APPENDIX A: FIGURES

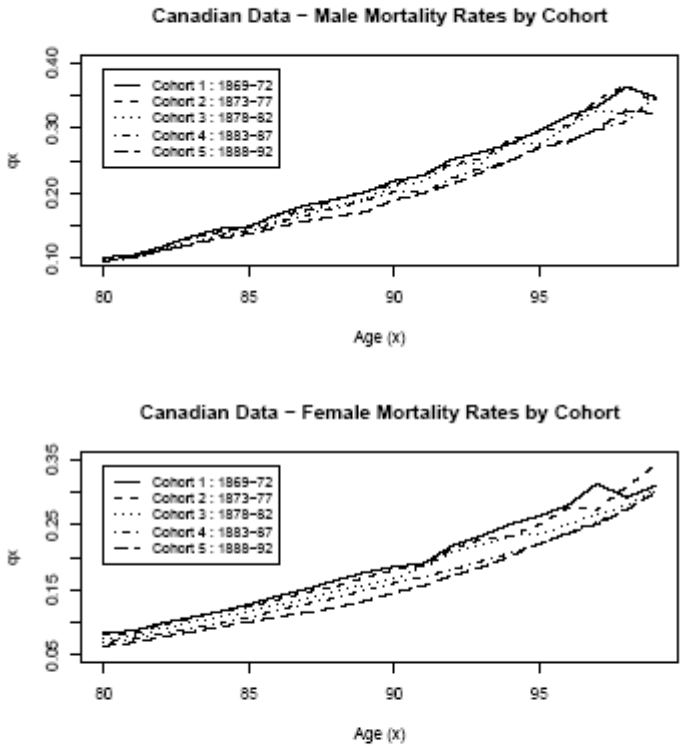

Figure 1: Canadian empirical cohort mortality rates.

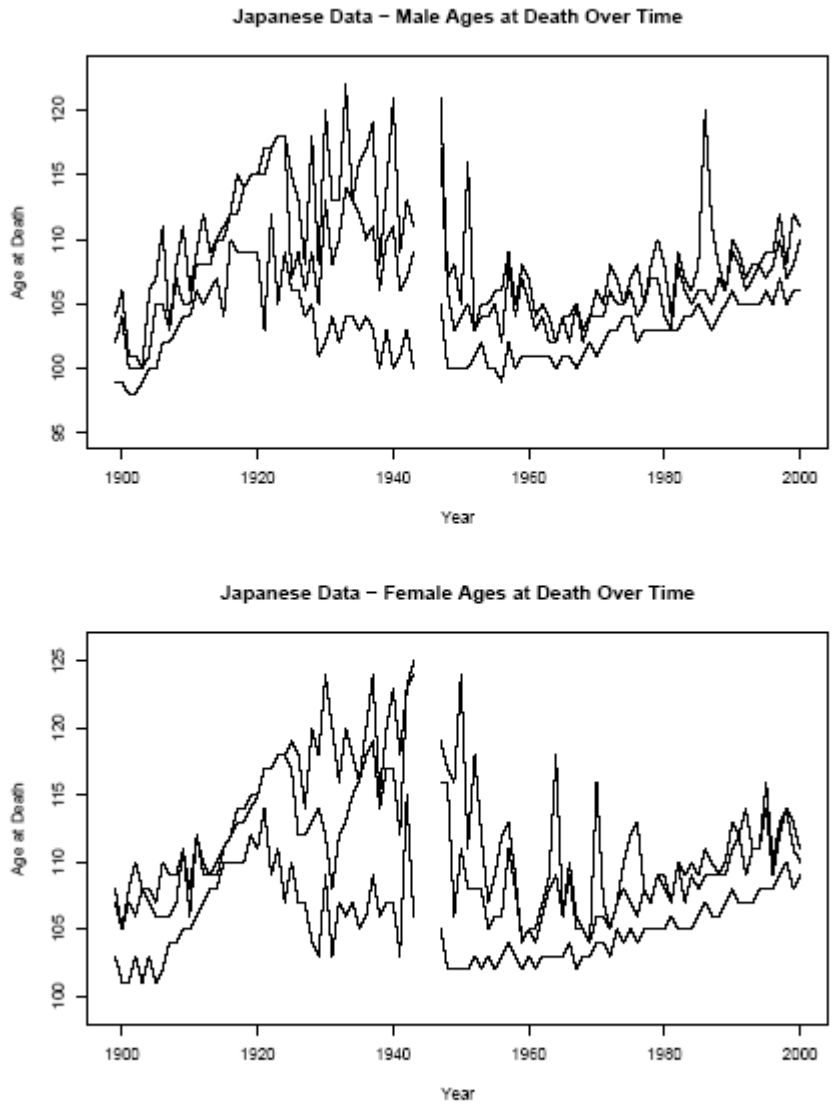

Figure 2: Largest, second-largest, and tenth-largest ages at death for the Japanese data, 1899-2000.

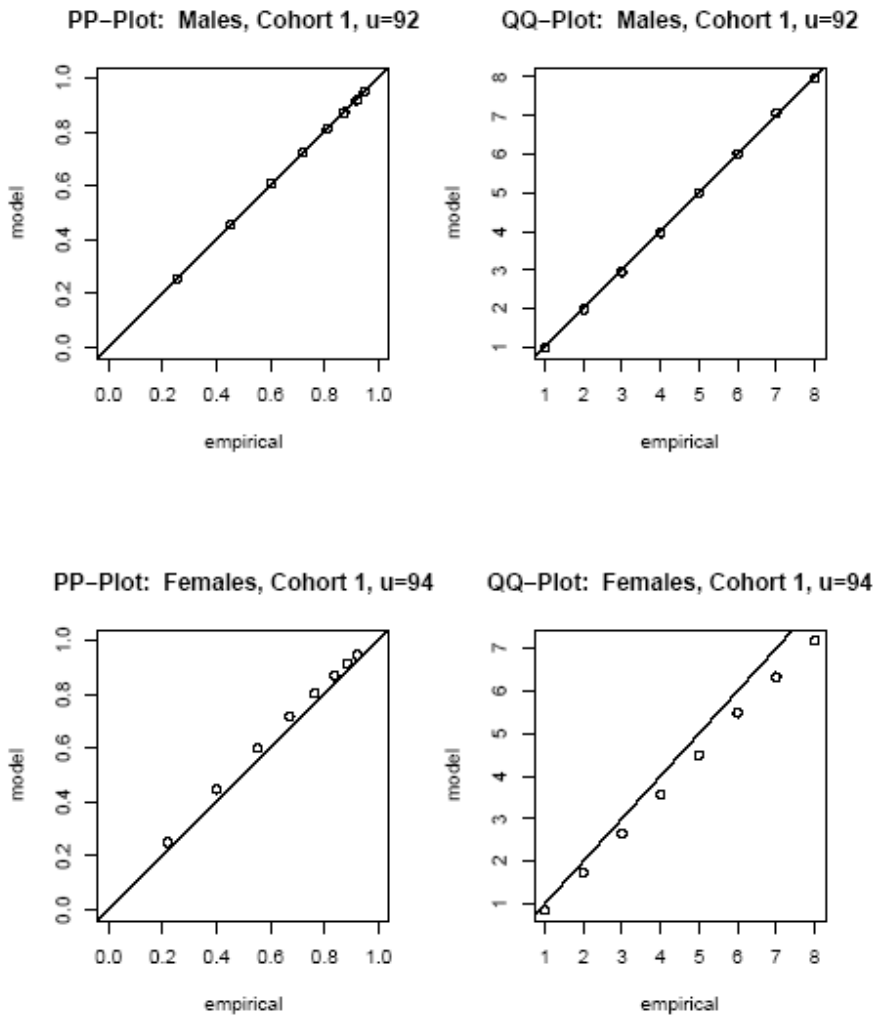

Figure 4: PP and QQ plots for the GP distributions fitted to exceedances of 92 (for males) and 94 (for females), for the Canadian data, Cohort 1.

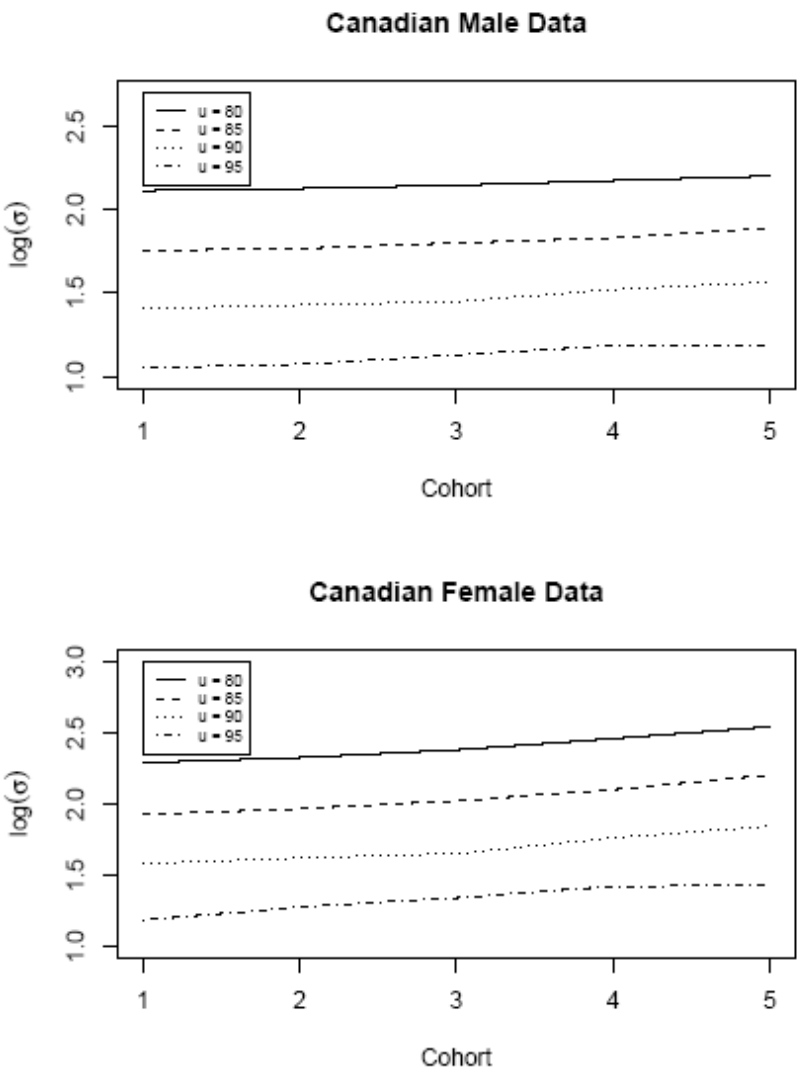

Figure 3: Maximum likelihood estimates of $\log(\sigma_y)$ for the Canadian male data.

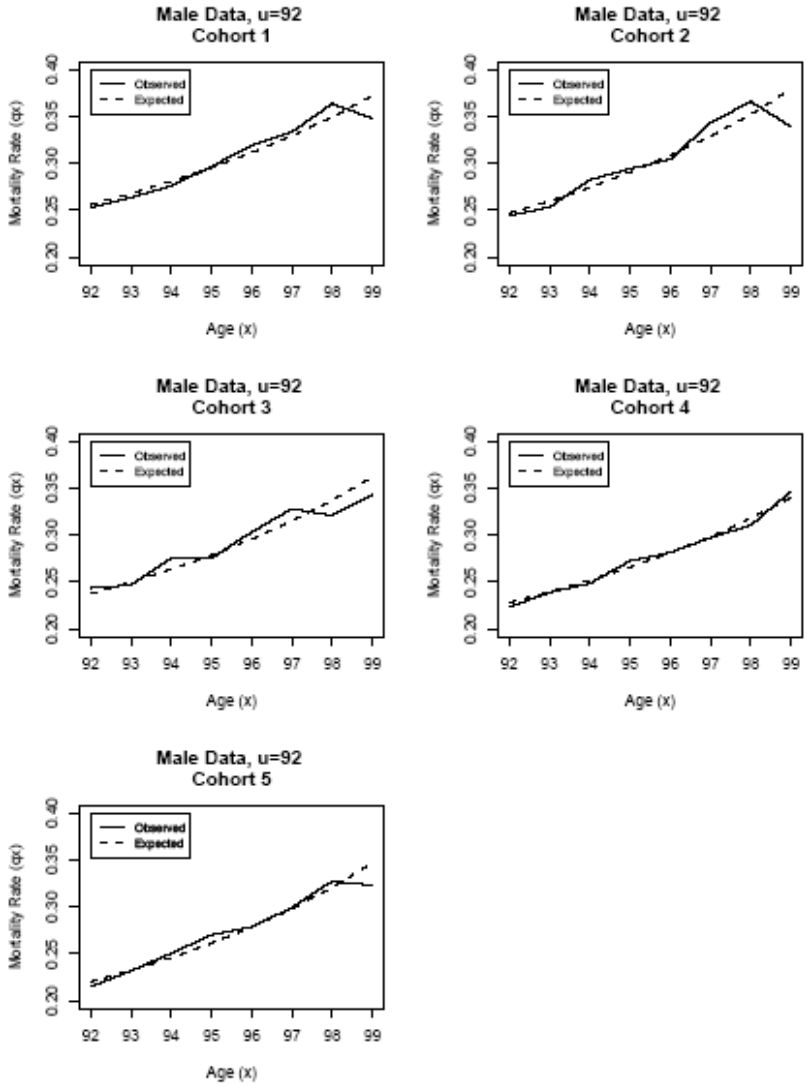

Figure 5: Fitted and observed mortality rates for the Canadian male data. The fitted rates are based on GP distributions fit to exceedances of 92.

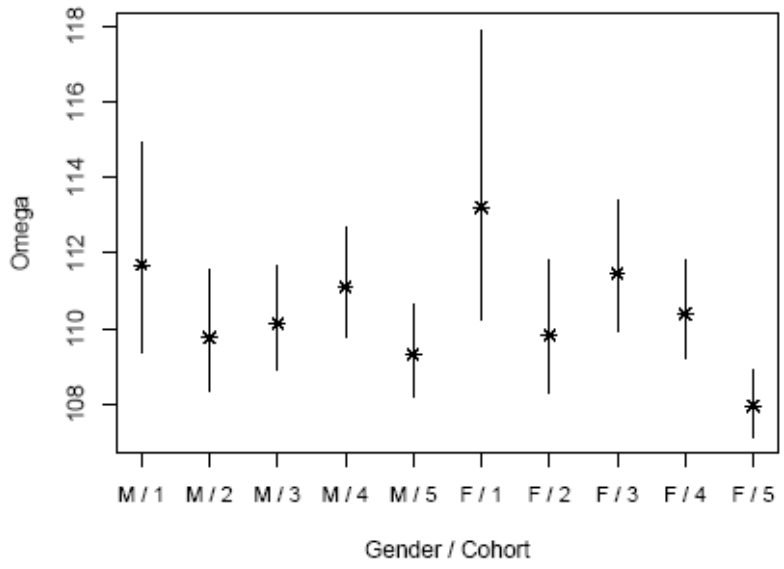

Figure 7: Estimates of ω with 95% profile confidence intervals for the Canadian data.

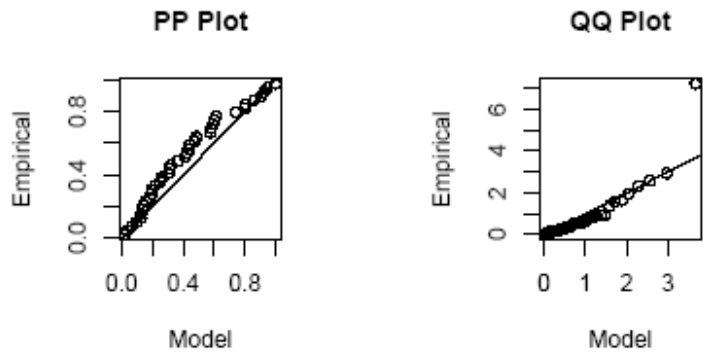

Figure 8: Diagnostic plots for the 10-largest model fit to the Japanese male data, with the potential outlier in 1986 included. Here, μ is a linear function of t, while σ is constant.

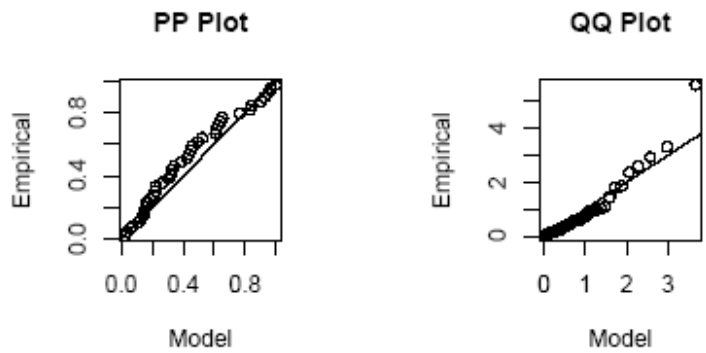

Figure 9: Diagnostic plots for the 10-largest model fit to the Japanese male data, with the potential outlier in 1986 replaced by a more plausible value. Here, μ is a linear function of t, while σ is constant.

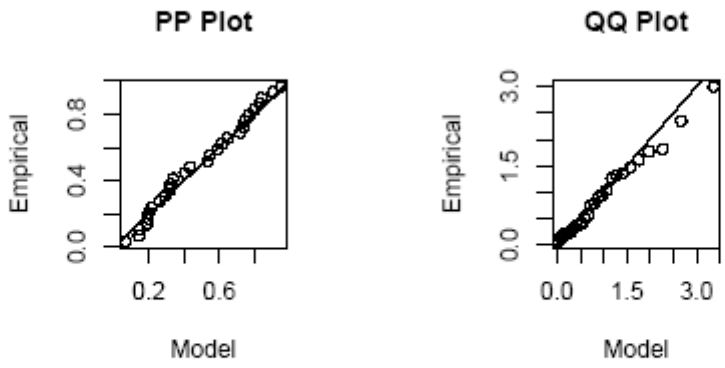

Figure 10: Diagnostic plots for the 10-largest model fit to the Japanese female data, with the potential outlier in 1986 included. Here, μ and σ are linear functions of t.

Threshold	Cohort (y)	Males		Females	
		σ_y	ξ_y	σ_y	ξ_y
80	1	8.27 (0.04)	-0.354 (0.003)	9.85 (0.04)	-0.419 (0.003)
	2	8.37 (0.03)	-0.358 (0.002)	10.21 (0.04)	-0.434 (0.003)
	3	8.53 (0.03)	-0.359 (0.002)	10.78 (0.04)	-0.454 (0.002)
	4	8.77 (0.03)	-0.362 (0.002)	11.67 (0.04)	-0.49 (0.002)
	5	9.02 (0.03)	-0.37 (0.002)	12.69 (0.04)	-0.54 (0.002)
85	1	5.77 (0.04)	-0.276 (0.005)	6.83 (0.04)	-0.328 (0.005)
	2	5.85 (0.03)	-0.28 (0.004)	7.13 (0.04)	-0.345 (0.004)
	3	6.04 (0.03)	-0.285 (0.004)	7.53 (0.03)	-0.36 (0.004)
	4	6.23 (0.03)	-0.283 (0.004)	8.15 (0.03)	-0.389 (0.004)
	5	6.61 (0.03)	-0.311 (0.004)	9.04 (0.03)	-0.454 (0.003)
90	1	4.08 (0.05)	-0.216 (0.011)	4.84 (0.05)	-0.265 (0.01)
	2	4.17 (0.04)	-0.225 (0.009)	5.04 (0.04)	-0.282 (0.008)
	3	4.26 (0.04)	-0.218 (0.008)	5.2 (0.04)	-0.271 (0.008)
	4	4.58 (0.04)	-0.237 (0.008)	5.8 (0.04)	-0.321 (0.007)
	5	4.78 (0.04)	-0.264 (0.008)	6.3 (0.04)	-0.383 (0.006)
95	1	2.87 (0.09)	-0.154 (0.039)	3.25 (0.08)	-0.144 (0.035)
	2	2.93 (0.08)	-0.168 (0.031)	3.57 (0.07)	-0.248 (0.027)
	3	3.09 (0.07)	-0.174 (0.029)	3.81 (0.07)	-0.229 (0.025)
	4	3.27 (0.07)	-0.186 (0.027)	4.13 (0.06)	-0.295 (0.022)
	5	3.27 (0.06)	-0.181 (0.025)	4.15 (0.05)	-0.294 (0.019)

Table 1: Parameter estimates (standard errors) for the GP distributions fit to the Canadian data, assuming that σ and ξ vary by cohort and gender.

Males				
Threshold	Cohort (y)	β_0	β_1	ξ_y
80	1	2.104(0.003)	0.023(0.001)	-0.349(0.002)
	2			-0.359(0.002)
	3			-0.362(0.001)
	4			-0.362(0.001)
	5			-0.367(0.002)
85	1	1.736(0.005)	0.035(0.002)	-0.266(0.004)
	2			-0.283(0.003)
	3			-0.289(0.002)
	4			-0.291(0.003)
	5			-0.302(0.003)
90	1	1.387(0.009)	0.043(0.003)	-0.202(0.009)
	2			-0.226(0.007)
	3			-0.235(0.005)
	4			-0.233(0.006)
	5			-0.259(0.007)
95	1	1.054(0.023)	0.036(0.008)	-0.154(0.031)
	2			-0.183(0.023)
	3			-0.170(0.019)
	4			-0.162(0.019)
	5			-0.196(0.022)

Table 2: Parameter estimates (standard errors) for the GP distributions fit to the Canadian male data, assuming that $\sigma_y = \exp[\beta_0 + \beta_1(y-1)]$ while ξ_y differs by cohort.

Females				
Threshold	Cohort (y)	β_0	β_1	ξ_y
80	1	2.262(0.003)	0.066(0.001)	-0.402(0.002)
	2			-0.438(0.002)
	3			-0.467(0.001)
	4			-0.494(0.001)
	5			-0.531(0.002)
85	1	1.893(0.004)	0.073(0.001)	-0.308(0.004)
	2			-0.346(0.003)
	3			-0.346(0.003)
	4			-0.399(0.002)
	5			-0.441(0.003)
90	1	1.544(0.008)	0.071(0.002)	-0.239(0.008)
	2			-0.279(0.006)
	3			-0.302(0.005)
	4			-0.321(0.005)
	5			-0.374(0.006)
95	1	1.216(0.017)	0.057(0.006)	-0.182(0.026)
	2			-0.247(0.020)
	3			-0.220(0.017)
	4			-0.254(0.015)
	5			-0.320(0.017)

Table 3: Parameter estimates (standard errors) for the GP distributions fit to the Canadian female data, assuming that $\sigma_y = \exp[\beta_0 + \beta_1(y-1)]$ while ξ_y differs by cohort.

Males				
Threshold	Cohort (y)	β_0	β_1	ξ_y
92	1	1.24(0.01)	0.044(0.004)	-0.176(0.013)
	2			-0.204(0.010)
	3			-0.209(0.008)
	4			-0.207(0.008)
	5			-0.239(0.010)
Females				
Threshold	Cohort (y)	β_0	β_1	ξ_y
94	1	1.26(0.01)	0.063(0.004)	-0.185(0.019)
	2			-0.239(0.014)
	3			-0.231(0.012)
	4			-0.262(0.011)
	5			-0.329(0.012)

Table 4: Parameter estimates (standard errors) for the GP distributions fit to the Canadian data, assuming that $\sigma_y = \exp[\beta_0 + \beta_1(y-1)]$ while ξ_y differs by cohort.

Group	μ_0	μ_1	σ	ξ
Males, Including Outlier	106.91(0.33)	3.70(0.38)	1.64(0.15)	0.008(0.04)
Males, Outlier Replaced	106.80(0.32)	3.53(0.38)	1.48(0.13)	-0.06(0.05)
Females	109.19(0.33)	3.96(0.47)	1.42(0.14)	-0.12(0.06)

Table 5: Parameter estimates (standard errors) for the 10-largest models fit to the Japanese data, assuming that $\mu(t) = \mu_0 + \mu_1 t^*$ while σ is constant.

Age(x)	Males, Including Outlier		Males, Excluding Outlier		Females	
	q_x	μ_x	q_x	μ_x	q_x	μ_x
114	0.4480	0.5957	0.5525	0.7861	0.4781	0.6179
115	0.4463	0.5927	0.5692	0.8225	0.5157	0.6849
116	0.4447	0.5897	0.5870	0.8625	0.5592	0.7683
117	0.4431	0.5868	0.6057	0.9066	0.6099	0.8747
118	0.4414	0.5839	0.6255	0.9555	0.6694	1.0154
119	0.4398	0.5810	0.6465	1.0099	0.7389	1.2099
120	0.4382	0.5781	0.6687	1.0709	0.8188	1.4967
121	0.4367	0.5753	0.6921	1.1397	0.9045	1.9617
122	0.4351	0.5725	0.7169	1.2180	0.9772	2.8458
123	0.4335	0.5697	0.7430	1.3078	$0.9^{4}80$	5.1804
124	0.4320	0.5670	0.7704	1.4120		28.8376
125	0.4305	0.5643	0.7990	1.5341		
126	0.4290	0.5616	0.8287	1.6795		
127	0.4274	0.5590	0.8591	1.8552		
128	0.4259	0.5563	0.8896	2.0719		
129	0.4244	0.5537	0.9193	2.3461		
130	0.4229	0.5511	0.9468	2.7038		
131	0.4215	0.5485	0.9703	3.1903		
132	0.4200	0.5460	0.9876	3.8902		
133	0.4185	0.5435	0.9971	4.9835		
134	0.4171	0.5410	0.9998	6.9315		
135	0.4157	0.5385	$0.9^{7}88$	11.3801		
136	0.4142	0.5360		31.7691		

Table 7: Mortality rates and forces of mortality for the Japanese data, based on the GEV parameter estimates from the r-largest approach. For males, $\mu(t) = \mu_0 + \mu_1 t^*$, while σ is constant. For females, $\mu(t) = \mu_0 + \mu_1 t^*$ and $\sigma(t) = \exp\{\sigma_0 + \sigma_1 t^*\}$.

Group	μ_0	μ_1	σ_0	σ_1	ξ
Males, Incl. Outlier	106.87(0.56)	3.77(1.04)	0.49(0.14)	0.01(0.24)	0.008(0.04)
Males, Outlier Repl.	106.77(0.51)	3.59(0.90)	0.38(0.14)	0.02(0.22)	-0.06(0.05)
Females	108.43(0.38)	5.47(0.77)	0.08(0.14)	0.45(0.19)	-0.15(0.06)

Table 6: Parameter estimates (standard errors) for the 10-largest models fit to the Japanese data, assuming that $\mu(t) = \mu_0 + \mu_1 t^*$ and $\sigma(t) = \exp\{\sigma_0 + \sigma_1 t^*\}$.