A Flexible and Robust Approach to Modelling Single Population Mortality

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Outline

- Motivation
- Model fitting
- Flexibility in mortality forecasting
- Bring closer industry practice and academic techniques

Motivation

Two strands of work in progress:

1: Can the model fitting process be simplified and made more robust?

2: *In practice:* Disconnect between central forecasts of mortality and models to assess uncertainty. Can we pull these together?

$\Rightarrow$ How to make stochastic mortality models more useable?

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Simplify the model fitting process

- Problem: interacting age, period and cohort effects
  $\Rightarrow$ identifiability problems
  $\Rightarrow$ arbitrary constraints
- If period and cohort effects are *random walks*
  $\Rightarrow$ no problem
- Otherwise choice of constraints makes a difference.

Simplify/break down the model fitting process

Potential fitting algorithms

Two Stage
Stage 1 (Poisson deaths)
Stage 2 (Time series models)

↙ ↘

Stage 1A, 1B, 1C	One Stage
Stage 2	Full Bayesian

Example: A CBD-X Model

$$\log m(t,x) = \alpha(x) + \sum_{k=1}^{3} \beta_i(x)\kappa_i(t) + \gamma(t-x)$$

$$\begin{aligned} \text{where} \quad \alpha(x) &= \text{non-parametric age effect} \\ \beta_1(x) &= 1 \quad \forall x \\ \beta_2(x) &= (x - \bar{x}) \\ \beta_3(x) &= (x - \bar{x})^2 - \sigma_x^2 \\ \kappa_k(t) &= \text{period effects} \\ \gamma(c) &= \text{cohort effect.} \end{aligned}$$

(Variation on Plat (2009, IME). Also see Hunt & Blake (2014, NAAJ).)
Suitable for a wider age range than CBD-M7.

Simplify/break down the model fitting process

$$\log m(t,x) = \alpha(x) + \sum_{k=1}^{3} \beta_k(x)\kappa_k(t) + \gamma(t-x)$$

- 1A: Estimate the base table, $\alpha(x)$
- 1B: Estimate the period effects, $\kappa_k(t)$, given $\alpha(x)$
- 1C: Estimate the cohort effect, $\gamma(c)$ given $\alpha(x), \kappa_1(t), \kappa_2(t), \kappa_3(t)$

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Pros and Cons

Cons:

- Not the perfect maximum likelihood
- Less good for small populations

Pros:

- Fexibility over how to calibrate $\alpha(x)$ (e.g. Lee & Miller, 2001)
- Can apply smoothing
- No identifiability problems
- Robust
- Enough period effects $\Rightarrow$
  $\gamma(c) =$ small residual effects *with a distinct pattern* (*)
  (Hunt & Blake, 2014)
  (*) No pattern/very random $\Rightarrow$ overfitting.
- Can import external $\tilde{\gamma}(c)$.
  (Then fit $\alpha(x)$ given $\tilde{\gamma}(c)$ etc.)

How to bring industry practice and academic approach together?

CMI: UK Continuous Mortality Investigation

Chart 4A: Mortality improvements (male)

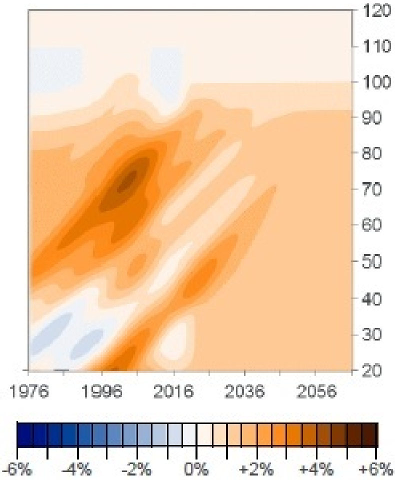
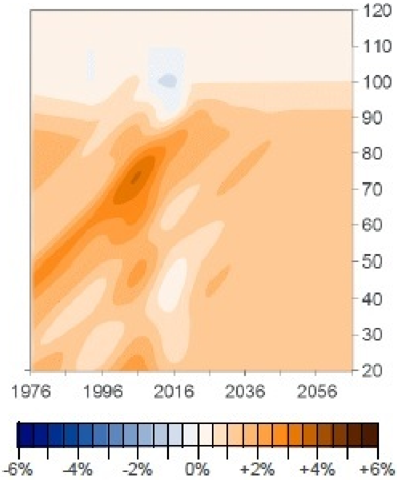


Chart 4B: Mortality improvements (female)



Source: CMI Working Paper 97

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CMI (2016): CMI_2016 v01 2017-03-27.xlsm projections

- Central forecast only
- Potentially lots of subjective inputs
  - Long term improvement rate (*)
  - Convergence period
- Default settings, but must set
  - Long term *flat* improvement rate
  - Base table

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CMI (2016) Model: Historical Data + APCI Model

$$\log m(t,x) = \alpha(x) + \beta(x)(t - \bar{t}) + \kappa(t) + \gamma(t-x)$$
$$\alpha(x), \beta(x) = \text{non-parametric age effects}$$

Optimise:

- Poisson log-likelihood; minus
- penalty for lack of smoothness in $\alpha(x)$, $\beta(x)$, $\kappa(t)$ and $\gamma(t-x)$

Smoothness $\Rightarrow$ smooth heat map

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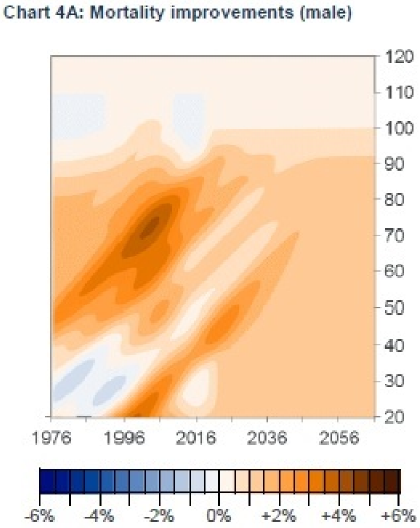
CMI Heat Map



Source: CMI Working Paper 97

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$$\log m(t,x) = \alpha(x) + \beta(x)(t-\bar{t}) + \kappa(t) + \gamma(t-x)$$

Historical improvement rate up to time T:

$$\begin{aligned} MI(t,x) &= -\beta(x) - \Big(\kappa(t) - \kappa(t-1)\Big) \\ &\qquad -\Big(\gamma(t-1-x) - \gamma(t-x)\Big). \end{aligned}$$

$$\begin{aligned} MI_{AP}(t,x) &= -\beta(x) - \Big(\kappa(t) - \kappa(t-1)\Big) \\ MI_{C}(t,x) &= \Big(\gamma(t-1-x) - \gamma(t-x)\Big) \end{aligned}$$

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CMI mortality improvements

Future improvement rates after time T:

$$MI(T+t,x) = MI_{AP}(T+t,x) + MI_C(T+t,x)$$

for $t = 1, 2, \ldots$.

Initial improvements $\longrightarrow$ long term rates:

$$MI_{AP}(T+t,x) \;\rightarrow\; L_{AP}(x) \;\text{ as } t \rightarrow \infty$$

$$MI_C(T+t,x+t) \;\rightarrow\; L_C(T-x) \;\text{ as } t \rightarrow \infty$$

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CMI mortality improvements

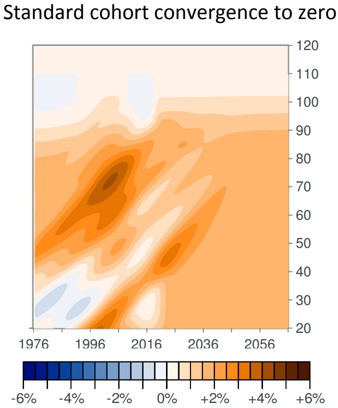
- Age-period convergence period: τ_{AP} (Default ≤ 20)
- Cohort convergence period: τ_C (Default ≤ 40)
- Define $u_{AP} = \min\{t/\tau_{AP}, 1\}$ and $u_C = \min\{t/\tau_C, 1\}$
- Then

$$\begin{aligned} MI_{AP}(T+t,x) &= L_{AP}(x) \\ &+ (MI_{AP}(T,x) - L_{AP}(x)) \left(1 - 3u_{AP}^2 + 2u_{AP}^3\right) \end{aligned}$$

$$\begin{aligned} MI_C(T+t,x+t) &= L_C(T-x) \\ &+ (MI_C(T,x) - L_C(T-x)) \left(1 - 3u_C^2 + 2u_C^3\right) \end{aligned}$$

CMI: default vs persistent cohort effect ($\tau_C = 1000$)

Projected mortality improvements: EW Males



(Source: outputs from CMI_2016 v01 2017-03-27.xlsm)

- Version 1:
  - Use CMI to generate central $\hat{m}(T+t,x)$
  - Use [CBD-X] to generate sample paths *relative to* $\hat{m}(T+t,x)$ $(k=1,\ldots,10000)$

$$m^{(k)}(T+t,x)=\frac{m_{CBD}^{(k)}(T+t,x)}{\bar{m}_{CBD}(T+t,x)}\hat{m}(T+t,x)$$

- Version 2 (many forms):
  - Use CMI to generate central $\hat{m}(T+t,x)\rightarrow\hat{a}(T,x)$
  - Use [CBD-X] to generate sample PV's *relative to* $\hat{a}(T+t,x)$ $(k=1,\ldots,10000)$

$$a^{(k)}(T,x)=\frac{a_{CBD}^{(k)}(T,x)}{\bar{a}_{CBD}(T,x)}\hat{a}(T,x)$$

CBD-X mortality improvements

Standard Random-Walk Model:

$$\kappa_k(T+t) = \kappa_k(T+t-1) + \mu_k + \epsilon_k(t)$$

Future improvement rates

$$MI(T+t,x) = MI_{AP}(T+t,x) + MI_C(T+t,x)$$

for $t = 1, 2, \ldots$.

$$\begin{aligned} MI_{AP}(T+t,x) &= -\beta_1(x)\mu_1 - \beta_2(x)\mu_2 - \beta_3(x)\mu_3 \\ MI_C(T+t,x+t) &= MI_C(T,x) \\ &= \gamma(T-1-x) - \gamma(T-x) \end{aligned}$$

CBD-X mortality improvements

New: Random-Walk Model with Time-Dependent Drift

$$\kappa_k(T+t) = \kappa_k(T+t-1) + \mu_k(t) + \epsilon_k(t)$$

Future improvement rates

$$MI(T+t,x) = MI_{AP}(T+t,x) + MI_C(T+t,x)$$

for $t = 1, 2, \ldots$.

$$MI_{AP}(T+t,x) = -\beta_1(x)\mu_1(t) - \beta_2(x)\mu_2(t) - \beta_3(x)\mu_3(t)$$

$$MI_C(T+t,x+t) = MI_C(T,x)$$

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Potential choices for $\mu_k(t)$

Version 1

$$\begin{aligned} \mu_k(t) &= \mu_k(\infty) + (\mu_k(0) - \mu_k(\infty))\phi^t \\ \tau &= -\ln 2 / \ln \phi = \text{half life} \end{aligned}$$

Version 2

$$\begin{aligned} \mu_k(t) &= \mu_k(\infty) + (\mu_k(0) - \mu_k(\infty))(1 - 3u_{AP}^2 + 2u_{AP}^3) \\ u_{AP} &= \min\{t/\tau_{AP}, 1\} \quad \text{(similar to CMI).} \end{aligned}$$

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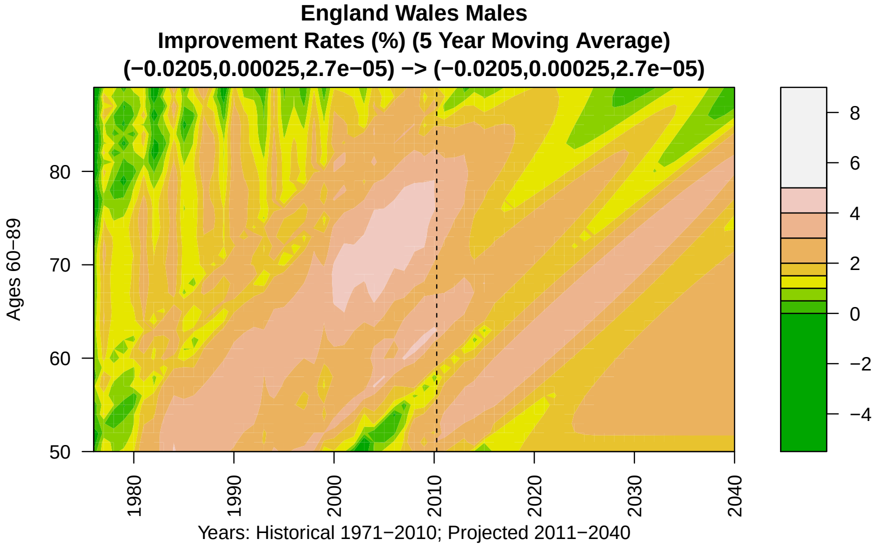
CBD-X: 1971-2010 + Standard Central Forecast



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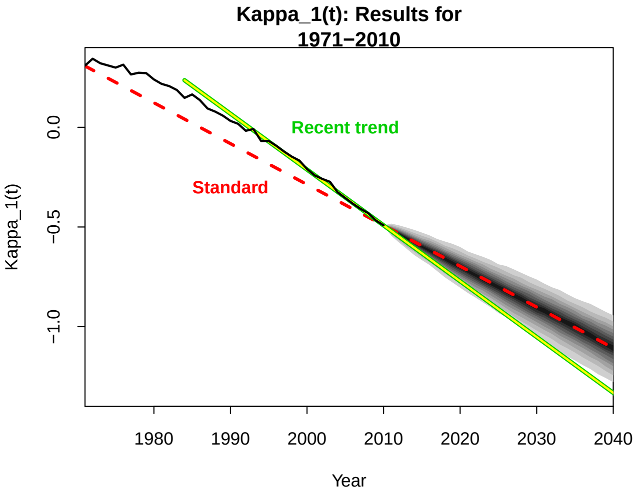
CBD-X: 1971-2010: Standard Stochastic Forecast, $\kappa_1(t)$



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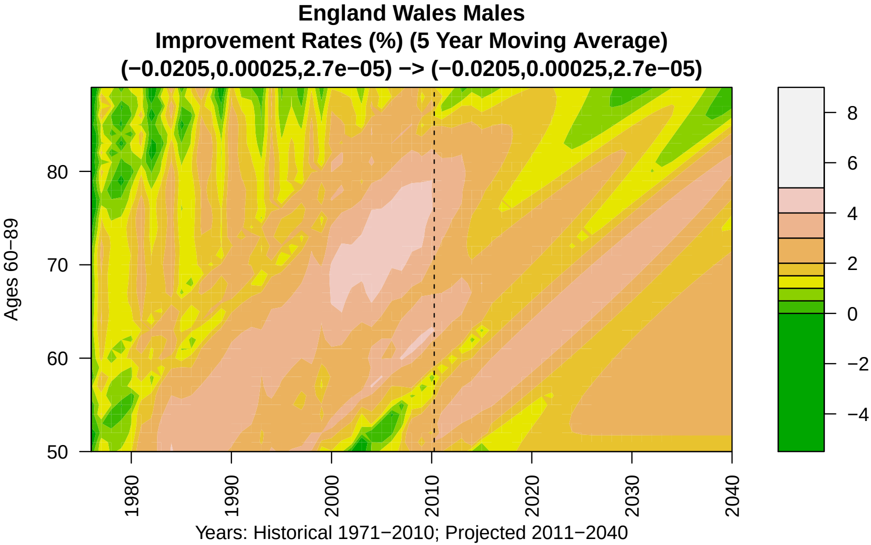
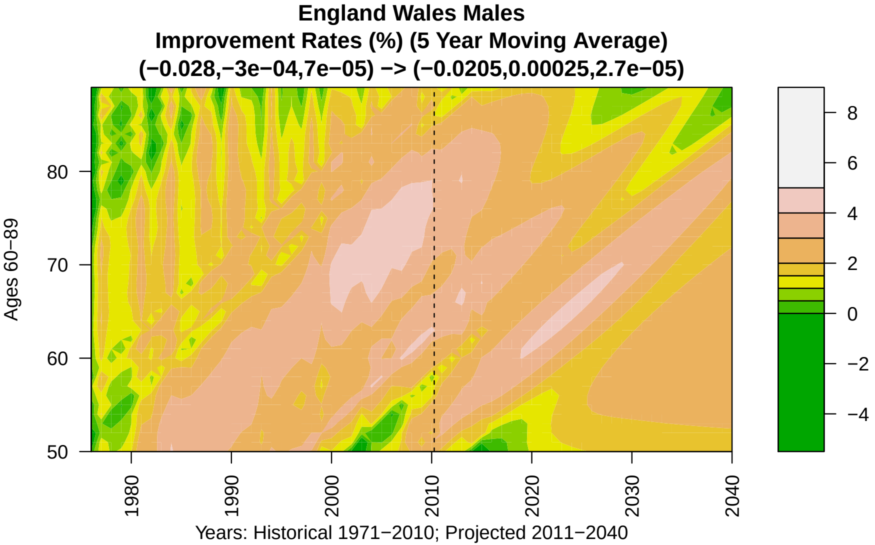
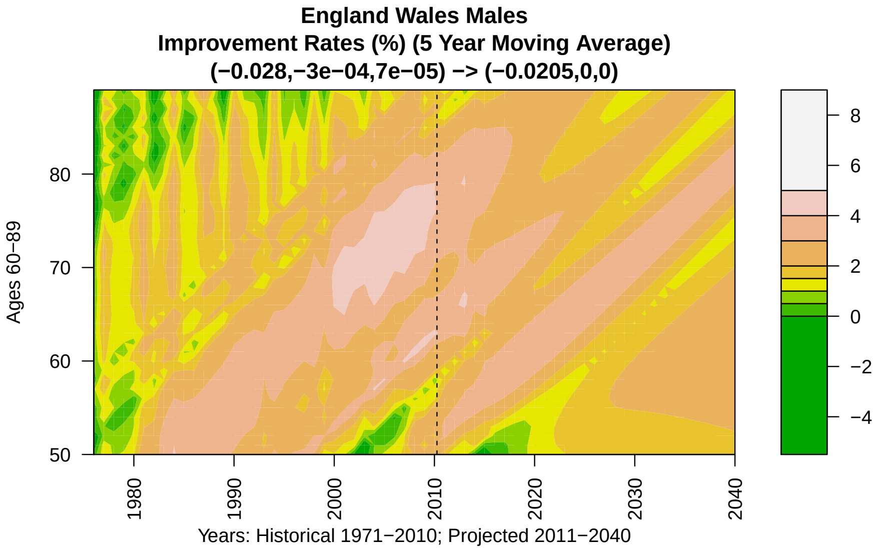
Central Improvement Rates Again



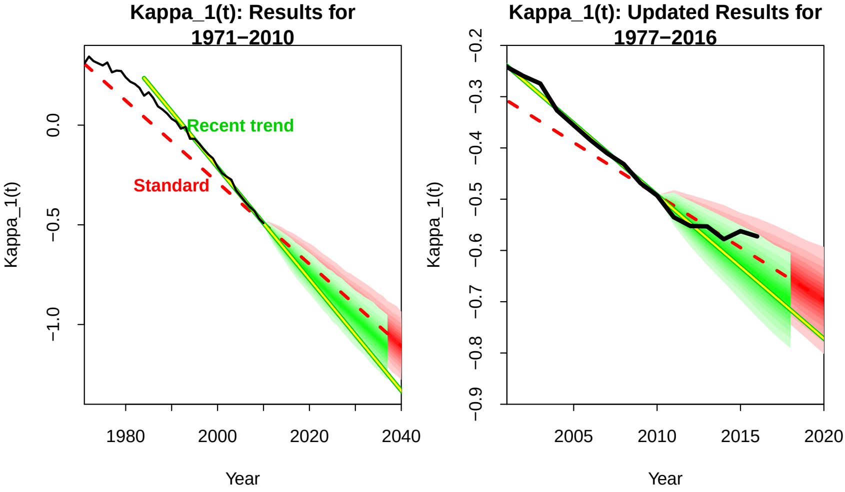
CBD-X: Time Dependent, Same Long Rates



CBD-X: Time Dependent, Flat Long Rates



$\kappa_1(t)$: Standard vs Time-Dependent Forecast 1971-2010 vs 1977-2016 outcome



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Discussion

- Exact CBD-X/CMI match requires:
  - CMI $\beta(x)$ quadratic in x
  - CMI long term improvement rate quadratic

    (data do not support flat long term improvement rate)

    (lower rate at high ages $\Rightarrow$ financial consequences)
  - Same age-period transition to long rate
  - CMI cohort convergence period $\tau_C = \infty$
- Default CMI: flat long-term AP mortality improvements $\Rightarrow$ requires CBD-X

$$\mu_2(\infty) = \mu_3(\infty) = 0$$

and $\mu_1(\infty) =$ CMI long-term AP improvement rate

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Consistency with the full stochastic model

- Alternatives to the random walk model:
  - Börger et al. (2013, ASTIN B.)
  - Liu and Li (2017, ASTIN B.)
  - Both attempt to capture local trend changes
  - Mavros et al. (2017, NAAJ)
- Both $\Rightarrow$ random walk around a stochastic drift, $\mu_k(t)$
  $\Rightarrow$ central forecast $\hat{\mu}(t)$
- Potential adjustments to both models (???)
  $\Rightarrow \hat{\mu}(t) = \mu(\infty) + \phi^t(\mu(0) - \mu(\infty))$
- Alternative approach: Richards et al. (2018, IFoA)
  Stochastic version of CMI APCI model
  ARIMA model for $\kappa(t) \Rightarrow$ time dependent $\hat{\mu}(t)$

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Summary

- Work in progress!
- Framework here: $CBD - X \longrightarrow$ other models
- Historical data: alternative sub-staged approach
- Central forecasts: adapt [CBD-X] to mimic CMI
- Common ground exists between [CBD-X] and *actively managed* CMI

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Thank You!

Questions?

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