# Construction and Evaluation of Actuarial Models Exam—June 2015

The Construction and Evaluation of Actuarial Models exam is a three-and-a-half hour exam that consists of 35 multiple-choice questions and is administered as a computer-based test.

For additional details, please refer to <u>Exam Rules</u>. The syllabus for this examination provides an introduction to modeling and covers important actuarial methods that are useful in modeling. A thorough knowledge of calculus, probability, and mathematical statistics is assumed.

The candidate will be introduced to a variety of useful frequency and severity models. The candidate will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The candidate should be able to: 1) analyze data from an application in a business context; 2) determine a suitable model including parameter values; and 3) provide measures of confidence for decisions based upon the model. The candidate will be introduced to a variety of tools for the calibration and evaluation of the models.

A variety of tables is available below for the candidate and will be provided to the candidate at the examination. These include values for the standard normal distribution, chi-square distribution, and abridged inventories of discrete and continuous probability distributions. Candidates will not be allowed to bring copies of the tables into the examination room.

Check the <u>Updates</u> section on this exam's home page for any changes to the exam or syllabus.

In the learning outcomes, weights have been provided to indicate the relative emphasis on different sections. The ranges of weights shown are intended to apply to the large majority of exams administered. On occasion, the weights of topics on an individual exam may fall outside the published range. Candidates should also recognize that some questions may cover multiple learning outcomes.

Each multiple-choice problem includes five answer choices identified by the letters A, B, C, D, and E, only one of which is correct. Candidates must indicate responses to each question on the computer.

As part of the computer-based testing process, a few pilot questions will be randomly placed in the exam (paper and pencil and computer-based forms). These pilot questions are included to judge their effectiveness for future exams, but they will NOT be used in the scoring of this exam.<sup>1</sup> All other questions will be considered in the scoring. All unanswered questions are scored incorrect. Therefore, candidates should answer every question on the exam. There is no set requirement for the distribution of correct answers for the multiple-choice preliminary

<sup>&</sup>lt;sup>1</sup> Beginning with the October 2013 examination there are some revised learning objectives and readings. Items covering them may appear as scored or pilot items in the same manner as items on continuing topics.

examinations. It is possible that a particular answer choice could appear many times on an examination or not at all. Candidates are advised to answer each question to the best of their ability, independently from how they have answered other questions on the examination.

Because the CBT exam will be offered over a period of a few days, each candidate will receive a test form composed of questions selected from a pool of questions. Statistical scaling methods are used to ensure within reasonable and practical limits that, during the same testing period of a few days, all forms of the test are comparable in content and passing criteria. The methodology that has been adopted is used by many credentialing programs that give multiple forms of an exam.

## LEARNING OUTCOMES

The candidate is expected to be familiar with survival, severity, frequency and aggregate models, and use statistical methods to estimate parameters of such models given sample data. The candidate is further expected to identify steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

Specifically, the candidate is expected to be able to perform the tasks listed below.

Sections A–E have a combined weight of 15-20%.

### A. Severity Models

- 1. Calculate the basic distributional quantities:
  - a) moments
  - b) Percentiles
  - c) Generating functions
- 2. Describe how changes in parameters affect the distribution.
- 3. Recognize classes of distributions and their relationships.
- 4. Apply the following techniques for creating new families of distributions:
  - a) Multiplication by a constant
  - b) Raising to a power
  - c) Exponentiation,
  - d) Mixing
- 5. Identify the applications in which each distribution is used and reasons why.
- 6. Apply the distribution to an application, given the parameters.
- 7. Calculate various measures of tail weight and interpret the results to compare the tail weights.
- 8. Identify and describe two extreme value distributions.

**B.** Frequency Models

For the Poisson, Mixed Poisson, Binomial, Negative Binomial, Geometric distribution and mixtures thereof:

- 1. Describe how changes in parameters affect the distribution,
- 2. Calculate moments,
- 3. Identify the applications for which each distribution is used and reasons why,
- 4. Apply the distribution to an application given the parameters.
- 5. Apply the zero-truncated or zero-modified distribution to an application given the parameters
- C. Aggregate Models
  - 1. Compute relevant parameters and statistics for collective risk models.
  - 2. Evaluate compound models for aggregate claims.
  - 3. Compute aggregate claims distributions.
- D. For severity, frequency and aggregate models
  - 1. Evaluate the impacts of coverage modifications:
    - a) Deductibles
    - b) Limits
    - c) Coinsurance
  - 2. Calculate Loss Elimination Ratios.
  - 3. Evaluate effects of inflation on losses.
- E. Risk Measures
  - 1. Calculate VaR, and TVaR and explain their use and limitations.

Sections F and G have a combined weight of 20-25%.

- F. Construction of Empirical Models
  - 1. Estimate failure time and loss distributions using:
    - a) Kaplan-Meier estimator
    - b) Nelson-Åalen estimator
    - c) Kernel density estimators
  - 2. Estimate the variance of estimators and confidence intervals for failure time and loss distributions.
  - 3. Apply the following concepts in estimating failure time and loss distribution:
    - a) Unbiasedness
    - b) Consistency
    - c) Mean squared error
- G. Estimation of decrement probabilities from large samples
  - 1. Estimate decrement probabilities using both parametric and nonparametric approaches for both individual and interval data
  - 2. Approximate the variance of the estimators

- H. Construction and Selection of Parametric Models (25-30%)
  - 1. Estimate the parameters of failure time and loss distributions using:
    - a) Maximum likelihood
    - b) Method of moments
    - c) Percentile matching
    - d) Bayesian procedures
  - 2. Estimate the parameters of failure time and loss distributions with censored and/or truncated data using maximum likelihood.
  - 3. Estimate the variance of estimators and the confidence intervals for the parameters and functions of parameters of failure time and loss distributions.
  - 4. Apply the following concepts in estimating failure time and loss distributions:
    - a) Unbiasedness
    - b) Asymptotic unbiasedness
    - c) Consistency
    - d) Mean squared error
    - e) Uniform minimum variance estimator
  - 5. Determine the acceptability of a fitted model and/or compare models using:
    - a) Graphical procedures
    - b) Kolmogorov-Smirnov test
    - c) Chi-square goodness-of-fit test
    - d) Likelihood ratio test
    - e) Schwarz Bayesian Criterion
- I. Credibility (20-25%)
  - 1. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.
  - 2. Perform Bayesian analysis using both discrete and continuous models.
  - 3. Apply Bühlmann and Bühlmann-Straub models and understand the relationship of these to the Bayesian model.
  - 4. Apply conjugate priors in Bayesian analysis and in particular the Poisson-gamma model.
  - 5. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.
- J. Simulation (5-10%)
  - 1. Simulate both discrete and continuous random variables using the inversion method.
  - 2. Simulate from discrete mixtures, decrement tables, the (a,b,0) class, and the normal and lognormal distributions using methods designed for those distributions
  - 3. Estimate the number of simulations needed to obtain an estimate with a given error and a given degree of confidence.
  - 4. Use simulation to determine the p-value for a hypothesis test.
  - 5. Use the bootstrap method to estimate the mean squared error of an estimator.
  - 6. Apply simulation methods within the context of actuarial models.

#### Reading Selections for learning outcomes A through H and J:

#### Text

- Loss Models: From Data to Decisions, (Fourth Edition), 2012, by Klugman, S.A., Panjer, H.H. and Willmot, G.E.
  - Chapter 3 Chapter 4 Chapter 5 Chapter 6 Chapter 8 Chapter 9, Sections 9.1–9.7 (excluding 9.6.1), Sections 9.8.1–9.8.2 Chapter 10 Chapter 11 Chapter 12 Chapter 12 Chapter 13 Chapter 14, Sections 14.1 – 14.4 and 14.6 Chapter 15 Chapter 16 (excluding 16.4.2) Chapter 20

#### Reading Selections for learning outcome I (Credibility):

- Foundations of Casualty Actuarial Science (Fourth Edition), 2001, Casualty Actuarial Society Chapter 8, Section 1 (background only) Sections 2–5
- <u>Topics in Credibility</u> by Dean, C.G.

**Note:** The choices of parameterization for several probability distributions differ in the *Foundations of Casualty Actuarial Science* and *Topics in Credibility* study notes when compared to *Loss Models: From Data to Decisions* by Klugman, Panjer and Willmot.

Various probability distributions are given on pages 132 to 134 of the *Foundations of Casualty Actuarial Science*. For example, the parameters of the gamma distribution are  $\alpha$  and  $\lambda$ , where  $E[x] = \alpha / \lambda$ . In the *Loss Models* text, the parameters of the gamma distribution are  $\alpha$  and  $\theta$ , where  $E[x] = \alpha \theta$ . When comparing the two distributions, one can see that  $\lambda = 1/\theta$ .

Please keep in mind that the tables given to candidates during the exam follow the format of Appendices A & B of *Loss Models: From Data to Decisions*, and therefore candidates are responsible for understanding how the parameterizations in these tables relate to the results derived in the *Foundations of Casualty Actuarial Science*.

## **Other Resources**

Tables for Exam C

All released exam papers since 2000, can be found at: Past Exam Questions and Solutions

Exam C Sample <u>Questions</u> and <u>Solutions</u>

Corrections and Comments for Loss Models, Fourth Edition