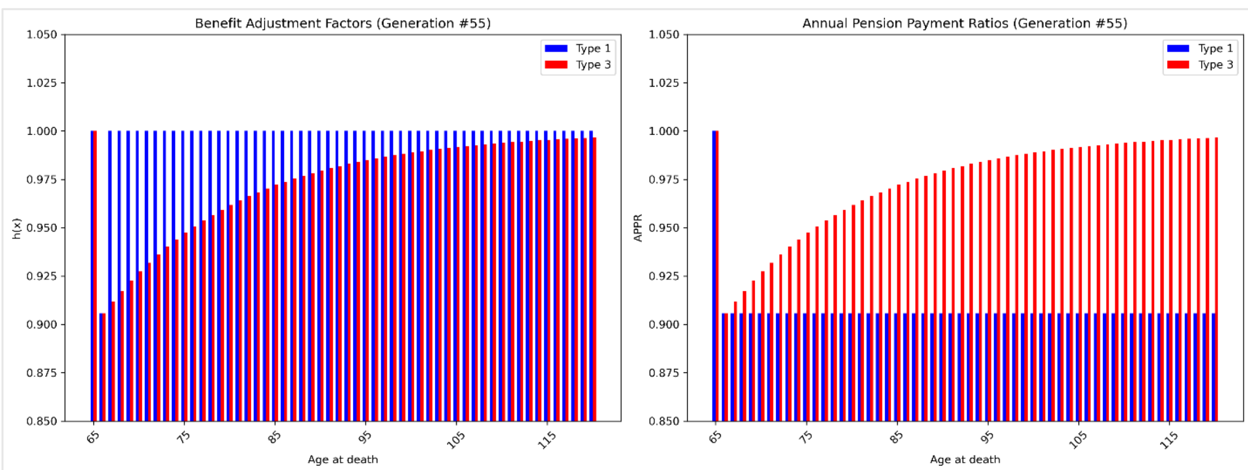
$$h_t = \frac{F_t}{L_t^T}$$

  where F_t is the value of the fund's assets, and L_t^T is the target benefit liability at time t. As new participants join the plan over time, this adjustment factor gradually increases from 0.906 and asymptotically approaches 1.0. This implies that future generations share the burden of the first-year loss by receiving pensions below their target amount (since future adjustment factors are less than 1), though this shortfall diminishes over time.

The charts below compare the benefit adjustment factors and the Annual Pension Payment Ratios (APPRs) for Generation #55 under both Type 1 and Type 3 adjustment mechanisms.

Figure 11.1

DISTRIBUTION OF BENEFIT ADJUSTMENT FACTORS AND ANNUAL PENSION PAYMENT RATIOS (GENERATION #55)



As shown, while both mechanisms reflect a reduction in benefits following the loss, the adjustments under Type 3 allow for partial recovery over time. For example, under Type 3, the APPR recovers to 0.972 by age 85, compared to the fixed 0.906 under Type 1. This demonstrates how the plan with Type 3 adjustments gradually absorbs and redistributes the impact of the loss. The average APPR over the lifetime of participants from Generation #55 is 0.951 under Type 3, compared to 0.910 under Type 1.

In the next section, the authors will explore the intergenerational risk-sharing effects of both Type 1 and Type 3 adjustment mechanisms in greater detail.

11.2 EXPLORATION OF INTERGENERATIONAL RISK-SHARING EFFECTS

The authors compared pension outcomes using the Annual Pension Payment Ratio (APPR) and the Repayment Ratio (RR), as defined in subsection 4.2, and applied to the open group membership model described in Section 7. A simulation study was conducted using 1,000 investment return scenarios generated by an economic scenario generator (Ma, 2023). Fund assets were allocated equally between equities (50%) and bonds (50%), with an assumed valuation interest rate of 6%. Mortality rates were drawn from the tables published by the Society of Actuaries, and the initial annual pension for a 65-year-old retiree from Generation #55 was estimated to be $39,400 (the plan was assumed to begin operation in 2012). The initial annual pension represents the target pension the plan aims to deliver.

The simulations covered a 55-year period beginning from the participation of Generation #55. Both Type 1 and Type 3 adjustments were modeled assuming initially fully funded plans, with mortality rates exactly realized throughout the simulation period.

- Under the Type 1 adjustment mechanism, the RR for Generation #55 remains fixed at 1 across all investment scenarios, indicating that participants recapture 100% of their contributions. However, the average APPRs range from 0.75 at the 5th percentile to 1.49 at the 95th percentile, with a median of 1.02, a mean of 1.06, and a standard deviation of 0.23. This indicates significant variability in annual pension payments due to the absence of intergenerational risk-sharing.
- Under the Type 3 adjustment mechanism, the RR for Generation #55 varies between 0.92 and 1.11, with a mean of 1.0 and a standard deviation of 0.06. This reflects the redistribution of investment gains and losses across generations. The average APPRs range from 0.82 at the 5th percentile to 1.32 at the 95th percentile, with a median of 1.02, a mean of 1.04, and a standard deviation of 0.15. The narrower distribution of APPRs demonstrates greater pension payment stabilization under Type 3, driven by intergenerational risk-sharing.

This comparison shows that, while Type 1 adjustments ensure participants fully recapture their contributions (as evidenced by a constant RR of 1), they do not provide the same level of stability in annual pension payments as Type 3. Type 3 adjustments smooth pension income by redistributing risk across generations, making them a more resilient mechanism for mitigating economic volatility in open group DP plans.

11.3 IMPLEMENTATION OF INTERGENERATIONAL RISK-SHARING IN DP PLANS

The Type 3 adjustment mechanism enables intergenerational risk-sharing, which stabilizes pension payments affected by retirement wealth transfers. To quantify the extent of this risk transfer, the authors assessed deviations of the Repayment Ratio (RR) from 1 for each generation of participants. For Generation #55, the simulation results show that RR fluctuates between 0.92 and 1.11, with a mean of 1.0 and a standard deviation of 0.06.

To systematically measure these deviations, we propose using the Root Mean Square Deviation (RMSD) of the RR as an indicator:

$$RR\ RMSD = \sqrt{\frac{1}{N}\left(\sum_{k}(RR_k - 1)^2\right)}$$

where N is the number of simulations, and k indexes the simulation paths. Although the Mean Absolute Deviation (MAD) could also be used, RMSD is preferred as it gives greater weight to larger deviations, an important consideration when it is desirable to maintain intergenerational fairness by avoiding significant variations in RR.

Building on the study by (Chen, Kanagawa, & Zhang, 2023), which explores intergenerational risk-sharing during the accumulation phase of CDC plans, the authors extended their approach to the decumulation phase in DP plans. Specifically, the benefit factor h_t at time t under Type 3 adjustment is determined by the target benefit funded ratio, denoted as α_t:

$$\alpha_t = \frac{F_t}{L_t^T}$$

where F_t represents the fund's assets at time t, and L_t^T is the target benefit liability. To allow for flexibility, the authors introduced an adjustment parameter θ that modifies the benefit adjustment factor to reflect only a fraction of the deviation of the target benefit funded ratio from 1.0:

$$h_t = 1 + \theta(\alpha_t - 1)$$

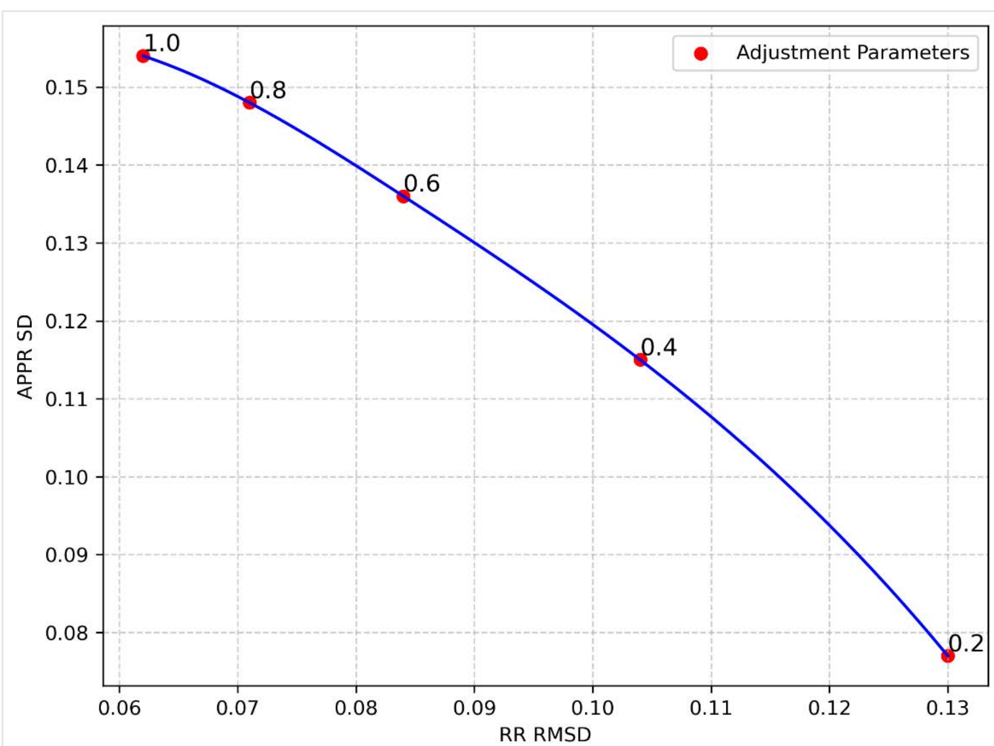
with $0 \leq \theta \leq 1$. If $\theta = 1$, then $h_t = \alpha_t$.

The mechanism described above extends the standard Type 3 adjustment by applying only a fraction (θ) of the calculated adjustment factor each period, rather than the full amount. Under standard Type 3, pension payments are scaled exactly to restore the target funded ratio at each valuation. In our parametric extension, only θ of that adjustment is enacted immediately, with the balance deferred to future valuations. This "partial adjustment" smooths the recognition of emerging gains and losses while preserving the core linkage to the target funded ratio.

The authors modeled pension outcomes using θ values ranging from 0.2 to 1.0. The results, illustrated in Figure 11.2, show the relationship between the standard deviation (SD) of APPR and the RMSD of RR for Generation #55 across different θ values. Smaller θ values lead to more stable pension payments, indicated by lower SD values for APPR, but this comes at the cost of greater variability in RR, reflecting increased intergenerational wealth transfers.

Figure 11.2

APPR SD VERSUS RR RMSD FOR DIFFERENT ADJUSTMENT PARAMETERS



It is also important to note that smaller adjustment parameters may elevate the risk of asset depletion during prolonged economic downturns. Therefore, selecting an appropriate adjustment parameter requires balancing pension stability with long-term sustainability.

11.4 IMPACT OF INVESTMENT STRATEGIES AND ADJUSTMENT PARAMETERS

In this section, the authors extended the analysis of benefit adjustment parameters by incorporating different investment strategies, specifically focusing on varying levels of equity exposure. Let π represent the equity allocation, w the standard deviation (SD) of the annual pension payment ratio (APPR), and v the root mean square deviation (RMSD) of the repayment ratio (RR).

Using Monte Carlo simulations, we estimated the outcomes for w and v across different combinations of π (0.2, 0.4, 0.5, 0.6, 0.8) and adjustment parameter θ (0.2, 0.4, 0.6, 0.8, 1.0). The simulations account for various valuation interest rates and target pensions, which are summarized in the table below:

Table 11.1
SUMMARY OF TARGET PENSION VALUES

π	Valuation Interest Rate	Target Pension
0.2	4.50%	$34,300
0.4	5.50%	$37,700
0.5	6.00%	$39,400
0.6	6.35%	$40,600
0.8	7.05%	$43,000

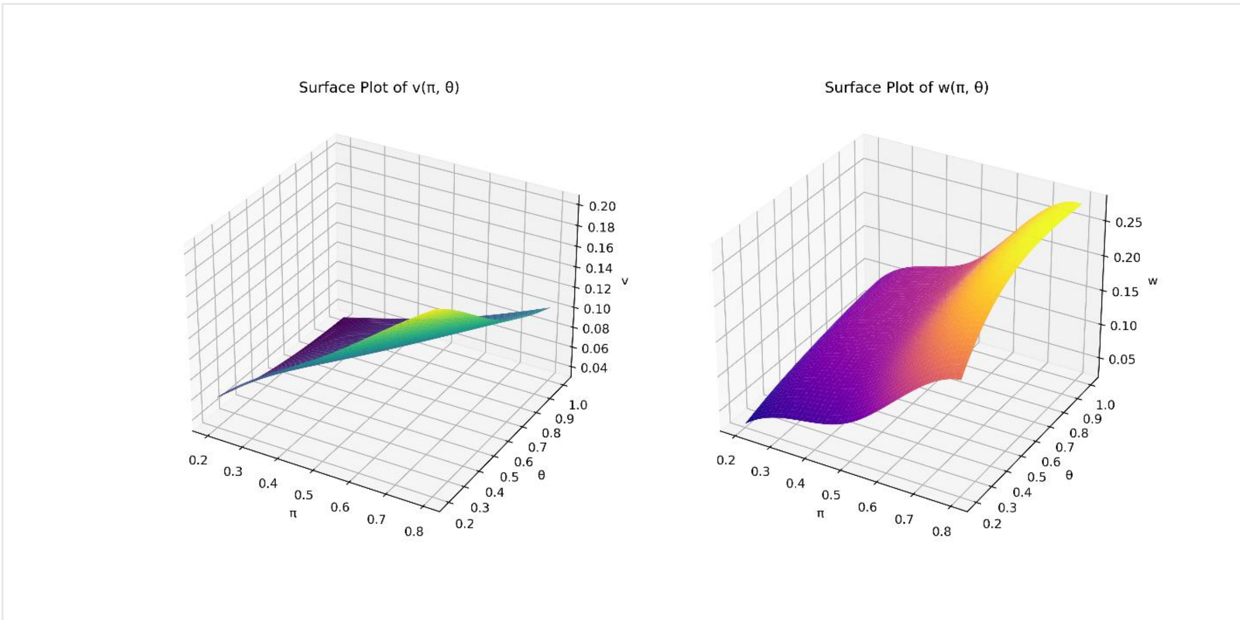
Table 11.2 provides a summary of the simulation results, illustrating the combined effects of different investment strategies and adjustment parameters on key pension outcomes.

Table 11.2
SUMMARY OF v AND w GRID VALUES

		RMSD of RR (v)					SD of APPR (w)				
θ		0.2	0.4	0.6	0.8	1.0	0.2	0.4	0.6	0.8	1.0
π	0.2	0.059	0.047	0.039	0.035	0.033	0.026	0.044	0.056	0.066	0.073
	0.4	0.109	0.085	0.069	0.058	0.051	0.061	0.094	0.114	0.126	0.132
	0.5	0.130	0.104	0.084	0.071	0.062	0.077	0.115	0.136	0.148	0.154
	0.6	0.155	0.125	0.101	0.085	0.074	0.117	0.166	0.191	0.201	0.204
	0.8	0.205	0.176	0.143	0.119	0.103	0.185	0.247	0.273	0.280	0.278

The table demonstrates a clear trend: higher equity allocations (π) result in greater pension payment volatility (higher w) and larger wealth transfers across generations (higher v). Conversely, for a fixed π, increasing θ increases pension volatility but reduces wealth transfers. These relationships are visualized in the following surface charts, showing how different combinations of π and θ influence pension volatility and intergenerational wealth transfer.

Figure 11.3
$\boldsymbol{v}$ AND $\boldsymbol{w}$ FOR DIFFERENT COMBINATIONS OF $\boldsymbol{\pi}$ AND $\boldsymbol{\theta}$

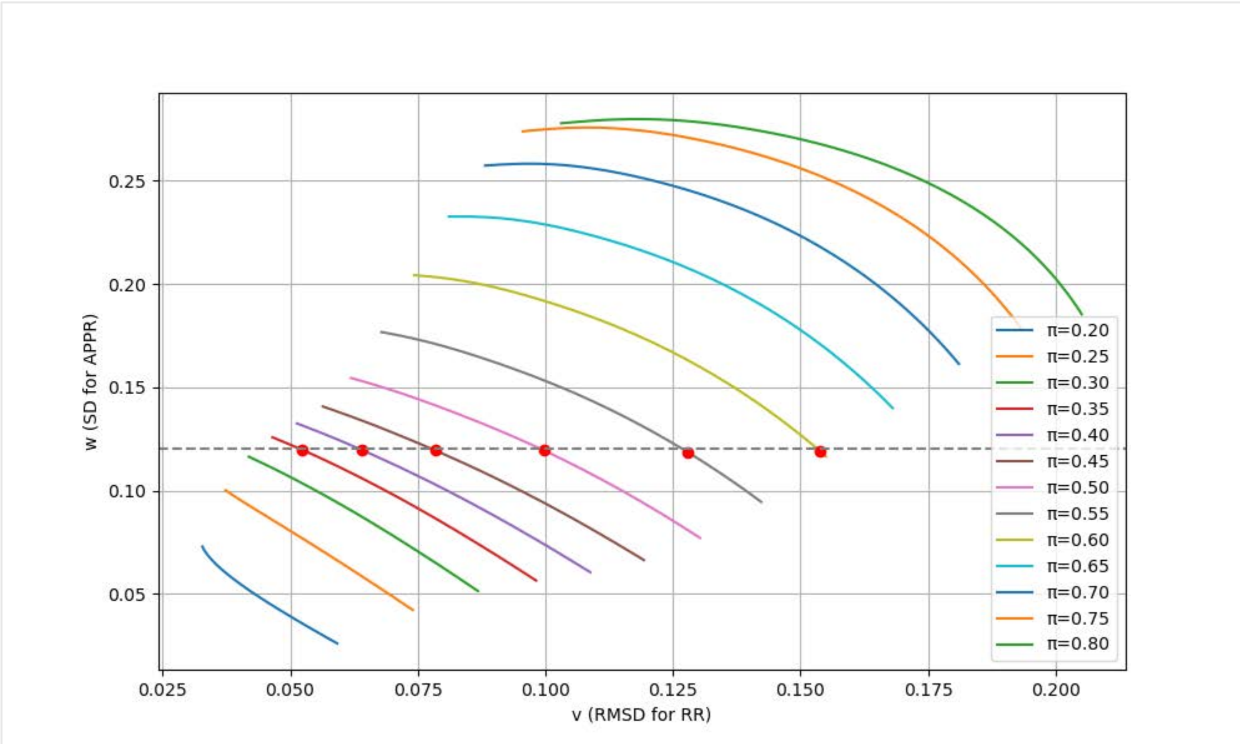


11.5 FINDING THE OPTIMAL INVESTMENT STRATEGY

In their study, (Chen, Kanagawa, & Zhang, 2023) sought to determine an optimal investment strategy that maximized the expected utility of plan participants, guided by the risk preferences of a social planner. The approach of the authors took a different direction by proposing an alternative objective function: *maximizing the equity allocation (π) while maintaining a target pension volatility (w) and adhering to a wealth transfer constraint (v).* Higher equity allocations are generally associated with increased expected pension payments; however, they also lead to greater pension volatility and intergenerational wealth transfers, making it essential to strike a balance between these goals.

The authors applied bivariate spline interpolation to the datasets for π and θ, generating smooth and precise interpolations for w and v. This method enables more accurate and continuous estimates of the relationship between these variables, which is helpful for identifying optimal strategies. The authors then plotted various curves representing the relationship between v and w for different π values. In this plot, the vertical axis represents w, and the horizontal axis represents v. By drawing a horizontal line at the target volatility level (w), the authors could determine the intersection points of θ and v along the π curves. Figure 11.4 illustrates these curves for a target pension volatility of w = 0.12, with π values ranging from 0.2 to 0.8 in increments of 0.05.

Figure 11.4
BIVARIATE SPLINE INTERPOLATION OF W VERSUS V FOR DIFFERENT Π VALUES (W=0.12)



The numerical results for pension volatility targets of w = 0.10, w = 0.12, and w = 0.15 are summarized in Table 11.3, which presents the optimal combinations of π (equity allocation) and θ (adjustment parameter) for each case. These findings shed light on how varying levels of pension volatility and wealth transfer constraints affect the optimal investment strategy.

For instance, when the pension volatility target is w = 0.12 and the wealth transfer constraint is v = 0.10, the optimal strategy involves an equity allocation of $\pi = 0.50$ and an adjustment parameter of $\theta = 0.43$. A Monte Carlo simulation confirms that these parameters achieve the desired outcomes, demonstrating the robustness of the proposed optimization approach.

Similarly, for other combinations of volatility targets and wealth transfer constraints, such as $(w, v) = (0.10, 0.10)$ and $(w, v) = (0.15, 0.10)$, the optimal equity allocation and adjustment parameter pairs are $(\pi, \theta) = (0.45, 0.39)$ and $(\pi, \theta) = (0.55, 0.49)$, respectively. These results highlight the adaptability of the optimization framework under varying targets and constraints.

The Python code used for these calculations is provided in Appendix D.

Table 11.3

INTERSECTION POINTS FOR SELECTED PENSION VOLATILITY TARGETS

Pension Volatility Target (w) = 0.10		
RR RMSD (v)	Equity Allocation (π)	Adjustment Parameter (θ)
0.04	0.25	0.99
0.06	0.30	0.60
0.07	0.35	0.50
0.08	0.40	0.44
0.10	0.45	0.39
0.12	0.50	0.31
0.14	0.55	0.22

Pension Volatility Target (w) = 0.12		
RR RMSD (v)	Equity Allocation (π)	Adjustment Parameter (θ)
0.05	0.35	0.81
0.06	0.40	0.68
0.08	0.45	0.57
0.10	0.50	0.43
0.13	0.55	0.30
0.15	0.60	0.21

Pension Volatility Target (w) = 0.15		
RR RMSD (v)	Equity Allocation (π)	Adjustment parameter (θ)
0.07	0.50	0.85
0.10	0.55	0.49
0.14	0.60	0.31
0.16	0.65	0.22

11.6 POTENTIAL EFFECTS OF COMPETITION IN PRIVATE PENSION SYSTEMS

Intergenerational risk-sharing is a fundamental aspect of the Type 3 adjustment mechanism, and its success relies on a steady inflow of new participants to help balance the pension fund across generations. However, the structure of the Type 3 adjustment introduces the potential for competition from new pension plans, which could divert prospective retirees from the existing plan. This concern also applies to the Type 2 adjustment mechanism, though to a lesser extent.

Competition is most likely when the target benefit funding ratio α_t for the existing plan at time t is less than 1. In such cases, a new plan, starting with $\alpha_t = 1$ by default, becomes more attractive to new retirees as they are not required to share in the accrued investment losses of the existing plan. For example, in the scenario outlined in subsection 11.1, where the existing plan experiences a 10% loss relative to the assumed valuation interest rate of 6%, a new plan formed after this loss—assuming no further gains or losses—would have a benefit adjustment factor of $h_{t,new} = 1$, compared to $h_{t,existing}$ for the existing plan, which would always be lower. As a result, the new plan would consistently offer higher benefits, making it more appealing to new retirees.

Conversely, when $\alpha_t > 1$ for the existing plan, new competing plans are less likely to form, as they would typically offer lower benefits, all else being equal. In this situation, the existing plan would attract a larger share of new retirees, who would then participate in the gains accrued by the plan. However, this variation in the number of new retirees could destabilize the intergenerational risk-sharing inherent in the Type 3 adjustment as the plan's ability to smooth gains and losses across generations would be affected by fluctuations in retiree inflow.

Section 12 Implications for the Design of DP Plans

The design elements of DP plans, including valuation interest rates, benefit smoothing approaches, pension annuity forms, and investment strategies, are essential in determining pension outcomes. An in-depth analysis of these elements, as discussed in Sections 8 to 11, reveals significant implications for the design and administration of DP plans. Understanding and optimizing these design elements can help DP plans achieve their primary goals of fair distribution of retirement wealth and benefit stability.

Valuation Interest Rates

Valuation interest rates are crucial in determining the present value of future pension benefits. These rates must be set to reflect realistic expected fund returns to increase the likelihood of an equitable distribution of benefits among participants. Overly optimistic rates can lead to underfunding, necessitating future reductions in participants' income, which can erode trust and the financial stability of the plan. Conversely, overly conservative rates can result in lower income for participants with shorter life expectancies, to the benefit of participants who live longer lives.

To mitigate these risks, setting rates that closely align with long-term market expectations is critical. Incorporating periodic reviews to adjust for significant changes in the economic environment can help maintain the plan’s solvency while increasing fairness across generations.

Benefit Smoothing Approaches and Intergenerational Risk-Sharing

Benefit smoothing mechanisms play a crucial role in stabilizing pension payments, which is essential for participants' retirement planning and security. However, the choice of smoothing approach must be carefully balanced to avoid unfair cross-subsidies among participants, particularly between current and future generations.

The analysis in Section 11 highlights the differences between the Type 1 and Type 3 benefit adjustment mechanisms. While the Type 3 adjustment provides a smoother pension payment stream, it comes at the cost of intergenerational wealth transfers. These transfers can either benefit or burden future generations, depending on market conditions. In unfavorable markets, higher benefits are maintained for current participants at the expense of future participants. Designers of DP plans must carefully consider the extent to which these wealth transfers are acceptable and ensure clear communication with participants about the rationale behind these adjustments. Participants should be made aware of the potential for cross-generational impacts to maintain transparency and trust in the system.

Pension Annuity Forms

The choice of pension annuity forms significantly affects wealth transfers among participants. A life-only annuity maximizes lifetime income for participants but creates a natural wealth transfer from those who die earlier to those who live longer. This can be perceived as unfair by participants with shorter life expectancies. On the other hand, annuities with guarantees (such as 20-year certain) can mitigate this transfer, providing some assurance to participants that their contributions will not be entirely lost. However, these guarantees come at the cost of lower initial pension payments. Offering a range of annuity forms allows participants to choose based on their individual circumstances and preferences, providing flexibility that can enhance participant satisfaction and trust in the plan.

Investment Strategies and Volatility Targeting

Investment strategy is another critical factor influencing the design of DP plans. Subsection 11.4 explores the interaction between investment strategies and the pension adjustment mechanism under a Type 3 adjustment and introduces the concept of volatility-targeted investment strategies. The analysis demonstrates that equity allocation significantly affects the stability of pension payments, with higher equity allocations leading to increased pension payment volatility and more pronounced wealth transfers across generations.

Subsection 11.5 presents a framework for finding optimal combinations of investment strategies and adjustment parameters based on predefined volatility targets and wealth transfer constraints. The use of bivariate spline interpolation allows for the identification of these optimal strategies, balancing the competing goals of maximizing annual pension levels while limiting benefit volatility and intergenerational cross-subsidies. This systematic approach can assist in setting investment policies that align with participants' risk preferences while ensuring the long-term sustainability of the plan.

Comprehensive Design Strategy

For DP plans to effectively meet their goals, a comprehensive design strategy that considers these key elements is essential. Specifically, plan administrators should consider the following issues:

1. **Set Realistic Valuation Interest Rates**: It is helpful to regularly review and adjust these rates to reflect market conditions, thereby facilitating an equitable distribution of benefits among participants. This is important in open group situations where realistic assumptions are critical for maintaining intergenerational fairness.
2. **Implement Balanced Benefit Smoothing Mechanisms**: Selecting smoothing approaches that provide stability without placing undue burdens on future generations can be beneficial. While intergenerational risk-sharing may be necessary for stability, clear communication about the rationale and impacts of these mechanisms is crucial.
3. **Offer Diverse Annuity Options**: Consider providing a range of annuity forms to accommodate different preferences and needs, enhancing fairness and satisfaction. The availability of flexible annuity options, coupled with clear explanations of their implications, can improve participant trust and engagement.
4. **Develop Volatility-Targeted Investment Strategies**: It can be helpful to use the Type 3 adjustment mechanism and the framework outlined in Section 11 to identify optimal combinations of equity allocations and adjustment parameters, balancing pension volatility with intergenerational wealth transfer. These strategies can help align the plan's investment approach with its risk management objectives.
5. **Conduct Regular Reviews**: It is beneficial to periodically review the plan's design elements and outcomes to ensure they continue to meet the goals of fairness, stability, and participant satisfaction. The dynamic nature of pension outcomes under open group plans requires ongoing monitoring and adjustment to maintain the plan's intended objectives.

By integrating these design principles, DP plans can better navigate the challenges of pension management while achieving long-term sustainability and fairness for all participants.

Section 13 Conclusion

This paper has applied two key measures—the annual pension payment ratio (APPR) and the repayment ratio (RR)—to analyze the mortality and investment risks inherent in Dynamic Pension (DP) plans. Through these measures, the authors have examined how these risks affect the stability of pension payments and the distribution of retirement wealth among participants.

The comparison between open and closed group plans highlights the importance of participant dynamics in plan design. Open group plans, which benefit from continuous contributions from new retirees, can spread financial risks over a larger pool and longer period, thereby enhancing stability. However, they must manage intergenerational equity carefully, as admission of new retirees to the plan can introduce unintended wealth transfers. In contrast, closed group plans face the challenge of maintaining stability without new entrants, often requiring more conservative investment strategies.

We have also examined how key design elements—such as valuation interest rates, benefit smoothing mechanisms, and pension annuity forms—play a crucial role in addressing concerns about benefit stability and equity. These features, along with well-designed investment strategies, can mitigate the potential loss of participants' retirement wealth due to premature deaths and other risks.

A central focus of this research has been on the impact of the Type 3 benefit adjustment mechanism, particularly under varying investment strategies. Section 11 explored how adjusting the equity allocation and benefit adjustment parameter can balance pension payment volatility and intergenerational wealth transfers. By applying bivariate spline interpolation, the authors identified optimal combinations of these parameters that meet specific pension volatility targets, providing plan administrators with actionable strategies to manage risk and equity across generations.

In conclusion, the design of DP plans requires a careful balancing act to achieve fairness, stability, and satisfaction for all participants. By setting realistic valuation interest rates, implementing balanced benefit smoothing mechanisms, offering diverse annuity options, optimizing equity allocations and adjustment parameters, and addressing the unique challenges of open versus closed group plans, plan administrators can help better meet the needs of their participants. The insights from the analysis provide a framework for refining DP plan designs to enhance sustainable and equitable pension outcomes.

Section 14 Acknowledgments

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Appendix A: Group-Based Benefit Adjustments in Closed Non-homogeneous Group and Open DP Plans

This appendix details the determination of benefit adjustment factors based on Type 1, Type 2 and Type 3 adjustments, as discussed in subsection 3.2, for both closed non-homogeneous groups and open group DP plans.

Existing Participants

Assume a plan has been operational for a number of years. At the present time, denoted as time 0, the group of existing participants comprises $n-1$ age cohorts $\{G_0^1, G_0^2, \ldots, G_0^{n-1}\}$. Each participant in G_0^j (where $j < n$) has the following profile:

- Entry age: η_j
- Amount contributed: C_j
- Initial pension: P_j^{η}
- Attained age: x_j
- Adjusted annual pension: $P_{j,0}$ payable on a lifetime basis

The liability of the plan at time 0 is:

$$L_0 = \sum_{j=1}^{n-1} \sum_{G_0^j} P_{j,0} \cdot \ddot{a}_{x_j}$$

where $\ddot{a}_{x_j}$ is a life annuity due based on a valuation interest rate i and the assumed mortality rates for participants in G_0^j.

The fund balance is F_0 , and the funded ratio of the plan is either at 1.0 or some other level.

New Participants

Let G_0^n denote a new set of participants who join the plan at time 0. All participants in G_0^n are of the same age η_n, which is their attained age x_n at time 0, and contribute the same amount C_n upon joining the plan. Their annual lifetime pension is:

$$P_{n,0} = \frac{C_n}{\ddot{a}_{\eta_n}}$$

where $\ddot{a}_{\eta_n}$ is a life annuity due based on the valuation interest rate i and the assumed mortality rates for participants in G_0^n. Note that G_0^n will be an empty set if the plan is a closed group plan.

The total amount of premiums paid by new participants and the total amount of pensions paid to all participants, including new participants if any, at time 0 are:

$$C = \sum_{G_0^n} C_n$$

$$P_0 = \sum_{j=1}^{n} \sum_{G_0^j} P_{j,0}$$

Fund and Liability Valuation

The expected value of the fund and the expected liability at time 1 are given by:

$$^{e}F_1 = (F_0 + C - P_0)(1 + i)$$

$$^{e}L_1 = \sum_{j=1}^{n} \sum_{G_0^j} P_{j,0} \cdot p_{x_j} \, \ddot{a}_{x_j+1}$$

where p_{x_j} is the expected survival probability for participants in G_0^j, and $\ddot{a}_{x_j+1}$ is the life annuity factor at time 1 for each surviving participant from G_0^j.

The actual fund value and the total liability at time 1 are:

$$F_1 = (F_0 + C - P_0)(1 + r_0)$$

$$\widetilde{L_1} = \sum_{j=1}^{n} \sum_{G_0^j} P_{j,0} \cdot p'_{x_j} \, \ddot{a}_{x_j+1}$$

where r_0 is the annual rate of return and p'_{x_j} is the actual survival probability for participants in G_0^j.

Type 1 Benefit Adjustment Factor

To ensure the funded ratio matches the expected funded ratio $\frac{^{e}F_1}{^{e}L_1}$, the pension payment upon which $\widetilde{L_1}$ is based must be adjusted by a factor h_1 such that:

$$\frac{F_1}{h_1 \widetilde{L_1}} = \frac{^{e}F_1}{^{e}L_1}$$

Thus,

$$h_1 = \frac{F_1}{^{e}F_1} \cdot \frac{^{e}L_1}{\widetilde{L_1}} \tag{7}$$

The adjustment factor h_1 can be decomposed into two adjustment factors:

1. **Investment Adjustment Factor:**

$$h_1^I = \frac{F_1}{{}^eF_1} = \frac{1+r_0}{1+i} \tag{8}$$

2. **Liability Adjustment Factor:**

$$h_1^L = \frac{{}^eL_1}{\widetilde{L_1}} = \frac{\sum_{j=1}^{n}\sum_{G_0^j} P_{j,0} \cdot p_{x_j}\, \ddot{a}_{x_j+1}}{\sum_{j=1}^{n}\sum_{G_0^j} P_{j,0} \cdot p'_{x_j}\, \ddot{a}_{x_j+1}} \tag{9}$$

In other words,

$$h_1 = h_1^I \cdot h_1^L$$

The adjusted pension payable at time 1 to a surviving participant from G_0^j is:

$$P_{j,1} = h_1 P_{j,0}$$

The total amount of adjusted pensions payable to all surviving participants from $\bigcup_{j=1}^{n} G_0^j$ at time 1 is:

$$\sum_{j=1}^{n}\sum_{G_1^j} P_{j,1} = h_1 \sum_{j=1}^{n}\sum_{G_1^j} P_{j,0}$$

where G_1^j denotes the set of participants from G_0^j who survive to time 1. The total number of participants in G_1^j is equal to $\sum_{G_0^j} p'_{x_j}$.

Type 2 Benefit Adjustment Factor

For this type of adjustment, it is assumed that the funded ratio of the plan at time 0 is equal to 1.0.

The adjustment factor at time 1 is determined based on an indexing rate that applies to the future pensions payable to all surviving participants from $\bigcup_{j=1}^{n} G_0^j$, ensuring that the liability of the plan equals the actual fund value F_1.

At time 1, the liability for a participant in G_1^j, incorporating a future indexing rate δ, is calculated as:

$$\sum_{t=0}^{120-x_j-1} (1+\delta)P_{j,0} \cdot {}_tp_{x_j+1} \cdot \left(\frac{1+\delta}{1+i}\right)^t$$

The authors denote the expression $\sum_{t=0}^{120-x_j-1} {}_tp_{x_j+1} \cdot \left(\frac{1+\delta}{1+i}\right)^t$ as $\ddot{a}_{x_j+1}^{\delta}$. Thus, the liability for the participant can be written as $(1+\delta)P_{j,0} \cdot \ddot{a}_{x_j+1}^{\delta}$.

The indexing rate δ can be solved from the following polynomial equation:

$$\sum_{j=1}^{n}\sum_{G_1^j} (1+\delta)P_{j,0}\, \ddot{a}_{x_j+1}^{\delta} = F_1$$

It follows that the benefit adjustment factor at time 1, h_1, is equal to $1+\delta$. Note that the indexing rate δ can be either positive or negative but must not be less than -1.

The total amount of adjusted pensions payable to all surviving participants from $\bigcup_{j=1}^{n} G_0^j$ at time 1 is:

$$h_1 \sum_{j=1}^{n} \sum_{G_1^j} P_{j,0} = (1+\delta) \sum_{j=1}^{n} \sum_{G_1^j} P_{j,0} \tag{10}$$

Type 2 adjustments operate with a fixed valuation interest rate. The plan modifies the future escalation rate on a fresh-start basis at each valuation for existing retirees. The updated escalation factor absorbs any gains or losses needed to bring assets and liabilities back into alignment. New entrants continue to be priced at the same fixed interest rate.

Type 3 Benefit Adjustment Factor

The benefit adjustment factor in this approach is tied to a funded ratio based on participants' target benefits, which are their initial pensions determined by the contributions made upon joining the plan. For a participant in G_0^j, their initial pension is calculated as:

$$P_j^{\eta} = \frac{C_j}{\ddot{a}_{\eta_j}}$$

where $\ddot{a}_{\eta_j}$ is a life annuity due based on the valuation interest rate i and the assumed mortality rates for participants in G_0^j. Note that P_j^{η} is the target pension the plan aims to deliver. It is the amount of pension paid at the participant's entry age η_j.

At time 1, the plan's target benefit liability is:

$$L_1^T = \sum_{j=1}^{n} \sum_{G_1^j} P_j^{\eta} \cdot \ddot{a}_{x_j+1}$$

where G_1^j represents the participants from G_0^j who survive to time 1.

The benefit adjustment factor at time 1, h_1, is determined by the plan's target benefit funded ratio:

$$h_1 = \frac{F_1}{L_1^T} \tag{11}$$

The adjusted pension payable at time 1 to a surviving participant from G_0^j is:

$$P_{j,1} = h_1 P_j^{\eta}$$

This adjustment ensures that the pensions payable after time 1 are aligned with the available fund assets. The total amount of adjusted pensions payable to all surviving participants from $\bigcup_{j=1}^{n} G_0^j$ at time 1 is:

$$\sum_{j=1}^{n} \sum_{G_1^j} P_{j,1} = h_1 \sum_{j=1}^{n} \sum_{G_1^j} P_j^{\eta}$$

Extension to Future Time Periods

The above benefit adjustment mechanisms can be extended to any future $t > 1$, ensuring the plan remains balanced and sustainable over time. For each cohort G_0^j (where $j \geq 1$), their evaluation measures, average $APPR$ and RR, can be calculated in the same manner as described in subsection 4.2.

Appendix B: Repayment Ratio in Closed Homogeneous Group DP Plans

In the case of a closed homogeneous group DP plan, the repayment ratio (RR) for the group of participants is defined in subsection 4.2 as follows:

$$RR = \sum_{t=0}^{120-x} {}_tp'_x \cdot q'_{x+t} \cdot RR_t$$

Given the characteristics of the group, the RR is identical to 1.0 regardless of the investment and mortality experience of the plan. This appendix provides mathematical proof of this property.

First, the authors decompose each term in the summation.

For $t = 0$,

$$q'_x \cdot RR_0 = (1 - p'_x)\frac{P_0}{C} = \frac{P_0}{C} - p'_x\frac{P_0}{C}$$

For $t = 1$,

$$p'_x \cdot q'_{x+1} \cdot RR_1 = (p'_x - {}_2p'_x) \cdot RR_1 = p'_x\frac{P_0}{C} + p'_x\frac{\frac{P_1}{1+r_0}}{C} - {}_2p'_x \cdot RR_1 = p'_x\frac{P_0\left(\frac{1+r_0}{1+i}\right)\left(\frac{p_x}{p'_x}\right)}{C(1+r_0)} + p'_x\frac{P_0}{C} - {}_2p'_x \cdot RR_1$$
$$= \frac{P_0}{C}\frac{p_x}{(1+i)} + p'_x\frac{P_0}{C} - {}_2p'_x \cdot RR_1$$

Likewise, for $t = 2$,

$${}_2p'_x \cdot q'_{x+2} \cdot RR_2 = ({}_2p'_x - {}_3p'_x) \cdot RR_2 = {}_2p'_x\left(RR_1 + \frac{\frac{P_2}{(1+r_0)(1+r_1)}}{C}\right) - {}_3p'_x \cdot RR_2$$
$$= {}_2p'_x\frac{P_0\left(\frac{1+r_0}{1+i}\right)\left(\frac{1+r_1}{1+i}\right)\left(\frac{{}_2p_x}{{}_2p'_x}\right)}{C(1+r_0)(1+r_1)} + {}_2p'_x \cdot RR_1 - {}_3p'_x \cdot RR_2 = \frac{P_0}{C}\frac{{}_2p_x}{(1+i)^2} + {}_2p'_x \cdot RR_1 - {}_3p'_x \cdot RR_2$$

In general, for $t = \tau$,

$${}_\tau p'_x \cdot q'_{x+\tau} \cdot RR_\tau = \frac{P_0}{C}\frac{{}_\tau p_x}{(1+i)^\tau} + {}_\tau p'_x \cdot RR_{\tau-1} - {}_{\tau+1}p'_x \cdot RR_\tau$$

Finally, for $t = 120 - x$,

$${}_{120-x}p'_x \cdot q'_{120} \cdot RR_{120-x} = \frac{P_0}{C}\frac{{}_{120-x}p_x}{(1+i)^{120-x}} + {}_{120-x}p'_x \cdot RR_{119-x}$$

since $q'_{120} = 1$.

When we add up all the terms for $t = 0, 1, \ldots, 120 - x$, it can be seen that, except for the first component of each term, all other components cancel each other out, leading to the following:

$$RR = \sum_{t=0}^{120-x} {}_tp'_x \cdot q'_{x+t} \cdot RR_t = \frac{P_0}{C}\left(\sum_{t=0}^{120-x}\frac{{}_tp_x}{(1+i)^t}\right) = \frac{P_0\ddot{a}_x}{C} = 1$$

Appendix C: Benefit Adjustment Rates under Three Sample Investment Scenarios

In subsection 9.3, the authors analyze the benefit outcomes for the sample DP plan under three distinct investment scenarios: consistent markets, unfavorable markets, and favorable markets. Figures C1, C2, and C3 below illustrate the annual fund returns and the corresponding benefit adjustment rates applied to the plan in these scenarios. The benefit adjustment factor, h_t, as discussed in Section 4, is applied to the pension paid at time $t-1$ to determine the pension payable at time t. This factor is converted to a rate by subtracting 1 and expressed as a percentage.

Figure C1

benefit adjustment rates under consistent markets

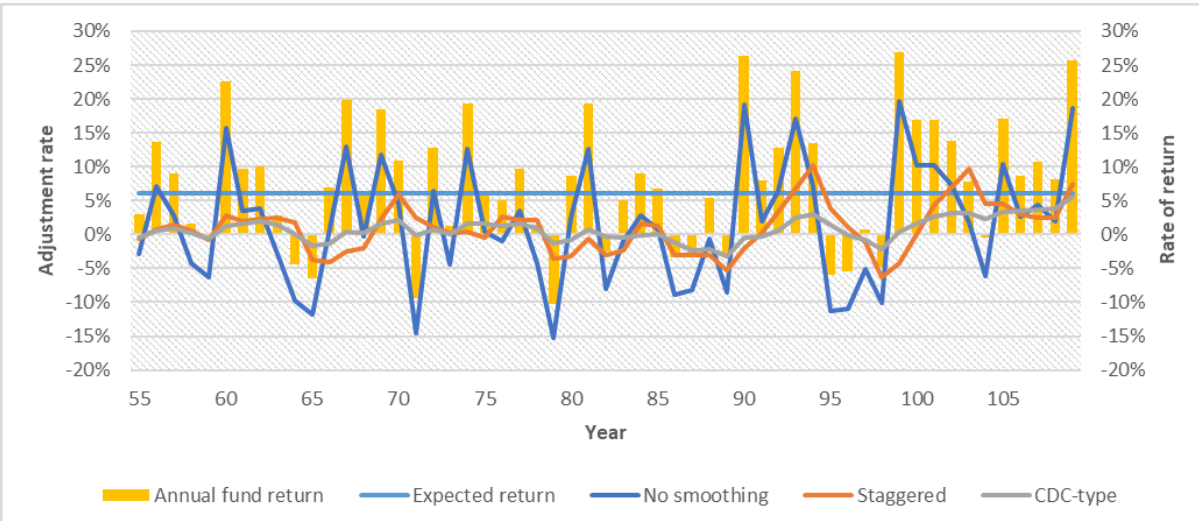


Figure C2

BENEFIT ADJUSTMENT RATES UNDER UNFAVORABLE MARKETS

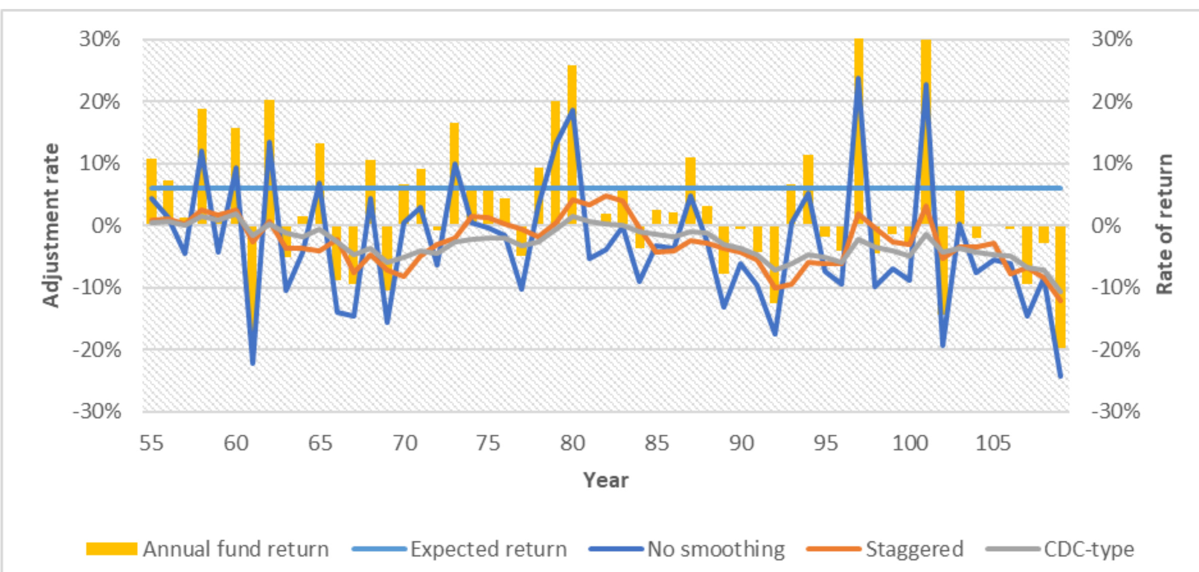
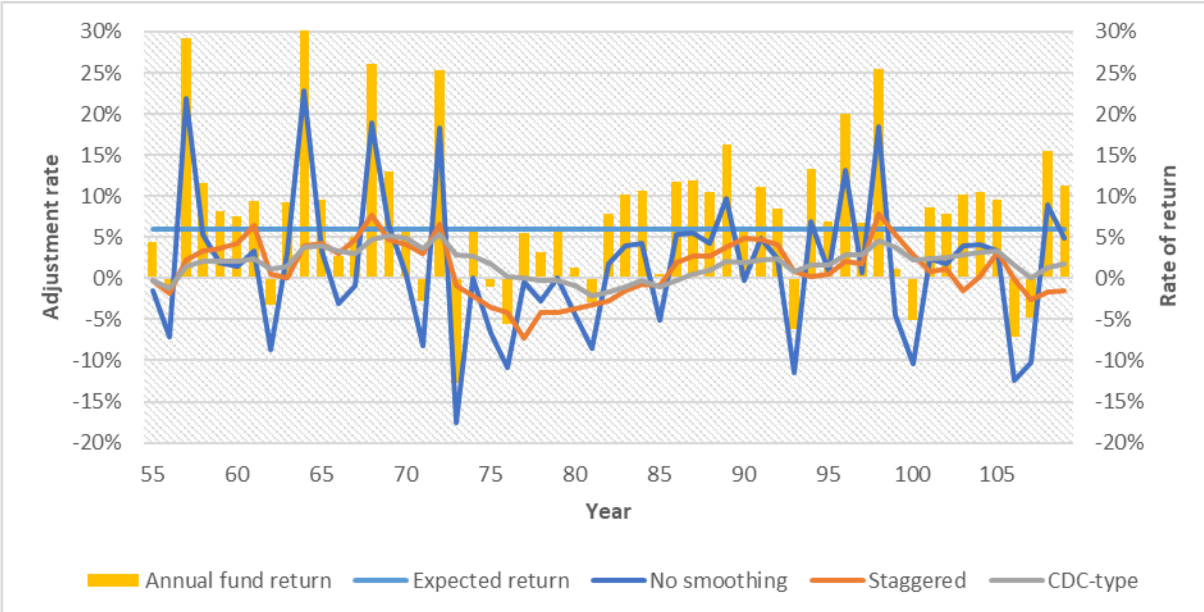


Figure C3

BENEFIT ADJUSTMENT RATES UNDER FAVORABLE MARKETS



Appendix D: Implementation of Bivariate Spline Interpolation in Python

In subsection 11.5, the authors demonstrate how bivariate spline interpolation is employed to identify optimal investment strategies for DP plans with Type 3 benefit adjustment. This appendix provides the Python implementation using the RectBivariateSpline class from the SciPy library.

Overview of the RectBivariateSpline Class

The RectBivariateSpline class is designed for fitting smooth surfaces to data points when both independent variables (x and y) lie on regular, structured grids. It then interpolates the dependent variable (z) at any point within the range of the grid, allowing for continuous evaluation of the smooth surface.

Key Concepts:

- Rectangular Grid: The independent variables (x and y) must be arranged in a regular grid. For example, if x-values range from 0 to 10 and y-values from 0 to 5, the grid should cover all combinations of these values.
- Cubic Spline Interpolation:
    - Cubic splines are used for interpolation, providing a smooth approximation of the data by fitting piecewise polynomials.
    - A cubic spline fits several lower-degree polynomials to different segments of the data, ensuring a smooth fit across the dataset.
    - The cubic spline is a popular choice because it guarantees continuous first and second derivatives, ensuring smooth transitions in both the x- and y-directions across the grid.
- How it Works: RectBivariateSpline breaks the 2D space into smaller segments based on the grid points and fits a cubic polynomial to each segment. This method results in smooth and precise interpolations, ideal for capturing complex relationships in the data.

Advantages of Bivariate Spline Interpolation:

- Smoothness: Bivariate spline interpolation ensures smooth and continuous surfaces, both in the function and its derivatives.
- Local Control: Changes in one part of the dataset affect the surface only locally, making it more stable than fitting a single polynomial to all data points.
- Accuracy: For well-behaved data, spline interpolation is often more accurate than linear methods because it captures data curvature more effectively.

Applications:

- RectBivariateSpline is commonly applied when data is structured in a grid, such as in image processing, geographical simulations, or outputs from simulations on predefined grids.

Python Code for Bivariate Spline Interpolation

Below is the Python code used in subsection 11.5 for interpolation. It utilizes the RectBivariateSpline class to create smooth interpolated surfaces for the variables v and w over different equity allocations (π) and adjustment parameters (θ).

```
import numpy as np

import matplotlib.pyplot as plt

import csv

from scipy.interpolate import RectBivariateSpline


# Define π (formerly x) and θ (formerly y) values based on your grid

pi_values = np.array([0.2, 0.4, 0.5, 0.6, 0.8])

theta_values = np.array([0.2, 0.4, 0.6, 0.8, 1.0])


# Updated data grids for v and w

v_grid = np.array([

    [0.0592, 0.0470, 0.0394, 0.0349, 0.0328],

    [0.1088, 0.0854, 0.0692, 0.0584, 0.0513],

    [0.1303, 0.1036, 0.0838, 0.0706, 0.0619],

    [0.1550, 0.1255, 0.1013, 0.0851, 0.0743],

    [0.2051, 0.1761, 0.1434, 0.1193, 0.1031]

])

w_grid = np.array([

    [0.0261, 0.0437, 0.0563, 0.0658, 0.0729],

    [0.0605, 0.0940, 0.1139, 0.1256, 0.1325],

    [0.0771, 0.1148, 0.1362, 0.1480, 0.1545],

    [0.1166, 0.1662, 0.1906, 0.2009, 0.2042],

    [0.1853, 0.2475, 0.2734, 0.2797, 0.2777]

])


# Create the interpolators for v and w

v_spline = RectBivariateSpline(pi_values, theta_values, v_grid)

w_spline = RectBivariateSpline(pi_values, theta_values, w_grid)
```

```
# Define the range for π
pi_range = np.arange(0.2, 0.85, 0.05)

# Plotting
plt.figure(figsize=(10, 6))

# Store interception points
interception_points = []

for pi in pi_range:
    theta_fine = np.linspace(0.2, 1.0, 100)
    v_fine = v_spline(pi, theta_fine)[0]
    w_fine = w_spline(pi, theta_fine)[0]

    # Plot the interpolated curves
    plt.plot(v_fine, w_fine, label=f'π={pi:.2f}')

    # Find interception point with w=0.12
    idx = np.where(np.diff(np.sign(w_fine - 0.12)))[0]
    if len(idx) > 0:
        idx = idx[0]
        v_intercept = v_fine[idx]
        w_intercept = w_fine[idx]
        theta_intercept = theta_fine[idx]
        interception_points.append((v_intercept, w_intercept, pi, theta_intercept))
        plt.plot(v_intercept, w_intercept, 'ro') # Mark interception point
# Draw horizontal line at w=0.12
plt.axhline(y=0.12, color='gray', linestyle='--')
```

```
# Add labels and legend
plt.xlabel('v (RMSD for RR)')
plt.ylabel('w (SD for APPR)')
plt.title('Bivariate Spline Interpolation of w vs. v for Different π Values')
plt.legend()
plt.grid(True)
# Save the plot
plt.savefig('G:\\My Drive\\Written papers\\Decumulation\\Outputs2\\wv_plot_w.12.png')
# Print interception points
print("Interception Points (v, w, π, θ):")
for point in interception_points:
    print(f"v={point[0]:.4f}, w={point[1]:.4f}, π={point[2]:.2f}, θ={point[3]:.4f}")
# Save interception points to a CSV file
file_path = 'G:\\My Drive\\Written papers\\Decumulation\\Outputs2\\
interception_points_w.12.csv'
with open(file_path, 'w', newline='') as csvfile:
    csvwriter = csv.writer(csvfile)
    # Write the header
    csvwriter.writerow(["RR RMSD", "APPR SD", "pi", "theta"])
    # Write the interception points data
    for point in interception_points:
        csvwriter.writerow([f"{point[0]:.4f}", f"{point[1]:.4f}", f"{point[2]:.2f}", f"{point[3]:.4f}"])
```

Explanation of Code:

- **Grid Definition**: The authors define grids for equity allocation (π) and adjustment parameter (θ), along with corresponding grids for wealth transfer (v) and pension volatility (w).
- **Bivariate Spline Interpolation**: The RectBivariateSpline creates smooth surfaces from these data points for both v and w.
- **Plotting**: The authors generate curves for different π values, highlighting the interception points where the pension volatility target w=0.12 is met.

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