# Fall 2018 LTAM

## Multiple Choice Solutions

Question 1: E

Question 2: B

$${}_5p_{30} - {}_{21}p_{30} = e^{-0.3} - e^{-(0.6+0.33)} = 0.35$$

Question 3: C

$$q_{50}^{(1)} = {}_{0.75}q_{50}^{(1)} + {}_{0.75}p_{50}^{(\tau)}\ {}_{0.25}q_{50.75}^{(1)}$$

$${}_{0.75}q_{50}^{(1)} = {}_{0.75}\ q_{50}^{\prime(1)} = 0.75(0.02) = 0.015$$

$$0.02 = {}_{0.75}\ q_{50}^{\prime(1)} + {}_{0.75}\ p_{50}^{\prime(1)}\ {}_{0.25}q_{50.75}^{\prime(1)} \Rightarrow {}_{0.25}\ q_{50.75}^{\prime(1)} = {}_{0.25}\ q_{50.75}^{(1)} = \frac{0.02 - 0.015}{{}_{0.75}p_{50}^{\prime(1)}}$$

$${}_{0.75}p_{50}^{(\tau)} = 0.9\,{}_{0.75}p_{50}^{\prime(1)}$$

$$\Rightarrow q_{50}^{(1)} = 0.015 + 0.9\,{}_{0.75}p_{50}^{\prime(1)}\left(\frac{0.02 - 0.015}{{}_{0.75}p_{50}^{\prime(1)}}\right) = 0.015 + 0.9(0.005) = 0.0195$$

Question 4: D

| Paths | Probability |
|---|---|
| $1 \to 1 \to 1 \to 1$ | $0.2^3$ |
| $1 \to 1 \to 3 \to 1$ | $(0.2)(0.5)(0.9)$ |
| $1 \to 3 \to 1 \to 1$ | $(0.2)(0.5)(0.9)$ |
| $1 \to 3 \to 1 \to 1$ | 0 |

$\Rightarrow$Total probability 0.1880

Question 5: C

$$\hat{S}(6) = \left(\frac{18}{20}\right)\left(\frac{15}{18}\right)\left(\frac{9}{14}\right) = 0.482$$

Question 6: B

$$\text{Prob} = 0.76(1 - 0.962 \times 0.25)(1 - 0.964 \times 0.965 \times 0.26) = 0.4376$$

Question 7: B

$$\hat{S}(15) = e^{-\left(\frac{2}{200} + \frac{2}{192} + \frac{2}{182} + \frac{17}{170}\right)} = 0.8769$$

Question 8: C

Question 9: A

$$\begin{aligned}\bar{A}^{02}_{60:\overline{10}|} &= \bar{A}^{02}_{60} - {}_{10}p^{00}_{60}\, v^{10}\, \bar{A}^{02}_{70} - {}_{10}p^{01}_{60}\, v^{10}\, \bar{A}^{12}_{70} \\ &= 0.164\end{aligned}$$

Question 10: E

$$\begin{aligned}EPV &= 9000\mathrm{v}^{10}\left({}_{10}p_{50}\,{}_{10}q_{60}\,\ddot{a}_{60} + {}_{10}p_{50}\,{}_{10}q_{60}\,\ddot{a}_{60}\right) \\ &= 9000\left({}_{10}E_{50}\,{}_{10}q_{60}\,\ddot{a}_{60} + {}_{10}E_{60}\,{}_{10}q_{50}\,\ddot{a}_{70}\right) \\ &= 5870\end{aligned}$$

Question 11: E

EPV Premiums: $P(1+0.99v) = 1.9519P$

EPV Benefits:

$$1000\, v^2\, {}_2p_{40}\left(1+1.04v\, p_{42} + 1.04^2 v^2\, {}_2p_{42} + \ldots\right) = 1000v^2\, {}_2p_{40}(1+e_{42})$$

$$e_{40} = 11.06 = p_{40} + {}_2p_{40} + {}_3p_{40} + \ldots = p_{40} + {}_2p_{40}(1+e_{42})$$

$$\Rightarrow (1+e_{42}) = 10.379$$

$$\Rightarrow P = \frac{9310}{1.9519} = 4770$$

Question 12: E

$$P = 100\frac{(1-d\ddot{a}^{ns}_x)}{\ddot{a}^{ns}_x} = 1.6896$$

EPV Premiums $= 1.6896(12.40) = 20.95$

EPV Benefits $= 100(1-d\ddot{a}^{s}_x) = 40.95$

EPV Loss at issue $= 40.95 - 20.95 = 20$

Question 13: D

EPV Annuity $= 50\,000\,{}_{10}E_{55}\,\ddot{a}_{65} = 402\,040$

EPV Survival Benefit $= 200\,000\,{}_{25}E_{55} = 45\,667$

EPV Premiums minus expenses $= P(0.95\,\ddot{a}_{55:\overline{10}|} - 0.15) = 7.4682P$

$\Rightarrow P = 59\,950$

Question 14: D

$$P\ddot{a}_{55:65:\overline{10}|} = 55{,}000\ {}_{10}E_{55}\ \ddot{a}_{65} + 55{,}000\ {}_{10}E_{65}\ \ddot{a}_{75} - 10{,}000\ {}_{10}E_{55:65}\ \ddot{a}_{65:75}$$

$${}_{10}E_{55:65} = (1+i)^{10}\ {}_{10}E_{55}\ {}_{10}E_{65} = 0.53459$$

$$\Rightarrow P = \frac{705{,}352}{7.7596} = 90{,}900.6$$

Question 15: A

$${}_{10}V = 200000\bar{A}^{1}_{75:\overline{10}|}v^{0.5} - 12P\ddot{a}^{(12)}_{75:\overline{10}|}$$

$$\ddot{a}^{(12)}_{75:\overline{10}|} = 7.3203 - \frac{11}{24}(1 - 0.44085) = 7.0640$$

$$\bar{a}_{75:\overline{10}|} = 7.3203 - \frac{1}{2}(1 - 0.44085) = 7.0407$$

$$\bar{A}^{1}_{75:\overline{10}|} = 1 - \delta\bar{a}_{75:\overline{10}|} - {}_{10}E_{75} = 0.21563$$

$$\Rightarrow {}_{10}V = 18{,}860$$

Question 16: C

$$\left({}_{5}V^{(0)} + P\right)(1+i) = p^{00}_{55}\ {}_{6}V^{(0)} + p^{01}_{55}(200{,}000) + p^{02}_{55}(100{,}000)$$

$$\Rightarrow {}_{6}V^{(0)} = 2998.3$$

Question 17: A

$$P^{*} = \frac{200{,}000A_{51}}{\ddot{a}_{51:\overline{14}|}} = \frac{39{,}560}{\ddot{a}_{51} - \frac{94579.7}{98457.2}v^{14}\ddot{a}_{65}} = 3851.23$$

$${}_{5}V^{FPT} = 200{,}000A_{55} - P^{*}\ddot{a}_{55:\overline{10}|} = 16{,}164$$

Question 18: B

$$\Pi_{10} = {}_{9}p_{35}Pr_{10}$$

$$\mathrm{Pr}_{10} = \left({}_{9}V + P - E\right)(1.06) - q_{44}(500{,}000)(1.06)^{(0.5)} - p_{44}\ {}_{10}V = 4070.5$$

$$\Rightarrow \Pi_{10} = 3989$$

Question 19: E

EPV at age 62 is:

$$5000(1.0268)^{32}\left\{1 + \left(1.0226\right)(1.0268)vp_{62} + \left(\left(1.0226\right)(1.0268)v\right)^{2}\ {}_{2}p_{62}\right\}$$

$$= 5000(1.0268)^{32}\left\{1 + p_{62} + {}_{2}p_{62}\right\} = 34812$$

$\Rightarrow$ EPV at age 30 is: $34{,}812\ {}_{32}p_{30}v^{32}_{5\%} = 7028$

Question 20: A

Projected total salary: $S_{35}\left(1+1.04+1.04^2+...+1.04^{29}\right)=S_{35}\dfrac{1.04^{30}-1}{0.04}=56.08S_{35}$

Projected final salary: $S_{35}(1.04)^{29}=3.12S_{35}$

Projected replacement rate: $\dfrac{0.02(56.08)}{3.12}=35.95\%$