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**SOCIETY OF ACTUARIES**  
**Quantitative Finance and Investment Advanced Exam**

# Exam QFIADV

## MORNING SESSION

**Date:** Thursday, April 26, 2018

**Time:** 8:30 a.m. – 11:45 a.m.

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### INSTRUCTIONS TO CANDIDATES

#### General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
  - a) The morning session consists of 9 questions numbered 1 through 9.
  - b) The afternoon session consists of 6 questions numbered 10 through 15.

The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

#### Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.





**\*\*BEGINNING OF EXAMINATION\***  
**Morning Session**

1. (5 points) Dave is the portfolio manager for XYZ Company’s equity portfolio. The portfolio invests in three major market sectors:

- I. Energy
- II. Financials
- III. Technology

Dave’s investment mandate is to track a benchmark portfolio, with some latitude to over-weight or under-weight particular stocks relative to the benchmark, in order to enhance portfolio returns.

Dave believes he has particular expertise investing in the Technology sector. He is bullish on the Technology sector – he expects the return in this sector will be higher. Dave constructs a portfolio consistent with this view.

You are an investment actuary working for the Chief Investment Officer (CIO) of XYZ Company. The CIO just received Dave’s portfolio performance report for the quarter. In the report, Dave states that his portfolio outperformed the benchmark by 5.5%.

The equity portfolio performance report contains the following information:

$$\text{Asset Allocation} = \sum_s (w_s^P - w_s^B) * R_s^B = 1.5\%$$

Sector (s)	Weight Difference ( $w_s^P - w_s^B$ )	Benchmark Return by Sector ( $R_s^B$ )
Energy	-20%	7%
Financials	-25%	10%
Technology	45%	12%
Asset Allocation		1.5%

$$\text{Security Selection} = \sum_s w_s^B * (R_s^P - R_s^B) = 4\%$$

Sector (s)	Benchmark Weight ( $w_s^B$ )	Return Difference ( $R_s^P - R_s^B$ )
Energy	40%	8%
Financials	50%	2%
Technology	10%	-2%
Security Selection		4%

# 1. Continued

- $R_s^B$  = Benchmark portfolio return by sector,  
 $R_s^P$  = Dave's portfolio return by sector,  
 $w_s^B$  = Sector Weight in Benchmark portfolio, and  
 $w_s^P$  = Sector Weight in Dave's portfolio

The CIO has asked you to review the equity portfolio performance report and provide commentary.

- (a) (2 points)
- (i) Recommend a correction to Dave's portfolio attribution model
  - (ii) Recalculate his outperformance.
- (b) (1 point) Explain whether you agree with Dave's belief that he has expertise in the Technology sector.

You are also asked to review the fixed-income portfolio performance.

The company's fixed-income portfolio manager provides you with the following report, which is based on the daily algorithm:

	Outperformance (bps)					
	Yield Curve					
			Carry	Change	Total	
Average			-0.5	14.6	14.1	
Key Rate & Cash			0.0	5.0	5.0	
Rest of Yield-Curve & Convexity			0.0	0.9	0.9	
Total			-0.5	20.5	20.0	

	Yield				Duration (Yrs)	
	Level (%)		Change (bps)		Average	
	Portfolio	Benchmark	Portfolio	Benchmark	Portfolio	Benchmark
Average	1.311	1.245	-31.6	-31.6	4.66	4.18

*Question 1 is continued on the next page.*

## 1. Continued

The 5 year point was the one where the yield curve changed the least. The portfolio was overweight at that point relative to the benchmark.

In the report, fixed income portfolio manager has commented:

- I. Our bond portfolio has outperformed the benchmark by 14.1 bps.
- II. Yield-curve change outperformance is mostly due to differences between the portfolio and benchmark durations.
- III. Fluctuations of the average duration exposure during the period had a negative impact on the outperformance.
- IV. The overweight of the 5 year point contributed positively to the outperformance.

(c) (2 points) Critique each of the fixed income portfolio manager's comments.

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2. (8 points) You are asked to help in the pricing and hedging of a long-term exotic interest-rate derivative.

In the past, the pricing and hedging have been done through Monte Carlo simulations using the Vasicek one-factor model for the short-rate, defined as:

$$dr(t) = k[\theta - r(t)]dt + \sigma dW(t), r(0) = r_0$$

where  $r_0, k, \theta$  and  $\sigma$  are positive constants

- (a) (1 point) Show that the absolute volatility of the instantaneous forward rate  $f(t, T)$  at time  $t$  for the Vasicek model is  $\sigma e^{-k(T-t)}$ .
- (b) (1 point) Assess whether a hump feature is possible in the term structure of the instantaneous forward rate volatilities.

Your team is still planning on using simulations, but they are now considering the use of a two-factor model (either the G2++ model or CIR2++ model) for the short rate. You require the model to allow for negative interest rates.

- (c) (1 point) Describe the advantages of the two-factor G2++ model versus the one-factor model in this situation.
- (d) (0.5 points) Describe the advantages of using the two-factor G2++ model over the CIR2++ model when the Brownian motions within each model are correlated.

## 2. Continued

Your team has calculated the empirical covariance matrix of the returns of bonds with maturities 3 months, 1 year, 2 years and 5 years. This matrix, denoted by  $A$ , can be decomposed as follows:

$$A = Q L Q'$$

where

- $Q$  is the matrix whose columns are the eigenvectors of the matrix  $A$
- $Q'$  is the transpose of the matrix  $Q$
- $L$  is a diagonal matrix whose diagonal elements are the eigenvalues of the matrix  $A$
- The trace of  $A$  is 4

The diagonal elements of matrix  $Q$  are:

$(2.93, \lambda_2, 0.12, 0.08)$  where  $\lambda_2$  is the eigenvalue corresponding to the second principal component.

Your team is assessing which model to use to value the interest rate derivative.

- (e) *(1.5 points)* Explain why, based on the matrices above, your team should consider using the two-factor  $G2++$  model instead of the one-factor model.

Moreover, your team is using only the market prices of European swaptions to calibrate the  $G2++$  model.

- (f) *(2 points)* Assess whether the scenarios can be used for the following tasks:
- (i) Pricing exotic interest rate derivatives
  - (ii) Calculation of the market-consistent (fair) value of an insurance liability
  - (iii) Calculation of the capital necessary to support worse-case scenarios such as CTE99
  - (iv) Estimation of the volatility of the earnings
- (g) *(1 point)* Explain the advantages of using  $G2++$  model for pricing out-of-the-money Bermudan swaptions after calibration to the corresponding at-the-money European swaption.

- 3.** (8 points) XYZ Bank is a domestic retail bank whose business model involves earning a yield on loans that are funded by consumer deposits. The bank wants to better understand their liquidity risks and has hired your consulting firm to provide guidance. XYZ's balance sheet contains the following:

Liabilities:

- Checking Accounts – deposit accounts earning no interest which allow the consumer unlimited access to their deposits
- Certificates of Deposit (CDs) – deposit accounts which provide consumers a low fixed return for a 1-year deposit with the bank. If the deposit is withdrawn early, a penalty fee is assessed.

Assets:

- Mortgages – 30-year loans at moderate interest rates backed by real estate, which allows pre-payment
- Credit Cards – revolving loans at high interest rates which allow consumers to draw on a letter of credit at any time

- (a) (3 points) Describe how each of the following scenarios would impact XYZ's liquidity position:
- (i) Increase in interest rates
  - (ii) Credit rating downgrade of XYZ
  - (iii) General downturn in economic activity that is not institution-specific

An analyst at your firm has decided to use deterministic scenario modeling to provide insight into XYZ's liquidity risk, and has proposed the following initial scenarios, each to be evaluated over 7 day and 30 day windows:

- Interest rates rise 1%
  - Credit rating downgrade (of XYZ) of one notch by any major rating agency
  - Domestic economic crisis
  - Emerging markets economic crisis
- (b) (2 points) Recommend and justify improvements and additions to the proposed scenarios and analysis framework.

### 3. Continued

XYZ has decided to address its liquidity risk by establishing a cash “liquidity reserve” that would ensure a cash flow cushion of at least 105% in any scenario over any period of time. This liquidity reserve could then be partially liquidated to meet any cash deficit(s).

Below are the monthly cash flows from one scenario:

<b>Cash Flow Source (millions)</b>	<b>Month 1</b>	<b>Month 2</b>	<b>Month 3</b>	<b>Month 4</b>
Mortgages	950	900	800	800
Credit Cards	150	100	-50	-100
CDs	-300	-350	-400	-50
Checking Accounts	-500	-1,000	-750	-150

- (c) (3 points) Calculate the minimum liquidity reserve that XYZ should immediately establish for this scenario.

4. (8 points) This question is concerned with one-factor short rate models.

You are given the following equation for the short-rate risk-neutral dynamics.

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW_t, \quad \sigma = 200\%, \quad a = 4, \quad r(0) = r_0$$

(a) (1 point) Interpret the coefficients  $a$  and  $\sigma$ .

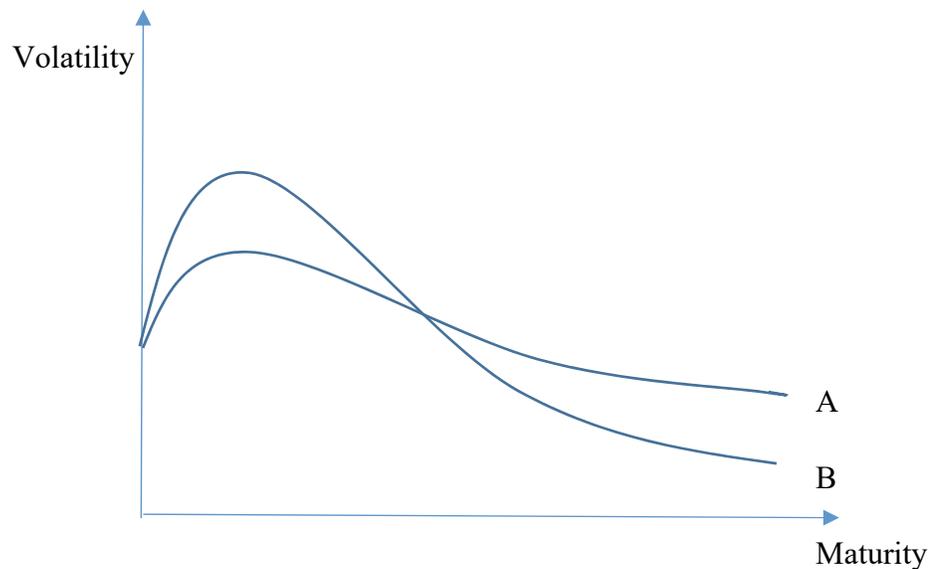
You calculated the market instantaneous forward rate at time 0 for the maturity in 3 years,  $f^M(0,3)$  to be 5%.

(b) (2 points) Calculate the risk-neutral probability of a negative rate at year 3.

(c) (2 points) Compare the Hull-White Extended Vasicek model and the Black-Karasinski Model.

(d) (1 point) Contrast market caplet volatility and model implied caplet volatility.

You are given a typical pattern for the term structure of caplet volatility and cap volatility below.



(e) (2 points) Justify whether A or B is the term structure for cap volatility.

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5. (7 points) You are an insurance company risk manager responsible for managing and monitoring the liquidity risks of an asset portfolio. Liquidity Cost Score (LCS) is used as the key risk measure to quantify the liquidity risk. You are reviewing several bond issues.

Consider the following attributes for bond issues A, B, C, and D:

Bond ID	Monthly Trading Volume (Millions)	Issue Size (Billions)	Issue Date	Duration (Years)	Spread (basis points)
Bond A	75	1.6	Jan 1, 2011	2.1	200
Bond B	100	1.5	Jan 1, 2015	1.5	230
Bond C	75	1.1	Jan 1, 2008	4.9	150
Bond D	50	1.2	Jan 1, 2016	5.8	110

- (a) (3 points)
- (i) Describe the relationship(s) between these attributes and LCS.
  - (ii) Design a ranking of these four bonds in terms of their liquidity attributes relative to each other.

Your manager has asked you to increase the liquidity of your portfolio by considering the two bonds shown in the table below (bps = basis points):

Bond ID	OAS Duration (Years)	Bid Price Range	Ask Price	Bid Spread Range (bps)	Ask Spread (bps)	Benchmark Bond?	LCS
Bond E		100 - 103	105			Unknown	X
Bond F	7.6			500 - 550	460	Yes	Y

You are also given:

The current LCS of the portfolio is 2.4%.  
The non-quoted adjustment factor is 1.3.

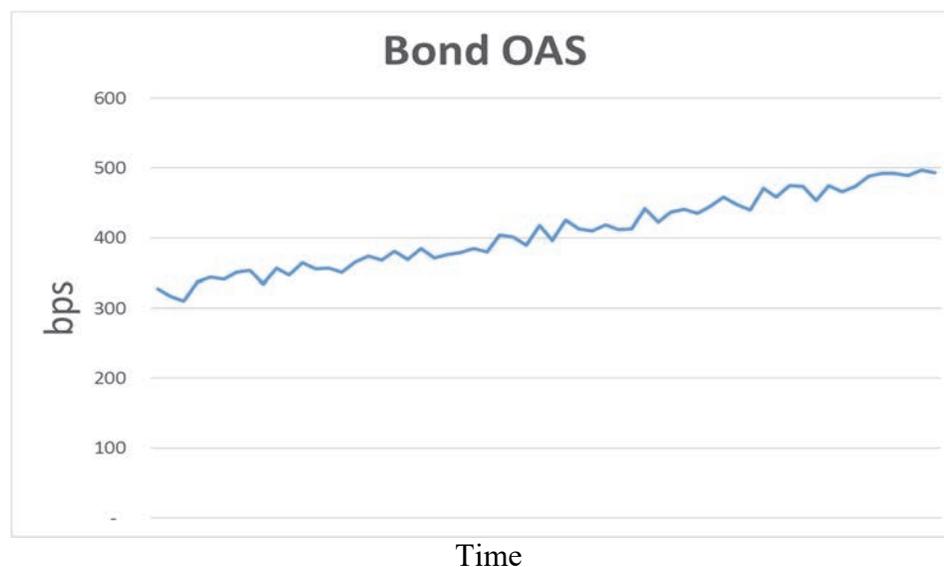
## 5. Continued

(b) (2 points)

- (i) Calculate the **maximum** value of X.
- (ii) Calculate the **minimum** value of Y.
- (iii) Evaluate which bonds, if any, you would add to the portfolio to increase liquidity. Justify your answer.

Your company owns Bond G within its current portfolio. Your junior analyst has given you a graph of Bond G's daily OAS over the last two months (see below) and stated:

*"I'm very concerned about the underlying default risk of this bond because the OAS has increased significantly over such a short period of time. I think it's time to sell the bond before it defaults or is downgraded."*



To evaluate your analyst's conclusion, you consider the decomposition of OAS into its three components.

(c) (2 points)

- (i) Describe and graph a situation that aligns with your analyst's statement using the components of a bond's OAS.
- (ii) Describe and graph a situation that contradicts your analyst's statement using the components of a bond's OAS.

6. (6 points) The Chief Investment Officer (CIO) of ABC Life Insurance Company is reviewing the company's current investment portfolio, summarized below:

	Annual return	Annualized std. dev.	Allocation %
Bonds	8%	8%	50%
Stocks	12%	12%	40%
REITs	10%	10%	10%

The CIO usually evaluates REITs using the NCREIF index, the most common benchmark of direct real estate performance used in capitalization rate calculations.

- (a) (0.5 points) Describe two key issues with the NCREIF index.

The standard specification of the capitalization ("cap") rate model includes several variables to relate rental fundamentals and a proxy for interest rates to explain cap rates and is given below:

$$\log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t}) + a_4 RTB_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

- (b) (1 point) Describe two key factors that are not included in the standard cap rate model and how both are expected to impact cap rates.

The CIO is proposing to replace ABC's indirect real estate exposures with direct real estate investments. In particular, he is considering a specific property, XYZ Building. He has received the following information:

- XYZ's annual return equals its capitalization rate of the current period,  $C_t$ , plus a margin of 1%
- Capitalization rates from previous periods:  $C_{t-1}=11\%$ ;  $C_{t-2}=10\%$ ;  $C_{t-3}=10\%$ ;  $C_{t-4}=9\%$ ;
- Treasury bond nominal yield: 5%;
- Inflation rate: 1%;
- Risk-free interest rate: 3%;
- Real Rent: 12,000;
- Mean Real Rent: 8,000;
- Fixed market-level effect: 1%;
- $a_0$  equals 2, and all other constants ( $a_n$ 's) equal to 1;
- No seasonality adjustments;
- The annualized standard deviation of this particular real estate investment is 12%.

**6. Continued**

(c) (1.5 points) Calculate the current annual return of XYZ using the standard cap rate model.

(d) (1 point) Calculate both the Sharpe ratio of XYZ and the Sharpe ratio of REITs.

The CIO has heard that most of the disadvantages of direct investment in real estate apply to individual investors only. You believe that these considerations do not differ substantially for an institutional investor like ABC and are just as important to consider.

(e) (2 points) Outline a short memo to the CIO summarizing why these disadvantages also apply to an institutional investor.

7. (7 points) You have been asked to help out the credit portfolio team to build credit models.

You joined a project to assign ratings to tranches in collateralized debt obligations (CDO). The project manager has asked you to research the four main categories of rating systems.

- (a) (1 point) Identify and explain which rating system you will recommend for the CDO business.
- (b) (1 point) Summarize in three steps the calibration involving regression of default probabilities to ratings.

You are assigned to peer review a calculation of Exposure at Default (EAD) for one borrower in a credit portfolio. For this borrower, the following information is provided:

- The total credit line is 30M.
- The borrower can draw 20M as cash and use the rest for contingent liabilities.
- The outstanding exposure is 15M.
- Assume the borrower will draw on the free part of the credit line in 40% of the cases and on average uses 50% of the available cash.
- The drawdown factor (DDF) for the contingent liability is 50%.
- The cash equivalent exposure factor (CEEF) is 80%.

The conclusion from the calculation showed that EAD is higher than the committed cash line.

- (c) (2 points) Evaluate whether the conclusion is correct based on your own calculation.

In the credit team's current model, Probability of Default (PD) is independent from Exposure at Default (EAD) as well as from Loss Given Default (LGD).

- (d) (1 point) Explain why these two assumptions can be invalid in a recession scenario.

In a newly established CDO credit portfolio, there are only a small number of positions. Your team has not been able to gather a significant amount of industry data and management is very concerned about the tail risk of the portfolio. You are asked to investigate whether to use Monte Carlo simulation or the Analytical Approximation method to generate loss distributions for the portfolio.

- (e) (0.5 points) Identify two considerations for choosing between the two methods.
- (f) (1.5 points) Recommend, based on the two considerations you have identified in (e), one method to generate loss distributions for the portfolio. Justify your answer.

**8.** (5 points) IVY, a health insurance company, has a fundamental value per common share of \$45. However, its price per share has declined over the last several months and is currently trading at \$20.

- (a) (2 points) Describe four reasons why the mispricing can persist for several months.
- (b) (1 point) Explain how IVY management might take advantage of its shares being undervalued and the barriers that could prevent successful implementation of this strategy.

Molly has decided to begin investing in stocks to fund her son's college expenses one year from now. She spent the last week researching IVY's stock on-line and decides to short the stock at \$20 per share because she believes the downward trend in its stock price over recent months will continue.

- (c) (2 points) Discuss the behavioral biases that led to Molly's belief.

9. (6 points) You, the Chief Risk Officer of ABC Life, are concerned about volatility smiles when pricing plain vanilla options.

- (a) (1 point) Explain issues with hedging plain vanilla options in the presence of a volatility smile.

Your colleague comments that in the presence of volatility smiles it is incorrect to calculate an option's delta simply by applying the Black-Scholes formula using the option's implied volatility.

- (b) (1 point) Derive an expression for the correct delta of a plain vanilla call option, in terms of Black-Scholes call option pricing function and its derivatives.

ABC Life decides to sell an annuity product. The crediting exposure of ABC is that it promises to grow the policyholder's investment at the same rate as the S&P500 index with a cap at 105% and a floor at 95% of the investment by year-end.

- (c) (1 point) Derive a hedging strategy to cover this crediting exposure using only call options.
- (d) (1 point) Discuss the sensitivity of initial hedging costs with respect to the shape of the volatility smile.

You model the volatility as a function  $g(S)$  of the current index value:

$$dS_t = rS_t dt + g(S_t) dW_t$$

where  $r$  is the risk-free interest rate and  $W$  is a Brownian motion under the risk-neutral measure.

You want to calculate  $\frac{\partial \sigma_{imp}(S, K)}{\partial S}$  using a Monte Carlo simulation.

- (e) (2 points) Describe the steps for this calculation.

**\*\*END OF EXAMINATION\*\***  
**Morning Session**

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