

## Appendix A - Select Longevity Risk and Annuity Market Literature Review

### *MORTALITY TRENDS AND OUTLOOK*

- A-1 Title: "Shapes and Limits of Longevity in Mexico"  
Author(s): Roberto Ham-Chande  
Publication: The Living to 100 and Beyond Symposium Sponsored by the Society of Actuaries, Orlando, Fla. January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-ii.pdf>

This paper discusses the levels and forms that mortality and survivorship are taking for the oldest-old in Mexico and the need for a better understanding of the characteristics and dynamics of longevity. The author cites essential factors that will be necessary to be able to predict future mortality increases: enhanced quality of population statistics and improved record keeping, including recording the cause of mortality and its link to morbidity.

### *RETIREMENT MARKETS DEVELOPMENT*

- A-2 Title: "Key Findings and Issues - Longevity: The Underlying Driver of Retirement Risk"  
Author(s): SOA Project Oversight Group  
Publication: Society of Actuaries, 2005 Risks and Process of Retirement Survey Report, July 2006  
URL: <http://www.soa.org/files/pdf/Longevity%20Short%20Report.pdf>

This presentation summarizes the results of a survey of 602 Americans which assesses their understanding and awareness of risks related to retirement. The results illustrate that the majority of both retirees and pre-retirees underestimate their life expectancy, and are therefore unlikely to be able to plan adequately for their remaining years. The results highlight that the majority of retirees try to manage their longevity exposure themselves, rather than purchasing an annuity.

- A-3 Title: "Longevity-Insured Retirement Distributions from Pension Plans: Regulatory and Market Issues"  
Author(s): Jeffrey R. Brown and Mark J. Warshawsky  
Publication: National Bureau of Economic Research (NBER), January 2001  
URL: <http://www.business.uiuc.edu/jbrown/My papers/w8064%20Brown%20warshawsky.pdf>

This paper explores the extent to which retirees can and do insure themselves against longevity risk in private pension plans. It discusses welfare benefits from annuitization and then explores possible explanations for the limited market for annuities. Among the

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most important factors are market imperfections, limited consumer understanding of the benefits of annuitization, and institutional and regulatory barriers to the provision of annuities. The authors also examine the current state of retirement plan distributions and likely future. They close by evaluating policy options to encourage annuitization. Some options discussed are mandatory annuitization, tax incentives, and government-sponsored life annuity agencies.

- A-4 Title: "The Market for Individual Annuities and the Reform of Social Security"  
Author(s): Mark J. Warshawsky  
Publication: Benefits Quarterly, Third Quarter 1997

This paper studies the Social Security reform proposals put forward by members of the 1994-96 Advisory Council on Social Security. The author examines the benefits of each proposal and discusses some of the drawbacks before presenting a view on the how effective each is likely to be. The author concludes that with regulatory oversight the private market could provide individual annuities efficiently, responding effectively to consumer's preferences for choices among investment types and payout features.

- A-5 Title: "Redistribution and Insurance: Mandatory Annuitization with Mortality Heterogeneity"  
Author(s): Jeffrey R. Brown  
Publication: The Journal of Risk and Insurance, 2003, Vol. 70, No.1, 17-41  
URL: <http://www.business.uiuc.edu/jbrown/Mypapers/JRI%20Feb%202003.pdf>

This article examines the distributional implications of mandatory longevity insurance when mortality heterogeneity exists in the population. This article finds that the degree of redistribution that occurs from the introduction of a mandatory annuity program is substantially lower on a utility adjusted basis than when evaluated on a purely financial basis. In a simple life-cycle model with no bequests, complete annuitization is welfare enhancing even for those with higher-than-average expected mortality rates, so long as administrative costs are sufficiently low.

### *PRODUCTS WITH LONGEVITY RISK EXPOSURE*

- A-6 Title: "Longevity Bonds: Financial Engineering, Valuation and Hedging"  
Author(s): David Blake, Andrew Cairns, Kevin Dowd and Richard MacMinn  
Publication: The Pension Institute, DISCUSSION PAPER PI-0617, December 2006  
URL: <http://www.pensions-institute.org/workingpapers/wp0617.pdf>

This paper examines different ways to financially engineer Longevity Bonds and considers the problems that exist in structuring such instruments. It analyses how Longevity Bonds can be used to hedge the longevity risk exposure of annuity writers.

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- A-7 Title: "Can Life Insurance be used to Hedge Payout Annuities? Part 1 : Modeling Longevity Risk"  
Author(s): M.A Milevsky and S.D Promislow  
Publication: Individual Finance and Insurance Decisions Centre, Summer 2003  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2003Summer.pdf](http://www.ifid.ca/pdf_workingpapers/WP2003Summer.pdf)

The purpose of this paper is to investigate the impact of unexpected longevity in payout annuity contracts on the profitability and return-on-equity of this business. Longevity risk is not captured by using a deterministic actuarial curve but by modeling the instantaneous hazard rates using a diffusion process. The main result is that profitability is quite sensitive to changes in mortality rates; however, the younger the at-issue age of the payout annuity, the less sensitive the portfolio is to an unexpected change in mortality.

- A-8 Title: "The Market for Mortality"  
Author(s): Paul Sweeting  
Publication: Reinsurance Section News November 2007 (Issue No. 61)  
URL: <http://www.soa.org/library/newsletters/reinsurance-section-news/2007/november/rsn-2007-iss61-sweeting.pdf>

This is a short article that summarizes the capital market solutions available to insurers to hedge and mitigate longevity risks. Among the solutions mentioned is the use of mortality bonds. The author presents the Swiss Re catastrophe bond and the BNP Paribas longevity bond, and discusses the reasons for the longevity bond's failure. The last section of the article is dedicated to the discussion of survivor swaps.

- A-9 Title: "Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities"  
Author(s): Moshe A. Milevsky  
Publication: Society of Actuaries, Managing Retirement Asset Symposium, 2004  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2004FEB\\_.pdf](http://www.ifid.ca/pdf_workingpapers/WP2004FEB_.pdf)

This paper introduces a concept product called an advanced-life delayed annuity (ALDA), which is a variant of a pure deferred annuity contract that is linked to, and adjusted for, consumer price inflation. The premiums for this insurance are designed to be very low and this is enabled by a long deferral period since the life-contingent income only kicks in at the advanced age of 80, 85 or even 90. This product contains zero cash value and as such has no survival or estate benefits and cannot be commuted for cash at any age. Therefore it provides retirees with mortality credits at advanced ages and insures them against the risk of outliving their assets, but does not offer any sort of death benefit. This drawback has been attributed, along with other reasons, to the lukewarm

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reception this concept has received. The paper discusses different design and pricing possibilities for the ALDA contract.

### QUANTIFICATION AND MANAGEMENT TECHNIQUES

- A-10 Title: "Credit Implications of the Payout Annuity Market on the Life Insurance Industry"  
Author(s): Arthur Fliegelman, Moshe Arye Milevsky, Scott A. Robinson  
Publication: Wharton/Pension Research Council Symposium, April 2002  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2002A.pdf](http://www.ifid.ca/pdf_workingpapers/WP2002A.pdf)

This paper deals with the risks and opportunities for an insurance company to convert qualified retirement or savings accounts into lifelong payout annuities. After revisiting the history of insurance failures and their link to the payout annuity market, the opportunities in this market are assessed. The authors go on to detail the role of an immediate annuity in retirement and the role of the law of large numbers in this market with a focus on the variable immediate annuity market. Finally, the risks inherent in the annuity market are discussed along with ideas on how best to manage them. The risks discussed include the impact of a decrease in mortality rates on profitability, investment risk, commutation rights, and variable immediate annuity risk.

- A-11 Title: "Survivor Bonds: Helping to Hedge Mortality Risk"  
Author(s): David Blake and William Burrows  
Publication: The Journal of Risk and Insurance, 2001, Vol 68, No. 2, 339-348

This article describes how Survivor Bonds (SBs) can be used to combat mortality risks in capital markets. The authors argue that the government should issue SBs for use as a hedge for aggregate mortality risk. So far insurance companies face the risk that no matching asset exits - SBs are a potential solution to that problem. These bonds are structured such that future coupon payments depend on the percentage of the whole population at retirement age on the issue date who remain alive on the future coupon payment dates.

- A-12 Title: "An Arbitrage-free Family of Longevity Bonds"  
Author(s): Daniel Bauer  
Publication: Independent, September 2006

This paper presents a stochastic mortality model that is defined via an exogenously given longevity bond market. It is different from other stochastic models in that it does not use the credit risk modeling framework and does not assume the independence of the term structure of mortality and the term structure of interest rates. A number of modeling approaches are suggested but one is discussed in more detail - the modeling

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of the forward force of mortality. This modeling framework is based on the other modeling approaches. Finally, the authors show how this framework can be employed to price more complex mortality derivatives.

### GENERAL

- A-13 Title: "Living with Mortality; Longevity Bonds and other Mortality-Linked Securities"  
Author(s): D. Blake, A. J. G. Cairns and K. Dowd  
Publication: Faculty of Actuaries, January 2006

This paper addresses the problem of longevity risk and discusses ways in which mortality-linked securities and over-the-counter contracts can be used to manage longevity risk exposure. It provides a detailed analysis of the Swiss Re Mortality Bond and the EIP/BNP Longevity Bond. The authors note that the success of the Mortality Bond came from the higher coupons being offered and the hedging opportunities it provided pension funds due to the bond's correlation with active pensioners' mortality risk. The EIP/BNP longevity bond, announced in November 2004, was not received well by investors. Suggested reasons for its slow take up include its short 25-year bond horizon; capital requirements; the degree of model and parameter risk; inherent basis risk; and its use of central death rates in calculating the index. The authors then provide readers with a discussion on a wide range of mortality-linked securities - some existing and others hypothetical. These include longevity bonds, mortality swaps, mortality futures and mortality options.

- A-14 Title: "Longevity Risk and Life Annuities"  
Author(s): Moshe A. Milevsky  
Publication: Independent, November 2006  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2006NOV20.pdf](http://www.ifid.ca/pdf_workingpapers/WP2006NOV20.pdf)

This paper primarily serves as a literature review on the topic of longevity risk and life annuities. It deals first with longevity risks' relation to individuals and their personal financial decisions, then it discusses the role of intermediaries and insurance corporations that take on and absorb longevity risk. Finally it introduces the role of governments, and of society as a whole, in managing longevity risk.

- A-15 Title: "Managing Longevity Risk in US Retirement Plans Through Mandatory Annuitization"  
Author(s): Beverly J. Orth  
Publication: North American Actuarial Journal (Vol 10, No. 3)  
URL: <http://www.soa.org/library/monographs/retirement-systems/managing-retirement-assets-symposium/2004/march/m-rs04-2-06.pdf>

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This paper analyzes the risks faced by individuals approaching retirement. The author notes that there has been a shift towards DC pension schemes which leave the beneficiary with a lump sum rather than a stream of payments, and that few people use this to purchase an annuity in the current environment. The author proposes several different possible structures for mandatory annuitization and compares the experience of other systems in other countries.

- A-16 Title: "Natural Hedging of Life and Annuity Mortality Risks"  
Author(s): Samuel H. Cox and Yijia Lin  
Publication: Independent, July 2004  
URL: [http://www.actuaries.org/AFIR/Colloquia/Boston/Cox\\_Lin.pdf](http://www.actuaries.org/AFIR/Colloquia/Boston/Cox_Lin.pdf)

This paper analyzes the effect of natural hedging of longevity risk as a risk management tool. It finds empirical evidence that companies that have a natural hedge between their life and annuity portfolios are able to charge lower annuity premiums than otherwise similar insurers. The paper also looks at how a mortality swap might be set up to provide the benefits of a natural hedge.

- A-17 Title: "Annuities: Private Solution to Longevity Risk"  
Author(s): Veronica Scotti, Dr Dirk Effenberger  
Publication: Swiss Re Sigma No 3/2007

This paper provides a general overview of the retirement markets in various countries, comparing how these markets have evolved, how they currently operate and the products available. The paper also analyses how product design impacts the profitability of annuity contracts. Finally the authors discuss the challenges of managing the longevity and financial risks embedded in annuity contracts, and suggest ways in which retirement markets might change in the future.

- A-18 Title: "Age of Retirement and Longevity"  
Author(s): Institute/Faculty Pension Provision Taskforce  
Publication: Institute/Faculty Pension Provision Taskforce  
URL: [http://www.actuaries.org.uk/data/assets/pdf\\_file/0017/33074/age.pdf](http://www.actuaries.org.uk/data/assets/pdf_file/0017/33074/age.pdf)

This paper analyzes the implications an increased life expectancy will have on the cost of providing State pensions in the UK. As a background, it discusses the observed and expected mortality and demographic trends in the UK. It then analyzes the principal State retirement benefits in light of these changes. An interesting finding is that Britain has managed to control its 'demographic time bomb' and is on track to suffer less from an ageing population than other countries. Reasons cited for this success are the increase in female pension age and the change from earnings growth-linked benefits to inflation-linked benefits.

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- A-19 Title: "Living to 100 and Beyond: Implications for Retirement"  
Author(s): Sam Gutterman, Colin England, Alan Parikh and Robert Pokorski  
Publication: Society of Actuaries, Living to 100 and Beyond: Implications for Retirement RECORD, Volume 28, No. 3  
URL: <http://www.soa.org/library/proceedings/record-of-the-society-of-actuaries/2000-09/2002/january/rsa02v28n395pd.pdf>

This transcript from the SOA Boston Annual Meeting held from October 27-30, 2002 has three main parts. The first part, presented by Dr. Robert Pokorski, dwells on trends observed worldwide on life expectancy and the relationship between population and insured lives mortality. Dr. Pokorski presents two life expectancy theories, the 'Limitless Lifespan' theory and the 'All Good Things Must End' theory as the optimistic and pessimistic view respectively. He then analyzes the uncertainties surrounding projections made about future life expectancy. Alan Parikh presents the second part on centenarian projections, factors that can be used to determine mortality rates, and the ramifications of using an outdated mortality table. He mentions that income and economic status influence mortality, and so does race; however he recognizes the potential public uproar on using the latter in underwriting. The last part is presented by Colin England and focuses on pension plan issues such as funding, disclosures, liabilities and assumption issues.

- A-20 Title: "Longevity in the 21st Century"  
Author(s): R. C. Willets, A. P. Gallop, P. A. Leandro, J. L. C. Lu, A. S. Macdonald, K. A. Miller, S. J. Richards, N. Robjohns, J. P. Ryan and H. R. Waters  
Publication: Faculty of Actuaries and Institute of Actuaries, March 2004

The objective of this paper is to offer a detailed analysis of mortality changes in the UK at the beginning of the 21st century. Underlying forces driving trends in longevity are discussed (i.e., the cohort effect) and the paper goes on to predict a rapid increase in longevity. International mortality statistics and trends are analyzed as is the pace of medical advances, specifically research into the ageing process and potential treatment for diseases. Finally, the financial impact of different mortality assumptions is explored. The paper asserts that future projections should be grounded in as solid an understanding of the past as possible.

- A-21 Title: "Financial Aspects of Longevity Risk"  
Author(s): Stephen Richards and Gavin Jones  
Publication: Pensions - An International Journal, Volume 11, Issue 2, February 2006, Palgrave Macmillan  
URL: [www.sias.org.uk/data/papers/LongevityRisk/DownloadPDF](http://www.sias.org.uk/data/papers/LongevityRisk/DownloadPDF)

This paper examines longevity risk and its implications for actuaries working in the private sector. The paper postulates that the greatest exposure to longevity risk resides

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with the shareholders of those industrial and service companies with the largest exposure to longevity risk through their defined benefit pension plans. The paper goes on to say that while regulators are doing a good job in certain areas, i.e., by requiring stochastic modeling of liabilities in certain situations, in others, there exists room for improvement.

- A-22 Title: "Retirement Vulnerability of New Retirees: The Likelihood of Outliving Their Assets"  
Author(s): Ernst & Young LLP  
Publication: Americans for Secure Retirement, July 2008  
URL: [http://www.paycheckforlife.org/uploads/2008\\_E\\_Y\\_RRA.pdf](http://www.paycheckforlife.org/uploads/2008_E_Y_RRA.pdf)

This report analyzes the likelihood of middle-income Americans, including those who are near retirement and those who have recently retired, outliving their financial asset. According to this paper, it is not realistic for retirees to envision a retirement where their lifestyle continues much as before. With the current state of savings and potentially very long life expectancies, many retirees will have to cut back far more on expenditures than they had ever expected. This study finds that retirees are much better prepared to have a financial secure retirement if they have guaranteed source of retirement income beyond Social Security, such as annuity and defined benefit plan. The degree of retirement vulnerability varies across 50 states due to differences in employer pension plan coverage, income distributions and demographics.

- A-23 Title: "Substandard Annuities"  
Author(s): Matthew Drinkwater, Joseph Montminy, Eric Sondergeld, Christopher Raham, Chad Runchey  
Publication: LIMRA International, Inc. and the Society of Actuaries, 2006  
URL: <http://www.soa.org/files/pdf/007289-Substandard%20annuities-full%20rpt-REV-8-21.pdf>

This report describes substandard annuity products and discusses the market opportunity for them, the associated risk management issues, and the issues companies address when competing in this market. The project included a literature review of substandard annuities (including those sold in the United Kingdom), an industry survey of companies offering them, and interviews of both home office staff and producers

- A-24 Title: "Stochastic Analysis of Long Term Multiple-Decrements Contracts"  
Author(s): Matthew Clark and Chad Runchey  
Publication: Society of Actuaries, 2008  
URL: <http://www.soa.org/files/pdf/research-stochastic.pdf>

This report examines the application of stochastic modeling techniques to non-market risks facing the life insurance industry. The focus of this paper is the modeling and quantification of mortality and lapse risks with consideration for the impact reinsurance

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has on these risks. Borrowing from the techniques employed with market risks and a careful understanding of the risk composition of mortality and lapse behaviors, stochastic generators can be created to reflect the non-market risks faced by insurance companies.

- A-25 Title: "Clause 20 of Financial Services Law (retirement plans) 2005 - Israel"  
Author(s): Maurice Brandman  
Publication: Brandman Actuarial Consultancy Services, January 2006  
URL: [http://www.watsonwyatt.com/images/database\\_uploads/15735/BACS\\_Note\\_Clause20.pdf](http://www.watsonwyatt.com/images/database_uploads/15735/BACS_Note_Clause20.pdf)

This document comprises a description of Clause 20 of the new Financial Services Law, actions for consideration, background notes and commentary. This clause introduces a radical change in the administration of retirement benefit plans in Israel. It appears that the employer's ability to control the investment and terms of severance pay assets held as a component of an individual retirement benefit arrangement for current hires will be severely restricted. This note includes clarifications based on interpretations which were not available at the time the regulation was first published.

- A-26 Title: "LifeMetrics – A Toolkit for Measuring and Managing Longevity and Mortality Risks"  
Author(s): Guy Coughlan, David Epstien, Alen Ong, Amit Sinha, Javier Havia-Portocarrero, Emily Gingrich, Marwa Khalaf-Allah, Praveen Joseph  
Publication: JPMorgan Pension Advisory Group, March 2007  
URL: [http://www.jpmorgan.com/directdoc/lifemetrics\\_technical.pdf](http://www.jpmorgan.com/directdoc/lifemetrics_technical.pdf)

LifeMetrics is a toolkit for measuring and managing longevity and mortality risk, designed for pension plans, their sponsors, insurers, reinsurers and investors. LifeMetrics enables these risks to be measured in a standardized manner, aggregated across different sources and transferred to other parties. It also provides a means to evaluate the effectiveness of longevity/mortality hedging strategies and the size of the residual risk. This toolkit comprises of three components; LifeMetrics Index, LifeMetrics Framework and LifeMetrics Software.

- A-27 Title: "The 2008 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds"  
Author(s): Social Security Administration  
Publication: Social Security Administration, March 2008  
URL: <http://www.ssa.gov/OACT/TR/TR08/tr08.pdf>

This report presents the current and projected status of trust funds. According to the report, annual cost will begin to exceed tax income in 2017 for the combined OASDI Trust Funds, which are projected to become exhausted and thus unable to pay scheduled benefits in full on a timely basis in 2041. For the trust funds to remain solvent throughout the 75-year projection period, the combined payroll tax rate could be increased during the period in a manner equivalent to an immediate and permanent

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increase of 1.70 percentage points, benefits could be reduced during the period in a manner equivalent to an immediate and permanent reduction of 11.5 percent, general revenue transfers equivalent to \$4.3 trillion in present value could be made during the period, or some combination of approaches could be adopted. Significantly larger changes would be required to maintain solvency beyond 75 years.

- A-28 Title: "Obesity – What it Means for Actuaries"  
Author(s): Sam Gutterman  
Publication: The Actuary Magazine – October/November 2008 Volume 5 Issue 5  
URL: <http://www.soa.org/library/newsletters/the-actuary-magazine/2008/october/act-2008-vol5-iss5-gutterman.aspx>

This article discusses the level and effect on mortality of being overweight or obese. The level of overweight and obesity may have an even more significant effect on morbidity and health care costs than on mortality. The level of obesity varies significantly by age, gender and ethnic group. The article also discusses the most common factors blamed for obesity and the contrasting views from other literature regarding the contribution of obesity to mortality experience.

- A-29 Title: "Tax-Efficient Pension Choices in the UK"  
Author(s): Paul Sweeting  
Publication: The Pension Institute, DISCUSSION PAPER PI-0708, June 2007  
URL: <http://www.pensions-institute.org/workingpapers/wp0708.pdf>

The special tax treatment of UK pensions means that the decision on how to use pension assets is more involved than in other tax jurisdictions. In this paper, the author compares the options available to retirees using stochastic modeling. The author finds that if the maximum amount of tax-free cash is available to be used to enhance retirement income, then phased retirement offers the best risk/reward trade off. The advantage is greatest for highest-rate tax payers. As the level of tax-free cash falls, income withdrawal becomes more attractive to those wishing to take greater risk.

- A-30 Title: "Amended and Restated Rules for the QxX Indices – April, 2008"  
Author(s): QxX Index Co., LLC  
Publication: QxX Index Co., LLC  
URL: [http://www.qxx-index.com/pdf/Amended\\_And\\_Restated\\_QxX\\_Rules.pdf](http://www.qxx-index.com/pdf/Amended_And_Restated_QxX_Rules.pdf)

QxX indices are designed to allow market participants to measure, manage and trade exposure to longevity and mortality risks in a standardized, transparent, and real-time manner. This document includes the rules that govern the index computation.

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- A-31 Title: "Credit Suisse Longevity Index Commentary"  
Author(s): Credit Suisse  
Publication: Credit Suisse  
URL: [http://www.credit-suisse.com/ib/doc/fixed\\_income/index\\_commentary.pdf](http://www.credit-suisse.com/ib/doc/fixed_income/index_commentary.pdf)

This document contains a high level description of the longevity index offered by Credit Suisse. It provides for key characteristics and data from the index, as well as sources for further information.

- A-32 Title: "Defined-benefit plans falling further out of favor, survey finds"  
Author(s): Jerry Geisel  
Publication: Financial Week May 22, 2008  
URL: <http://www.financialweek.com/apps/pbcs.dll/article?AID=/20080522/REG/205713784/1036>

This brief article highlights the findings of a recent Watson Wyatt survey on defined benefit plans.

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Title: "Joint Life Annuities and the Demand for Annuities by Married Couples"  
Author(s): Jeffrey R. Brown and James M. Poterba  
Publication: The Journal of Risk and Insurance, 2000, Vol. 67, No. 527-554  
URL: <http://www.business.uiuc.edu/jbrown/My papers/JRI%20Dec%202000.pdf>

The article is divided into five sections. The first describes the structure of joint annuity products and develops expressions for actuarially fair joint-life annuity products. The second section presents the authors' algorithm for evaluating the utility gains that flow from participating in an actuarially fair joint annuity market. The third section discusses a range of difficult calibration issues that arise in modeling household rather than individual annuity valuation. The fourth section reports the authors' basic results on "annuity equivalent wealth," which measures the household utility gain from access to an actuarially fair joint annuity market. It also reports evidence on how the authors' findings depend on assumptions about the correlation between mortality rates for husbands and wives and the degree of pre-existing annuitization. The fifth section concludes and suggests directions for further research

Title: "Longevity Risk: Measurement and Application Perspectives"  
Author(s): Emilia Di Lorenzo and Marinela Sibillo  
Publication: Proceedings in 2nd Conference in Actuarial Science and Finance on Samos, Karlovassi (Grecia)  
URL: <http://kpmiit.wl.dvqu.ru/library/samos2002/proceedSibillo.pdf>

This paper addresses the issue of longevity risk on a life insurance business and suggests the use of projected mortality tables to control this risk. Along with longevity risk, it recognizes the existence of insurance risk and financial risk as the two other risk components in a portfolio. In exploring the impact these risks have on a company's solvency, the authors recommend a European Economic Area (EEA) system of settling a minimum solvency margin to avert this risk. They also recognize the need for risk profiling to effectively overcome solvency problems. For the majority of the paper, the authors focus on projection risk, i.e., the model risk deriving from the choice of the projected mortality table to use for portfolio valuations. Projection risk in a random mortality and interest rate environment are considered and its impact on reserves is analyzed.

Title: "Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities"  
Author(s): Peng Chen and Moshe A. Milevsky  
Publication: Journal of Financial Planning, June 2003  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2003JUN.pdf](http://www.ifid.ca/pdf_workingpapers/WP2003JUN.pdf)

The authors in this paper focus on the risks faced by investors during retirement. Particularly, they address financial market risk and longevity risk. The modern portfolio theory (MPT) is presented as a solution for financial market risk. To insure against longevity risk, the authors recommend lifetime payout annuities and discuss its

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advantages and disadvantages. They end by discussing optimal asset allocation mix with payout annuities and develop numerical results.

Title: "Assessing Investment and Longevity Risks within Immediate Annuities"  
Author(s): Daniel Bauer, Frederik Weber  
Publication: University of Munich, Munich School of Management, July 2007  
URL: <http://epub.ub.uni-muenchen.de/1982/1/wp2007-07.pdf>

This paper provides a long-term analysis of the risks inherent within an insurers' annuity book. In particular it looks at the joint impact of systematic mortality risk and investment risk, as well as their respective influences on the insurer's financial position. The authors find that, using data from the UK, the risk premium charged for the aggregate longevity risk is very large relative to the risk characteristics.

Title: "Risk-Adjusted Retirement"  
Author(s): Kwok Ho, Moshe Arye Milevsky and Chris Robinson  
Publication: Canadian Investment Review, Spring 1996  
URL: <http://www.yorku.ca/milevsky/Papers/CIR1996A.pdf>

This paper addresses the issue of retirement planning and how individuals and families should allocate their assets between risky and risk-free assets in order to minimize their shortfall risk.

Title: "Mortality Risk, Inflation Risk and Annuity Products"  
Author(s): Jeffrey R. Brown - Olivia S. Mitchell - James M. Poterba  
Publication: National Bureau of Economic Research (NBER), July 2000  
URL: <http://www.business.uiuc.edu/jbrown/Mypapers/w7812%20PRC.pdf>

This paper presents the trend in the pricing of single-premium individual annuity products in the US. After describing the "money's worth" of the different types of nominal annuity products, the few inflation-protected US annuity products are introduced. As this market is effectively not developed in the US, similar products which exist in other countries are presented. The study concludes by summarizing data pricing of both nominal and inflation-linked annuities in the United Kingdom and other nations.

Title: "A Different Perspective on Retirement Income Sustainability: Introducing the Ruin Contingent Life Annuity (RCLA)"  
Author(s): H. Huang, M. A. Milevsky and T.S. Salisbury  
Publication: York University, Toronto, September 2007  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2007SEPT15\\_RCLA.pdf](http://www.ifid.ca/pdf_workingpapers/WP2007SEPT15_RCLA.pdf)

This paper introduces a new concept of product: the ruin-contingent life annuity (RCLA). This product offers an advanced life delayed annuity if both the individual is alive and if he is experiencing a bear market during the early critical years of retirement. After

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describing in more detail how such a product would work, a simplified approach of pricing is introduced. Then the need for such a product is discussed.

Title: "Variable Payout Annuities with Downside Protection: How to Replace the Lost Longevity Insurance in DC Plans"  
Author(s): Moshe A. Milevsky, Anna Abaima  
Publication: IFID Centre Research Report, October 2005  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2005OCT12.pdf](http://www.ifid.ca/pdf_workingpapers/WP2005OCT12.pdf)

This report deals with the introduction of variable payout annuities with downside protection inside an individually-controlled defined contribution plan. After reviewing the pure longevity insurance mechanics of variable or fixed payout annuity, this paper presents the general structure of variable payout annuities which provide downside protection. Then some simulation results regarding the value of having downside protection are presented.

Title: "Static and Dynamic Valuation of Guaranteed Minimum Withdrawal Benefits "  
Author(s): Moshe A. Milevsky and Thomas S. Salibury  
Publication: York University and the Fields Institute of Toronto, August 2004  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2004Aug25.pdf](http://www.ifid.ca/pdf_workingpapers/WP2004Aug25.pdf)

This paper deals with the valuation of a Guaranteed Minimum Withdrawal Benefit (GMWB), guarantee available on many variable annuity products. First, for the static approach, it is assumed that individuals behave passively in utilizing the guarantee. Then, for the dynamic approach, it is assumed that individuals seek to maximize the embedded option value by lapsing the product at an optimal time, i.e. once the expected present value of fees exceeds the present value of benefits. The main conclusion is that GMWB are usually underpriced for individuals who know how to utilize these options in a rational manner.

Title: "The New Retirement Challenge"  
Author(s): Jeffrey R. Brown  
Publication: University of Illinois at Urbana-Champaign and National Bureau of Economic Research, September 2004  
URL: [http://www.business.uiuc.edu/jbrown/Mypapers/ASR\\_whitepapers.pdf](http://www.business.uiuc.edu/jbrown/Mypapers/ASR_whitepapers.pdf)

This paper presents the changing retirement landscape in the US and concludes these changes will force future retirees to learn how to accumulate assets to last their lifetime. After highlighting the importance of life annuities to individual retirees, it explains the importance of annuitization for reducing the fiscal strain on federal, state and local means-tested programs for the aged. However, many people who could avoid dependence on these programs do not fully appreciate the financial implications of living a long life or the value of life annuitizations. It is to be noted that no tax or other policy

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incentives exist to encourage households to convert their nest eggs into streams of lifetime retirement income.

Title: "Living to 100 and Beyond: Implications for Retirement"  
Author(s): (Boston Annual Meeting) Moderator: Sam Gutterman, Panelists: Colin England, Alan Parikh, Robert Polorski  
Publication: Society of Actuaries. RECORD, Volume 28. No 3, October 2002  
URL: <http://www.soa.org/library/proceedings/record-of-the-society-of-actuaries/2000-09/2002/january/rsa02v28n395pd.pdf>

This paper discusses the two prevalent life expectancy theories: (1) there is a limitless lifespan as mortality improvements are going to carry on for a long time; and (2) there is a biological limit namely because of age-related diseases. The panelists then explain that pricing products given mortality improvements requires effective underwriting. It is postulated that while frailty may not be the cause of death, it is a major contributor in the deterioration of health. Therefore, it is important to consider frailty when underwriting elderly risks. Finally, this paper discusses the movement of retirement plans towards Defined Contribution type system.

Title: "Survivor Bonds: A Comment on Blake and Burrows"  
Author(s): Kevin Dowd  
Publication: The Journal of Risk and Insurance, 2003, Vol. 70, No.2, 339-348

The paper discusses how insurance companies should hedge aggregate mortality risk. The company profitability on annuity portfolios is heavily dependent on realized mortality and companies stand to lose a great deal if mortality improves unexpectedly. One way of hedging is through Survivor Bonds (SBs). SBs will provide coupon payments that are highly correlated with annuity payments thus providing a very good hedge against mortality improvement risk. The paper also provides alternate hedging means such as reinsurance, sale of life contracts and dynamic hedging. Reinsurance and dynamic hedging are not as cost effective as SBs while life contracts may not hedge the mortality risk that well. Other survivor derivatives mentioned in this paper are European put/call options, futures and swaps. Though the concept of SBs is in its infancy, the idea is a very sound one and may be a great way to manage longevity risk.

Title: "Living to 100 and Beyond: An Extreme Value Study"  
Author(s): Zhongxian (Jerry) Han  
Publication: The Living to 100 and Beyond Symposium Sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-vii.pdf>

This paper analyzes different statistical models and concludes that the transformed exponential model fits increasing age population data best. Han feels that current life

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tables do not accurately reflect the rapid acceleration in mortality rates for the elderly population. To conduct his analysis Han uses life tables from 1901-1999.

Title: "Ending the Mortality Table"  
Author(s): Edwin C. Husted  
Publication: The Living to 100 and Beyond Symposium Sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-ix.pdf>

This paper explores options available to calculate the ultimate age for a mortality table without having a significant financial impact on large pension plans. Four methods have been used to end mortality tables: forced method; blended method; pattern method and less-than-one method. The author has also put forth theories concerning the shape of the mortality tables: (1) mortality rates continue to increase with age until 1; (2) there is a natural limiting wall to the life span and (3) rates are asymptotic to an ultimate rate that is below 1. The author concludes that 120 is a reasonable ultimate age for tables today. This conclusion is based upon analysis of factual data (verification that people have lived between ages 115-122). Finally, the author concludes that there is no significant difference in pension reserves when the three mortality tables are applied.

Title: "Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities "  
Author(s): Peng Chen and Moshe A. Milevsky  
Publication: Journal of Financial Planning - June 2003 pgs. 64-72  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2003JUN.pdf](http://www.ifid.ca/pdf_workingpapers/WP2003JUN.pdf)

The shift in retirement funding from DB to DC requires investors to decide which products should be bought for retirement. There are two factors to consider: financial market risk and longevity risk. Lifetime annuities (payout annuities) will grow in popularity as longevity risk becomes prominent. The article analyzes the optimal asset allocation mix using three cases: (1) total altruism and complete bequest motives; (2) no bequest motives and (3) 20% bequest motives and 80% consumption motives.

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Title: "The Great Debate on the Outlook for Human Longevity: Exposition and Evaluation of Two Divergent Views"  
Author(s): Jacob S. Siegel  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xv.pdf>

This article presents two sides (1) life expectancy will reach 100 years in industrialized countries by 2060 and (2) life expectancy will not exceed 85 years by 2060 as the extension of life is greatly dependant upon the role of medical and other human interventions and biological considerations. The first theory is based on previous demographic trends for projecting mortality but the author does not find arguments as to why these trends will continue to be sound and convincing. The second argument is based on biological theory but lacks a formal statistical basis for developing projections of age-specific mortality.

Title: "Summary of Panel Discussion on Implications of Increasing Life Spans for the Private Sector"  
Author(s): Steven C. Siegel  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxviii.pdf>

This paper deals with the implications of an aging society. After presenting the societal and governmental implications, the effects on the retirement system are discussed. There is a focus on women, who are more impacted than men by the increase of the life expectancy. The implications on private-sector products are then introduced. The paper concludes with the thought that educating people on this subject will be crucial.

Title: "Search for Predictors of Exceptional Human Longevity: Using Computerized Genealogies and Internet Resources for Human Longevity Studies"  
Author(s): Natalia S. Gavrilova, Leonid A. Gavrilov  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-v.pdf>

This paper deals with research into the predicators of people living to 100 and beyond. After presenting the method of choosing and validating the underlying data, the assumptions which have been tested are presented. It seems that exceptional longevity is linked to family factors and early-life conditions. In particular, a person's birth order and the location of early growth of a person may be a determining factor.

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Title: "Reinsurers Turn Attention to Longevity Risks"  
Author(s): Jim Connolly  
Publication: National Underwriter Life & Health, January 2008

This article is a brief discussion of how reinsurers are managing longevity. It will continue to be both their opportunity and their challenge in 2008.

Title: "The Effects of Advanced Age Mortality Improvement on the Valuation of Variable Annuities with Guaranteed Death Benefits"  
Author(s): Lijia Guo  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxxv.pdf>

This paper studies the impact of improved advanced age mortality on the pricing and valuation issues of variable annuity contracts with guaranteed minimum death benefits (GMDBs). The contingent claim analysis is applied to evaluate the GMDBs, and the impact of the mortality improvement is then examined. The implications of the advanced age mortality improvement on GMDBs for various mortality models are discussed. A new hybrid mortality model is presented for the valuation of GMDBs.

Title: "Longevity Risk and Annuities: Methods for Modeling and Forecasting Mortality"  
Author(s): Steven Haberman, Richard Verrall and Marwa Abd El-Rahman Khalaf-Allah  
Publication: Cass Business School, 2003  
URL: <http://www.cass.city.ac.uk/faculty/s.haberman/files/milanconf2003-1.pdf>

In this presentation, the authors take a look at a few methodologies available to model and forecast mortality. They base their analyses on English life tables and discuss observed trends in mortality tables, namely, a downward trend in mortality rates, the rectangularization of the survival function and expansion. They end the presentation with an analysis of the differences in Annuity Values with age, interest rate, and level of mortality improvement as variables.

Title: "Valuation and Hedging of Life Insurance Liabilities with Systematic Mortality Risk"  
Author(s): Mikkel Dahl and Thomas Moller  
Publication: Independent, February 2006  
URL: <http://www.math.ku.dk/~tmoller/papers/systematic2006.pdf>

The authors in this paper propose the application of techniques from incomplete markets in order to hedge and value contracts with systematic insurance risk that cannot be diversified. Two possibilities are explored: risk-minimization techniques and utility

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indifference valuation and hedging. All technicalities are explained, so readers can simulate the seemingly complicated model.

Title: "The Impact of Medical Advances and Lifestyles on Mortality"  
Author(s): Tushar Chatterjee, Angus Macdonald, Catriona Macdonald, Edward Roche, and Howard Waters  
Publication: Faculty of Actuaries, January 2008  
URL: [http://www.actuaries.org.uk/\\_data/assets/pdf\\_file/0003/101892/fac\\_sm20080114.pdf](http://www.actuaries.org.uk/_data/assets/pdf_file/0003/101892/fac_sm20080114.pdf)

This paper summarizes the research resulting from two initiatives aimed at helping actuaries gain a better understanding of trends in morbidity and mortality: the Actuaries Panel on Medical Advances (APMA) and the Mortality Scoping Project. APMA developed a stochastic model to quantify effects of treatment with statins, smoking and obesity on expected future lifetimes and the prevalence of other medical conditions. Among other major conclusions was that smoking reduces expected future lifetime from age 20 by approximately 7 years for males and 6.3 years for females. The Mortality Scoping Project on the other hand attempts to bring broad and overlapping research programs on mortality from different disciplines to the attention of actuaries, and promotes joint work wherever possible.

Title: "New Evidence on the Money's Worth of Individual Annuities"  
Author(s): Olivia S. Mitchell, James M. Poterba, Mark J. Warshawsky, Jeffrey R. Brown  
Publication: The American Economic Review, December 1999  
URL: <http://www.business.uiuc.edu/jbrown/Mypapers/AER%20December%201999.pdf>

This paper presents data on the value of individual life annuities that were available in the private US market in 1995. The analysis shows that prices for a single premium immediate life annuity varies widely, and that the expected present discounted value of the annuity stream per dollar of premium averages between 80 and 85 cents for the population as a whole, and between 90 and 94 cents for annuitants.

Title: "The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program"  
Author(s): Jeffrey R. Brown, Olivia S. Mitchell and James M. Poterba  
Publication: National Bureau of Economic Research (NBER), March 1999  
URL: <http://www.business.uiuc.edu/jbrown/Mypapers/w7005%20real%20annuities.pdf>

The authors note the difference in the market for real annuities between the US and the UK, where index-linked government bonds are more widely available. They then look to specific products that offer inflation protection in the US market. The study finds that inflation protection appears to provide only modest value, and that people would be expected to value a variable payout equity-linked annuity more highly because the additional returns more than compensate for the volatility of prospective payouts.

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Title: "Asset Allocation, Life Expectancy and Shortfall"  
Author(s): Kwok Ho, Moshe Arye Milevsky and Chris Robinson  
Publication: Financial Services Review 3(2), pg. 109-126, 1994  
URL: <http://www.yorku.ca/milevsky/Papers/FSR1994A.pdf>

This article focuses on the question of asset allocation and how a retiree should allocate his/her wealth between risky and risk-free asset. The considerations for the allocation decision are the mortality tables, desired consumption, initial investable wealth and the age of a retiree. Equity is considered to be superior to bonds for holding periods greater than 10 years.

Title: "Efficiency Dynamics of the Canadian SPIA Market"  
Author(s): Moshe A. Milevsky and Flora Li  
Publication: Individual Finance and Insurance Decisions Centre, Summer 2007  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2007SUMMER\\_SPIA.pdf](http://www.ifid.ca/pdf_workingpapers/WP2007SUMMER_SPIA.pdf)

There has been a significant improvement in the relative efficiency of the Canadian SPIA market. There has been a decline in the spread or gap between the "highest" and "lowest" quotes offered by Canadian insurance companies. This is mainly because the cost of acquiring information about competing companies has declined dramatically and has generated an overall more efficient SPIA market.

Title: "Optimal Asset Allocation and The Real Option to Delay Annuitization: It's Not Now-or-Never"  
Author(s): Moshe A. Milevsky and Virginia R. Young  
Publication: Individual Finance and Insurance Decisions Centre, April 2002  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2002B.pdf](http://www.ifid.ca/pdf_workingpapers/WP2002B.pdf)

Voluntary annuitization amongst the public is not very common, nor is it well understood. There exists a real option (similar to the word usage in corporate terms) in the decision to annuitize and, given the move to DC pension plans, increasingly Americans are faced with the dilemma of when to annuitize. It is determined that from a practical point of view, most individuals in their 60s and 70s should hold a substantial portion of their wealth in non-annuitized assets since the option value attached to waiting is quite large. Finally the author concludes that the availability of low-cost variable immediate annuities reduces the value of the option to defer since the alternative asset class yields a similar pre-mortality return (ex. TIAA-CREF).

Title: "Private Pensions, Mortality Risk, and the Decision to Annuitize"  
Author(s): Jeffrey R. Brown  
Publication: National Bureau of Economic Research (NBER), June 1999  
URL: <http://www.business.uiuc.edu/jbrown/Mypapers/w7191%20JPubE.pdf>

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The author has found that a one-percentage point increase in annuity equivalent wealth corresponds to a one-percentage point increase in the probability of planning to annuitize. Marital status appears to be a particularly important in the decision to annuitize. Married couples are less likely to annuitize than are single individuals, presumably due to the ability to pool mortality risk. The results also indicate that individuals in very poor health are significantly less likely to annuitize.

Title: "Asset Allocation and the Transition to Income: The Importance of Product Allocation in the Retirement Risk Zone"  
Author(s): Moshe A. Milevsky and Thomas S. Salisbury  
Publication: Individual Finance and Insurance Decisions Centre, September 2006  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2006OCT3.pdf](http://www.ifid.ca/pdf_workingpapers/WP2006OCT3.pdf)

The first decade of retirement is a sensitive and fragile period regardless of who you are and how much you are spending. The factors that effect income during retirement depend on spending, investing or living too long. Poor market performance during the retirement risk zone can decimate an investment portfolio. As Canadian baby boomers approach the retirement risk zone they must realize that asset allocation alone is not enough to generate a sustainable income stream for their retirement of unknown length. Product allocation – namely buying instruments that can protect a portfolio against negative adverse returns early in retirement will become as important as asset allocation within the financial services industry over the next few years.

Title: "Optimal Asset Allocation and Ruin-Minimization Annuitization Strategies"  
Author(s): Moshe A. Milevsky, Kristen S. Moore, and Virginia R. Young  
Publication: Individual Finance and Insurance Decisions Centre, May 2004  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2004May10.pdf](http://www.ifid.ca/pdf_workingpapers/WP2004May10.pdf)

This paper derives the optimal investment and annuitization strategy for a retiree whose objective is to minimize the probability of ruin. The author found that ruin probability and optimal strategies respond in an intuitive and predictable way to changes in the model parameters. The impact of the mortality assumption, in particular the shape of the hazard rate function is significant.

Title: "Ruined Moments in Your Life: How Good Are the Approximations?"  
Author(s): H. Huang, M. Milevsky and J. Wang  
Publication: Individual Finance and Insurance Decisions Centre, February 2004  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2004FEB28.pdf](http://www.ifid.ca/pdf_workingpapers/WP2004FEB28.pdf)

This is a quantitative paper. The authors determine that a 65 yr old retiree requires 30 times their desired annual consumption to generate a 95% probability of sustainability, which is equivalent to a 5% probability of lifetime ruin. A Reciprocal Gamma approximation provides an accurate fit to estimating the probability of ruin.

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Title: "Annuitization and Asset Allocation"  
Author(s): Moshe A. Milevsky and Virginia R. Young  
Publication: Individual Finance and Insurance Decisions Centre, October 2003  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2003OCT15.pdf](http://www.ifid.ca/pdf_workingpapers/WP2003OCT15.pdf)

This article examines the optimal annuitization, investment and consumption strategies of a utility-maximizing retiree. The author presents 4 key points: (1) A person who is placed in an irreversible decision to annuitize has an incentive to delay. This is valuable a younger ages. (2) In an "all or nothing" framework (retirement) where one is forced to annuitize, the optimal age to annuitize is the age at which the 'option to delay' (utility) is zero. (3) In the event that asset allocation flexibility is available, the optimal age to annuitize is earlier. (4) In an open system (annuitize anytime) utility maximizing investors should acquire a base amount of annuity income and then annuitize additional amounts if and when their wealth-to-income ration exceeds a certain level.

Title: "Asset Allocation within Variable Annuities: The Impact of Guarantees"  
Author(s): Moshe A. Milevsky and Vladyslav Kyrychenko  
Publication: Individual Finance and Insurance Decisions Centre, June 2007  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2007JUNE28\\_AAVA.pdf](http://www.ifid.ca/pdf_workingpapers/WP2007JUNE28_AAVA.pdf)

This article examines policyholder behavior when certain insurance riders, giving investors the option to annuitize at a favorable rate, are available for variable annuity products. It can be seen that individuals will invest more aggressively when they are granted this type of put option. These put options are contingent on annuitizing so it will be interesting to see whether those who have the tendency to avoid annuitization will, in fact, exercise this option.

Title: "What to do about Retirement Ages"  
Author(s): Martin Sullivan  
Publication: Institute of Actuaries, January 2002  
URL: [http://www.actuaries.org.uk/\\_data/assets/pdf\\_file/0017/19331/sullivan\\_presentation.pdf](http://www.actuaries.org.uk/_data/assets/pdf_file/0017/19331/sullivan_presentation.pdf)

This paper looks at the impact on the state pension benefits of different socioeconomic groups as a result of rising state retirement ages. The author concludes that those with lower life expectancies experience a larger reduction in the value of their benefits as do citizens with below average earnings. The author suggests that an alternative, more equitable solution for curtailing large pension liabilities would be to create a financial incentive to postpone retirement: either by increasing the monthly pension benefit or by reducing the income taxes payable.

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Title: "High-Age Implications of Postretirement Risks"  
Author(s): Anna M. Rappaport and Monica Dragut  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xvii.pdf>

This paper focuses on retirement risks and special issues for retirees, and those close to retirement age, as private schemes move from defined benefit to defined contribution. The authors present a list of the primary post-retirement risks, identify the stakeholder who bears the risk, and propose ways to mitigate the risk where possible. Survey data showing which risks retirees are most worried about is presented and gaps in understanding, in particular with respect to longevity risk and the primary sources of retirement income, are highlighted. Finally the authors propose several improvements that can be adopted by private pension schemes to help mitigate some of these issues going forward.

Title: "Optimal Asset Allocation in Life Annuities: A Note"  
Author(s): Narat Charupat and Moshe A. Milevsky  
Publication: Insurance: Mathematics and Economics 30 (2002) 199–209  
URL: <http://www.yorku.ca/milevsky/Papers/IME2002B.pdf>

This paper looks at the optimal retirement asset mix (between the risky and risk-free asset) as compared to its preretirement counterpart in order to determine whether the annuitization option has any effect on the optimal mix. The author is interested in the interaction between financial risk, mortality risk and consumption, towards the end of the life cycle. The result is that the optimal asset allocation during the decumulation phase is the same as that during the accumulation phase: the classical Merton solution. This analysis did not consider the issues of when or how much to annuitize.

Title: "Self-Annuitization and Ruin in Retirement"  
Author(s): Moshe Arye Milevsky and Chris Robinson  
Publication: York University, September 1999  
URL: <http://www.yorku.ca/milevsky/Papers/NAAJ2000A.pdf>

This paper looks at the lifetime and eventual probability of ruin for an individual who wishes to self-annuitize (i.e., employ discretionary management of assets with systematic withdrawals). The authors believe that the probability of ruin, as a proxy for longevity risk, should help individuals assign a 'risk metric' to their investment and consumption portfolio. The analysis yields an approximation for net present value under stochastic return models. The result of the analysis shows that the lowest probability of ruin occurs with a well diversified portfolio.

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Title: "The Timing of Annuitization: Investment Dominance and Mortality Risk"  
Author(s): Moshe A. Milevsky and Virginia R. Young  
Publication: Individual Finance and Insurance Decisions Centre, June 2005  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP\\_WhyNotNow20JUNE2005.pdf](http://www.ifid.ca/pdf_workingpapers/WP_WhyNotNow20JUNE2005.pdf)

This study asserts that only a small number of retirees are using variable annuity contracts in the US. The authors postulate that the following two characteristics of the marketplace create a situation where self-annuitization dominates a payout annuity: all insurers charge a longevity risk fee to protect against population-wide reductions in mortality; and the set of options available within variable payout annuities is limited. The authors determine the optimal age at which a retiree should purchase an irreversible life annuity in the presence of these criteria. They show that annuitization prior to age 65-70 is dominated by self-annuitization; after age 70, the optimal age depends on the retiree's willingness to accept risk in exchange for retaining the benefits of liquidity. This study has implications for Social Security reforms in the US that will mandate annuitization at a set age.

Title: "Making Retirement Income Last a Lifetime"  
Author(s): John Ameriks, Robert Veres and Mark Warshawsky  
Publication: Journal of Financial Planning, December 2001  
URL: <http://www.tiaa-crefinstitute.org/research/articles/docs/120101.pdf>

This article models the portfolio liquidation process in order to explore the maximum withdrawal rate for a number of portfolio asset mixes. Historically, both the level and variability of the maximum sustainable withdrawal rate vary directly with the riskiness of the portfolio. However, for shorter retirement periods, the less volatile portfolios offer a better chance of survival. The authors demonstrate that the volatility of the withdrawal rates can be dampened by converting a portion of wealth at retirement into a fixed, immediate annuity thereby creating a longevity hedge. The downside of this approach is that it reduces the upside potential of the portfolio.

Title: "Survivor Swaps"  
Author(s): Kevin Dowd, David Blake, Andrew J. G. Cairns, and Paul Dawson  
Publication: Journal of Risk and Insurance, Vol. 73, No. 1, pp. 1-17, March 2006

This article discusses the possible uses of survivor swaps (SSs) as instruments for managing, hedging, and trading mortality-dependent risks including, for insurers, exploiting natural hedges in lines of business, diversifying mortality risk exposures, and rearranging mortality exposures across the mortality term structure. Capital market institutions may also be interested in SSs as low beta investment assets. Vanilla SSs are investigated in some detail including a discussion of how premiums and values may be set in an incomplete market.

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Title: "Recent Developments in Life Annuity Markets and Products"  
Author(s): Mark Warshawsky  
Publication: Benefits Quarterly, 2007, VOL 23; NUMB 2, pages 46-57

This article describes the different annuity products that are available to effectively reduce the risk of outliving one's assets. The article describes Single Premium Immediate Annuities (SPIAs), Variable Annuities and Inflation-Indexed Annuities. Another recent innovation in individual immediate life annuity products is the offering of enhanced liquidity, the ability to convert some of the value of the stream-of-income payments into a lump-sum payment after the start of payment flows. Individual DB Pensions are deferred annuities but are distributed as a life annuity and not a lump sums. Lifelong distribution is an increasingly common option offered on individual deferred variable annuities. The author has come up with an innovative product called Life Care Annuity that would integrate a SPIA with a significant pop-up benefit (ideally a cash and not an indemnity approach) when the annuitant becomes disabled with the common triggers found in Long Term Care Insurance policies.

Title: "Annuities and Individual Welfare"  
Author(s): Thomas Davidoff, Jeffrey R. Brown and Peter A. Diamond  
Publication: American Economic Review, 2005 Vol 95,  
URL: [http://www.business.uiuc.edu/jbrown/My Papers/AER\\_version\\_DBD.pdf](http://www.business.uiuc.edu/jbrown/My Papers/AER_version_DBD.pdf)

This article surrounds the idea of when is an ideal time for an individual to annuitize. The authors believe that certain consumers should fully annuitize all of their savings. A second contribution is to examine annuity demand in some incomplete market settings. If some desired consumption paths are not available when all wealth is annuitized in an incomplete annuity market, full annuitization may no longer be optimal. With incomplete markets, the full annuitization result can break down when there is a sufficient mismatch between the optimal consumption path and the income stream offered by the annuity market.

Title: "Asset Allocation Via The Conditional First Exit Time or How To Avoid Outliving your Money"  
Author(s): Moshe Arye Milevsky, Kwok Ho, and Chris Robinson  
Publication: Review of Quantitative Finance and Accounting, Vol. 9(1), July 1997  
URL: <http://www.yorku.ca/milevsky/Papers/RQFA1997A.pdf>

The authors use a stochastic mortality model with random rates of return, fixed initial wealth and desired level of consumption as an analytical tool to assess shortfall risk against different investment strategies. A unique property of the model developed is that consumption is endogenized and therefore a person can trade off changes in shortfall risk against changes in desired consumption or expected bequest. Principal findings are that retirees in most cases should hold a higher proportion of high risk, high return assets in their portfolios than traditionally recommended by planners. This is especially important for women as they live longer than men on average and therefore need to earn more from their retirement funds.

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Title: "The Implied Longevity Yield: A Note on Developing an Index for Life Annuities"  
Author(s): Moshe A. Milevsky  
Publication: Individual Finance and Insurance Decisions Centre, September 2004  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP\\_2004Sept15.pdf](http://www.ifid.ca/pdf_workingpapers/WP_2004Sept15.pdf)

This paper uses the idea of self-annuitization as the basis for constructing a life annuity index to track and explain the benefits of life annuity payouts over time. The Implied Longevity Yield (ILY) index is defined equal to the internal rate of return (IRR) that an individual would have to earn on their financial portfolio during deferral period, if they choose to self-annuitize, instead of purchasing a life annuity. As a result, the ILY index measures how much higher the payments are from annuities as compared to products that do not pool risk. One of the main advantages of the index developed in this paper is that it does not require any knowledge of the company's mortality tables, rates, loads or pricing basis. The authors also simplify the non-linear implied longevity yield equation into a neat quadratic approximation. By tracking the ILY values over time and across countries, one can obtain a better measure of the investment return from annuities without making distributional assumptions regarding mortality.

Title: "International Equity Diversification and Shortfall Risk"  
Author(s): Kwok Ho, Moshe Arye Milevsky and Chris Robinson  
Publication: Financial Services Review 8 (1999), pg 11-25  
URL: <http://www.yorku.ca/milevsky/Papers/FSR1999A.pdf>

The authors in this paper focus on the potential benefits for Canadian investors by investing in a portfolio that includes international equity and how such a portfolio can substantially reduce shortfall risk. On the other hand, such a strategy would not benefit American investors as much since the equity market in the United States is a large proportion of the international equity market. Moreover returns from the United States equity market are highly correlated with other markets.

Title: "How to Completely Avoid Outliving Your Money: An Introduction to Variable Payout Annuities for Retiring Canadians"  
Author(s): Moshe A. Milevsky  
Publication: Individual Finance and Insurance Decisions Centre, October 2002  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2002OCT1.pdf](http://www.ifid.ca/pdf_workingpapers/WP2002OCT1.pdf)

This paper focuses on how Canadian retirees should invest for retirement such that they do not outlive their money. Contrary to popular thought, the author argues, as he did in his 1994 paper, that a broadly diversified portfolio of equities is a prudent asset allocation even during retirement and performs better than a portfolio purely invested in Guaranteed Investment Certificates (GICs) and Treasury Bills. However, in the latter half of the paper, he introduces variable payout annuities (VPAs) as an even more efficient method to allocate assets in order to avoid outliving money for retirees solely concerned with maintaining a standard of living during retirement. The different aspects

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of VPAs, such as asset allocation mix, and ideal purchasing and annuitization ages are then discussed.

Title: "Supercentenarians: slower ageing individuals or senile elderly?"  
Author(s): Jean-Marie Robine and James W. Vaupel  
Publication: Experimental Gerontology 36 (2001) 915-930

Supercentenarians are growing rapidly. They are frail people with little resistance to environmental hazard. The death rate appears to be more a measure of the quality of the environment brought about by humans than a measure of biological aging. The maximum age has increased considerably since the 1980s (from 112 to 122). The quote "There is not only a necessity of building on the new concept of plasticity of ageing or of longevity describing how small modifications in environment or genomes may result in large changes in lifespan, we need also a model to explain the mortality trajectory taking into consideration the protected environment in which we place our frail older people." is an accurate synopsis of the entire paper.

Title: "Emergence of Supercentenarians in Low Mortality Countries"  
Author(s): Jean-Marie Robine and James W. Vaupel  
Publication: The North American Actuarial Journal (NAAJ), July 2002  
URL: [http://www.soa.org/library/journals/north-american-actuarial-journal/2002/july/naaj0207\\_6.pdf](http://www.soa.org/library/journals/north-american-actuarial-journal/2002/july/naaj0207_6.pdf)

The centenarian population is well documented but this is not the case for extremely old people having reached the age of 105 years and 110 yrs (semicentenaries and supercentenaries). Using alternative models other than Gompertz's curve to fit mortality experience at extreme ages will result in large numbers of supercentenaries in the future. There are large fluctuations in the distribution of deaths according to seasons of the year due to frailty.

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Title: "The Interdependency of Increasing Life Expectancy and Driving Life Expectancy of Elderly Populations"  
Author(s): Chao-Chun Leng and Min-Ming Wen  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-x.pdf>

Mortality dependency is an estimate of the number of surviving years an individual does not drive arrived at by comparing driving life expectancy to life expectancy. Mobility independence can be used as a measure of the quality of life of the elderly. Older drivers are at risk of social isolation; however, advocating elderly mobility is likely to create increased accident rates. To combat the degree of dependency this paper provides several options. One option is caregiving which will be a viable option for pre-baby boomers given the vast number of baby boomers, but is not a viable solution for the baby boomers themselves.

Title: "What if Mortality Was to Diminish Much More than Was Forecast? Implications for Financing Social Security"  
Author(s): Robert Bourbeau, Bertrand Desjardins, and Jacques Légaré  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-iv.pdf>

Aging is raising concerns among policy-makers over the funding of Social Security programs: especially pension and health care. Changes in mortality, in combination with changes in fertility, lead to unexpected consequences for public pension plans as the rise in retirement age has not paralleled the rise in individual life expectancy. Improvements in longevity increase the burden of pension benefits especially those funded by a pay-as-you-go system. This article sheds light on the consequences of longevity for these plans and provides a tool for policy makers to make informed decisions.

Title: "Trajectories of Disability and Mortality Among the U.S. Elderly Population: Evidence from the 1984–1999 NLTCS"  
Author(s): Eric Stallard  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxvii.pdf>

This is a very technical paper which introduces the reader to a broad range of research on survival at advanced ages, and integrates the findings of that research into a coherent model of the trajectories of changes in health and survival characteristics of individuals. The Grade of Membership (GoM) model is used to specify and estimate a

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multivariate model of the trajectories of disability and mortality. This model can be fitted to existing data and the results are interpretable as generalizations of fixed frailty with linearly declining vitality.

Title: "Analysis of Trends in the Age-Specific Shape of Mortality Curves for Populations in the United States and Japan"  
Author(s): Christine Dugan, Hande Gulumser, Richard Humble, and Daniel Ryan  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xx.pdf>

When age-specific mortality curves for population mortality experience are investigated, Japan has the highest population life expectancy of any developed nation over the last 20 years. The authors have developed models that attempt to relate the age-specific mortality curves for individual calendar years by means of time-dependent variables. The main conclusions are as follows: the Weibull distribution provides a valuable model of age at death; a logistic function provides a valuable model of mortality rates for ages 50 and over; and finally, the models chosen show that mortality rates will continue to decline.

Title: "IDL, the International Database on Longevity"  
Author(s): Jean-Marie Robine, Amandine Cournil, Jutta Gampe, and James W. Vaupel  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxiii.pdf>

This paper uses data in the International Database on Longevity, an international database which gathers longevity records from 28 developed countries for people who have reached at least their 110th birthday, to provide an estimate of the mortality trajectory up to age 114. The impetus was to test the hypothesis that mortality in the very aged is lower than expected using traditional models. This paper shows that mortality does not increase, or increases only very slightly, after 110 years of age implying that there is no biologically controlled limit on age linked to natural selection.

Title: "Hedging Brevity and Longevity Risk with Mortality-based Securities"  
Author(s): Richard MacMinn and Andreas Richter  
Publication: Munich School of Management, October 2006  
URL: [http://epub.ub.uni-muenchen.de/1219/1/MacMinnRichter\\_10\\_2006.pdf](http://epub.ub.uni-muenchen.de/1219/1/MacMinnRichter_10_2006.pdf)

The goal of this paper is to prove the usefulness (in terms of creating value) of mortality based securities in hedging risk. The determination of value considers the trade-off an insurer or reinsurer faces in optimizing a hedging strategy for its life insurance or annuity

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exposure. The primary interest is in the trade-off between the credit risk and the basis risk associated with an index or indemnity based security given the moral hazard problem faced in underwriting.

Title: "Variable Annuities versus Mutual Funds: A Monte Carlo Analysis of the Options"  
Author(s): Moshe A. Milevsky and Kamphol Panyagometh  
Publication: Financial Services Review 10 (2001) pg. 145-161  
URL: <http://www.yorku.ca/milevsky/Papers/FSR2002B.pdf>

The authors in this paper quantify the impact of investment return uncertainty when comparing variable annuities to mutual funds. These two products mainly differ in their tax treatment, causing their relative appeal to be a subject of intense debate in the industry. By using a Monte Carlo stochastic simulation model and computing their certainty equivalent of utility (CEU), the authors make a risk-adjusted comparison of the two. They conclude that although low-cost variable annuities - with insurance expense lower than 10 basis points - are superior to low-cost mutual funds with a long time horizon, the critical threshold is at least 10 years for typical levels of risk aversion.

Title: "Mortality at Advanced Ages in the United Kingdom"  
Author(s): Adrian P. Gallop and Angus S. Macdonald  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxi.pdf>

The paper gives an overview of the data available to GAD and CMIB and discusses the problems encountered in estimating mortality rates at old ages in the UK for both the general population and those taking out insurance. It describes the current and past methodologies used to construct mortality rates at advanced ages for official population life tables and the CMIB mortality tables of insured lives. Possible methods for projecting mortality rates at advanced ages are discussed. This analysis is especially relevant given the increase in the elderly population in the UK and the interest of the insurance regulators in stochastic assessment of mortality risk.

Title: "Seasons and Longevity: Mortality Trajectories among the Oldest Old"  
Author(s): Jean-Marie Robine, Siu Lan K. Cheung, and Fred Paccaud  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxiv.pdf>

This study looks at the use of season-related life tables to study mortality trajectories of the oldest old. The authors assert that these tables can be used to explore the plasticity of longevity by opposing winter and summer trajectories. Deleterious mortality conditions

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can artificially concentrate the ages at death of the oldest old in winter, therefore, they further assert that summer-related tables are most relevant to the study of the secular change in human longevity as they summarize the best conditions of mortality (i.e., lowest, most stable values).

Title: "The Cohort Effect: Insights and Explanations"  
Author(s): R. C. Willets  
Publication: Institute of Actuaries  
URL: [http://www.actuaries.org.uk/\\_data/assets/pdf\\_file/0003/31647/sm20040426\\_cohort.pdf](http://www.actuaries.org.uk/_data/assets/pdf_file/0003/31647/sm20040426_cohort.pdf)

The purpose of this report is to understand the UK 'cohort' effect (i.e., the phenomenon that people born in the UK between 1925 and 1945 have experienced more rapid improvement in mortality than generations born either side of the period) by analyzing mortality data by cause of death. It is suggested that the experience of different generations (before birth, in childhood and adulthood) is a powerful determinant of experience in later life and has predictive possibilities.

Title: "Increased Longevity and the Challenge of Determining qx at Extreme Ages—Part 1"  
Author(s): Allen M. Klein  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-tr1.pdf>

This transcript discusses two papers presented at the 'Living to 100 Symposium' and presents the results of an industry-wide mortality study (the results of this study have been since revised). The speaker asserts that the decision on how to first develop and then end a mortality table centers on three basic issues: available data, purpose of the table and whether changes will be needed based on future expectations (i.e., with respect to changes in lifestyle, environment, etc.). The Husted paper speaks to the first two issues, and the Dugan, et. al. paper explores the last.

Title: "Modeling Mortality: Empirical Studies on the Effect of Mortality on Annuity Markets"  
Author(s): Norbert Hari  
Publication: Tilburg University, August 2006  
URL: <http://arno.uvt.nl/show.cgi?fid=55820>

This paper considers the macroeconomic aspects of ageing (impact on productivity, labor supply, capital intensity, etc). The author concludes that the elderly are dying in an age interval which is getting narrower and that the survival function is expanding via a shift of the death curve and an upward shift in the max attainable age. The author characterizes longevity risk as: micro (uncertainty in the lifetime of an individual and the

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realized number of deaths) and macro (the added risk that defies the law of large numbers).

Title: "Differential Mortality and the Value of Individual Account Retirement Annuities"  
Author(s): Jeffrey R. Brown  
Publication: National Bureau of Economic Research, February 2000  
URL: <http://www.business.uiuc.edu/jbrown/Mypapers/w7560%20differential%20mortality.pdf>

This paper examines redistributions that would occur under various annuity and bequest options as part of an individual accounts retirement program. The author estimates mortality between different groups and estimates transfers based on mortality assumptions. The article measures the magnitude of the expected transfers that would result under various annuity options in an individual accounts system. The results are as follows: 1) Low-income age groups have higher mortality but also higher expected bequests than higher income groups. Moreover, pre-retirement bequests from individual accounts are more important to groups with high mortality and younger ages. 2) The degree of dispersion in the money's worth ratio is very sensitive to the structure of annuity programs.

Title: "Optimal Annuity Purchasing"  
Author(s): Virginia R. Young and Moshe A. Milevsky  
Publication: IFID Center, January 2003  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2003JAN26.pdf](http://www.ifid.ca/pdf_workingpapers/WP2003JAN26.pdf)

This article looks at the optimal annuity purchasing scheme for an individual who seeks to maximize her expected utility of lifetime consumption and bequest. The individual is allowed to possess pre-existing annuities, to annuitize only a portion of the wealth at a given time, to buy annuities more than once and to consume something other than the annuity income after annuitization. The article finds that if the marginal utility of annuity income is larger than the adjusted marginal utility of wealth, then the individual will annuitize a lump sum. Finally the article examines the annuity-purchasing problem for the specific case of an individual with preferences that exhibit constant relative risk aversion.

Title: "Mortality Derivatives and the Option to Annuitize"  
Author(s): Moshe A. Milevsky and S. David Promislow  
Publication: Insurance: Mathematics and Economics 29 (2001) pg. 299–318  
URL: <http://www.yorku.ca/milevsky/Papers/IME2001D.pdf>

Most US-based insurance companies offer holders of their tax-sheltered savings plans (VAs), the long-term option to annuitize their policy at a pre-determined rate over a pre-specified period of time. There are two risks inherent in these contracts: investment risk and longevity risk. This paper attempts to value mortality-contingent claims by stochastically modeling the hazard-plus-interest rate. The main qualitative observation is

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that both mortality and interest rate risk can be hedged, and the option to annuitize can be priced by locating a replicating portfolio involving insurance, annuities and default-free bonds.

Title: "Mortality Swaps and Tax Arbitrage in the Canadian Insurance and Annuity Markets"  
Author(s): Narat Charupat and Moshe A. Milevsky  
Publication: Journal of Risk and Insurance, June 2001  
URL: <http://www.yorku.ca/milevsky/Papers/JRI2000A.pdf>

This paper analyzes tax arbitrage opportunities that result from a mortality swap (i.e., the acquisition of a fixed immediate life annuity and the subsequent use of a periodic annuity to fund a term-to-life insurance policy). The leniency of Canadian tax law implies that the before-tax rate of return on this product will be the same as a bank deposit (risk free rate) while the after-tax rate of return will be higher. Tax arbitrage is less likely under the U.S. tax rules because the taxable portion increases to 100% after annuitants reach their life expectancies.

Title: "Valuation of Mortality Risk via the Instantaneous Sharpe Ratio: Applications to Life Annuities "  
Author(s): Erhan Bayraktar, Moshe A. Milevsky, S. David Promislow and Virginia R. Young  
Publication: IFID Center, March 2008  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2008MAR3.pdf](http://www.ifid.ca/pdf_workingpapers/WP2008MAR3.pdf)

This research challenges the assumption that mortality risk is diversifiable when pricing a life insurance or annuity contract. The authors purport that uncertainty in a hedging portfolio translates into a non-zero standard deviation per policy (law of large numbers does not apply). The paper describes how to use the instantaneous Sharpe ratio to value the life annuity.

Title: "Optimal Annuitization Policies: Analysis of the Options"  
Author(s): Moshe Arye Milevsky  
Publication: North American Actuarial Journal, January 2001  
URL: <http://www.yorku.ca/milevsky/Papers/NAAJ2001B.pdf>

The authors conclude that annuitization provides valuable longevity insurance and should be actively encouraged at higher ages. Utility-based arguments indicate that consumers would be willing to pay a substantial "loading" in order to gain access to a life annuity. Secondly, the large adverse selection costs associated with life annuities might serve as a strong deterrent to full annuitization. Thirdly, retirees with a (strong) bequest motive might be inclined to self-annuitize during the early stages of retirement. Lastly, variable immediate annuities are currently underutilized (and not available in certain jurisdictions) and can only grow in popularity.

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Title: "Exchanging Variable Annuities: An Optional Test for Suitability"  
Author(s): M.A. Milevsky and K. Panyagometh  
Publication: IFID Center, December 2003  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2003DEC19.pdf](http://www.ifid.ca/pdf_workingpapers/WP2003DEC19.pdf)

The authors postulate that variable annuity options can be decomposed into three options: lapse value, death value and guaranteed annuity value. Therefore, a VA policy can be decomposed into a portfolio of financial guarantees and then the difference in aggregate value between the embedded options can be computed using an option pricing model. An example of a particular transaction or exchange might result in a reduction in the lapse value component but a significant increase in the mortality component. The approach is to add-up the value of the options that are being surrendered and compare them to the value of options that are being acquired to determine if it is beneficial.

Title: "Patterns of Mortality Improvement over Age and Time in Developed Countries: Estimation, Presentation and Implications for Mortality Forecasting"  
Author(s): K. Andreev and J. Vaupel  
Publication: Independent, February 2005  
URL: <http://paa2005.princeton.edu/download.aspx?submissionId=51061>

This article uses extreme value techniques to analyze Canadian and Japanese mortality data at high levels. The author fits a generalized Pareto distribution to future lifetimes of survivors to a fixed age obtained from the Canadian data. The data suggests that there is a finite upper bound on the distribution of human life spans. A similar fit of generalized extreme value distributions on Japanese data suggests that an unbound future lifetime may be possible.

Title: "Lifetime Financial Advice: Human Capital, Asset Allocation, and Insurance"  
Author(s): Roger G. Ibbotson, Moshe A. Milevsky, Peng Chen and Kevin X. Zhu  
Publication: Research Foundation of CFA Institute, April 2007  
URL: <http://www.cfapubs.org/doi/pdfplus/10.2470/rf.v2007.n1.4580?cookieSet=1>

This paper uses the principles of life-cycle finance to optimize an individual's asset allocation by including human capital and human capital-contingent assets (life insurance and annuities) with the more traditional assets (bonds, equity). The aim is to go beyond the traditional aims of minimizing the likelihood of falling below a target to guarantee a standard of living.

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Title: "Portfolio Choice with Puts: Evidence from Variable Annuities"  
Author(s): Moshe A. Milevsky and Vladyslav Kyrychenko  
Publication: IFID Center, January 2008  
URL: [http://www.ifid.ca/pdf\\_workingpapers/WP2008JAN29.pdf](http://www.ifid.ca/pdf_workingpapers/WP2008JAN29.pdf)

The authors investigate the asset allocation behavior of individuals who select an out-of-the-money long-dated "longevity put" option on their investment funds against those who do not. The conclusion is that these investors take-on 5% to 30% more risky/equity exposure when protected. Finally, the authors present a rudimentary model of utility-maximizing behavior that justifies the increased allocation to risk, provided the investor is willing, able and understands to exercise the longevity put option if-and-when it matures in the money.

Title: "Gerontology: Implications for Future Mortality Experience"  
Author(s): Alex Comfort  
Publication: Faculty of Actuaries, February 1970  
URL: [http://www.actuaries.org.uk/data/assets/pdf\\_file/0015/26160/0157-0179.pdf](http://www.actuaries.org.uk/data/assets/pdf_file/0015/26160/0157-0179.pdf)

Until now, medicine has concentrated on the treatment of specific diseases and this has contributed notably to reductions in mortality rates in early and middle life. This progress has, however, not materially increased the total life span. Future improvements in longevity may therefore come from methods of delaying the aging process, rather than from specific advances in preventative medicine. This paper discusses the possibilities of, and need for research into these factors.

Title: "An Extreme Value Analysis of Advanced Age Mortality Data"  
Author(s): Kathryn A. Robertson, Debbie J. Dupuis, and Bruce L. Jones  
Publication: North American Actuarial Journal, October 2006  
URL: [http://www.soa.org/library/journals/north-american-actuarial-journal/2006/october/naaj0604\\_10.pdf](http://www.soa.org/library/journals/north-american-actuarial-journal/2006/october/naaj0604_10.pdf)

This paper presents a statistical analysis of advanced age mortality data, using extreme value models to quantify the upper tail of the distribution of human life spans. The analysis focuses on mortality data from Canada and Japan. The authors fit the generalized extreme value and generalized Pareto distributions to the life span data. Changes in distribution by birth cohort or over time are modeled through the use of covariates. The appropriateness of the fitted models and any shortcomings are discussed. Finally, the findings are used to discuss the existence of a finite upper bound and the behavior of the force of mortality at advanced ages.

Title: "Perspectives on Mortality Forecasting: Causes of Death"  
Author(s): Tommy Bengtsson and Kaare Christensen  
Publication: Försäkringskassan, Swedish Social Insurance Agency, 2006  
URL: [http://www.fk.se/filer/publikationer/pdf/sis\\_4.pdf](http://www.fk.se/filer/publikationer/pdf/sis_4.pdf)

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The authors argue that subjective judgments are always made when projecting future rates of mortality. Therefore, in order to understand trends in aggregate mortality, trends in individual causes of death need to be analyzed. The authors purport that it is necessary to form a view on the forces driving historic trends. Cause-of-death modeling can also be a good methodology to test “extreme scenarios”, which are becoming of increasing interest to insurance regulators and capital markets.

Title: "Modelling and Management of Mortality Risk: A Review"  
Author(s): Andrew J. G. Cairns, David Blake and Kevin Dowd  
Publication: The Pensions Institute, May 2008  
URL: <http://www.pensions-institute.org/workingpapers/wp0814.pdf>

This paper considers the range of extrapolative stochastic mortality models that have been proposed in the last 20 years. The authors discuss how these models can be evaluated, compared and contrasted. The authors discuss a discrete-time market model that facilitates valuation of mortality-linked contracts with embedded options. Several approaches to modeling mortality in continuous time are presented and used to examine the potential for dynamic hedging of mortality risk. Finally, a range of financial instruments (traded and over-the-counter) that could be used to hedge mortality and risk are reviewed.

Title: "Securitization, Structuring and Pricing of Longevity Risk"  
Author(s): Samuel Wills and Michael Sherris  
Publication: The Pensions Institute, June 2008  
URL: <http://www.pensions-institute.org/workingpapers/wp0816.pdf>

This paper considers the securitization of longevity risk focusing on the structuring and pricing of a longevity bond using techniques developed in the financial markets, particularly for mortgages and credit risk. A model based on Australian mortality data and calibrated to insurance risk linked market data is used to assess the structure and market consistent pricing of a longevity bond. Age dependence in the securitized risks is shown to be a critical factor in structuring and pricing longevity linked securitizations.

Title: "Longevity Risk Pricing"  
Author(s): Jiajia Cui  
Publication: SOA Living to 100 Symposium, January 2008  
URL: <http://www.soa.org/files/pdf/2008-orlando-cui-05.pdf>

Longevity-linked securities are not traded in financial markets due to the pricing difficulties. This paper proposes a method to price the longevity risk premia, which are helpful for market participants to negotiate. Based on the equivalent utility pricing principle, the minimum risk premium required by the longevity insurance seller and the maximum acceptable risk premium by the longevity insurance buyer are obtained. The longevity risk premia are within a narrow range and consistent with the financial risk

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premia (e.g. inflation risk premium). The authors show that the size of the risk premium depends on the payoff structure of the security, the financial strength of the longevity insurance seller and buyer, and the availability of the natural hedges.

Title: "Pricing the Risk on Longevity Bonds"  
Author(s): Andrew J.G. Cairns, David Blake, Paul Dawson, Kevin Dowd  
Publication: Independent, February 2005  
URL: <http://www.ma.hw.ac.uk/~andrewc/papers/ajgc43.pdf>

The impact of increasing longevity on pension provision has become a major concern recently. More effective management of longer-term mortality risk has been made possible following a recent and innovative bond issue. This paper analyses the risk premium embedded in this instrument and argues that demand for further such instruments and innovations can be expected.

Title: "Coping with Longevity: The New German Annuity Valuation Table DAV 2004 R"  
Author(s): Ulrich Pasdika and Jürgen Wolff  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xvi.pdf>

The German annuity market is dominated by immediate and deferred annuities. Neither product contains a significant death benefit, so industry practice is to take explicitly into account the number of insured lives that will die before the benefit payment period. This exposes insurers to longevity risk. The boom in the German annuity market since 1994 (the publication date of the last mortality table) has incited the a study of the DAV1994R table and the resulting exposure of its inadequacy. This paper discusses the derivation of the new DAV2004R.

Title: "Some Background from Census 2000"  
Author(s): W. Ward Kingkade  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxxiv.pdf>

This paper presents certain data from Census 2000 as background to the research and discussion that follows in subsequent volumes. Of particular note are the following: there is a growing number of women at advanced ages; and nearly 1/3rd of all US women aged 65 and older are living alone vs. 1/6th of men in the corresponding age bracket.

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Title: "Number of Centenarians in the United States Jan. 1, 1990, Jan. 1, 2000, and Jan. 1, 2010 Based on Improved Medicare Data"  
Author(s): Bert Kestenbaum and B. Reneé Ferguson  
Publication: Living to 100 and Beyond Symposium sponsored by the Society of Actuaries, January 2005  
URL: <http://www.soa.org/library/monographs/retirement-systems/living-to-100-and-beyond/2005/january/m-li05-1-xxvi.pdf>

This article examines two assertions: the centenarian population in a developed country in 1985 was about 1/20th of its population over 75 (Kannisto, 1990); and the number of people in a country achieving the century mark in a given year is about double the number who achieved that milestone 10 years earlier (Vaupel and Jeune, 1995). The conclusion is that the size of the centenarian population at Jan 1, 1990 and Jan 1, 2000 and projected to Jan 1, 2010 are in strong conflict with both of these assertions. The demographic composition of the centenarian population on Jan 1, 2000 is also presented and contrasted with the US 1990 decennial census.

Title: "Facing Up to Uncertain Life Expectancy: The Longevity Fan Charts"  
Author(s): Kevin Dowd, David Blake, and Andrew J. G. Cairns  
Publication: The Pensions Institute Discussion Paper PI-0703, June 2008  
URL: <http://www.pensions-institute.org/workingpapers/wp0703.pdf>

This paper first addresses longevity risk and the changing landscape around mortality in the UK. The authors then examine the quantifiable uncertainty associated with expected future lifetimes using longevity fan charts. The fan charts are based on English and Welsh mortality data and reference a stochastic mortality model developed in a paper by the same authors in 2006. The fan charts are used to show life expectancy improvements over time and carry out stress tests. A comparison of the parameter certain model and the parameter uncertain model confirms that longevity risk is above all a trend risk: getting the trend right is the key to successful forecasting. Even though highly uncertain, the main findings confirm that expected future lifetimes are projected to increase strongly.

Title: "On Systematic Mortality Risk and Risk-Minimization with Survivor Swaps"  
Author(s): Mikkell Dahl, Martin Melchior and Thomas Moller  
Publication: Independent, March 2007  
URL: <http://www.actuaries.org/AFIR/Colloquia/Stockholm/Dahl.pdf>

Systematic mortality risk inherent in life insurance contracts is discussed in this paper. The authors note that this risk can be addressed in two ways: via mortality-linked contracts, or by hedging with financial assets. The paper focuses on the latter, particularly on survivor swaps. A stochastic representation of survivor swaps is developed and applied to insurance contracts. The authors then determine risk-minimizing strategies in different financial markets. Lastly, the authors develop investment strategies using survivor swaps and/or bonds. The authors note that the

## Appendix B - Complete Longevity Risk and Annuity Market Literature Review

most significant source of risk in insurance contracts is the interest rate, which can be considerably reduced using bonds. An increased portfolio size diversifies away the unsystematic mortality risk, and survivor swaps help reduce systematic mortality risk.

Title: "On the Pricing of Longevity-Linked Securities"  
Author(s): Daniel Bauer, Matthias Borger and Jochen Russ  
Publication: Independent, August 2008  
URL: <http://www2.toulouse.inra.fr/lerna/congres/egrie/papers/Borger,%20Matthias%20-Paper.pdf>

This paper reviews, compares and comments on the methods that have been proposed to price longevity derivatives. It uses these techniques to price the EIB/BNP Longevity Bond that was designed (but not issued) in 2004. The authors propose a different design structure which has the potential to avoid some of the problems of the EIB/BNP Bond.