

QFI ADV Model Solutions

Spring 2018

1. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

- (4c) Describe and assess performance measurement methodologies for assets portfolios.
- (4f) Calculate and interpret performance attribution metrics for a given asset, portfolio.

Sources:

Ch 69: Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012.

Ch 70: Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012.

CAIA Level II, Advanced Core Topics in Alternative Investment, 2nd Edition, 2012, Ch. 16.

Commentary on Question:

This question tests candidates' understanding in performance attribution techniques for equity and fixed income portfolio

Solution:

- (a)
 - (i) Recommend a correction to Dave's portfolio attribution model
 - (ii) Recalculate his outperformance.

Commentary on Question:

Candidates performed as expected in this question. Most candidates were able to identify the correct security selection formula to be used and calculate correct outperformance. Only a few candidates were able to identify that the interaction term was missing and should be calculated separately or incorporated into security selection.

1. Continued

Dave's formula is missing interaction term. He should add interaction term or incorporate interaction. This can be corrected by calculating term explicitly

$$\sum_s (w_s^P - w_s^B) * (R_s^P - R_s^B) \text{ (which is -3\%)}$$

OR by incorporating the correction into the security selection term

$$\sum_s w_s^P * (R_s^P - R_s^B) \text{ (which - is 1.0\%)}$$

Dave's outperformance is actually $1.5\% + 4\% - 3.0\% = 2.5\%$ (or $1.5\% + 1\% = 2.5\%$)

- (b) Explain whether you agree with Dave's belief that he has expertise in the Technology sector.

Commentary on Question:

Most candidates performed as expected on this question. Most candidates correctly disagree and identify the underperformance in technology compared to the benchmark. Only a few candidates identified that Dave is correct on outperformance overall but failed to pick top performers in the sector.

I disagree with Dave's belief. His return is lower than the benchmark and he fails to pick top performers in the sector; however, he has a correct view on outperformance overall

- (c) Critique each of the fixed income portfolio manager's comments.

Commentary on Question:

Candidates performed below average on this question. Most candidates were correct on Statement (I), but did not get full mark on the rest of the statements.

Statement I - disagree

Tina's portfolio outperformed by 20 bps

Statement II - agree

The yield-curve change outperformance of 14.6 bps can be estimated by taking the yield change at the average maturity point of the benchmark of -31.6 bps, then multiplying by the (negative of) the average duration overweight of 0.48 years ($4.66 - 4.18$), which equal 15.17 bps outperformance contribution The 14.6 bps also takes into account the duration exposure fluctuated during the period.

Statement III - agree

$14.6 - 15.17 = -0.57\%$, which is the impact of duration exposure fluctuated during the period

Statement IV - disagree

The overweighting of 5 year point cannot be justified with info given from Tina Also, an increase in yield should result in an underperformance

2. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

- (1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

- (1h) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

- (4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.

- (4i) Demonstrate an understanding of the general uses and techniques of stochastic modeling.

Sources:

Brigo, D and Mercurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 4.1, p.137-138,142

Brigo, D and Mercurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 4.1, p.140-141.

Brigo, D and Mercurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 4.1, p.139.

Brigo, D and Mercurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 4.1, p.59, 69, 91

QFIA-125-16 Market Models A Guide for Financial Data Analysis, Ch. 6

QFIA-124-16, StochModeling Theory and Reality from an Actuarial Perspective, I-7, I-8, I-9, I-10

2. Continued

Commentary on Question:

This question tests the candidate's understanding of the G2++ and CIR2++ models, the use of PCA for model selection, as well as the reasons for using, or not using, market prices to calibrate a model.

Solution:

- (a) Show that the absolute volatility of the instantaneous forward rate $f(t, T)$ at time t for the Vasicek model is $\sigma e^{-k(T-t)}$.

Commentary on Question:

Candidates performed as expected on this section. Most candidates were able to express the forward rate $f(t, T)$ as a partial derivative of P as a function of T , and then a partial derivative of B as a function of T . Successful candidates were those who were able to proceed to the next step showing $\frac{\partial\left(\frac{1}{k}[1-e^{-k(T-t)}]\right)}{\partial T}$.

$$f(t, T) = -\frac{\partial \ln P(t, T)}{\partial T}$$
$$\sigma_f(t, T) = \frac{\partial B(t, T)}{\partial T} \sigma(t, r(t))$$
$$\sigma_f(t, T) = \frac{\partial\left(\frac{1}{k}[1-e^{-k(T-t)}]\right)}{\partial T} \sigma$$
$$\sigma_f(t, T) = \sigma e^{-k(T-t)}.$$

- (b) Assess whether a hump feature is possible in the term structure of the instantaneous forward rate volatilities.

Commentary on Question:

The candidates performed above average on this section. Unsuccessful candidates argued that even $\sigma_f(t, T)$ is monotonic decreasing function, a hump is still possible in some situations which is incorrect.

As $e^{-k(T-t)}$ is a decreasing function of $T-t$, $\sigma_f(t, T) = \sigma * e^{-k(T-t)}$ is monotonic decreasing.

As $\sigma_f(t, T) = \sigma * e^{-k(T-t)}$ is monotonic decreasing, hump is not possible.

- (c) Describe the advantages of the two-factor G2++ model versus the one-factor model in this situation.

2. Continued

Commentary on Question:

The candidates performed below average on this section. A majority of the candidates correctly mentioned the short-comings of most one-factor models in terms of correlation modeling (perfect correlation). Successful candidates were those who mentioned the pricing of exotic interest rate derivatives might involve the correlation between spot rates of different maturities, and recognized that such a flexibility is needed in the modeling of this correlation.

- In order to price exotic interest rate derivatives accurately, it is often useful to model the correlation between spot rates for different maturities.
 - The observed correlation between two such rates is not usually equal to 1. Therefore, we need some flexibility in the modeling of the yield curve to reflect this fact.
 - Most one factor models do not allow for realistic modeling of the correlation between spot rates for different maturities. For example, this correlation is always 1 in the one-factor Vasicek model.
 - Two-factor models such as the G2++ model allow for more realistic modeling of the correlation between spot rates for different maturities, while keeping a good level of analytical tractability.
- (d) Describe the advantages of using the two-factor G2++ model over the CIR2++ model when the Brownian motions within each model are correlated.

Commentary on Question:

The candidates performed below average on this section. Successful candidates explained why the restriction on ρ in the CIR2++ model can be a problem.

- The G2++ model is analytically tractable for any value ρ , while the analytical tractability of the CIR2++ model is lost when ρ is not equal to 0.
 - This restriction on ρ reduces the flexibility of the CIR2++ model. For example, with ρ equal to zero, it cannot reproduce a humped volatility curve.
- (e) Explain why, based on the matrices above, your team should consider using the two-factor G2++ model instead of the one-factor model.

Commentary on Question:

The candidates performed above average on this section. Most candidates were able to quantify the explanatory power of the “first” and “first and second” components. Successful candidates were those who mentioned principal component analysis, as well as the interpretation of the differences in the explanatory powers of the “first” and “first and second” components. For example, one should explain something along the line “the use of a two-factor model instead of a one-factor one allow to explain an additional 20% of the variation”.

2. Continued

The candidate must use the eigenvalues provided to calculate the variation explained by one- and two-factor models. If no numerical analysis was provided partial credit was given.

- Studying the matrix of covariance of the returns on bonds of different maturities can help identify the number of factors needed to accurately model the variation and covariation in the series of returns. The decomposition of the matrix presented in the question is called the spectral decomposition and is useful for principal component analysis (PCA). The eigenvalues forming the diagonal of the matrix L can help select the number of factors necessary to accurately reproduce the variations in the bond returns using a model where the short rate is a linear combination of factors.
 - Using the data from the question, we know that the first principal component explains $2.93/4 = 73.25\%$ of the total variation.
 - A model using two components would explain $(2.93+0.87)/4 = 95\%$ of the total variation where $0.87=4-2.93-0.12-0.08$ as the sum of eigenvalues (trace of correlation matrix).
 - Therefore, the use of a two-factor model instead of a one-factor one increases allow to explain an additional 20% of the variation. This is a good indication that a two-factor model should be used.
- (f) Assess whether the scenarios can be used for the following tasks:
- (i) Pricing exotic interest rate derivatives
 - (ii) Calculation of the market-consistent (fair) value of an insurance liability
 - (iii) Calculation of the capital necessary to support worse-case scenarios such as CTE99
 - (iv) Estimation of the volatility of the earnings

Commentary on Question:

The candidates performed above average on this section. Successful candidates identified that the model is calibrated under the risk-neutral measure and identified the type of tasks (market-value related calculations) for which this calibration is suitable. Unsuccessful candidates failed to identify that the model is not suitable for calibration involving real-world probabilities.

2. Continued

- When it is calibrated using market prices of swaptions, the model is calibrated under the risk-neutral measure.
 - Therefore only be used for assessing the market values of different quantities. This calibration could be used to price exotic interest rate derivatives (i) and to calculate the market consistent (fair) value of an insurance liability (ii).
 - When it is calibrated using market prices of swaptions, the model is not suitable for calculations that should be done under the real-world measure.
 - Therefore, it is not suitable for the estimation of the volatility of the earnings (iii) and the calculation of the capital necessary to support worse-case scenarios (iv).
- (g) Explain the advantages of using G2++ model for pricing out-of-the-money Bermudan swaptions after calibration to the corresponding at-the-money European swaption.

Commentary on Question:

The candidates performed poorly on this section. Successful candidates explained the presence of the smile effect in the market, identifying that it may be captured by a distribution with a significant mass around zero. Generally unsuccessful candidates did not attempt this section of the question.

- Under G2++ model, a non-perfect correlation between rates of different maturities results in a more precise calibration to correlation-based products like European swaptions. Such a model can also be helpful when pricing out-of-the-money exotic instruments after calibration to at-the-money plain-vanilla products.
- The smile effect that is present in the market can be better captured by a distribution with a significant mass around zero.

3. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

- (3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.
- (3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.
- (3f) Apply liquidity scenario analysis with various time horizons.

Sources:

QFIA-106-13: Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices, Matz, Leonard & Neu, Peter, 2006, Ch. 3

QFIA-105-13: Report of the Life Liquidity Work Group of the American Academy of Actuaries to the Life Liquidity Risk Working Group of the NAIC

Commentary on Question:

This question tested the candidate's understanding of liquidity risks in different environments, proper liquidity stress testing, and how to interpret a specific liquidity stress scenario. Candidates should keep in mind that liquidity risk is not the same as ALM risk. Using XYZ as an example, increased checking account balances would improve liquidity due to the cash inflow, but increase their liabilities. Similarly, mortgage demand would present a liquidity need, even though new assets would be added. Many candidates did not understand this distinction.

Solution:

- (a) Describe how each of the following scenarios would impact XYZ's liquidity position:
 - (i) Increase in interest rates
 - (ii) Credit rating downgrade of XYZ
 - (iii) General downturn in economic activity that is not institution-specific

3. Continued

Commentary on Question:

Candidates performed above average on this section. Successful candidates described how each scenario would impact the cash flows of the bank's products, and formed a general conclusion about the impact to liquidity risk based on that. Partial credit answers lacked discussion about XYZ specifically, provided shallow explanation for the impact, or failed to provide a general assessment of the liquidity impact to XYZ. Few candidates identified ways liquidity would be positively impacted. Several candidates justified impacts that are relevant to banks in general (i.e. lack of ability to raise funds) which also received credit.

This question is open-ended; justified interpretations different than as illustrated below were acceptable.

Part (i):

- Withdrawals from checking accounts in search of higher yields increases liquidity risk
- Older CDs may withdraw for higher return, but new CD deposits should increase for a minimal liquidity impact
- Lower income from prepayments (assuming fixed-rate mortgages), but slight offset from lower mortgage demand due to high rates, slight increase to liquidity risk
- Overall impact is a moderate increase to liquidity risk for XYZ

Part (ii):

- Consumer panic may trigger "run-on-the-bank" as XYZ's reputation deteriorates, resulting in checking and CD withdrawals and a significant liquidity crunch
- Mortgage and credit card payments may be unaffected as they are borrowers from XYZ instead of lenders
- Overall impact is a significant increase in liquidity risk

Part (iii):

- Checking and CD balances may decline as consumers become less wealthy, increasing liquidity needs
- Mortgage and credit card defaults may increase as consumers lack funds to pay, reducing cash income
- Less mortgage demand as a result of general wealth decrease would somewhat reduce the liquidity demand
- Overall impact is a moderate increase to liquidity risk

- (b) Recommend and justify improvements and additions to the proposed scenarios and analysis framework.

3. Continued

Commentary on Question:

Candidates performed as expected on this question. Successful candidates provided improvements covering several aspects with sufficient justification for each. Partial credit answers may not have fully justified their recommendations or addressed a sufficiently broad set of characteristics. Unsuccessful candidates received no credit for simply listing recommendations. To receive credit any recommendation required support or justification.

Similar to (a), this is an open-ended question, and reasonable responses with good justification received credit even if they differ from the below.

- Recommend adding more stress levels, such as interest rates +/- 3% and credit downgrade by 2 notches. Even extreme scenarios are often not as bad as reality, so many extreme scenarios should be tested.
 - Recommend including shorter and longer timeframes, such as 3 days and 1 year. Liquidity crises can erupt rapidly or develop slowly over many months, so several timeframes are required.
 - Recommend removing the emerging market scenario. Liquidity scenarios need to be relevant, and as a domestic bank XYZ should not include an emerging market crisis scenario.
 - Recommend including scenarios around consumer behavior, such as a run-on-the-bank. XYZ is sensitive to changes in changes in consumer behavior independent of broader financial/economic events.
- (c) Calculate the minimum liquidity reserve that XYZ should immediately establish for this scenario.

Commentary on Question:

Candidates performed below average on this question. Successful candidates separated flows into cash sources and cash needs, correctly rolled forward positive cash balances, and determined the correct minimum reserve. Very few candidates understood how to perform this calculation.

*Partial credit was awarded for demonstrating an understanding of the cash flow cushion by splitting flows into sources and needs, calculating the cushion each month, and understanding that the reserve = 105%*Needs – Sources.*

Unsuccessful candidates received no credit for simply adding all the cash flows, or not understanding the reserve.

Candidates needed to recognize that positive liquidity on hand accumulates in scenarios but needs to be assessed period-by-period.

3. Continued

Month 1:

Cash Sources = $950 + 150 = 1,100$

Cash Needs = $300 + 500 = 800$

Cash Flow Cushion = Sources/Needs = $1,100/800 = 1.38$ – Pass

End of Month Cash = $1,100 - 800 = 300$, which gets rolled to month 2.

Month 2:

Cash Sources = 300 (rolled over) + $900 + 100 = 1,300$

Cash Needs = $1,000 + 350 = 1,350$

Cash Flow Cushion = $1,300/1,350 = 0.96$ – Fail

Month 2 Reserve Need = $1,350*(105\%) - 1,300 = 117.5$

End of Month Cash with Reserve = $(1,300 + 117.5) - 1,350 = 67.5$, which gets rolled to month 3

Month 3:

Cash Sources = 67.5 (rolled over) + $800 = 867.5$

Cash Needs = $50 + 400 + 750 = 1,200$

Cash Flow Cushion = $867.5/1,200 = 0.72$ – Fail

Month 3 Reserve Need = $1,200*(105\%) - 867.5 = 392.5$

End of Month Cash with Reserve = $(867.5 + 392.5) - 1,200 = 60$, which gets rolled to month 4

Month 4:

Cash Sources = $60 + 800 = 860$

Cash Needs = $100 + 50 + 150 = 300$

Cash Flow Cushion = $860/300 = 2.87$ – Pass

Final Reserve Calc:

Minimum reserve = required month 2 reserve + required month 3 reserve

Minimum reserve = $117.5 + 392.5 = 510$

If we have a reserve of 510, then 50 is used in month 2 to satisfy the liquidity needs, and 460 gets rolled to month 3 for a cash flow cushion of $(460 + 800)/1200 = 105\%$

Alternative Calculation

You can arrive at 510 also by working backwards. Once we identify month 3 as having the worst cash flow cushion, we find that we need 460 at month 3

$(1,200*105\% - 800)$ to pass it. To have an additional 460 in month 3, we need to have $460+X$ in month 2, where X gets liquidated to help survive the scenario.

With month 2 flows only, this need is $1,350 - 1,000 = 350$, so we need $460+350 = 810$ of additional cash going into month 2. Month 1 generates a cash surplus of 300, so we need an initial liquidity reserve of $810 - 300 = 510$ to pass month 3.

4. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1c) Explain the dynamics of and motivation for the Hull-White extension of the Vasicek model.
- (1d) Explain the features of the Black-Karasinski model.
- (1e) Understand and explain the relationship between market-quoted caplet volatilities and model volatilities.

Sources:

Brigo, D and Mercurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.3, 3.5, 3.6

Commentary on Question:

This question tested knowledge of stochastic interest rate models and use and structure of given formulas.

Solution:

- (a) Interpret the coefficients a and σ .

Commentary on Question:

Candidates performed at an outstanding level showing an understanding of both the mean reversion and standard deviation parameters, usually achieving full credit.

The coefficient σ is the standard deviation of the interest rate per unit time
The coefficient a is the strength of mean reversion.

- (b) Calculate the risk-neutral probability of a negative rate at year 3.

4. Continued

Commentary on Question:

Candidates performed as expected, typically scoring most grading points. Common reasons for not achieving full marks were not showing their work by omitting to write down the formulas and then following through with a miscalculation of the solution.

$$\alpha(t) = f^M(0, t) + \frac{\sigma^2}{2a^2}(1 - e^{-at})^2 = 0.175$$

$$Q\{r(t) < 0\} = \Phi\left(-\frac{\alpha(t)}{\sqrt{\frac{\sigma^2}{2a}(1 - e^{-2at})}}\right) = 0.4$$

- (c) Compare the Hull-White Extended Vasicek model and the Black-Karasinski Model.

Commentary on Question:

Candidates performed as expected on this question, demonstrating an understanding of the differences between each model and understood the calibration of the models, often achieving full credit

Both are exogenous and match the yield curve using time-dependent coefficients. Hull-White uses the normal distribution, is analytically tractable and allows negative rates, BK is lognormal, non-tractable, and always positive.

- (d) Contrast market caplet volatility and model implied caplet volatility.

Commentary on Question:

Candidates performed below average on this question, usually scoring half credit as a result of not understanding the implied volatility of the caplet. They did well in calculating the market price of the caplet.

The market caplet sigma is the parameter that must be plugged into the Black caplet formula to achieve the market price, the implied volatility is the deterministic solution to the caplet formulas.

4. Continued

- (e) Justify whether A or B is the term structure for cap volatility.

Commentary on Question:

Candidates performed as expected, usually getting the correct answer with some supporting details. Candidates who did not get full credit often excluded supporting details in their answer.

A is the term structure of the market cap volatility, this volatility assumes that all concurrent caplets share the same volatility with the market price equal to average caplet price.

5. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

- (3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.
- (3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.
- (3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.

Sources:

Quantitative Credit Portfolio Management, Ben-Dor, et.al., Ch 5, Ch 6

Commentary on Question:

This question tests candidates' understanding on concept of Liquidity Cost Score and OAS decomposition.

Solution:

- (a)
 - (i) Describe the relationship(s) between these attributes and LCS.
 - (ii) Design a ranking of these four bonds in terms of their liquidity attributes relative to each other.

Commentary on Question:

For part i) candidates performed as expected. Most candidates identified 3 of 4 attributes such as Trading Volume, Issue Size, and Aging. While 3 of 4 was sufficient to attain full marks, most candidates did not identify the DTS attribute and in some cases treated Duration, and Spread separately as 2 of their 3 components which resulted in them not achieving full marks.

For part ii) candidates performed poorly. Very few provided a ranking with sufficient support in terms of an explanation or justification. Candidates were not required to identify all attributes needed to establish a valid ranking, appropriate credit was given to candidates who took this approach (the solution below shows a ranking with all attributes used).

5. Continued

(i)

Trading Volume: LCS is negatively correlated to a bond's trading volume. Higher trading volume indicates lower liquidity costs (LCS). Traders are willing to make narrow bid-ask markets for bonds with large trading volume and many market participants. ~~to offload any undesired risks.~~

Issue Size: LCS is negatively correlated to bond issue size. Bid-ask spreads are typically tighter for issues with large amounts outstanding; Traders are more likely to find buyers and sellers as issue size increase.

Aging: LCS is positively correlated with age of issue. LCS for longer time since issue is higher than newly issued bonds since many portfolios are buy-and-hold strategy.

DTS: LCS is positively correlated to DTS (Duration Times Spread)
Higher DTS bonds tend to have more mark-to-market volatility, which means more risk for the market maker to bear until the trade is covered.

(ii) For ranking we must utilize all attributes such Volume, Issue Size, Aging and DTS which represent the product of Duration X Spread.

The value of DTS for each bond is:

$$\text{Bond A} = 2.1 \times 200 = 420,$$

$$\text{Bond B} = 1.5 \times 230 = 345,$$

$$\text{Bond C} = 4.9 \times 150 = 735,$$

$$\text{Bond D} = 5.8 \times 110 = 638.$$

Then higher DTS reflect lower liquidity.

For each attribute we assign a rank from 1 to 4 to each bond with higher number as highest liquidity.

Volume Bond A=2, Bond B=4, Bond C=2, Bond D=1.

Issue Size Bond A=4, Bond B=3, Bond C=1, Bond D=2.

Aging Bond A=2, Bond B=3, Bond C=1, Bond D=4.

DTS Bond A=3, Bond B=4, Bond C=1, Bond D=2.

5. Continued

For each attribute, we assign a weighting. My recommendation is an equal weight for each attribute. So, the total for each bond is:

Total Bond A=11, Bond B=14, Bond C=5, Bond D=9.

Then the highest to lowest liquid bond is Bond B, Bond A, Bond D and Bond C.

- (b)
- (i) Calculate the **maximum** value of X.
 - (ii) Calculate the **minimum** value of Y.
 - (iii) Evaluate which bonds, if any, you would add to the portfolio to increase liquidity. Justify your answer.

Commentary on Question:

For this question candidates performed above average. *For part i) some candidates justified that the adjustment factor should not be used because of non-quoted bonds. Full credit was given with this justification. Many candidates calculated the minimum LCS without the adjustment factor, resulting in this value being less than the minimum LCS of the portfolio and in turn providing the wrong justification in part iii). For part ii) some candidates incorrectly applied the adjustment factor in the formula and failed to achieve full credit. For part iii) some candidates did not achieve full marks as they selected a bond with the lowest LCS without considering the impact on the portfolio other candidates wrongly calculated the LCS in their analysis. Very few calculated the minimum for bond E. In general, candidates failed to recognize the choice not to invest was based on the fact that minimum and maximum LCS are higher than the LCS of the portfolio.*

i) Since bond E may or may not be a non-benchmark bond, we must assume it is not. Then we need to apply the adjustment factor to it when calculating the LCS. To calculate the maximum LCS, the lower the bid price the larger the LCS so we chose the minimum bid price in the range.

$$\begin{aligned}\text{LCS} &= \text{AdjustmentFactor} \times (\text{AskPrice} - \text{BidPrice}) / \text{BidPrice} \\ &= 1.3 \times (105 - 100) / 100 \\ &= 6.5\%\end{aligned}$$

This is the maximum LCS for this bond.

ii) Bond F is a benchmark bond so there is no need to apply the adjustment factor. When using spread, the lower the ask spread the higher the LCS so we want to choose the maximum ask spread in the range to calculate the minimum LCS.

5. Continued

$$\begin{aligned}\text{LCS} &= \text{OAS Duration} \times (\text{Bid Spread} - \text{Ask Spread}) \\ &= 7.6\% \times (500 - 460)/100 \\ &= 3.04\%\end{aligned}$$

This is the minimum LCS for this bond.

iii) Both bonds have an LCS higher than the portfolio LCS of 2.4% so adding them to the portfolio would only make the portfolio less liquid. Thus neither bond should be added to the portfolio.

(c)

- (i) Describe and graph a situation that aligns with your analyst's statement using the components of a bond's OAS.
- (ii) Describe and graph a situation that contradicts your analyst's statement using the components of a bond's OAS.

Commentary on Question:

For this question candidates below average with more than 25% of candidates receiving no credit because they chose to not answer this part. Candidates also received minimal credit as they did not mention the 3 components of OAS. Some candidates omitted the graph and only received partial credit.

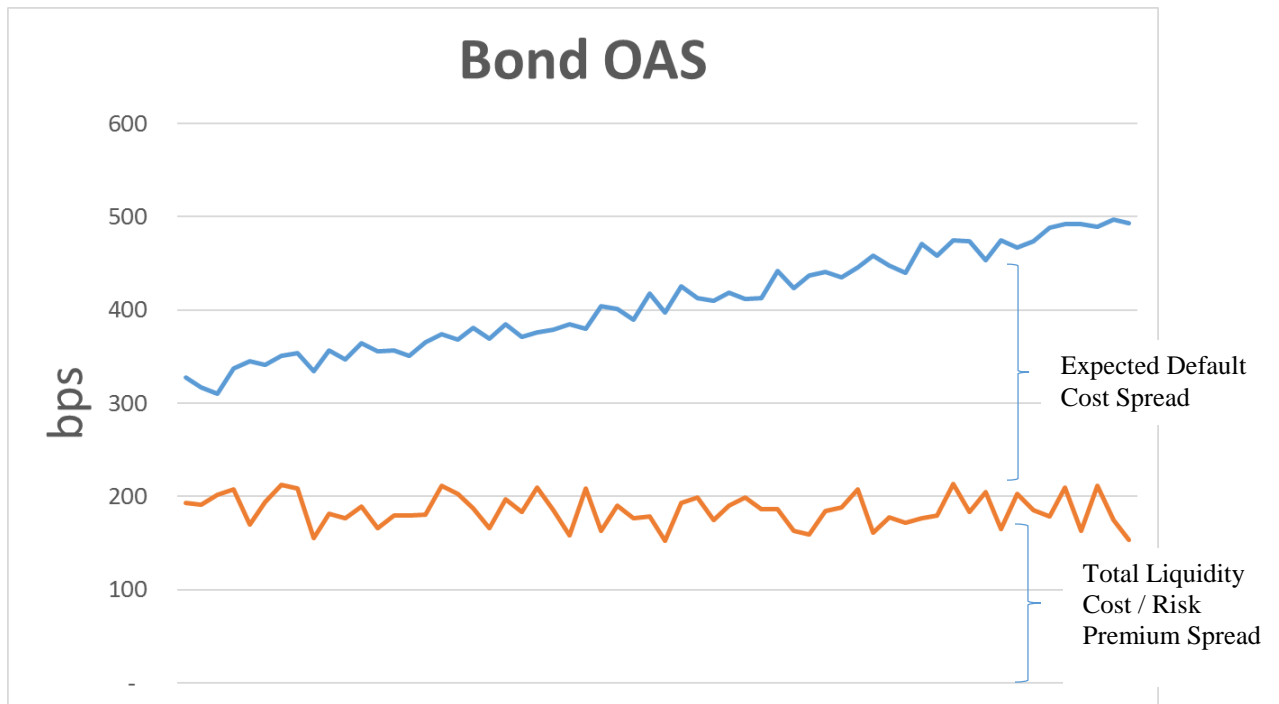
Part i)

The OAS of a bond can be split into three components (pieces):

- i) Expected Liquidity Cost (LCS)
- ii) Expected Default Cost (Can be estimated using CDS spreads)
- iii) Risk Premium (Alpha) – Represents market level risk premium

If the analyst's statement is correct then the Expected Default Cost portion of the OAS would be growing over time faster than the combined liquidity cost and risk premium. This can be diagrammed as shown in the graph below and shows the change in OAS is due to an increase in the expected credit risk and not to changing liquidity/risk premium.

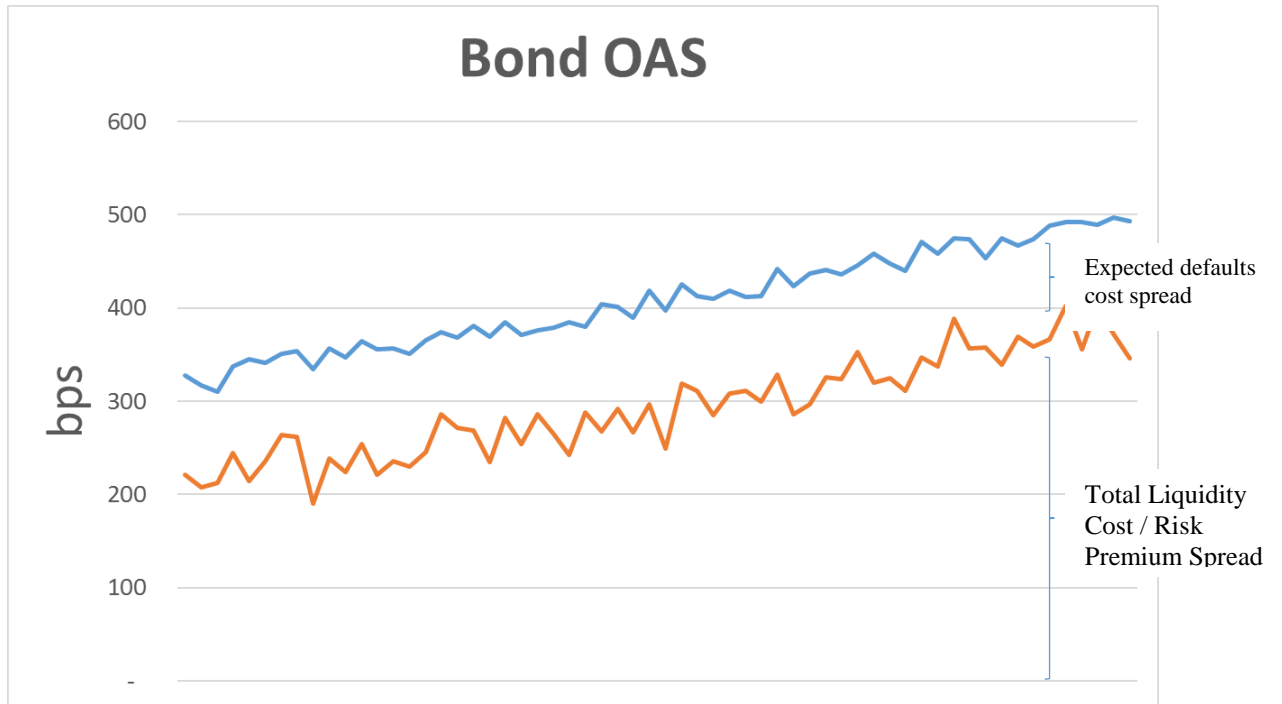
5. Continued



Part ii)

Here we graph the OAS (blue line) with the risk premium and liquidity (red line) and show that the red line follows the same slope as the blue line indicating that the liquidity and risk premium are actually increasing over time (sum of them) but that the expected credit risk (difference between the two) is relatively constant suggesting that the credit risk is not increasing.

5. Continued



6. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
- Real Estate
 - Private Equity
 - Commodities
 - Hedge Funds
 - Managed Futures
 - Distressed Securities
 - Infrastructure

Learning Outcomes:

- (6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.
- (6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.
- (6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.
- (6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

QFIA-111-13: Maginn & Tuttle, *Managing Investment Portfolios*, 3rd Ed. 2007, Ch. 8

QFIA-113-13: *Secular and Cyclic Determinants of Capitalization Rates: The Role of Property Fundamentals, Macroeconomic Factors and “Structural Changes”*

Commentary on Question:

This question tests candidates understanding of alternative assets.

Solution:

- (a) Describe two key issues with the NCREIF index.

Commentary on Question:

The candidates performed above average on this section. Most candidates were able to identify that the NCREIF index is appraisal based. Candidates failed to achieve full marks if they did not identify that the index would underestimate volatility of underlying values.

6. Continued

Appraisal-based values reported in this benchmark, and property appraisal conducted infrequently;
Exhibit remarkable inertia - underestimate volatility in underlying values.

- (b) Describe two key factors that are not included in the standard cap rate model and how both are expected to impact cap rates.

Commentary on Question:

The candidates performed above average on this section. Most candidates were able to identify two key factors; however, some failed to describe how they are expected to impact cap rates, resulting in them not attaining full credit.

Spread/Risk Premium is expected to have a positive effect on cap rates;
Debt flow is expected to have a positive effect on asset values and thus a negative effect on cap rates.

- (c) Calculate the current annual return of XYZ using the standard cap rate model.

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to recognize and use the correct formula from the formula sheet to calculate cap rate and annual return; however, many did not appear to demonstrate an understanding of the components of the formula resulting them not achieving full marks.

$$\text{RRI} = \text{RR}/\text{Mean}(\text{RR}) = 12000/8000 = 1.5$$

$$\text{RTB} = \text{nominal yield} - \text{inflation} = 5\% - 1\% = 4\%$$

$$\text{Cap rate} = \exp(2 + 1 * \ln(11\%) + 1 * \ln(9\%) + 1 * \ln(1.5) + 1 * 4\% + 1 * 1\%)$$
$$= 11.54\% \text{ based on formula (1) in the formula sheet page 10}$$

$$\text{Annual Return} = \text{cap rate} + \text{margin} = 11\% + 1\% = 12.54\%$$

- (d) Calculate both the Sharpe ratio of XYZ and the Sharpe ratio of REITs

Commentary on Question:

The candidates performed above average on this section and were able to use the correct formula to calculate the Sharpe ratio.

$$\text{Sharpe Ratio of XYZ} = (12.54\% - 3\%)/12\% = 0.795$$

$$\text{Sharpe Ratio of REIT} = (10\% - 3\%)/10\% = 0.70$$

- (e) Outline a short memo to the CIO summarizing why these these disadvantages also apply to an institutional investor and risks in support of your position.

6. Continued

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to identify that expenses and political risk applied to both individual and institutional investors. Some candidates pointed out liquidity issues applied to both investors, but failed to recognize that property can't be easily divided into pieces.

Most of the advantages and disadvantages of direct investment in real estate apply to both individual and institutional investors. These include:

Parcels of real estate are not easily divided into pieces which has nothing to do with whether an investor is individual or institutional but with the property itself.

Real estate requires special management expertise which does not necessarily mean that the institution can obtain it more easily.

Expenses (such as information acquisition, commissions, and operating and maintenance costs) may be much higher than for other investments which may be just as material for an institutional investor for a percent allocation.

The tax benefits that real estate may convey carry political risk that they may be discontinued which applies more to the government than whether the taxable entity is an individual or an institution.

7. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.
- (2c) Demonstrate an understanding of credit valuation models.

Sources:

Introduction To Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Ch. 1-3 and 6

Commentary on Question:

This question tests basic concepts of credit risk modeling and practical considerations in selecting modeling methods.

Solution:

- (a) Identify and explain which rating system you will recommend for the CDO business.

Commentary on Question:

The candidates performed below average on this question. Many candidates identified the causal rating system, however very few candidates explained specific considerations for the CDO business.

Causal rating system should be used. The causal rating systems rely in their mechanism on a causal relationship between underlying credit risk drivers and the default event of an asset or borrower. CDO model requires a fully-fledged model for both the cashflow structure of the CDO as well as the credit risk of the underlying reference portfolio. Causal models force the modeling team to really understand how defaults can happen and how losses will accumulate under certain circumstances.

7. Continued

- (b) Summarize in three steps the calibration involving regression of default probabilities to ratings.

Commentary on Question:

The candidates performed below average on this question. It appeared that many candidates wrote down an algorithm that they had memorized from a previous exam that while on the same topic was not applicable for this question. This isn't meant to be a tricky question, and those who correctly wrote down intuitive thoughts about the process received credits.

Step 1: Calculate the historic default frequency of rating class R for year i. Compute the mean value and the standard deviation of these frequencies over the years.

Step 2: Plot the mean values into a coordinate system where the x-axis refers to the rating classes. On a logarithmic scale mean default frequencies can be fitted by a regression line.

Step 3: Use regression equation for the estimation of default probabilities PD(x) assigned to rating classes x.

- (c) Evaluate whether the conclusion is correct based on your own calculation.

Commentary on Question:

The candidates performed as expected on this question. Candidates could have received more credit if they showed their process in writing down formulas and explanations for their solutions, especially if their conclusion was incorrect.

$$P(D) = 40\% \quad E[X] = 50\%, \quad \text{DDF}_{\text{cash}} = P(D) * E[X] = 40\% * 50\% = 20\%$$

$$E[\text{EAD}_{\text{cash}}] = P(D) * E[X] * (20M - 15M) = \text{DDF}_{\text{cash}} * 5M = 40\% * 50\% * 5M = 1M$$

$$E[\text{EAD}_{\text{cont}}] = \text{DDF}_{\text{cont}} * \text{CEEF} * (30M - 20M) = 50\% * 80\% * 10M = 4M$$

$$E[\text{EAD}] = [15m] + E[\text{EAD}_{\text{cash}}] + E[\text{EAD}_{\text{cont}}] = 15M + 1M + 4M = 20M$$

The expected EAD is the same as the committed cash line (20M); the junior analyst's conclusion is incorrect.

- (d) Explain why these two assumptions can be invalid in a recession scenario.

Commentary on Question:

The candidates performed as expected on this question. Most can identify the positive correlations between PD, EAD, and LGD. Some candidates did not provide explanations of why the correlations exist which were required to receive full credits.

7. Continued

In a recession scenario, default rates increase and firms tend to draw on their open credit lines. This increases EADs in times where default rates are going high systematically. EAD cannot be considered as independent from default rates and default rates are the basis for PD estimation.

As default rates increase, institutions will be forced to sell collateral securities related to defaulted loans or assets to the market. This will increase supply for certain goods. The principle of supply and demand leads to a price drop of such collateral securities, which are now over-supplied in the market. This leads to higher LGDs.

- (e) Identify two considerations for choosing between the two methods.

Commentary on Question:

The candidates performed above average on this question. Most identified or described at least two of the considerations.

Consideration 1: Time and computing requirements

Consideration 2: Requirements of assumptions (especially regarding dependencies)

Consideration 3: Model risk / Tail Risk

Consideration 4: Homogeneity of portfolio

- (f) Recommend, based on the two considerations you have identified in (e), one method to generate loss distributions for the portfolio. Justify your answer.

Commentary on Question:

The candidates performed below average on this question. Many did not answer this question based on their answers in part (e). The grading was done together with part (e) for those who provided a combined answer including part (f) in part (e). Sufficient justification based on only two of the four considerations in the model solution are required for full credits.

Consideration 1: Time and computing requirements

Full Monte Carlo simulation of a large portfolio can be time consuming. Using the analytical approximation method, because the true loss distribution is substituted by a closed form, analytical and well known distributions, all necessary calculations can be done in fractions of a second. Given that the portfolio is new and small, time and computing requirements are not a hurdle for implementing Monte Carlo simulation. Both approaches can be appropriate.

7. Continued

Consideration 2: Requirements of assumptions

Using the analytical approximation approach, assumptions regarding default correlations are required based on practical experience. The main advantage of a MC simulation is that it accurately captures the dependencies inherent in the portfolio instead of relying on a whole bunch of assumptions. Monte Carlo simulation should be used.

Consideration 3: Model risk / Tail Risk

All calculations in analytical approximation are subject to significant model risk. The selection of an appropriate family of distributions for an analytical approximation is a remarkable source of model risk. Monte Carlo simulation does not involve selection of the underlying distribution; therefore there is less of such model risk. Monte Carlo simulation should be used.

Consideration 4: Homogeneity of portfolio

In practice, analytical approximation techniques can be applied quite successfully to homogeneous portfolios. Monte Carlo simulation takes into account all the different risk characteristics of the loans in the portfolio. Given the small number of positions and lack of industry data, homogeneity cannot be assumed, it's prudent to model using Monte Carlo simulation.

8. Learning Objectives:

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

- (5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.
- (5b) Describe how behavioral finance explains the existence of some market anomalies.
- (5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

Sources:

QFIA-109-13: A Survey of Behavioral Finance, by Barberis & Thaler

Commentary on Question:

This question tests the understanding of concepts of mispriced stock prices and how investors make choices about products. Overall, the candidates performed as expected on this question.

Solution:

- (a) Describe four reasons why the mispricing can persist for several months.

Commentary on Question:

The candidates performed as expected on this question. Most candidates were able to list out several drivers of a persistent mispricing. However, many candidates did not provide explanations of how the drivers contribute to a mispricing and, hence, only received partial credit.

The mispricing has persisted for several months due to a variety of risks:

1. Noise trader risk - Pessimistic investors that caused the stock to be undervalued in the first place has become even more pessimistic, lowering the price further.
2. Fundamental risk – if an arbitrageur buys stock at \$20, bad news about the fundamental value of the stock will cause its price to fall further.
3. Implementation costs – Legal constraints, the cost of finding and learning about a mispricing, as well as the cost of the resources needed to exploit a mispricing makes it less attractive to exploit a mispricing.
4. Arbitrageurs are risk-averse and have short horizons, which prevent the mispricing from being wiped out by a single arbitrageur taking a large position.

8. Continued

- (b) Explain how IVY management might take advantage of its shares being undervalued and the barriers that could prevent successful implementation of this strategy.

Commentary on Question:

The candidates performed as expected on this question. Most candidates recommended a share repurchase program but did not explain why. Most candidates were also able to explain the barriers to implementing the recommendation.

Management can start a share repurchase program. This causes a shortage of supply in shares, resulting in share price increase. Barriers include implementation costs, difficulty to know for certain that shares are truly undervalued, and possible pressure from shareholders to instead deploy capital elsewhere.

- (c) Discuss the behavioral biases that led to Molly's belief.

Commentary on Question:

The candidates performed as expected on this question. Like part (a), most candidates were able to list out the behavioral biases but did not always provide explanations of how it affects Molly's beliefs. Thus, most candidates only received partial credit for this question.

It's expected that Molly will underperform the average investor for several reasons:

Overconfidence – Molly is trading online and has access to more information and is more likely to act irrationally as a result. She's only been researching the stock for a week and is likely to overestimate her investment ability.

Sample size neglect/Representativeness – Molly has only researched IVY's stock for one week and is putting significant weight on a small sample of stock performance.

Anchoring – Molly is anchored on an initial estimate of IVY's value and believes that historical performance over the last several months is indicative of future performance.

Belief perseverance – Molly is hoping to use this investment to pay for necessary college expenses. She's formed an opinion that the stock will not do well and is clinging on to it too tightly and for too long.

9. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

Learning Outcomes:

- (1l) Define and explain the concept of volatility smile and some arguments for its existence.
- (1m) Calculate the hedge ratio for a call option given the dependency of the Black-Scholes volatility on the underlying.

Sources:

Rebonato, Volatility Correlation – The Perfect Hedger and the Fox, 2nd Edition, Ch. 6

Commentary on Question:

This question tests candidate's knowledge on volatility smile with a real world application. To receive maximum points, candidates are required to not only understand what a volatility smile is, but also analyze the effect of volatility smiles on the prices of hedging strategies.

Solution:

- (a) Explain issues with hedging plain vanilla options in the presence of a volatility smile.

Commentary on Question:

Candidates performed as expected on this part. Candidates would have received more credit if they had pointed out that hedge ratio would be impacted by the change in volatility. Unsuccessful candidates also did not illustrate replicating the option payoff.

Black-Scholes is being used as a quoting mechanism for vanilla options but it is not the correct valuation/hedging model.

The deltas calculated using the volatility at the money might not be the same volatility for other money-ness.

If the valuation formula is wrong, then the hedge ratio is wrong, therefore the hedging strategy does not replicate the option payoff.

- (b) Derive an expression for the correct delta of a plain vanilla call option, in terms of Black-Scholes call option pricing function and its derivatives.

Commentary on Question:

Candidates performed above average on this part. Most candidates were able to recall the formula for delta and apply the chain rule to derive the correct expression.

9. Continued

$$\Delta = \frac{\partial \text{Black}(S, T, K, \sigma_{\text{impl}}(S, K))}{\partial S} = \frac{\partial P(\sigma_{\text{impl}}(S, K))}{\partial S}$$

Because of the dependence of the implied volatility on S,

$$\begin{aligned} \Delta &= N(h_1) + \frac{\partial \text{Black}(S, T, K, \sigma_{\text{impl}}(S, K))}{\partial \sigma_{\text{impl}}(S, K)} \times \frac{\partial \sigma_{\text{impl}}(S, K)}{\partial S} \\ &= N(h_1) + \text{BlackVega}(S, T, K, \sigma_{\text{impl}}(S, K)) \times \frac{\partial \sigma_{\text{impl}}(S, K)}{\partial S} \end{aligned}$$

- (c) Derive a hedging strategy to cover this crediting exposure using only call options.

Commentary on Question:

Candidates performed as expected on this part. Unsuccessful candidates identified the guarantee as a short position on the put option, while true for a VA product with a separate account, this is not the case here. The majority of these unsuccessful candidates did not understand the payoff pattern for this product where the question was looking for a call spread, they immediately assumed a put option.

The account value crediting scheme can be replicated by a call spread
The trader needs to buy a ITM ($K = 0.95 * S$) call and sell a OTM ($K = 1.05 * S$) call option with the same maturity

- (d) Discuss the sensitivity of initial hedging costs with respect to the shape of the volatility smile.

Commentary on Question:

Candidates performed below average on this part. Most candidates were not able to explain that even though the cost of both options would increase, depending on the shape of the volatility smirk, the magnitudes of the increase were different.

Hedging cost = Price of ITM call – Price of OTM call

Volatility increases as strike prices move away from the current underlying price.
So the price of both calls will increase.

The hedging cost depends on the observed vol shape. If the vol smile is symmetrical, the hedge cost will likely be relatively the same due to the change of the price for both call options will cancel out each other. The hedging cost will increase when vol smile is more like a vol smirk (ITM costs more than OTM).

9. Continued

- (e) Describe the steps for this calculation.

Commentary on Question:

Candidate performed below average on this part. Many candidates pointed out ways to estimate the volatility function through statistical methods; however, the question specifically asked for methods using Monte Carlo simulations.

Step 1: Start with $S_0 + \delta S$ and $\sigma(S_0 + \delta S)$

Step 2: Average over discounted expectations and calculate the prices of several calls for different strikes using Monte Carlo simulation

Step 3: Convert these prices into an implied volatility function: $\sigma_{impl}(S_0 + \delta S, K)$

Step 4: Repeat the exercise with today's price for the underlying shifted down and the volatility. $S_0 - \delta S$ and $\sigma(S_0 - \delta S)$

Step 5: Calculate the prices for the same set of strikes and convert them into a new function: $\sigma_{impl}(S_0 - \delta S, K)$

Step 6: For each strike K , the trader can compute the expression:

$$\frac{\partial \sigma_{impl}(S, K)}{\partial S} \approx \frac{\sigma_{impl}(S_0 + \delta S, K) - \sigma_{impl}(S_0 - \delta S, K)}{2 \delta S}$$

Step 7: Verify $\frac{\partial \sigma_{impl}(S, K)}{\partial S}$ with market observation

10. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

- (4a) Understand the concept of a factor model in the context of financial time series.
- (4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.

Sources:

QFIA-119-14 Tsay, *Analysis of Financial Time Series*, 3rd Ed., Ch. 9

Commentary on Question:

This question tests the concept surrounding factor models and their application.

Solution:

- (a) Compare characteristics of the three types of factor models:
 - (i) Macroeconomic factor models
 - (ii) Fundamental factor models
 - (iii) Statistical factor models

Commentary on Question:

The candidates performed as expected on this section. To receive full credit, candidates should mention the main characteristics of each factor model. Unsuccessful candidates did not explain what a macroeconomic or fundamental factor is.

Macroeconomic factor models:

- Macroeconomic variables, such as growth rate of GDP, interest rate, inflation rate and unemployment rate, are used to describe the common behavior of asset returns.
- The factors are observable and the model can be estimated via linear regression methods.

Fundamental factor models:

- Firm- or asset-specific attributes, such as firm size, book and market values and industrial classification, are used to construct common factors to explain the excess returns.
- Two approaches are BARRA Factor Model and Fama-French Approach.

10. Continued

Statistical factor models:

- Common factors are treated as unobservable or latent variables to be estimated from the return series.
- A few factors that can account for most of the variations in the covariance or correlation matrix of the data are identified.
- Data is assumed to have no serial correlation. If this assumption is violated, remove the linear dynamic dependence of the data and apply factor analysis to the residual series.

(b) Compare the two approaches to fundamental factor models:

- (i) BARRA approach
- (ii) Fama-French approach

Commentary on Question:

The candidates performed poorly on this section. To receive credit, candidates must demonstrate some understanding of the two approaches. Unsuccessful candidates did not demonstrate an understanding that both BARRA and Fama-French models are fundamental factor models by describing the characteristics of a macroeconomic model. Numerous candidates did not even attempt this part of the question.

BARRA approach – In the BARRA approach the observed asset fundamentals are treated as the factor betas, and the factor realizations are assumed to be mean corrected and are estimated in two steps:

- Use ordinary least squares to find the first estimate of the factor realization.
- Solve for the residual covariance matrix and use this matrix to perform a refined estimate of the factor realization using weighted least squares.

Fama-French approach – In the Fama-French approach the factor betas and the factor realizations are determined in a two-step process:

- For each asset fundamental, assets are sorted and a hedge portfolio is created which is long in the top quintile and short in the bottom quintile of the sorted assets. The observed return of this portfolio is the factor realization at time t . This step is repeated for each asset fundamental and for each time step t from $1, \dots, T$.
- The betas of the factor realizations are estimated using time series regression.

10. Continued

- (c) Interpret the information shown above.

Commentary on Question:

The candidates performed as expected on this section. Successful candidates mentioned the main observations in the bar chart and provide appropriate interpretations.

Based on the beta bar chart:

- All excess returns are negatively related to the unexpected changes in CPI growth rate.
- Most of the excess returns are negatively related to the unexpected changes in CE16, except AA and F.
- R2 is low for all excess returns, indicating that the two macroeconomic variables used have very little explanatory power regarding the excess returns of the 13 stocks.

- (d) Outline and justify an approach for further analyzing these 13 stock returns.

Commentary on Question:

The candidates performed below average on this section. Candidates should identify that the model is a macroeconomic factor model, suggest a fundamental factor model and/or a statistical factor model, and provide justifications. Unsuccessful candidates only provided one approach that would not be sufficient for the full analysis and full credit.

- This is a macroeconomic factor model.
- One approach would be to use macroeconomic factors which have a stronger relation to all the industry sectors where the 13 stocks come from.
- Even though the 13 stocks came from different industry sectors, you can still try a fundamental factor model with attributes such as firm size, book value, and market value.
- Even without knowing details about the 13 stocks other than they come from different industry sectors, you can always try a statistical factor model since the factors do not need to be observable but purely based on statistical relation.
- A statistical factor model assumes the data have no serial correlation. If this assumption is violated, remove the linear dynamic dependence of the data and apply factor analysis to the residual series.

11. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

- (7b) Demonstrate understanding of risks associated with guarantee riders including: market, insurance, policyholder behavior, basis, credit, regulatory and accounting.
- (7c) Demonstrate understanding risk management and dynamic hedging for existing GMXB and it embedded options – including:
 - (i) Hedgeable components including equity, interest rate, volatility and cross Greeks
 - (ii) Partially Hedgeable or Unhedgeable components include policyholder behavior, mortality and lapse, basis risk, counterparty exposure, foreign bonds and equities, correlation and operation failures
 - (iii) Static vs. dynamic hedging

Sources:

QFIA-115-13: IAA, Stochastic Modeling, Theory and Reality from an Actuarial Perspective IV.A.1-8.

Commentary on Question:

This question tests the candidate's understanding of the Monte Carlo simulations used in variable annuities hedging programs.

Solution:

- (a) Describe the major steps of simulating a Greeks-based hedging strategy.

Commentary on Question:

The candidates performed below average on this section. Unsuccessful candidates failed to show an understanding of the general process for simulating the hedging strategy. Many candidates outlined the technical calculations involved which the question did not ask for.

A hedge simulation is consisted of the following major steps

1. At the beginning of each time step, summarize the hedge profit and loss (P&L) from the previous hedge positions
2. Calculate the Greeks for the liability on an economic (market-consistent) basis
3. Calculate the Greeks for units of hedge assets
4. Determine the number of units required to match the liability Greeks and establish the hedge positions

11. Continued

- (b)
- (i) Identify three risks that cannot be managed by such a strategy.
 - (ii) Propose a way to reduce exposure to each of these three risks.

Commentary on Question:

The candidates performed above average on this section. Most candidates were able to identify the risks that are not hedged under the hedging program.

Examples of the risks the hedging program may omit:

- Lapse risk and other policyholder behavior risks. Dynamic lapse dependent on moneyness can be used to reduce exposure.
- Basis Risk – the underlying investment portfolio is an actively managed mutual fund. Over time, it may not follow the proxy fund mapping. The proxy fund mapping can be updated more frequently to better align with the investment portfolio. Another basis risk is in the dynamic hedging itself i.e. hedge errors.
- Other Greeks (gamma, vega) – the liability is not necessarily linear. As a result, delta and rho hedging may not be sufficient. Gamma hedging with equity options /swaptions and vega hedging with variance swaps may be more added.

- (c) Describe the simulations required to obtain the values in the table above.

Commentary on Question:

The candidates performed poorly on this section. A majority of candidates failed to recognize the underlying nested stochastic simulations and the nature of those simulations. Many candidates outlined obvious calculations which this question did not ask for.

Nested stochastic simulation is required to obtain the balance sheets. Stochastic real-world scenarios (outer loop) are used to produce the 5 scenarios. Risk-neutral paths (inner loop) are used to calculate the market-consistent asset and liability values. Real-world paths (inner loop) are used to calculate the statutory capital based on the CTE measure.

- (d) Describe the likely impacts that a highly effective hedging program would have on the projections above.

Commentary on Question:

The candidates performed below average on this section. Again, many candidates attempted to quantify their answers differing from successful candidates who described the impacts qualitatively.

11. Continued

A highly effectively hedging program would have the following impacts:

- Reduce the statutory capital required for all scenarios.
- Reduce the volatility of market-consistent liability value.
- Increase the distributable earnings or surplus.
- Reduce distributable earnings or surplus volatility.

12. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.
7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

- (4i) Demonstrate an understanding of the general uses and techniques of stochastic modeling.
- (7f) Demonstrate understanding of projection methods of Greeks (for embedded options in variable annuities) based on:
 - (i) Fully nested stochastic simulation
 - (ii) Fitted proxy functions based on the Least Square Monte Carlo method.

Sources:

QFIA-124-16: IAA, Stochastic Modeling, Theory and Reality from an Actuarial Perspective, sections I-1 to I-29 and II-1 to II-24

QFIA-127-16: Proxy functions for the projection of Variable Annuity Greeks

Commentary on Question:

This question tests the candidates' knowledge of several stochastic modeling techniques, such as binomial lattice and Monte Carlo and how the models are used in practice, including improvements and simplifications.

Solution:

- (a) Describe how the binomial lattice and Monte Carlo methods can be used to price derivative securities.

Commentary on Question:

The candidates performed as expected on this section. Many candidates were able to sufficiently describe how each of the methods can be used to price derivative securities but failed to provide sufficient support to attain maximum points. Candidates who performed poorly compared and contrasted the two methods without describing support on how the methods would be used.

Binomial Lattice:

Price is assumed to move either up or down at each time interval.

Probability price follows up move is p while probability of down move is $(1-p)$

Under risk neutral valuation, all assets are assumed to earn risk-free rate, so the derivative instrument price is assumed to equal the expected value of the derivative discounted at the risk-free rate.

12. Continued

In the risk-neutral, binomial tree world, the approach to valuing an option price is to determine the probabilities of up and down movement in the price that sets the discounted expected value of the option equal to the current price.

Monte Carlo:

Generate a number of sample paths of the underlying asset/option price under the risk-neutral measure.

Calculate discounted cash flows on the sample paths

Average the discounted cash flows to arrive at option price.

The accuracy of the estimate is inversely proportional to the square root of the number of runs.

- (b) Recommend one of the above methods to use to value the GMAB at ABC Insurance.

Commentary on Question:

The candidates performed above average on this section. Many candidates were able to sufficiently justify the recommendation of Monte Carlo. Some candidates received no credit for providing a recommendation without any justification whatsoever.

Recommend using the Monte Carlo method.

Due to the ability of the Monte Carlo method to simulate multiple factors (i.e. exchange rate and index values for non-currency hedged foreign indices).

Due to the Monte Carlo method being better able to accommodate complex options.

- (c) Describe why the use of a nested stochastic model is preferred in this context and the challenges of doing so.

Commentary on Question:

The candidates performed below average on this section. Many candidates were able to describe some challenges of the use of nested stochastic modeling. Many candidates failed to achieve marks as they did not describe why nested stochastic modeling was preferred in this context.

Estimate variable annuity Greeks by estimating market-consistent values before and after the corresponding risk factor is 'bumped'

A nested simulation problem results, with a large number of inner risk-neutral simulation scenarios emerging from each time-step in each outer real-world scenario

12. Continued

The total number of inner simulations required to accurately model a real hedge program using this approach is likely to be unfeasibly large for most firms to run on a daily basis given model run-time

Any practical nested simulation is likely to adopt simplifying assumptions in order to reduce run-time to acceptable levels

- (d) Describe how proxy functions can be used as an alternative to nested simulations.

Commentary on Question:

The majority of candidates performed poorly on this section. Some candidates were able to partially describe how proxy functions could be used as an alternative. Many candidates described how to fit the proxy function instead of describing how the proxy function could be used which was required to achieve full marks.

Proxy functions can be used in the projection of market-consistent value of liabilities

Proxy functions can be used for the purpose of calculating 1-year VaR

Proxy functions provide analytical approximations to market-consistent value that can be evaluated more quickly than a risk-neutral simulation

Proxy functions can be used for projection of market-consistent value at a single future time horizon or across entire paths over the liability lifetime

Proxy functions can be used to evaluate various variable annuity Greeks

Proxy functions for variable annuity Greeks allows for dynamic hedge programs to be projected at daily frequencies

- (e) Outline one approach to fit the proxy function to Monte Carlo estimates using the least squares approach.

Commentary on Question:

The candidates performed below average on this section. Some candidates were able to fully outline one approach to fitting the proxy function. Some candidates referred to their answer to part (d) and received credit not earned in (d). Many candidates did not provide a response to this section.

Credit awarded for outlining one of the three methods.

Method 1

Estimate values → Fit value proxy function → Differentiate

Straightforward generalization of the multi-step proxy functions developed for market-consistent value

Fit a set of polynomial proxy functions using least squares regression

12. Continued

Given a set of Monte Carlo estimates of market consistent value $\hat{V}_{i,t}$ (corresponding to fitting scenario i at time t with equity index $S_{i,t}$ and risk-free spot rate $R_{i,t}$), choose the polynomial coefficients to minimize the sum of squared residuals over all fitting scenarios $\sum_i (\hat{V}_{i,t} - V_t^{proxy}(S_{i,t}, R_{i,t}))^2$

Repeat the fitting process for each time t of interest.

Differentiate the value proxy functions to estimate proxy functions for the Greeks

$$\Delta_t^{proxy}(S, R) = \frac{\partial}{\partial S} V_t^{proxy}(S, R), \rho_t^{proxy}(S, R) = \frac{\partial}{\partial R} V_t^{proxy}(S, R)$$

Method 2

Estimate the Greeks directly and fit proxy functions using global regression

Use a bump and revalue approach that involves bumping/stressing the relevant risk factor by a small amount δx , revaluing the option and estimating the

derivative (sensitivity) as the finite difference $\frac{\partial V}{\partial x} \cong \frac{V(x+\delta x) - V(x)}{\delta x}$

Bumping/stressing a risk factor creates an additional scenario and would have to be done for a large number of different points in time, which will increase the number of required scenarios significantly

To avoid the large increase in scenarios, the same bump/stress scenarios can be re-used at different time steps by creating clusters of scenarios.

Clusters of scenarios can be created by first creating an independent set of base scenarios, where each base scenario has different initial parameters for the initial equity index level and yield curve level

For each base scenario, a single risk factor is bumped/stressed at $t=0$, while leaving all other risk factors unchanged. The scenario is then regenerated under the stressed initial condition using the same random numbers and is repeated for each risk factor.

A cluster is the base scenario plus stresses for equity index level and yield curve level. All scenarios within a particular cluster can be expected to be close to each other in risk factor space, if the stresses are small enough and the same random scenarios are used for all scenarios within a cluster.

Given a particular cluster i at time t , the Delta and Rho Greeks can be estimated by solving the system of linear equations

Once the Delta and Rho Greeks have been estimated, two separate least squares fits to estimate polynomial proxy functions for Δ_t^{proxy} and ρ_t^{proxy} can be performed $\sum_i (\hat{\Delta}_{i,t} - \Delta_t^{proxy}(S_{i,t}, R_{i,t}))^2, \sum_i (\hat{\rho}_{i,t} - \rho_t^{proxy}(S_{i,t}, R_{i,t}))^2$

The proxy function for the market-consistent value can be estimated by regressing on the base scenario values – choosing the coefficients of V_t^{proxy} to minimize

$$\sum_i (\hat{V}_{i,t}^{base} - V_t^{proxy}(S_{i,t}, R_{i,t}))^2$$

12. Continued

Method 3

Estimate the Greeks directly and fit proxy functions using local regression

Local regression is a better fit for certain shapes of functions, for example, as the time to maturity of a put option tends to zero, the shape of the Delta approaches a step function, which is not well described by a low order polynomial

To estimate the option value at time t, the polynomial V_t^{proxy} whose coefficients have been chosen so as to minimize the weighted sum of squared residuals

$$\sum_i w\left(\frac{S_{i,t}-S}{h^S}, \frac{R_{i,t}-R}{h^R}\right) (\hat{V}_{i,t}^{base} - V_t^{proxy}(S_{i,t}, R_{i,t}))^2$$

The weight $w\left(\frac{S_{i,t}-S}{h^S}, \frac{R_{i,t}-R}{h^R}\right)$ depends on the distance of the fitting points from the point at which the proxy function is evaluated and the weight function is typically chosen so as to put less weight on fitting points that are further away from the evaluation point

Since the fitting space is adjusted dynamically to reflect a small range around the chose point, the function can be accurately estimated using only low-order polynomials, such as linear function or constants

Local regression is more complex / harder to communicate / computationally more expensive

13. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.
- (2i) Demonstrate an understanding an understanding of CDS valuations

Sources:

Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012 , Ch. 66

Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012 , Ch. 67

QFIA-103-13: Bond-CDS Basis Handbook

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a)
 - (i) Describe a financial instrument that the above agreement resembles.
 - (ii) Sketch a graph showing each participant and indicate their respective roles within this financial agreement.

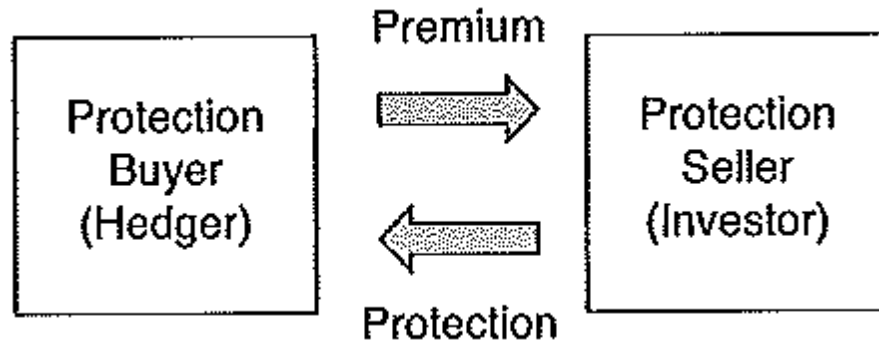
Commentary on Question:

Candidates performed as expected on this section. The majority of the candidates recognized that the agreement resembles a CDS. For full credit, a sketch/graph was required.

The financial instrument is similar to a Credit Default Swap deal. Eric is the issuer of a coupon paying bond to Adam who is the investor. Adam is also the protection buyer and Caroline is the protection seller.

It is similar to Adam buying an insurance contract from Caroline to protect him against a possible default from Eric.

13. Continued



- (b) Recommend a deal that David could propose to either side for his own benefit.

Commentary on Question:

This part was poorly answered by majority of the candidates. Most of the candidates failed to recognize the negative basis trade. Merely noting that David could make a swap with Eric and keeping the difference was missing the point of this part.

Negative basis trade

$$\text{Basis} = \text{CDS spread} - \text{Bond spread measure} \\ = 0.5 - 2 = -1.5$$

Negative basis, thus opportunity for a negative basis trade

Buy bond and buy CDS protection

- David could borrow 1000 marbles at his preferred rate of 3%
- Then lend those 1000 marbles to Eric at any rate above 3.5%
- Seek Caroline to get the same deal from her that she proposed to Adam

- (c) Calculate the profit and loss for Adam and Caroline showing the flow of marbles for all participants.

Commentary on Question:

Most candidates were able to recognize the regular interest and coupon payments, and/or the marble flow at the time of default. Candidates struggled to identify the correct timing, i.e. that interest is paid at the end of the period while CDS coupons are at the beginning, resulting in candidates failing to achieve maximum points.

13. Continued

End of Week	Eric	Adam		Caroline
	Debt payment	Interest received	CDS	CDS
0			-5	5
1	-50	50	-5	5
2	-50	50	-5	5
3	-50	50	-5	5
4	-50	50	-5	5
5	-50	50	-5	5
6	-50	50	-5	5
7	-50	50	-5	5
8	-50	50	-5	5
9	-50	50	-5	5
10	-50	50	-5	5
11	-50	50	-5	5
12	-50	50	-5	5
13	-50	50	-5	5
14	-50	50	-5	5
15	-600	0	1000 = return of principle from Eric and Caroline	-400
Total profit		700	-75	-325
Profit/(loss)		625		-325

- (d) Calculate how many marbles Eric left to Adam when he left the game.

Commentary on Question:

This part was answered above average by the majority of candidates, with many receiving maximum points.

CDS payment received by Caroline = $18 \times 5 = 90$

She's breaking even, thus she must have handed back 90 marbles to Adam.

To cover the difference from 1000

Thus Eric must have had $1000 - 90 = 910$ marbles

- (e) Calculate, from this information, the probability that Eric will fail on his commitment sometime from the end of week 17 to end of week 20.

Commentary on Question:

While a few candidates achieved full marks, most candidates answered this part poorly. While some candidates did not attempt this part, those that did failed to identify the correct formulas or show their work.

13. Continued

$$Q(t) = {}_t p_0 \times q_t$$

You're looking for $Q(17) - Q(20) = 1 - {}_3 p_{17} = q_{18} + p_{18} \times q_{19} + p_{18} \times p_{19} \times q_{20} =$
 Probability of default in year 18th, 19th or 20th, where $Q(17) = {}_{17} p_0 = 1$

$$\begin{aligned} -U(T) &= V_{\text{premium}} - V_{\text{protection}} \\ -U(T) &= C \times \sum_{t=T+1}^{20} (1+i)^{-(t-T)} Q(t) - (1-R) \times \sum_{t=T+1}^{20} (1+i)^{-(1+t-T)} (Q(t-1) - Q(t)) \\ -U(T) &= C \times \sum_{t=T}^{19} (1+i)^{-(t-T)} {}_{t-T} p_T - (1-R) \times \sum_{t=T}^{19} (1+i)^{-(1+t-T)} {}_{t-T} p_T \times q_{t+1} \end{aligned}$$

The compensation ask is equivalent to the negative upfront premium if bought the CDS at the end of Tth year.

$$-U(17) = 6$$

$$-U(18) = 5$$

$$-U(19) = 3$$

From the question:

$C(T) = C = \text{coupon} = 5$ for all period

Recovery rate = 90%, or assume to recover 900 marbles, thus $1 - R = 100$ marbles or 10%

Thus:

$$\begin{aligned} -U(19) &= C - (1-R) \times q_{20} / (1+i) = C \\ 3 &= 5 - 100 \times q_{20} / 1.03 \\ \Rightarrow q_{20} &= (5 - 3) \times 1.03 / 100 = 0.0206 \\ \Rightarrow Q(19) - Q(20) &= {}_2 p_{17} \times q_{20} \end{aligned}$$

$$\begin{aligned} -U(18) &= 5 = C + (1-R) \times q_{19} / (1+i) + (1 - q_{19}) \times -U(19) / (1+i) \\ (-U(18) - C) \times (1+i) &= -(1-R) \times q_{19} - U(19) - q_{19} \times U(19) \\ \Rightarrow q_{19} &= [(1+i) \times (-U(18) - C) + U(19)] / -[(1-R) - U(19)] \\ \Rightarrow q_{19} &= [(1.03) \times (5 - 5) + -3] / -[(100) - -3] \\ \Rightarrow q_{19} &= 3 / 103 \\ \Rightarrow q_{19} &= 0.02912621 \\ \Rightarrow Q(18) - Q(19) &= {}_1 p_{17} \times q_{19} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{Given } q_{18} &= 0.03780952 \\ \Rightarrow [(1+i) \times (-U(17) - C) + U(18)] &= -[(100) - -5] \times q_{18} \\ \Rightarrow [(1+i) \times (-U(17) - C) + U(18)] &= 3.97 \end{aligned}$$

Background

$$\begin{aligned} -U(17) &= 6 = C + (1-R) \times q_{18} / (1+i) + (1 - q_{18}) \times -U(18) / (1+i) \\ (-U(17) - C) \times (1+i) &= -(1-R) \times q_{18} - U(18) - q_{18} \times U(18) \\ \Rightarrow q_{18} &= [(1+i) \times (-U(17) - C) + U(18)] / -[(1-R) - U(18)] \\ \Rightarrow q_{18} &= [(1.03) \times (6 - 5) + -5] / -[(100) - -5] \\ \Rightarrow q_{18} &= 3.97 / 105 \\ \Rightarrow q_{18} &= 0.03780952 \\ \Rightarrow Q(17) - Q(18) &= {}_0 p_{17} \times q_{18} = q_{18} \end{aligned}$$

13. Continued

P = probability of default

$$P = 3.97/105 + ((105 - 3.97)/105) \times (3/103) + ((105 - 3.97)/105) \times (100/103) \times (2.06/100)$$

$$P = 0.0850783$$

Or

$$P = Q(17) - Q(20) = [(Q(17) - Q(18)) + (Q(18) - Q(19)) + (Q(19) - Q(20))]$$

$$= q_{18} + {}_1p_{17} \times q_{19} + {}_2p_{17} \times q_{20}$$

$$= q_{18} + (1 - q_{18}) \times q_{19} + (1 - q_{18}) \times (1 - q_{19}) \times q_{20}$$

$$= 3.97/105 + ((105 - 3.97)/105) \times (3/103) + ((105 - 3.97)/105) \times (100/103) \times (2.06/100)$$

$$= 0.0850783$$

14. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
 - One and two-factor short rate models
 - LIBOR market modelsThe candidate will understand approaches to volatility modeling.
2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.
- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.
- (2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

Sources:

Blum Ch 6

Brigo Interest Rate Models – Theory and Practice Ch 3

Commentary on Question:

This question tests candidates' understanding of default probability term structure and features of the CIR model.

Solution:

- (a) Calculate the implied risk-neutral default intensity based on the assumptions above.

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to determine the default probability. Unsuccessful candidates did not recognize that in order to receive full credit, they were required to determine the default intensity.

The value of risky bond is:

$$\sum_{T_i > 0} B(0, T_i) \Delta_i S(T_i) + V \left[B(0, T_n) S(T_n) + \text{REC} \int_0^T B(0, t) F(dt) \right]$$

$$\therefore \text{Bond Value} = (\text{Face Value})(e^{-rT})(1 - \text{PD}) + (\text{Face Value})(e^{-rT})(\text{REC})(\text{PD})$$

14. Continued

$$\begin{aligned} \Rightarrow 88.5 &= (100)(e^{-0.02(0.5)})(e^{-\lambda(0.5)}) + (100)(e^{-0.02(0.5)})(0.65)(1 - e^{-\lambda(0.5)}) \\ &\Rightarrow \lambda = \mathbf{72.24\%} \end{aligned}$$

(b) Calculate:

- (i) The real cumulative default probability from time 0 to time 2
- (ii) The risk-neutral cumulative default probability from time 0 to time 2

Commentary on Question:

The candidates performed above average on this section. Successful candidates recognized that to receive full credit, they must identify the correct formulas, substitute the correct values, and calculate the correct final answers. Candidates who evaluated the cumulative normal distribution function using linear interpolation, answers are acceptable as well.

$$PD_t^{\text{Real}} = N\left(-\frac{\ln\left(\frac{A_0}{C}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right) = N\left(-\frac{\ln\left(\frac{100\text{m}}{75\text{m}}\right) + \left(0.07 - \frac{0.3^2}{2}\right)(2)}{0.3\sqrt{2}}\right)$$

$$= N(-0.79592) = \mathbf{0.21304}$$

$$PD_t^{\text{RN}} = N\left(-\frac{\ln\left(\frac{A_0}{C}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right) = N\left(-\frac{\ln\left(\frac{100\text{m}}{75\text{m}}\right) + \left(0.02 - \frac{0.3^2}{2}\right)(2)}{0.3\sqrt{2}}\right)$$

$$= N(-0.56022) = \mathbf{0.28766}$$

(c) Show that:

$$PD_t^{\text{RN}} = N\left(N^{-1}(PD_t^{\text{Real}}) + \rho_{a,m} \frac{\pi}{\sigma_m} \sqrt{t}\right)$$

Commentary on Question:

The candidates performed as expected on this section. Candidates would also receive full credit if they used other versions of the CAPM equation.

$$PD_t^{\text{Real}} = N\left(-\frac{\ln\left(\frac{A_0}{C}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right) \Rightarrow -\frac{\ln\left(\frac{A_0}{C}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} = N^{-1}(PD_t^{\text{Real}})$$

14. Continued

$$\begin{aligned} PD_t^{RN} &= N\left(-\frac{\ln\left(\frac{A_0}{C}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right) = N\left(-\frac{\ln\left(\frac{A_0}{C}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} + \frac{(\mu - r)}{\sigma}\sqrt{t}\right) \\ &= N\left(N^{-1}(PD_t^{\text{Real}}) + \frac{(\mu - r)}{\sigma}\sqrt{t}\right) \end{aligned}$$

Using the CAPM, $\mu - r = \beta\pi$, where $\beta = \frac{\rho_{a,m}\sigma}{\sigma_m}$. Thus, we have:

$$PD_t^{RN} = N\left(N^{-1}(PD_t^{\text{Real}}) + \frac{\beta}{\sigma}\pi\sqrt{t}\right) = N\left(N^{-1}(PD_t^{\text{Real}}) + \frac{\rho_{a,m}}{\sigma_m}\pi\sqrt{t}\right)$$

- (d) Calculate the variance of λ_1 as seen from time 0.

Commentary on Question:

The candidates performed above average on this section. Full credit can be obtained even if the steps are not as detailed as the solution given below.

Unsuccessful candidates did not correctly identify all four required parameters.

Based on the given process, identify that $k = 0.8$.

From “long-term mean of λ_t as t tends to infinity is 2%,” identify that:

$$\theta = 0.02$$

From “long-term variance of λ_t as t tends to infinity is 0.0125%,” identify that:

$$\begin{aligned} \frac{\theta\sigma^2}{2k} &= 0.000125 \\ \Rightarrow \frac{0.02\sigma^2}{2(0.8)} &= 0.000125 \\ \Rightarrow \sigma &= 0.10 \end{aligned}$$

From “mean of λ_1 as seen from time 0 is 2.4493%,” identify that:

$$\begin{aligned} \lambda_0 e^{-k} + \theta(1 - e^{-k}) &= 0.024493 \\ \Rightarrow \lambda_0 e^{-0.8} + 0.02(1 - e^{-0.8}) &= 0.024493 \\ \Rightarrow \lambda_0 &= 0.03 \end{aligned}$$

The desired variance

$$\begin{aligned} &= \lambda_0 \frac{\sigma^2}{k} (e^{-k} - e^{-2k}) + \frac{\theta\sigma^2}{2k} (1 - e^{-k})^2 \\ &= 0.03 \frac{0.10^2}{0.80} (e^{-0.8} - e^{-2(0.8)}) + 0.000125(1 - e^{-0.8})^2 \\ &= \mathbf{0.0001307} \end{aligned}$$

14. Continued

- (e) Explain the impact of k and σ on the variance of λ_t .

Commentary on Question:

The candidates performed above average on this section. Successful candidates identified the correct variance formula and appropriately commented on the impact of k and σ on the variance.

The variance is:

$$\text{Var}(\lambda_t) = \lambda_0 \frac{\sigma^2}{k} (e^{-kt} - e^{-2kt}) + \frac{\theta\sigma^2}{2k} (1 - e^{-kt})^2$$

From the equation above, note that:

- As k increases, the variance decreases.
 - As σ increases, the variance increases.
- (f) Evaluate the use of the CIR model for estimating the default probabilities.

Commentary on Question:

The candidates performed below average on this section. Successful candidates explained how the CIR model was suited to modeling default probabilities. Unsuccessful candidates received minimal credit for just listing the facts without any explanation.

- The intensity λ_t remains positive all the time since
$$2k\theta > \sigma^2 ; 2 * 0.8 * 0.02 \geq 0.10^2$$
- Intensity models allow us to apply the results for interest rate modeling in the default model setting. With CIR processes, one can derive many closed-form formulas that are useful in pricing credit derivatives.
- It can be shown that the risk-neutral survival probability is given by
$$A(t, T) = e^{B(t, T)\lambda_t}$$
- The model is analytically tractable.
- Default intensity is mean-reverting, which means that the expected value of the default probability tends to a long-run constant value of 0.02 as time goes to infinity.
- Variance of the intensity grows with its magnitude rather than being constant.

15. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

- (2a) Demonstrate an understanding of events and causes of the 2008 global credit crisis.
- (2l) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

QFIA-101-13: Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, John B., et. al., 2nd Edition, 2008, Ch. 20

Commentary on Question:

This question tests the understanding of credit valuation models and portfolio optimization

Solution:

- (a) Describe the CreditMetrics approach and its key assumptions.

Commentary on Question:

Candidates did poorly on this section. A significant number of candidates either did not provide a clear description of the CreditMetrics approach, did not identify key assumptions of the CreditMetrics approach or did one or both of the aforementioned items in an incomplete manner by failing to provide a sufficient description.

CreditMetrics estimates the distribution of portfolio value and portfolio credit risk subject to changes in credit quality and default through Monte Carlo simulation

15. Continued

key assumptions (candidates only needed to provide two key assumptions, in addition to the description above, to receive full credit for this section)

- Econometric estimates of parameters will continue to prevail in the future
- Credit quality correlations are approximated by asset value correlations
- Asset value correlations are approximated by equity correlations
- Assets in same credit rating but different vintages have the same credit transition probabilities and default risk

(b)	Verify that	(units in millions)
(i)	the expected portfolio value is	105
(ii)	the standard deviation is	2
(iii)	the 1% value is	10

Commentary on Question:

Candidates did above average on this section. Most candidates were able to show the work that mathematically verified the expected portfolio value and the standard deviation.

Some candidates misidentified the 1% value as 115 and some candidates attempted to calculate a 1% value using a normal distribution approximation rather than from the given data.

$$E[x] = ((95 \times 128 + 100 \times 313 + 105 \times 9146 + 110 \times 299 + 115 \times 114)) / 10,000 = 104.979$$

$$\text{Standard deviation} = \sqrt{E[x^2] - E[x]^2}$$

$$E[x^2] =$$

$$((95^2 \times 128) + (100^2 \times 313) + (105^2 \times 9146) + (110^2 \times 299) + (115^2 \times 114)) / 10,000$$

$$= 11,024.54$$

$$\sigma = \sqrt{11,024.54 - 104.979^2} = \sqrt{3.949559} = 1.9873$$

15. Continued

1% value:

There is $128/10,000 = 1.28\% > 1\%$ chance the value of the portfolio is 95, hence the 1st percentile is 95.

The 1 percent value = Expected portfolio value – 1st percentile value
= $104.979 - 95$
= 9.979

- (c)
- (i) Define the efficient frontier.
 - (ii) Assess if the current portfolio is efficient.

Commentary on Question:

Candidates demonstrated a strong understanding and did brilliantly on this section.

The efficient frontier shows the portfolios which maximize expected return for given level of risk or minimize risk for given level of expected return.

The current portfolio has a return of 5% and standard deviation of 2%.

It is below the efficient frontier.

The portfolio risk-return trade-off of the current portfolio can be improved.

- (d) Recommend two approaches that could be applied to improve the current fixed income portfolio.

Commentary on Question:

Candidates did below average on this section. While many candidates identified that adjusting the portfolio was necessary, often candidates provided proposed solutions that were not consistent with the restriction that ABC Company's investment policy only allow fixed income investments.

The portfolio can be improved by adjusting the weight of the 10 bonds.

The following approaches can be considered,

- Altman's optimization to find the optimal weights of each asset to maximize the portfolio ratio.
- Z''-Score model to determine the unexpected losses.

(Other reasonable approaches also received credit)