

# GI IRR Model Solutions

## Spring 2018

### 1. Learning Objectives:

1. The candidate will understand the key considerations for general insurance actuarial analysis.

### Learning Outcomes:

- (11) Adjust historical earned premiums to current rate levels.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 12.

### Commentary on Question:

*This question tests the candidate's understanding of adjusting premium to current rate level for purpose of ratemaking.*

### Solution:

- (a) State one advantage and one disadvantage of using the extension of exposures method to adjust historical premiums for prior rate changes.

#### Commentary on Question:

*Other advantages and disadvantages are possible.*

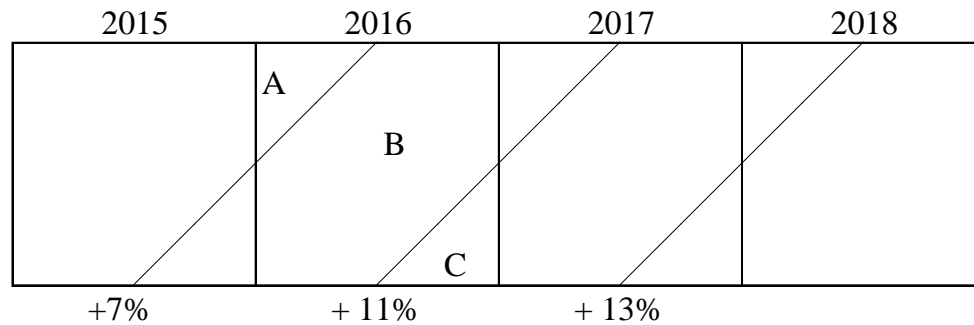
Advantage: It is the most precise method of adjusting historical premiums for prior rate changes.

Disadvantage: It is not viable if new rating variables have been introduced for which historical data are not available.

- (b) State two key assumptions for the parallelogram method to be appropriate with policies of any term.
  - Exposures are uniformly distributed over time.
  - Policies are written evenly over the experience period.

# 1. Continued

- (c) Calculate the on-level factor to use for ratemaking for calendar year 2016 earned premium using the parallelogram method.



Region	Area	Rate Level Relative Value	
A	0.125	1.0000	
B	0.750	1.0700	
C	0.125	1.1877	
Weighted average rate level:		1.0760	
Current rate level:		1.3421	$= 1.07 \times 1.11 \times 1.13$
2016 on-level factor:		1.2473	$= 1.3421 / 1.0760$

- (d) Describe one shortcoming.

Insurers make changes in the rates for rating factors for certain classes or territories which are aggregated into one estimate of the overall rate change. The premiums for specific class or territory would not be on-level if the overall average rate change is not the same as the specific class or territory rate change.

- (e) Describe how you can adapt the analysis to address this shortcoming.

This shortcoming has led many actuaries to rely on the extension of exposures method instead of the parallelogram method for classification and territory ratemaking, especially for personal lines, automobile and homeowners insurance.

## 2. Learning Objectives:

1. The candidate will understand the key considerations for general insurance actuarial analysis.
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

### Learning Outcomes:

- (1j) Create a claims development triangle from claims transaction data.
- (2a) Use loss development triangles for investigative testing.
- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 10, 13, 14, and 19.

### Commentary on Question:

*This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data. This question also tests the candidate's ability to estimate ultimate claims using Berquist-Sherman adjustments when there has been a change in case reserve adequacy.*

### Solution:

- (a) Calculate the calendar year 2017 reported claims.

Paid claims in calendar year 2017 = Paid from Jan. 1, 2017 through Nov. 30, 2017 + paid during December 2017

$$= 30 + 25 + 26 + 15 \text{ (from second claim)} = 96.$$

Case estimate as of Dec. 31, 2017 = Case estimate as of Nov. 30, 2017 + Changes in case estimates in December 2017

$$= 22 + 62 + 30 + 20 \text{ (case from first claim)} - 25 \text{ (case change from second claim)} + 10 \text{ (case from reopened third claim)} = 119.$$

Case estimate as of Dec. 31, 2016 =  $55 + 35 = 90$  (last diagonal of case estimates triangle).

2017 calendar year reported claims = Paid in 2017 + Case estimate as of Dec. 31, 2017 – Case estimate as of Dec. 31, 2016

$$= 96 + 119 - 90 = 125.$$

## 2. Continued

(b) Calculate ultimate claims using the paid development method.

Accident Year	Cumulative Paid Claims (000)		
	12	24	36
2015	18	48	78 <sup>(1)</sup>
2016	24	64 <sup>(2)</sup>	
2017	26 <sup>(3)</sup>		

Age-to-age Development Factors:

Accident Year	12-24
2015	2.667
2016	2.667

Notes: (1) 78 = 48 + 30 (AY 2015 incremental paid claims Jan. 1, 2017 through Nov. 30, 2017)

(2) 64 = 24 + 25 (AY 2016 incremental paid claims Jan. 1, 2017 through Nov. 30, 2017) + 15 (from second claim)

(3) 26 (AY 2017 incremental paid claims Jan. 1, 2017 through Nov. 30, 2017)

Development Factors	12-24	24-36	36-ult
Selected age-to-age	2.667	1.50	1.20
Age-to-ultimate	4.80	1.80	1.20

Accident Year	(1) Cumulative Paid Claims to Dec. 31, 2017	(2) Age-to-ultimate Development Factor	(3) = (1)(2) Ultimate Claims
2015	78	1.20	94
2016	64	1.80	115
2017	26	4.80	125

## 2. Continued

(c) Construct a triangle of average case estimates as of December 31, 2017.

Accident Year	Case Estimates (000)		
	12	24	36
2015	45	35	32 <sup>(4)</sup>
2016	55	37 <sup>(5)</sup>	
2017	50 <sup>(6)</sup>		

Notes: (4) 32 = 22 (AY 2015 case estimate as of Nov. 30, 2017) + 10 (from third claim)

(5) 37 = 62 (AY 2016 case estimate as of Nov. 30, 2017) – 25 (from second claim)

(6) 50 = 30 (AY 2017 case estimate as of Nov. 30, 2017) + 20 (from first claim)

Accident Year	Open Counts		
	12	24	36
2015	85	65	46 <sup>(7)</sup>
2016	115	74 <sup>(8)</sup>	
2017	121 <sup>(9)</sup>		

Notes: (7) 46 = 45 (AY 2015 open counts as of Nov. 30, 2017) + 1 (from reopened third claim)

(8) 74 = 75 (AY 2016 open counts as of Nov. 30, 2017) – 1 (from closed second claim)

(9) 121 = 120 (AY 2017 open counts as of Nov. 30, 2017) + 1 (from new first claim)

Average Case Estimates = Case Estimates / Open Counts

Accident Year	Average Case Estimates (000)		
	12	24	36
2015	0.529	0.538	0.696
2016	0.478	0.500	
2017	0.413		

## 2. Continued

- (d) Construct a triangle of paid to reported ratios as of December 31, 2017.

Reported Claims = Cumulative Paid Claims (from part (b)) + Case Estimates (from part (c))

Accident Year	Reported Claims		
	12	24	36
2015	63	83	110
2016	79	101	
2017	76		

Paid to Reported Ratios = Cumulative Paid Claims / Reported Claims

Accident Year	Paid to Reported Ratios		
	12	24	36
2015	0.286	0.578	0.709
2016	0.304	0.634	
2017	0.342		

- (e) Explain whether or not the two triangles in parts (c) and (d) indicate any change in case adequacy.

Paid to reported ratios are increasing down each column (by AY). Average case estimates are decreasing down each column (by AY). Therefore, both diagnostics indicate decreasing case reserve adequacy.

## 2. Continued

- (f) Calculate an adjusted triangle of reported claims using the Berquist-Sherman methodology.

Accident Year	Adjusted Average Case Estimate (000)		
	12	24	36
2015	0.424 <sup>(13)</sup>	0.534 <sup>(11)</sup>	0.696 <sup>(10)</sup>
2016	0.397 <sup>(12)</sup>	0.500 <sup>(10)</sup>	
2017	0.413 <sup>(10)</sup>		

Notes: (10) Latest diagonal values from part (c) average case estimates triangle.

(11)  $0.534 = 0.500 / (1.04 \times 0.9)$

(12)  $0.397 = 0.413 / 1.04$

(13)  $0.424 = 0.397 / (1.04 \times 0.9)$

Adjusted Reported Claims = Cumulative Paid Claims + (Adjusted Average Case Estimate)(Open Counts)

Accident Year	Adjusted Reported Claims (000)		
	12	24	36
2015	54	83	110
2016	70	101	
2017	76		

i.e.,  $70 = 24 + 0.397 \times 115$

### 3. Learning Objectives:

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

#### Learning Outcomes:

- (4b) Describe the influences on frequency and severity of changes in deductibles, changes in policy limits, and changes in mix of business.
- (4c) Choose trend rates and calculate trend factors for claims.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 25.

#### Commentary on Question:

*This question tests the candidate's understanding of various influences on frequency and severity trends, as well as considerations in choosing trend rates for claims.*

#### Solution:

- (a) Describe the effect, if any, that increasing policy limits can have on frequency trend and on severity trend.

Changing policy limits will not affect frequency trend.

Changing policy limits will increase severity due to higher policy limits.

- (b) Recommend two approaches for the trend analysis to consider the industry reform.

Any two of the following are acceptable:

- Exclude pre-reform data (year 1 to year 5) if recent data are considered credible.
- Adjust pre-reform data (year 1 to year 5) for the level that is expected post-reform.
- Adjust a multi-variable regression model to include an extra variable that reflects industry reform.

- (c) Describe two other approaches that can be used to adjust the trending analysis to account for seasonality.

- Analyze the data separately for the first part of each accident half year and for the second part of each accident half year.
- Combined data into full accident years.



### 3. Continued

- (d) Critique the appropriateness of each model with respect to selecting a frequency trend to use for ratemaking.

Model 1 (all years) would not model the tort change properly and estimates too negative a trend.

Model 2 (1-5) only models the first part and gives no consideration to later years. Therefore it is not representative to recent trend experience and likely not consistent with future expected trend.

Model 3 (6-11) models only the experience since the tort change and is more likely reflective of future expected trend.

#### 4. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.
6. The candidate will understand the need for monitoring results.

#### Learning Outcomes:

- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.
- (2e) Assess the appropriateness of the projection methods cited in (2b) in varying circumstances.
- (2f) Evaluate and justify selections of ultimate values based on the methods cited in (2b).
- (3d) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.
- (6b) Analyze actual claims experience relative to expectations.

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 17, 21, 23, and 36.

#### Commentary on Question:

*This question tests the candidate's understanding of reported and unreported salvage using the Bornhuetter Ferguson method. In addition, this question tests the understanding of expected reported salvage for an interim period between actuarial analyses.*

#### Solution:

- (a) Estimate projected ultimate salvage for accident year 2017 using the Bornhuetter Ferguson method.

$$\begin{aligned} & \text{12 month to ultimate cumulative development factor (CDF)} \\ & = 0.980 \times 0.995 \times 0.999 = 0.974 \end{aligned}$$

$$1.0 - (1/\text{CDF}) = -0.0267$$

$$\text{Projected Ultimate Salvage} = 65,000 + (67,000)(-0.0267) = 63,211.$$

- (b) Calculate unreported salvage for accident year 2017 as of December 31, 2017.

$$\begin{aligned} \text{Unreported salvage} & = \text{Projected ultimate salvage} - \text{Actual salvage reported} \\ & = 63,211 - 65,000 = -1,789. \end{aligned}$$

## 4. Continued

- (c) Calculate the difference between accident year 2017 actual and expected reported salvage recoveries as of December 31, 2017, using the a priori expected salvage recoveries.

$$\begin{aligned}\text{Expected reported salvage} &= (\text{A priori expected salvage}) \times (1/\text{CDF}) \\ &= 67,000 \times (1/0.974) = 68,789\end{aligned}$$

$$\begin{aligned}\text{Difference between actual and expected salvage} \\ &= 65,000 - 68,789 = -3,789, \text{ or } -5.5\% \text{ of expected reported salvage.}\end{aligned}$$

- (d) Assess the reasonableness of the inputs for the Bornhuetter Ferguson method using your results from part (c).

Any one of the following is acceptable:

- Actual versus expected difference is unreasonable at  $-5.5\%$
- A priori expected salvage is higher than actual to-date which is inconsistent with the observed downward development
- A priori expected salvage looks odd relative to actual reported salvage since development is less than 1.0, but a priori expected salvage implies development greater than 1

- (e) Identify two issues to investigate from your salvage analysis.

Any two of the following are acceptable:

- What (or who) was the source of the a priori expected salvage?
- Are there any operational changes that could be influencing 2017?
- Are there any environmental (internal/external) changes that could be influencing 2017?
- Is the books volume relatively stable or changing in 2017?
- Can we examine other diagnostics for reasonability, for example, salvage to claim ratios?

#### 4. Continued

- (f) Calculate the difference between accident year 2017 actual and expected reported salvage from December 31, 2017 through June 30, 2018, using linear interpolation and your results from part (a).

$$\text{AY 2016 expected \% reported as of Dec. 31, 2017} = 1/(0.999 \times 0.995) = 100.6\%$$

$$\text{AY 2017 expected \% reported as of Dec. 31, 2017} = 1/0.974 = 102.7\%$$

$$\text{AY 2017 expected \% reported as of June 30, 2018}$$

$$= 0.5 \times 100.6\% + 0.5 \times 102.7\% = 101.7\%$$

$$\text{Actual reported salvage as of June 30, 2018} = 64,600 - 65,000 = -400$$

$$\text{Expected reported salvage as of June 30, 2018}$$

$$= (63,211 - 65,000) \frac{(101.7\% - 102.7\%)}{(1 - 102.7\%)} = -663$$

$$\text{Difference} = (-400) - (-663) = 263.$$

- (g) Describe one situation where you would not want to use linear interpolation for estimating expected development between annual evaluations.

For immature periods where development factors are high.

## 5. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

### Learning Outcomes:

- (2d) Explain the effect of changing conditions on the projection methods cited in (2b).

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 20.

### Commentary on Question:

*This question tests the candidate's understanding of changing conditions on different projection methods.*

### Solution:

- (a) Explain the expected effect on projected ultimate claims under each of the projection methods (I) through (IV) for line of business A when there is a *reduction in claim costs*.

### Commentary on Question:

*Candidates need to specify whether the reduction in claim costs will affect the projected ultimate claims and not just if the method is responsive to the change.*

- I. Expected method is not responsive to changes in overall claims without an explicit change in the expected ratio. Therefore, the ultimate claim would be overstated.
- II. Changes in claim amounts will be present in the latest diagonal with the reported development method. As a result, this method should respond to the change in claim amounts producing an accurate estimate of ultimate claims.
- III. The reported Bornhuetter Ferguson projection is a function of observed experience and expected unobserved experience. The projection will be overstated without an explicit change in the expected ratio.
- IV. The reported Cape Cod projection is also a function of observed experience and expected unobserved experience. Unlike the reported Bornhuetter Ferguson method, reported claims are a function of the expected, so the Cape Cod method will adjust to changes in claims but the ultimate claims will still be overstated.

## 5. Continued

- (b) Rank the accuracy of projection methods (I) through (IV) from most accurate to least accurate when there is a *reduction in claim costs* for line of business A.

II (reported development method) is the most accurate, followed by IV (reported Cape Cod method), followed by III (reported Bornhuetter Ferguson method), followed by I (expected method) as the least accurate.

- (c) Explain the expected effect on projected ultimate claims under each of the projection methods (I) through (IV) for line of business B when there is a *deterioration in case reserve adequacy*.

I. The expected method is unaffected by the change in reserve adequacy because there is no change in the expected ratio. Therefore, projected ultimate claims will be accurate.

II. The reserve deterioration will impact the development pattern, which will under estimate ultimate claims.

III. The reported Bornhuetter Ferguson projection is a function of observed experience and expected unobserved experience. The projection will be understated due to the change in development pattern. The impact will be less than the development method because it includes expected claims in the projection.

IV. The reported Cape Cod projection is also a function of observed experience and expected unobserved experience. The Cape Cod method is influenced by the cumulative development factors, which do not accurately reflect the slower reporting, and also by the lower values for reported claims, which lead to an understated expected claim ratio.

- (d) Describe the difference in accuracy between projection methods (II) and (III) for line of business B when there is a *deterioration in case reserve adequacy*.

The reported development method is less accurate. With the reported development method, a lower proportion of ultimate claims are now reported earlier than in the past, thus higher cumulative development factors would be required to adequately project the reported claims to an ultimate basis. The Bornhuetter Ferguson method is only influenced by the cumulative development factor which is multiplied by an estimate of expected claims and then added to the actual reported claims.

## 6. Learning Objectives:

1. The candidate will understand the key considerations for general insurance actuarial analysis.
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (1b) Identify different types of data used for actuarial analysis.
- (1k) Estimate written, earned and unearned premiums.
- (3e) Describe the components of premium liabilities in the context of financial reporting.
- (3f) Evaluate premium liabilities.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 11, and 24.

### Commentary on Question:

*This question tests the candidate's understanding of net unpaid claim liabilities, unearned premium liabilities, and premium liabilities.*

### Solution:

- (a) Calculate the following liabilities as of December 31, 2017:

- (i) Net unpaid claims, excluding ULAE
- (ii) Net unearned premium
- (i) Net unpaid claims, excluding ULAE, as of December 31, 2017:

2017 Incurred Claims = Unpaid Claims as of December 31, 2017 – Unpaid Claims as of December 31, 2016 + 2017 Paid Claims  
Therefore, Unpaid Claims as of December 31, 2017 = 2017 Incurred Claims – 2017 Paid Claims + Unpaid Claims as of December 31, 2016

2017 Gross Incurred Claims = (Gross Earned Premium × Gross Incurred Claims Ratio)  
= (96,000 × 70%) = 67,200

Therefore, Gross Unpaid Claims as of December 31, 2017  
= 67,200 – 61,000 + 300,000 = 306,200

Net Unpaid Claims as of December 31, 2017, excluding ULAE  
= 306,200 × 0.90 = 275,580.

## 6. Continued

(ii) Net unearned premium as of December 31, 2017:

2017 Earned Premium = 2017 Written Premium + Unearned Premium as of December 31, 2016 – Unearned Premium as of December 31, 2017

Therefore, Gross Unearned Premium as of December 31, 2017 = 2017 Written Premium – 2017 Earned Premium + Unearned Premium as of December 31, 2016  
 $= 100,000 - 96,000 + 36,000 = 40,000$

Net unearned premium as of December 31, 2017 =  $40,000 \times 0.90 = 36,000$ .

(b) Determine either the premium deficiency reserve or the equity in the unearned premium as of December 31, 2017 on a net of reinsurance basis, and label accordingly.

Selected claim ratio = 69% (justification is the average of the most recent two years to reflect the increasing ratios)

Calendar Year	(1) Gross Written Premium	(2) General Expense	(2)/(1) General Expense Ratio
2015	80,000	16,000	0.200
2016	90,000	18,630	0.207
2017	100,000	20,900	0.209

Recommended general expense ratio = 0.208 (justification is the average of the most recent two years to reflect the increasing ratios)

Net unearned premium (from part (a))	36,000
Net expected claims = $69\% \times 36,000 =$	24,840
Expected ULAE = $10\% \times 24,840 / (1 - 0.10) =$	2,760
<i>{note: based on gross expected claims so need to divide by 0.90}</i>	
Selected maintenance expenses = $20.8\% \times 30\% \times 36,000 / (1 - 0.10) =$	2,496
<i>{note: based on gross unearned premium so need to divide by 0.90}</i>	
Equity in unearned premium = $36,000 - (24,840 + 2,760 + 2,496) =$	5,904
<i>{equity in unearned premium since <math>36,000 &gt; (24,840 + 2,760 + 2,496)</math>}</i>	



## 6. Continued

- (c) Identify any two of these types of premium development.

Any two of the following are acceptable:

- Provision for retrospectively-rated policies
- Earned but not recorded premiums
- Audit premiums
- Development on reinsurance assumed or ceded contracts (or retro-rated reinsurance contracts)

- (d) Select either one of the two types of premium development you identified in part (c) and:

- (i) Describe how the premium development arises.
- (ii) Describe how this development would be reflected in estimating premium liabilities.

For retrospectively-rated policies:

- (i) Development arises because premium adjustments are calculated after policy expiration based on claim experience.
- (ii) Premium liabilities should include the difference between estimated final premium and premium collected through the accounting date which could be positive or negative.

## 7. Learning Objectives:

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (4a) Identify the time periods associated with trending procedures.
- (4c) Choose trend rates and calculate trend factors for claims.
- (5d) Calculate loadings for catastrophes and large claims.
- (5e) Demonstrate the use of credibility in ratemaking.
- (5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 25, 30, and 31.

### Commentary on Question:

*This question tests the candidate's understanding of loadings for large claims as well as basic ratemaking.*

### Solution:

- (a) Describe two ways that catastrophe claims are different than large claims.

Any two of the following are acceptable:

- Catastrophes typically result in GI claims for multiple insurers providing coverage in an affected area, whereas large losses are limited to a few claims for an individual insurer.
- Catastrophes are associated with an event which is infrequent and results in unusually large aggregate losses.
- Catastrophes typically result in a significant number of GI claims for multiple insurers providing coverage in the area affected by the event. Large claims do not typically affect the entire GI industry, or even all GI companies operating in a specific area.

## 7. Continued

(b) Calculate the expected wildfire claims in State C to use for ratemaking.

Accident Year	(1) Earned House Years (EHY)	(2) Ultimate Wildfire Claims (000)	(3) Trending Period in Months	(4) = $1.08^{(3)/12}$ Severity Trend @ 8.0%	(5) = (2)(4) Trended Ultimate Wildfire Claims (000)
2008	14,000	400	135	2.377	950.80
2014	13,600	275	63	1.498	411.95
Total	170,000	675			1,362.75

Note: (3) Trending period for 2008: Trend from average accident date in AY 2008 (July 1, 2008) to average accident date in future rating period (October 1, 2019) = 135 months.

(6) Trended Ultimate Wildfire Pure Premium per 100 EHY = $(5) \times 1,000 / [(1)_{\text{Total}} / 100]$	801.62
(7) Region W Pure Premium per 100 EHY	1,000.00
(8) Credibility for State C	20%
(9) Credibility for Region W = $1 - (8)$	80%
(10) Credibility weighted wildfire pure premium per 100 EHY = $(6)(8) + (7)(9)$	960.32
(11) 2017 EHY for State C	14,050
(12) Expected Claims = $(10)(11) / 100$	134,925

(c) Calculate the indicated rate level change.

Accident Year	(13) Earned House Years	(14) Trended Earned Premiums at Current Rate Level	(15) Trended Ultimate Claims	(16) Accident Year Weights	(17) = (15)/(14) Claim Ratio
2015	13,400	10,454,300	6,482,000	20%	62.0%
2016	13,800	10,647,600	6,772,000	30%	63.6%
2017	14,050	10,511,300	6,204,000	50%	59.0%
Total	41,250	31,613,200	19,458,000	100%	

## 7. Continued

(18)	Weighted average claim ratio = $\text{sumproduct}[(16),(17)]$	61.0%
(19)	Wildfire claim ratio = $[(12)/(14)_{2017}]$	1.28%
(20)	ULAE to claim ratio	14%
(21)	Total claim ratio including ULAE = $[(18) + (19)] \times [1 + (20)]$	71.0%
(22)	Credibility of experience period = $\text{Squareroot}[(13)_{\text{Total}}/80,000]$	71.8%
(23)	Region W trended, adjusted ultimate claim, including ULAE, ratio	75%
(24)	Credibility-weighted experience claim, including ULAE, ratio = $(21)(22) + [1 - (22)](23)$	72.13%
(25)	Selected fixed expenses to premiums ratio	4%
(26)	Selected variable expenses to premiums ratio	12%
(27)	Selected profit and contingencies to premiums ratio	6%
(28)	Indicated rate level change = $[(24) + (25)] / [1 - (26) - (27)] - 1$	-7.16%

## 8. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

### Learning Outcomes:

- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 2, 15, 17, and 18.

### Commentary on Question:

*This question tests the estimation of ultimate claims using the Cape Cod method, as well as understanding situations where the Cape Cod, Bornhuetter Ferguson and frequency-severity methods are well suited to projecting ultimate claims.*

### Solution:

- (a) Describe the meaning of the term “used-up premiums” when using the Cape Cod method.

Earned premiums that have been adjusted to reflect that portion of the exposure that has been used-up at the valuation date based on paid or reporting patterns.

- (b) State one way in which actuaries can incorporate professional judgment into the Cape Cod method.

Either of the following are acceptable:

- Determination of the experience period
- Determination of the decay factor

## 8. Continued

- (c) Describe the purpose of a decay factor in the Generalized Cape Cod method.

The decay factor allows different weighting of the years in the experience period with the greatest weight being applied to the year under consideration (origin year) and then decreasing weights to the years preceding and subsequent to the origin year.

- (d) Calculate the expected pure premium for accident year 2017 using the Generalized Cape Cod method applied to reported claims and a decay factor of 90%.

Accident Year	(1) Earned Exposures	(2) Reported Cumulative Development Factors	(3) = 1/(2) Expected % Developed	(4) = (1)(3) Used-Up Earned Exposures	(5) Actual Reported Claims	(6) Claim Trend @ 4%	(7) = (5)(6) Adjusted Claims	(8) 90% Decay Factors
2015	1,000	1.500	0.667	667	320,000	1.082	346,240	0.81
2016	1,100	3.400	0.294	324	150,000	1.040	156,000	0.90
2017	1,200	7.000	0.143	171	85,000	1.000	85,000	1.00

$$\begin{aligned} \text{Expected pure premium} &= \text{sumproduct}[(7),(8)] / \text{sumproduct}[(4),(8)] \\ &= (346,240 \times 0.81 + 156,000 \times 0.90 + 85,000 \times 1.00) / \\ &\quad (667 \times 0.81 + 324 \times 0.90 + 171 \times 1.00) = 504. \end{aligned}$$

- (e) Describe one other situation for each of the following projection methods, such that the method is well-suited for the situation. Do not repeat any situations.

- (i) Cape Cod method applied to reported data
- (ii) Bornhuetter Ferguson method applied to paid data
- (iii) Frequency-severity closure method
- (i) Immature experience period
- (ii) New product (or area) with limited experience
- (iii) Environmental changes (internal or external) that cause historical patterns to be unreliable

## 8. Continued

- (f) Provide three reasons why the claim experience of a reinsurer may have greater variability than the claim experience of a primary insurer.

Any three of the following are acceptable:

- Reinsurance data often has less credibility
- Longer lags are common for reporting and settlement
- Reinsurance severity is often higher
- Reinsurance frequency is often low
- Case estimates are set by numerous claim adjusters at various companies with difference reserving philosophies
- Limited detailed data from primary insurers

## 9. Learning Objectives:

7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

### Learning Outcomes:

- (7a) Describe the structure of catastrophe models.

### Sources:

Catastrophe Modeling: A New Approach to Managing Risk, Grossi, P. and Kunreuther, H., Chapter 3.

### Commentary on Question:

*This question tests the candidate's understanding of the risk assessment process in catastrophe modeling.*

### Solution:

Explain why each of the following statements is either correct or incorrect.

- (i) In developing the model domain (the geographic extent of the region to be modeled) it is sufficient to know the location and likely magnitude of future earthquakes.
- (ii) The return period (time until the next earthquake) and magnitude of the earthquake can be separately modeled.
- (iii) To model IFTEM's specific portfolio for the *inventory* module, detailed information will be required for each building in the portfolio.
- (iv) To model IFTEM's specific portfolio for the *vulnerability* module, detailed information will be required for each building in the portfolio.
- (v) The total loss from an event (prior to applying the terms of the insurance coverage) could come from either the vulnerability module or the loss module.



## 9. Continued

- (i) This is incorrect. It is also essential to know the regions geological features and how an earthquake will propagate through the regions soil structure.
- (ii) This is incorrect. The Gutenberg-Richter relationship indicates that the longer the time until the next earthquake, the higher the magnitude.
- (iii) This is correct. While public data may be sufficient for aggregate estimates it is essential that risk-specific structural details be known.
- (iv) This is incorrect. It is too difficult to do this and have a model that works with individual buildings. Instead, buildings are divided into classes.
- (v) This is correct. One approach is to link the results in the vulnerability module directly to a monetary loss. A second option is to use the loss module to perform the translation of physical loss to monetary loss.

## 10. Learning Objectives:

1. The candidate will understand the key considerations for general insurance actuarial analysis.
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

### Learning Outcomes:

- (1) Adjust historical earned premiums to current rate levels.
- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.
- (3b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.
- (3d) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 12, 16, 22, and 23.

### Commentary on Question:

*This question tests the candidate's ability to estimate ultimate claims using the expected method. This question also tests the calculation of unallocated loss adjustment expenses using the classical paid-to-paid method, as well as determining the total claim liability for a book of business.*

### Solution:

- (a) Recommend the 2017 cost and rate level expected claim ratio to be used to estimate expected claims. Justify your recommendation.

2015	2016	2017
A = 1.0		B = 1.05

+5%

## 10. Continued

Calendar Year (CY)	Area at Rate Level:		Average Rate Level	On-Level to CY 2017
	A	B		
2015	100.0%	0.0%	1.00000	1.04375
2016	87.5%	12.5%	1.00625	1.03727
2017	12.5%	87.5%	1.04375	1.00000
Rate Level	1.00	1.05		

Notes: e.g., Average rate level for 2016 =  $87.5\% \times 1.00 + 12.5\% \times 1.05 = 1.00625$   
e.g., On-Level to CY 2017 for 2016 =  $1.04375 / 1.00625 = 1.03727$

Accident Year	(1) Earned Premiums (000)	(2) Premium On-Level Factor	(3) Paid Claims as of Dec. 31, 2017 (000)	(4) % of Claims Paid	(5) Claim Trend Factor	(6) = [(3)(5)/(4)] / [(1)(2)]
						Claim Ratio
2015	1,520	1.04375	650	79%	1.042	54.0%
2016	1,790	1.03727	530	56%	1.021	52.0%
2017	2,050	1.00000	240	24%	1.000	48.8%

Notes: (4) = sum of incremental paid pattern (e.g., for 2015 = 24% + 32% + 23%)

Recommended claim ratio = (A) = 53.0%.

Justification is the average of 2015 and 2016 as 2017 is highly leveraged.

- (b) Calculate the expected claims for each accident year.

Accident Year (AY)	(1) Earned Premiums (000)	(7) = [(A)(2)] / (5) Claim Ratio at Each AY Cost Level	(8) = (1)(7) Expected Claims (000)
2015	1,520	53.1%	807
2016	1,790	53.8%	963
2017	2,050	53.0%	1,087
			2,857

## 10. Continued

- (c) Calculate the unpaid ULAE as of December 31, 2017 using the classical paid-to-paid method.

Total ultimate claims (from part (b))	2,857,000
Total paid claims = 650,000 + 530,000 + 240,000 =	1,420,000
Total unpaid claims = 2,857,000 – 1,420,000 =	1,437,000
Case reserve	536,000
IBNR = 1,437,000 – 536,000 =	901,000

$$\begin{aligned}\text{Unpaid ULAE} &= (\text{ULAE ratio} \times \text{IBNR}) + (\text{ULAE ratio} \times \text{multiplier} \times \text{case} \\ &\text{estimates}) \\ &= (11.5\% \times 901,000) + (11.5\% \times 35\% \times 536,000) = 125,189.\end{aligned}$$

- (d) Calculate the total claim liability for this book of business.

$$\begin{aligned}\text{Claim Liability} &= \text{Case Estimates} + \text{IBNR} + \text{Unpaid ULAE} \\ &= 536,000 + 901,000 + 125,000 = 1,562,189.\end{aligned}$$

## 11. Learning Objectives:

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5f) Calculate overall rate change indications under the claims ratio and pure premium methods.  
(5k) Calculate rates for claims-made coverage.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 31 and 34.

### Commentary on Question:

*This question tests the candidate's understanding of claims-made ratemaking.*

### Solution:

- (a) Describe the difference between an occurrence policy and a claims-made policy with respect to the trigger for coverage.

The trigger for occurrence policies is the accident (or incident) date, and the trigger for claims-made policies is the report date.

- (b) Calculate the report year 5 earned exposures (i.e.,  $E_{i,5}$ , where  $i$  = accident year lag).

Policy A contributes one earned exposure to  $E_{2,5}$ .

Policy B contributes 2/12 earned exposure to cells  $E_{0,5}$ ,  $E_{1,5}$ ,  $E_{2,5}$ ,  $E_{3,5}$ , and  $E_{4+,5}$ .

Policy C contributes one earned exposure to cells  $E_{0,5}$ ,  $E_{1,5}$ ,  $E_{2,5}$ ,  $E_{3,5}$ , and  $E_{4+,5}$ .

- (c) Calculate the indicated rate for a mature claims-made policy, effective January 1, year 5.

Pure premium for a report year 5 mature claims-made policy =  $60 + 110 + 130 + 110 + 90 = 500$

Indicated rate =  $(500 + 10) / (1 - 0.15 - 0.04) = 630$ .

- (d) Calculate the tail factor for a second year claims-made coverage, effective January 1, year 5.

Numerator =  $C_{1,6} + C_{2,7} + C_{3,8} + C_{4+,9} + C_{2,6} + C_{3,7} + C_{4+,8}$   
=  $116 + 144 + 128 + 110 + 137 + 122 + 105 = 862$

Denominator =  $C_{0,5} + C_{1,5} = 60 + 110 = 170$

Tail factor =  $862 / 170 = 5.071$ .

## 11. Continued

- (e) Explain why a tail factor determined from a reporting pattern that is equal across each of the five accident year lags (i.e., 20% per year) is less than the tail factor calculated in part (d).

The denominator is higher because more are reported earlier (i.e., 40% instead of 34%). The numerator is lower because less claims are reported in the later AY lags (i.e., not AY lag 0). Therefore, the factor should be lower.

## 12. Learning Objectives:

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5j) Perform individual risk rating using standard plans.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

### Commentary on Question:

*This question tests the candidate's understanding of retrospective rating.*

### Solution:

- (a) Define retrospective rating.

Retrospective rating is not an insurance product but an approach to determining the premium for an insurance product (i.e., the insurance policy) that incorporates the insured's own claims into the determination of the cost of the product (i.e., the premium).

- (b) Explain how a retrospectively-rated policy works.

The deposit premium that is due at the beginning of the policy term is often based on the premium developed through prospective experience rating. Periodic retrospective adjustments, either refunds to the policyholder or additional payments to the insurer, will be made following the conclusion of the policy term based on the terms and conditions set out in the policy or the rules of the retrospective rating program.

- (c) Describe a benefit of retrospective experience rating.

Retrospective risk rating programs provide meaningful incentives for increased risk management and risk control efforts to help reduce claims and hence premiums.

- (d) Evaluate whether or not you would recommend this client for retrospective rating.

Retrospective rating works best for:

1. Sufficiently large and stable volume of claims – meets this criteria
2. Lines characterized by high frequency and low to medium severity – meets this criteria
3. Better than average claim experience – does not meet this criteria

Retrospective rating can be recommended for this situation as most criteria are met.

### 13. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

#### Learning Outcomes:

- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.
- (2e) Assess the appropriateness of the projection methods cited in (2b) in varying circumstances.
- (2f) Evaluate and justify selections of ultimate values based on the methods cited in (2b).

#### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 19 and 21.

#### Commentary on Question:

*This question tests the candidate's ability to estimate ultimate claims using Berquist-Sherman adjustments.*

#### Solution:

- (a) Recommend one method to project ultimate claims for this book. Justify your recommendation.

Any reserving method that takes into account the internal changing conditions. Examples include, but are not limited to: the expected method, the paid development method, the paid Bornhuetter Ferguson method, or the paid Cape Cod method.

- (b) Describe the steps involved in adjusting data for changes in case reserve adequacy.

Step 1: Create a triangle of average adjusted case estimates, by using the latest diagonal and detrend using severity trend selection.

Step 2: Calculate the adjusted reported claim triangle

= (step 1 adjusted case estimates)(open counts) + cumulative paid claims.

Step 3: Project ultimate claims from adjusted reported claim triangle.



### 13. Continued

- (c) Describe the steps involved in adjusting data for changes in claim settlement rate.

Step 1: Select disposal ratios by maturity age (disposal ratio = cumulative closed counts to selected ultimate counts).

Step 2: Find mathematical relationship curve to approximate relationship between closed counts and paid claims (e.g., linear regression).

Step 3: Calculate adjusted closed counts triangle based on selected disposal ratios

Step 4: Adjusted paid claims triangle = restated closed counts triangle  $\times$  mathematical curve.

Step 5: Project ultimate claims from adjusted paid claim triangle.

- (d) Assess whether or not there has been a change in the overall adequacy of case estimates.

Change in Average Case (down the column)

Accident Year	12	24	36
2014-15	-24%	30%	21%
2015-16	54%	31%	
2016-17	11%		

e.g.,  $2,800 / 3,690 - 1 = -24\%$

The change in average case (down each column) should be consistent with severity trend (i.e., 3%). It is much higher than 3% for the most part, which therefore indicates change in case adequacy.

## 14. Learning Objectives:

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

### Commentary on Question:

*This question tests the candidate's understanding of deductible pricing and the use of external data.*

### Solution:

- (a) Calculate the indicated deductible factor for a deductible of 1,000.

Indemnity eliminated at 500 deductible =  $1,147,000 + 7,900 \times 500 = 5,097,000$

Total Indemnity at 500 deductible =  $21,039,000 - 5,097,000 = 15,942,000$

Indemnity eliminated at 1,000 deductible =  $1,147,000 + 1,610,000 + 5,600 \times 1,000 = 8,357,000$

Total Indemnity at 1,000 deductible =  $21,039,000 - 8,357,000 = 12,682,000$

1,000 deductible relativity =  $12,682,000 / 15,942,000 = 0.796$ .

- (b) Recommend a factor for a deductible of 1,500. Justify your recommendation.

Indemnity eliminated at 2,000 deductible =  $1,147,000 + 1,610,000 + 2,982,000 + 3,500 \times 2,000 = 12,739,000$

Total Indemnity at 2,000 deductible =  $21,039,000 - 12,739,000 = 8,300,000$

2,000 deductible relativity =  $8,300,000 / 15,942,000 = 0.521$

Therefore, the factor for a deductible of 1,500 needs to be between 0.796 and 0.521, and we can use the consistency test to find the appropriate range for a factor.

Let  $x$  = relativity for a 1,500 deductible.

Based on consistency test, the marginal difference in the deductible factors should decrease as the deductibles increase.

## 14. Continued

For the 500 to 1,000 deductible compared to the 1,000 to 1,500 deductible:

$$\frac{1-0.796}{1,000-500} > \frac{0.796-x}{1,500-1,000}, \text{ which solves for } x > 0.592.$$

For the 1,000 to 1,500 deductible compared to the 1,500 to 2,000 deductible:

$$\frac{0.796-x}{1,500-1,000} > \frac{x-0.521}{2,000-1,500}, \text{ which solves for } x < 0.659.$$

Therefore, any deductible factor that satisfies  $0.592 < x < 0.659$  is acceptable.

- (c) Describe two challenges in using industry data for your company's deductible analysis.

Any two of the following are acceptable:

- The industry data may not be available for all the deductibles you need for your company (e.g., 1,500)
- There would not be a precise mapping in the treatment of ALAE in the industry data and that used by the company
- The data may be incomplete in that losses that fall within insureds' deductibles are likely not included in the data underlying the size of loss distribution
- Claims aggregated on an industry basis are generally not adjusted to an ultimate value or adjusted for trend

## 15. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.
6. The candidate will understand the need for monitoring results.

### Learning Outcomes:

- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.
- (3d) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.
- (6b) Analyze actual claims experience relative to expectations.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 15, 23, and 36.

### Commentary on Question:

*This question tests the frequency-severity closure method of estimating ultimate claims. In addition, this question tests the estimation of claim liabilities as well as the understanding of expected paid claims for an interim period between actuarial analyses.*

### Solution:

- (a) Calculate the indicated ultimate frequency at the 2017 cost level using a simple average of accident years 2014 to 2017.

Accident Year	(1) Earned Exposures	(2) Projected Ultimate Counts from Development Method	(3) = (2)/(1) Indicated Ultimate Frequency	(4) Frequency Trend Factor	(5) = (3)(4) Indicated Ultimate Frequency @ 2017 Level
2014	55,000	1,507	0.0274	0.9127	0.0250
2015	56,000	1,500	0.0268	0.9409	0.0252
2016	57,100	1,465	0.0257	0.9700	0.0249
2017	58,000	1,435	0.0247	1.0000	0.0247
				Average:	0.0250

e.g., (4) 2015 Frequency Trend Factor:  $0.9409 = (1 - 0.03)^2$

## 15. Continued

- (b) Calculate the selected ultimate counts for accident years 2014 to 2017 using the indicated ultimate frequency from part (a).

$$(6) = 2.50\% \times (1)/(4)$$

Accident Year	Projected Ultimate Counts
2014	1,507
2015	1,488
2016	1,472
2017	1,450

- (c) Calculate the proportion of closed counts at 24 months maturity using a simple average of accident years 2014 to 2016 and the selected ultimate counts from part (b).

Accident Year	(7) Selected Ultimate Counts	(8) Closed Counts at 12 months	(9) = (7) – (8) Outstanding at 12 months	(10) Closed Counts at 24 months	(11) = (10)/(9) Proportion Closed at 24 months
2014	1,507	910	597	305	0.511
2015	1,488	905	583	300	0.515
2016	1,472	890	582	295	0.507
				Average	0.511

- (d) Calculate the incremental paid severity at the 2017 cost level for the 12 and 24 month maturities using a weighted average of all accident years.

Incremental Paid Severities @ 2017 Level		
Accident Year	12	24
2014	9,111	18,223
2015	9,085	18,387
2016	9,048	18,408
2017	9,000	
Average	9,061	18,339

e.g., 2015 @ 12 months:  $9,085 = 8,400 \times 1.04^2$   
 2014 @ 24 months:  $18,223 = 18,000 \times 1.04^3 \times 0.90$

## 15. Continued

- (e) Calculate the ultimate claims for accident years 2015 to 2017.

Accident Year	Incremental Paid Severity			
	12	24	36	48
2015	8,400	17,000	23,000	27,737
2016	8,700	17,700	24,038	28,846
2017	9,000	18,339	25,000	30,000

e.g.,  $24,038 = 25,000 / 1.04$

Accident Year	Incremental Closed Counts				Ultimate Counts
	12	24	36	48	
2015	905	300	225	58	1,488
2016	890	295	227	60	1,472
2017	870	296	224	60	1,450
Proportion closed		0.511	0.79	1.00	

e.g., 2016 @ 36 months:  $227 = 0.79 \times (1,472 - 890 - 295)$   
 2016 @ 48 months:  $60 = 1,472 - 890 - 295 - 227$

Accident Year	Projected Incremental Claims = Incremental Paid $\times$ Increment Closed Counts				Ultimate Claims
	12	24	36	48	
2015	7,602,000	5,100,000	5,175,000	1,608,746	19,485,746
2016	7,743,000	5,221,500	5,456,626	1,730,760	20,151,886
2017	7,830,000	5,428,344	5,600,000	1,800,000	20,658,344

e.g., 2016 @ 36 months:  $5,456,626 = 24,038 \times 227$

- (f) Calculate the unpaid claims for accident years 2015 to 2017.

Accident Year	Paid to Date	Unpaid =
		Ultimate – Paid to Date
2015	17,877,000	1,608,746
2016	12,965,000	7,186,886
2017	7,830,000	12,828,344

## 15. Continued

- (g) Complete the table by calculating the expected paid claims on closed counts for accident years 2015 to 2017.

Actual versus Expected Paid Claims on Closed Counts (000)  
from Dec. 31, 2017 through Dec. 31, 2018

Accident Year	Actual	Expected	Difference
2015	1,600	1,609	-9
2016	5,440	5,457	-17
2017	5,450	5,428	22

e.g., Expected claims from the next year diagonal of the projected incremental claims determined in part (e).

- (h) Explain what the result of part (g) implies about using the frequency-severity closure method to estimate ultimate claims in this case.

The results are reasonably close to actual so this implies that the frequency-severity closure method is reasonable in this case.

## 16. Learning Objectives:

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

### Learning Outcomes:

- (4a) Identify the time periods associated with trending procedures.
- (4d) Describe the influences on exposures and premiums of changes in deductibles, changes in policy limits, and changes in mix of business.
- (4e) Choose trend rates and calculate trend factors for exposures.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 26.

### Commentary on Question:

*This question tests premium trending due to vehicle rate group drift for a self-insurer.*

### Solution:

- (a) Explain why vehicle rate group drift should be reflected in the ratemaking for an automobile book of business.

Premiums in a portfolio of insureds tend to increase over time as insureds trade in old vehicles, with low model year factors, for new vehicles, with higher model year factors.

- (b) Assess the reasonableness of the student's recommendation.

Vehicle Rating Group	Earned Vehicles by Vehicle Rating Group				Current Differentials
	2013	2014	2015	2016	
1	400	385	285	250	1.00
2	300	385	380	450	1.90
3	300	330	285	300	2.50
Total	1,000	1,100	950	1,000	
Average differential:	1.720	1.765	1.810	1.855	
Change in differential:		2.62%	2.55%	2.49%	

e.g., 2014 average:  $1.765 = ((1.00 \times 385) + (1.90 \times 385) + (2.50 \times 330)) / 1,100$   
2013 to 2014 change:  $2.62\% = 1.765 / 1.720 - 1$

The assessment is that the student's recommendation seems a little low as all three years are somewhat higher than the 2.3% recommendation.



## 16. Continued

- (c) Calculate the trend factor applicable to calendar year 2015, using the annual premium trend of 2.3% recommended by the student.

Average earned date in calendar year 2015 (self-insured): July 1, 2015

Average earned date in future rating period (self-insured): July 1, 2018

Trending period (years): 3

Trend factor =  $1.023^3 = 1.071$

## 17. Learning Objectives:

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5e) Demonstrate the use of credibility in ratemaking.
- (5g) Calculate risk classification changes and territorial changes.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

### Commentary on Question:

*This question tests the candidate's understanding of classification ratemaking.*

### Solution:

- (a) Explain why implied development factors are required to estimate ultimate claims.

Implied development factors are required to take into account the additional methods used in determining ultimate values beyond simply the development method.

- (b) Explain why the pure premium method and the claim ratio method might not provide the same result for a risk classification analysis.

Differences arise due to the approximations that are made for premium adjustment factors (e.g., premium trend and on-level factors) that are required for the claim ratio approach.

## 17. Continued

- (c) Calculate the indicated relativities to the base Territory A using the claim ratio approach.

	(1)	(2)	(3)	(4)	(5)	(6)
Terr	Ultimate Counts	Calendar Year 2017 Earned Premium at Current Rates	Initial Indicated Claim Ratio Relativity	Current Relativity	Industry Relativity	Industry Credibility
A	1,200	25,000	1.025	1.000	1.000	100%
B	900	18,000	1.035	1.100	1.050	100%
C	400	13,000	1.011	0.900	0.850	50%
D	800	15,000	0.907	0.850	0.900	75%
Total	3,300	71,000	1.000	0.975	0.964	

	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Terr	Credibility	Industry Relativity Rebalanced	Current Relativity Rebalanced	Industry Credibility	Balance of Credibility	Credibility Weighted Indicated Relativity	Rebalanced to Base Territory
A	52.6%	1.037	1.026	47.4%	0.0%	1.031	1.000
B	45.6%	1.089	1.128	54.4%	0.0%	1.064	1.032
C	30.4%	0.882	0.923	50.0%	19.6%	0.929	0.901
D	43.0%	0.934	0.872	57.0%	0.0%	0.922	0.894

Notes: (7) =  $\text{Min}(\text{squareroot}[(1) / 4,331], 1)$

(8) =  $(5)_{\text{terr}} / (5)_{\text{Total}}$

(9) =  $(4)_{\text{terr}} / (4)_{\text{Total}}$

(10) =  $\text{Min}[(1 - (7)), (6)]$

(11) =  $1 - (7) - (10)$

(12) =  $(3)(7) + (8)(10) + (9)(11)$

(13) =  $(12)_{\text{terr}} / (12)_A$

## 18. Learning Objectives:

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

### Learning Outcomes:

- (5c) Incorporate underwriting profit and contingency margins into ratemaking.
- (5d) Calculate loadings for catastrophes and large claims.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 29 and 30.

### Commentary on Question:

*This question tests loadings for large claims as well as the importance of fixed and variable expenses in ratemaking.*

### Solution:

- (a) Provide an example of what an insurer could do to continue to provide consumer access to homeowners insurance coverage in hurricane prone areas.

The insurer could implement a catastrophe model to better predict losses.

- (b) Explain the appropriateness of using this noninsurance data for a ratemaking analysis.

It is the actuary's responsibility to ensure that the use of the non-insurance data results in ratemaking procedures that appropriately reflect the expected frequency and severity distribution of catastrophes, as well as anticipated class, coverage, geographic, and other relevant exposure distributions.

- (c) Describe whether the expense associated with each incentive structure above should be categorized as fixed or variable.

I. Fixed expense – because it varies by policy rather than premium.

II. Variable expense – increase to commission expense varies with written premium.

III. Fixed expense – Other acquisition expense does not vary with premium.

- (d) Describe a consequence of an insurer treating fixed expenses as variable expenses when determining rates.

Treating all expenses as variable can lead to inadequate expense provisions for insureds with low premium and excessive expense provisions for insureds with high premium.

## 18. Continued

- (e) Describe whether the expense associated with each incentive structure above should be related to earned premium or written premium when calculating an expense ratio.
- I. The expense ratio should be shown relative to written premium because it is incurred at policy inception.
  - II. The expense ratio should be shown relative to written premium because it is incurred at policy inception.
  - III. The expense ratio should be shown relative to earned premium because it is incurred throughout the year.
- (f) Describe the result of selecting an inappropriate premium type to calculate an expense ratio in times of significant growth.

If written premium is selected instead of earned premium, the ratio will be lower than it should be.

If earned premium is selected instead of written premium, the ratio will be higher than it should be.

## 19. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

### Learning Outcomes:

- (2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

### Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 14.

### Commentary on Question:

*This question tests the candidate's understanding of estimating ultimate claims using the development method.*

### Solution:

- (a) State two potential limitations of using benchmark data for tail factors.

Any two of the following are acceptable:

- differences in how claims are adjusted or reserved
- differences in initial reporting pattern
- differences in adjudication process for litigated claims
- differences in the potential for long-developing high value claims
- statistical reliability of the benchmark triangle

- (b) Explain how you would decide whether or not to use this benchmark data in the selection of a tail factor.

Compare age-to-age factors of benchmark data against own data. If the patterns are similar, consider using the data. If not, then adjust or reject if the data cannot be adjusted.

## 19. Continued

- (c) Calculate the age-to-age factors for paid claims using the geometric method.

Accident Year	Cumulative Paid Claims			
	12	24	36	48
2014	29,000	35,000	38,000	39,000
2015	21,000	30,000	33,000	
2016	23,000	29,000		
2017	27,000			

Paid development factors:

Accident Year	12-24	24-36	35-48
2014	1.207	1.086	1.026
2015	1.429	1.100	
2016	1.261		

Geometric  
average:

$$\text{e.g., } 1.296 = (1.207 \times 1.429 \times 1.261)^{(1/3)}$$

- (d) Calculate the accident year 2017 ultimate claims using the original Bondy method for the tail factor.

Repeat final age-to-age factor:

$$1.296 \times 1.093 \times 1.026 \times 1.026 = 1.491$$

$$\text{Ultimate claims} = 27,000 \times 1.491 = 40,257.$$

- (e) State one advantage and one disadvantage of Boor's algebraic method.

Advantage: The method is based entirely on data in triangles, so no other data is needed.

Disadvantage: Reliable estimates of ultimate claims for most mature periods is needed and they are not always available.

## 19. Continued

- (f) Calculate paid claims tail factors for accident years 2014 and 2015 using Boor's algebraic method.

Ultimate claims (based on reported):

$$\text{AY 2014: } 41,000 \times 1.02 = 41,820 \quad (1)$$

$$\text{AY 2015: } 36,000 \times 1.05 \times 1.02 = 38,556 \quad (2)$$

Paid claims developed to 48 months:

$$\text{AY 2014: } 39,000 \quad (3)$$

$$\text{AY 2015: } 33,000 \times 1.026 = 33,858 \quad (4)$$

Paid claim tail factors:

$$\text{AY 2014} = (1) / (3) = 1.072$$

$$\text{AY 2015} = (2) / (4) = 1.139$$