Advanced Approaches to Improving Model Performance Through Technology and Proxy Modeling

Aubrey Clayton
Trevor Howes, FSA, FCIA, MAAA
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The Challenges of Improving Model Performance

» Need for better model performance
  – Larger, more complex models
  – Increasing use of stochastics
  – Peak loads and narrow reporting windows
  – Need faster close
  – Capacity for unexpected demands

» Need to limit total actuarial costs
  – Costs of IT infrastructure and support
  – Actuarial HR costs

» These needs apply throughout the full modeling process

[Diagram showing ETL, Modeling, and Reporting stages]
Handle conflicting priorities in model design

**Better performance**
- Reduce total resource costs
- Use technology efficiently
- Reduce runtime and increase analysis time

**Increase quality**
- More detail, granularity
- Asset/liability interaction
- Holistic, integrated models
- Reduce and control approximations

**Increase flexibility**
- Multiple applications
- React to changes in products and risks
- Sustainable regulatory compliance

**Improve control**
- Reduce model risk
- Assure run-time reliability
- Support auditability and transparency standards
Taxonomy of Modeling Efficiency Techniques

Actuarial and Modeling Techniques
- Scenario Design & Selection
- Mathematical and/or Model Design
- Model Data Building Techniques

(Conceptual Model Design) *(some impact on model results)*

Technology Solutions
- Software Design
- Hardware Design

(Model Implementation) *(no impact on model results)*
Technology Layers within a Modeling System

- **Application Software**
  - User written & compiled code or user configured vendor code

- **System Software**
  - Middleware
  - Standardized communications interface between:
    - Application to hardware
    - Networks/grid/internet
  - OS and Networking
  - Device drivers

- **Hardware**
  - Processor(s)
  - Storage
Model Efficiency in Application Software Layer

- User written & compiled code or user configured vendor code

Opportunities to improve performance by software design:

1. Design or coding of algorithms within programs
2. Minimizing or optimizing disk and DB access (e.g. using more RAM)
3. Using faster programming languages (e.g. APL vs C++)
4. Optimized or profile-guided compilation techniques
Model Efficiency in Application Software Layer

Pros and Cons

» If vendor-supplied system, may be dependent on vendor

» If user coded
  – Requires programmer’s skill and available time
  – May impact transparency and future maintenance costs

» Same or lower IT costs subject to hardware and systems software dependencies (e.g. PC’s/Microsoft Windows and .Net)

Model Risk?

» Risk of software changes (thorough testing and validation)

» Risk of new untried techniques, inadequate design work

» If model conceptual design unaffected, should not impact result
Model Efficiency in System Software Layer

- System Software enables independent upgrades of other layers, but offers little opportunity for efficiency.

- Standardized communications interface between:
  - Applications
  - Networks/grid/internet
  - Processing and storage hardware
Model Efficiency in Hardware Layer

Hardware upgrade approaches:

- Upgrading speed of processors, disks, network connections
- Adding more resources (expanding server farms/grids, using public Cloud services)
- Exploring new or unconventional technology (e.g. GP GPUs)
- Quantum computing?

- Processor(s)
- Storage
- Network
Model Efficiency in Hardware Layer

Pros and Cons

» Hardware upgrade may be the simplest strategy with immediate benefit

» Requires $ investment and cost/benefit analysis

» Adding processing power depends on effective task distribution
  – Reduce total runtime
  – Minimize idle time
  – Minimize application and human overhead

Model Risk?

» Risk of infrastructure change impact (thorough testing and validation)

» Risk of new untried techniques, inadequate design work

» If model conceptual design unaffected, should not impact results
Task Distribution by Creating Mini-models

» Creating multiple mini-models is simplest and most common

Scenarios

Segments of business data

Distribute single scenarios over grid

Distribute small segments of business over grid
Demands of actuarial modeling

» Production applications with complex, multi-step applications require full end-to-end automation
  – File ETL processing, possibly data compression, & other single thread steps before actuarial calculations
  – Large models with seriatim or near seriatim granularity, or possibly compressed models
  – Stochastic processing over many scenarios implies data storage, retrieval, aggregation as well as computation
  – ALM processes usually require aggregate level roll forward period to period

» Efficient use of IT infrastructure including large Grids or the public cloud requires:
  – Appropriate specification and configuration of hardware
  – Intelligent Grid management to efficiently allocate processing steps and manage data across available resources
Application-aware Automated Task Distribution

Dedicated software component for task distribution and grid management
- Communicates with modeling software and IT infrastructure
- Manages:
  - subdivision of tasks
  - allocation to available cores
  - monitoring of task completion
  - error recovery/reallocation
  - dynamic rebalancing
  - aggregation of results
Application-aware Automated Task Distribution

Pros and Cons

» Automatically handles distribution and aggregation
» Intelligently reacts to changing demand and resources
» Improves efficiency of IT usage for both simple and complex applications
» Requires complex and coordinated programming by team with both actuarial and IT expertise

Model Risk?

» Low impact on results provided change control and revalidation is co-ordinated over both modeling and task distribution software
» Reduced risks of node failure and inaccurate distribution and aggregation
Distribution Challenges of Complex Applications

» Example of CCAR Stress Testing or similar projection application applied to a large block of Variable Annuities with hedging strategy

» Large portfolio of policies modeled on a seriatim basis

» Multiple scenarios testing adverse trends and management actions over 2 year period
  – Require realistic projection of earnings and capital impact on statutory basis, plus projection of GAAP earnings
  – Reflect dynamic hedging in cash flows

» Scalability demands for largest applications imply a need to distribute over public cloud permitting thousands of cores
CCAR Stress Testing for Variable Annuities

Each outer loop scenario requires:
1. asset and liability cashflows
2. hedging & reinvestmt strategy

Projection over next 8 quarters which requires a nested calculation of Greeks

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<td>Short Term Int Rates</td>
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<td>$66,643</td>
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<td>Net Income after tax</td>
<td>$147</td>
<td>$149</td>
<td>$156</td>
<td>$164</td>
<td>$171</td>
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<td>$195</td>
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<td>$46.21</td>
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<td>Req Capital (C3 Phil)</td>
<td>$7,791</td>
<td>$8,144</td>
<td>$8,502</td>
<td>$8,877</td>
<td>$9,269</td>
<td>$9,679</td>
<td>$10,108</td>
<td>$10,556</td>
<td>$11,024</td>
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<td>After tax Surplus</td>
<td>$12,209</td>
<td>$11,856</td>
<td>$11,498</td>
<td>$11,123</td>
<td>$10,731</td>
<td>$10,321</td>
<td>$9,892</td>
<td>$9,444</td>
<td>$8,976</td>
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CCAR Stress Testing for Variable Annuities

### Projection Dates:
- Dec-14: 0.75%
- Mar-15: 0.85%
- Jun-15: 1.00%
- Sep-15: 1.10%
- Dec-15: 1.25%
- Mar-16: 1.45%
- Jun-16: 1.65%
- Sep-16: 1.85%
- Dec-16: 2.10%

### Economic Scenario:
- Short Term Int Rates: 2059.0
- US Broad Equity Index: 2151.7

### Net Cashflows:
- Net Premiums: $7,359
- Net Benefits: $4,415
- Reinv’t & Hedging: $1,462
- Net Cashflow: $2,944

### Hedging & Reinvestment:
- Delta on G’tes: $68

### US GAAP Financials:
- DAC Balance: $10,500
- SOP 03-1 Liability: $488
- FAS 133 Derivative: $760
- Net Fas 97 Liability: $58,492
- Net Income after tax: $147

### Statutory Financials:
- AG 43 Aggreg Reserve: $64,926
- Net Income: -$182.52
- Req Capital (C3 Phil): $7,791
- After tax Surplus: $12,209

3. US GAAP earnings involves a complex DAC projection...requiring two more nested stochastic calc’ns
### CCAR Stress Testing for Variable Annuities

**Projection Dates:**
- Dec-14: 
- Mar-15: 
- Jun-15: 
- Sep-15: 
- Dec-15: 
- Mar-16: 
- Jun-16: 
- Sep-16: 
- Dec-16: 

**Economic Scenario:**
- Short Term Int Rates: 0.75%, 0.85%, 1.00%, 1.10%, 1.25%, 1.45%, 1.65%, 1.85%, 2.10%
- US Broad Equity Index: 2059.0, 2151.7, 2248.5, 2349.7, 2455.4, 2565.9, 2681.4, 2802.0, 2928.1

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**Hedging & Reinvestment:**
- Greeks: $68, $72, $75, $78, $81, $85, $89, $93, $97

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- Net Income: $46,21, $46,141, $147,54, $153,38, $159,35, $165,46, $171,70, $178,06, $185,46
- Reg Capital (C3 Phl): $7,791, $8,144, $8,502, $8,877, $9,269, $9,679, $10,108, $10,556, $11,024
- After tax Surplus: $12,209, $11,856, $11,498, $11,123, $10,731, $10,321, $9,892, $9,444, $8,976

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4. Statutory requires projecting AG43 & C3P2

Nested stochastics with ALM roll fwd
Challenges of CCAR Stress Testing

» For each outer loop, distribute first by model points to project liability CFs

» Left with complex ALM process with nested stochastics to project AG43 and C3P2 along main projection

Job phases
- Job Initiation on master
- Task distribution to helpers
- Calculations
- Merging to master

Job time
- Scenario n
- ALM with repeated nested stochastics

By policies or model points

Moody's Analytics
Advanced Approaches to Improving Model Performance Through Technology and Proxy Modeling 19
A More Complex Distribution Approach

- Distributing first by model points to project liability CFs including nested stochastics for Greeks and US GAAP
- Distribute each nested stochastic process in two phases, first by model point, then by scenario

Job time

Master

Helpers

Job phases

One stress test Scenario

For each nested stochastic calculation

By policies or model points

Liabilities by policies or model points

Assets and ALM roll fwd by scenario

Job Initiation on master

Task distribution to helpers

Calculations

Merging to master

A/L roll forward

Nested Stochastic ALM

Generating reports

Job Termination
Effective Task Distribution over a Large Grid

» Keep all cores busy; minimize idle time

» Explore various approaches to dividing up parallel tasks and assigning to independent cores

» Watch for bottlenecks or inefficiencies especially when large models and large numbers of cores are required
  – Time consumed in distributing data out to helpers
Challenges of CCAR Stress Testing – Task Distribution

» Distributing first by model points to project liability CFs including nested stochastics for Greeks and US GAAP

» Distribute each nested stochastic process in two phases, first by model point, then by scenario

Job phases:
- Job Initiation on master
- Task distribution to helpers
- Calculations
- Merging to master
- A/L roll forward
- Nested Stochastic ALM
- Generating reports
- Job Termination

Look at:
- Amount of data being distributed
- Number of cores in Grid
- Disk and network speed

Job time:
- One stress test Scenario
- For each nested stochastic calculation

Master

Helpers
Effective Task Distribution over a Grid

» Keep all cores busy; minimize idle time

» Explore various approaches to dividing up parallel tasks and assigning to independent cores

» Watch for bottlenecks or inefficiencies especially when large models and large numbers of cores are required
  – Time consumed in distributing data out to helpers
  – Time consumed in aggregating results back from helpers
Challenges of CCAR Stress Testing – Result Aggregation

» Distributing first by model points to project liability CFs including nested stochastics for Greeks and US GAAP

» Distribute each nested stochastic process in two phases, first by model point, then by scenario

Job time

Master

Helpers

Job phases

One stress test Scenario

For each nested stochastic calculation

Job Initiation on master

Task distribution to helpers

Calculations

Merging to master

A/L roll forward

Nested Stochastic ALM

Generating reports

Job Termination

Look at:
• Memory use
• Result volume
• Disk and DB capacity for aggregation by multiple cores at once
Effective Task Distribution over a Grid

» Keep all cores busy; minimize idle time

» Explore various approaches to dividing up parallel tasks and assigning to independent cores

» Watch for bottlenecks or inefficiencies especially when large models and large numbers of cores are required
  – Time consumed in distributing data out to helpers
  – Time consumed in aggregating results back from helpers
  – Time wasted when hardware failures occur in grid
Challenges of CCAR Stress Testing – Fault Recovery

» Distributing first by model points to project liability CFs including nested stochastics for Greeks and US GAAP

» Distribute each nested stochastic process in two phases, first by model point, then by scenario

Job time

Master

Helpers

Job phases

When are failures detected?

What is failure impact?

Job Initiation on master
Task distribution to helpers
Calculations
Merging to master
A/L roll forward
Nested Stochastic ALM
Generating reports
Job Termination
Effective Task Distribution over a Grid

» Keep all cores busy; minimize idle time

» Explore various approaches to dividing up parallel tasks and assigning to independent cores

» Watch for bottlenecks or inefficiencies especially when large models and large numbers of cores are required
  – Time consumed in distributing data out to helpers
  – Time consumed in aggregating results back from helpers
  – Time wasted when hardware failures occur in grid

» Advanced grid management tools are essential to optimize use of large Grids or Cloud
  – Monitor and analyze performance of the entire Grid or Cloud
  – Warn about pending or actual hardware failures
  – Automatic spin up of large number of “instances” in Cloud
  – Automatic release of idle instances if warranted
Conclusions

» Technology-based model efficiency can effectively reduce runtimes and/or costs, without significant model governance concerns

» Effective exploitation of technology requires:
  – Collaboration between IT and Actuarial software experts
  – Common understanding of what the model needs to do
  – Ability to react to the specific demands of each application and its implications for utilization of required IT resources

» Technology-based approaches alone are unlikely to be sufficient;
  – A combination of independent model efficiency techniques will be most effective likely to include some mathematical or scenario based techniques…
Proxy Functions for Value & Capital Projection

Aubrey Clayton, Moody’s Analytics
Types of nested stochastic problems

Projecting mark-to-market value

Projecting hedge positions

Projecting required run-off capital/reserves

Projecting capital net of hedging
Large scenario requirements with limited resources

Scenario requirements for a complex capital planning exercise:

- 6 planning scenarios * 9 quarters * 1,000 run-off scenarios * 480 months
- * 10,000 valuation scenarios * 3 sensitivities * 1 million policies

= 777.6 quadrillion policy-scenarios

Is a full revaluation really needed?
Why use proxy functions?

Proxy methods are a scenario reduction technique

» Similar **outer scenarios** should produce similar **inner results**
   → smooth functional relationship

\[
Value_t = f(RiskFactors_t) \\
Capital_t = g(RiskFactors_t)
\]

» It can be more efficient to **pre-compute** this function before it’s needed.

» Fewer total scenarios

» Scenarios run before the reporting date
Least Squares Monte Carlo (LSMC)

» Instead of doing full nested simulation, do only a few inner simulations

Regression through inaccurate valuations to get function which approximates true nested stochastic valuation
Proxy fitting procedure

There are four main steps followed to derive the liability proxy function:

1. **Identify risks and generate fitting points**
2. **Run regression to fit proxy functions**
3. **Validate proxy function**
4. **Outside model**

**Inaccurate valuation**

**“Heavy” model**

**Approximate Valuations**

**LSMC fit**

**Outside model**
Ingredients for a proxy function

» A definition of **outer scenarios** in terms of a “smallish” number of key variables

» A way of calculating **unbiased estimates** of the metric of interest

» A **functional form** that allows for the right kinds of dependency

» **Automated interaction** between stress scenario definitions and actuarial models
Evolution and adoption of LSMC

- **2000**
  - American Option Pricing
  - Longstaff Schwartz

- **2006**
  - LSMC Application to Nested Simulation

- **2009**
  - LSMC applied to GMWB w/ ratchet
    - Morrison Cathcart
  - Solvency II and nested simulations
    - Bauer et al.

**2012**
- PG v1
- First UK insurer to adopt PG
- Banks adoption of LSMC for projection of counterparty exposure
- Risk magazine

**2013**
- PG v2
- European insurer implements LSMC using PG
- Tier 1 Asian insurer adopts PG for economic capital
- North American insurer adopts PG for EC and ORSA
- Apr: Multi-year projection of MC liabilities
- July: Multi-year projection of CTE

**2014**
- PG v3.0
- Generali implements LSMC for internal model
- VA provider uses LSMC for assessing hedge effectiveness
- Jan: Multi-period modeling of Greeks using LSMC
- Sep: Multi-year projection of 1-year VaR

**2015**
- PG v3.1
- Japanese LSMC project for valuation and ORSA projection
- Nov: Dynamic hedge projection for vanilla option

**2016**
- PG v3.2
- North American insurer adopts PG for EC and ORSA
- PG v4.0
- Tier 1 Asian insurer adopts PG for economic capital

- PG v4.0
- Oct: Run-off CTE Capital: Life Insurance Case Study
- Sep: Proxy Model Validation
- May: Hedge Projection: VA Case Studies

- Proxy Functions for Value & Capital Projection 36
What about path-dependency?

» For complex products, the value will depend on the path of economic risk factors up to that point.

» **Summarise** important features of the path in a small number of variables
  – Lookback option value depends on ‘running maximum’
  – VA guarantee value depends on moneyness of guarantee, etc.

» May need to develop proxy functions at the **policy-level**
Example: VA Greeks

» Flexible premium deferred VA in waiting period
  – GLWB and annual ratchet, deferral bonuses
  – Three possible fund allocations: Conservative, Moderate, Aggressive

» Policy variables (gender/moneyness/etc.) & market risk variables (yield curves/etc.)

Proxy Methods for Hedge Projection: Two Variable Annuity Case Studies (June 2016)
What about tail percentiles?

» Capital could be defined by a percentile of value distribution, e.g., VaR(99.5)

» Quantile regression can extract functional behavior from 1 inner scenario

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<th>Instead of this…</th>
<th>Use this…</th>
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<td>( \arg\min_{\beta} \sum (Y_i - X_i \cdot \beta)^2 )</td>
<td>( \arg\min_{\beta} \sum \rho_\tau(Y_i - X_i \cdot \beta) )</td>
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<tr>
<td></td>
<td>( \rho_\tau(y) = y \cdot (\tau - I_{y&lt;0}) )</td>
</tr>
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Example: VaR(99.5) for a fixed annuities portfolio

» Liabilities: Fixed cash flows with longevity risk
» Assets: Corp & Govt bonds, chosen to approximately match liability cash flows
» 1-year VaR(99.5) as a function of leverage, allocation, avg. corporate credit rating
What about CTE measures?

» Reserves/capital typically defined by run-off Conditional Tail Expectation

» Requires a change in scenario allocation and bias-corrected CTE estimators
  − Can also apply clever scenario filtering…

Instead of this… | Use this…
---|---
\[ \overline{CTE}_\alpha = \frac{1}{n(1 - \alpha)} \sum_{i=n\alpha+1}^{n} X(i) \] | \[ \overline{CTE}_\alpha = \sum_{i=1}^{n} w_i X(i) \]

Example: Life insurance case study

» Capital = CTE(99) of accumulated deficiencies in 40 year run-off of assets and liabilities

» Products:
  – Participating Whole Life (“OL”)
  – Fixed Deferred Annuities (“FDA”)
  – Aggregate

Double-nested stochastic capital proxy functions

- Layer 1: Replace market-consistent scenario calculation with proxy function for value and Greeks

  \[
  \text{Market-consistent value} = f(\text{risk factors})
  \]
  \[
  \text{Greek} = g(\text{risk factors})
  \]

- Layer 2: Replace capital scenario calculation with proxy function

  \[
  \text{Capital} = F(\text{risk factors})
  \]
Summary

» Proxy methods can make dramatically more efficient use of available computing resources.

» The proxy calibration effort can be pre-computed at a more advantageous time.

» All assumptions present in actuarial models will still be there!

» Calibration requires good automation of stress/scenario generation.

» Different methods of function fitting may be required for different nested stochastic problems.