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Full Funding of Traditional State and Local Government Pensions: The Entry-Age-Service-Cost Method¹

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Abstract

Traditional defined-benefit pension plans like the California Public Employees' Retirement System (CalPERS) use the entry-age-normal-cost method to determine required contributions. The method uses a single contribution rate, the plan-normal rate, that applies to all employees. The plan-normal rate is based on a variety of assumptions, including an assumption about the pay raises that employees will receive. Unfortunately, when an actual pay raise differs even modestly from what is assumed, the plan-normal rate does not accurately reflect the impact of that raise on pension liability. That causes a mismatch between budgeted pension costs and true pension costs.

We propose instead that the employer should be required to contribute, on an employee-by-employee basis, the amount that correctly funds the liability attributable to that employee's actual pay raise. We call our proposed method the "entry-age-service-cost" method. The method ensures that the impact of actual pay raises on liability is paid for in the budget that grants the raises. This provides the employer with a lever to control pension costs, because small changes in pay raises can have a big impact on budgeted pension costs, for better or worse. It also solves funding problems that arise from pension spiking, delayed retirement, and employee transfers.

1. Introduction

State and local governments often use traditional defined benefit pension plans to help ensure that their employees will have adequate pensions throughout their retirement years. For example, in a typical final-average-pay pension plan, long-service employees can expect to receive a pension benefit (B) equal to 2 percent times years of service (yos) times final average pay (fap). Under that plan, an employee with a 30-year career and final average pay of \$100,000 could expect to receive a pension of \$60,000 a year for life ($\$60,000 = 2\% \times 30\ yos \times \$100,000\ fap$).

Pension actuaries help employers with defined-benefit pension plans determine the annual contributions that those employers are required to make to fully fund the retirement benefits that those employers have promised their employees. When pension actuaries are accurate in their

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forecast of future pension costs and employers actually make their annual required contributions (ARCs), pension plans should be very close to fully funded. On the other hand, if the actuarial forecast turns out to be inaccurate or if employers do not make their annual required contributions, then plans can become significantly over- or underfunded. In the real world, far more state and local government pension plans are underfunded than overfunded (Public Plans Database, 2018; Munnell & Aubry, 2016). In some plans, that underfunding can be attributed to state and local governments not making their ARCs (Brainard & Brown, 2015). But in a significant number of these plans, employers have made their required contributions, and yet their plans are underfunded. It follows that in these underfunded plans that have made the required contributions, the problem is that the actuarial forecast turned out to be inaccurate.

In this paper, we show how the failure of pension plan actuaries to properly account for the effects of individual pay raises on pension liabilities can lead to significant underfunding of state and local pension plans.² We then show how properly accounting for individual pay raises would help ensure full funding of these plans.

Our proposal is to modify the method that traditional defined-benefit pension plans like CalPERS use to determine their annual required contributions (ARCs) (CalPERS, 2017). Currently, most traditional defined-benefit pension plans (i.e., final-average-pay pensions) use the *entry-age-normal-cost method* (Actuarial Standards Board, 2017). That actuarial-funding method results in a single contribution rate (the plan-normal rate) that applies to the employer's aggregate payroll. The plan-normal rate is based on a variety of actuarial assumptions, including an assumption (or assumptions) about the projected salary growth of the employer's covered workers.

Instead of using an assumed pay raise for each employee to determine the amount to contribute on behalf of that employee and then aggregating the result into a plan-normal rate, we believe that the employer should be required to contribute, for each employee, the amount that is actually necessary to fully fund the increase in the employee's accrued pension liability that is directly attributable to that employee's *actual* annual pay raise. Accordingly, rather than calculating a plan-normal rate based on assumed pay raises and then applying that rate to the actual payroll, the plan actuary would determine the annual required contribution on an employee-by-employee basis based on the pay raise that the employer actually gives to each employee. We call our proposed method the "entry-age-service-cost" method.

Pertinent here, we cannot help but notice that CalPERS cities and other public agencies are in a tough spot. Their annual required contributions have risen significantly in recent years due to past underfunding, and that has put a great deal of stress on annual budgets (Linn, 2017). However, since these employers cannot lower promised benefits, nor control their required contributions, they may feel that they have no solution—and indeed, that is largely true with the entry-age-normal-cost method. Under our proposed entry-age-service-cost method, however, these employers would have a lever by which they could exert control over their pension costs,

² As used here, the term *pay raise* means any increase in salary that an employee experiences for any reason. In particular, it includes the salary increase an employee might receive if promoted to a higher-level position. It also includes increases due to seniority, merit, inflation, etc. We caution that the term *pay raise* is sometimes used more narrowly than it is here, to refer only to an increase in salary for a particular job description and experience level.

and that lever is employee salary. As more fully explained later, the marginal effect of a change in salary on accrued pension liability can be quite high, particularly with long-time employees. Put simply, *a modest increase (or decrease) in an employee's salary in a given year can lead to a much larger increase (or decrease) in the pension contribution required for that employee.*

To be sure, the current entry-age-normal-cost funding method has the laudable goal of simplicity and predictability, but the method masks the potentially large impact that even modest salary changes can have on an employer's accrued pension liability. As a result, the entry-age-normal-cost method does not provide employers with the right incentives, as the annual budget of the employer does not take much of a hit when pay raises cause a large increase in accrued pension liability, nor does the budget get much relief when salary discipline yields a big decrease in accrued pension liability. While changes in an employer's accrued pension liability do show up in the pension plan's valuation and on the employer's financial statements, those changes do not show up in the employer's annual budget, and that absence all but eliminates any incentive the employer has to currently consider the impact of salary on pension costs.

The entry-age-service-cost method that we propose is primarily targeted toward multiple-employer traditional defined-benefit pension plans for which state law: (1) prohibits the reduction of both accrued benefits and future benefit accruals; and (2) requires that employers make annual contributions as determined by the plan's actuary.³ We are particularly interested in California cities (and other public agencies) with pension plans that are administered by CalPERS; however, our entry-age-service-cost method could be applied to any multiple-employer pension plan in other state and local systems that operates under a framework similar to CalPERS. For that matter, our entry-age-service-cost method could apply to almost any final-average-pay pension plan.

While implementing our entry-age-service-cost method might require some state and local governments to enact legislative changes, we believe that many pension plans could implement the change administratively. CalPERS, for example, already has the authority to determine the amount that each of its covered employers must contribute to their pension plans each year (CalPERS, 2018), and, therefore, CalPERS should be able to use the entry-age-service-cost method to determine that amount.⁴

This paper develops the technical details of how to implement the entry-age-service-cost method. Instead of using an actual defined-benefit pension plan, the paper uses a simplified model

³ See, e.g., CalPERS (2014) (noting that CalPERS administers "multiple-employer" plans).

⁴ While the employee-by-employee nature of our proposed entry-age-service-cost method is slightly more complicated than applying a single plan-normal rate to the aggregate payroll, it would be easy enough for the employer and plan actuary to manage. The plan actuary does not need any salary data beyond what is already needed for the entry-age-normal method. But instead of calculating a plan-normal rate and providing that to the employer, the plan actuary calculates a service-rate formula for each employee and provides that formula to the employer. The employer implements the formula in the payroll-processing software, just as it does any other tax or benefit withholding. The employer also uses the service-rate formula to budget pension costs. This is only slightly more complicated than how the employer currently budgets pension costs under entry-age-normal-cost method—the spreadsheet gets a little more complicated, but not much, because it already involves employee-by-employee recordkeeping. The point is that the actual pension cost for the year will be included in the employer's budget—and that cost might be very different than the plan-normal cost, for better or worse.

pension plan that sidesteps cluttering details. At the outset, part 2 of the paper creates the simple model pension plan. Part 3 then shows how the entry-age-normal-cost method fails to properly account for individual pay raises that differ from the pension plan’s assumed salary growth rate. Part 4 then develops the entry-age-service cost method and explains how it promotes full funding each year. Part 5 then explains how the entry-age-service-cost method would influence employer determinations about pay raises and budgets, and part 5 also discusses how the method would apply to pension spiking, delayed retirement and employee transfers. Finally, part 6 offers some concluding remarks.

2. A Model Pension Plan

America’s state and local governments operate thousands of traditional final-average-pay pension plans (U.S. Census Bureau, 2018; Public Plans Database, 2018). Indeed, CalPERS alone is really a system that includes hundreds of similar final-average-pay benefit pension plans for the numerous cities, water districts, and other public employers in California (CalPERS, 2017). Instead of using an actual final-average-pay pension plan for our analysis, we create a simplified model pension plan that sidesteps cluttering details, and, later in this Paper we use that model pension plan to show how our proposed method for using individual salary, pay raise, and pension information to determine annual required contributions on an employee-by-employee basis would work and how it would promote full funding of state and local pensions.

2.1 Overview

For simplicity, we develop the proposed entry-age-service-cost method by creating a model pension plan that is similar to—but less complicated than—the kind of traditional final-average-pay pension plans that state and local employers like CalPERS use. The model plan allows us to focus on the essence of the entry-age-service-cost method, without getting bogged down in cluttering details.

Benefit description. The model pension uses a “2-percent-at-60” benefit rule, as follows:

- An employee who enters service at age 59 or younger is eligible to retire at age 60 with an initial pension amount equal to 2 percent times the number of years of service (*yos*) times the final year’s pay (*fp*) ($B = 2\% \times yos \times fp$).⁵
 - For example, an employee who begins service at age 30 and is eligible to retire at age 60 will receive an initial pension amount equal to 60 percent of the salary earned during age 59 ($B = 60\% \times fp = 2\% \times 30 yos \times fp$). If the employee defers retirement until age 65, the initial pension amount would be 70 percent of the salary earned during age 64 ($B = 70\% \times fp = 2\% \times 35 yos \times fp$).
- An employee who enters service at age 60 or older is not eligible for a pension benefit.

⁵ For simplicity, we base the model pension amount on the final year’s salary, as is done by some plans within CalPERS and elsewhere. Other plans, however, base the pension amount on the average salary over several final years, rather than on the single final year. We acknowledge that by picking the single-year approach for our model plan, we are picking the more expensive of the two possibilities, but we choose it because it is less complicated to discuss.

- There is no vesting period. An employee is eligible for a pension benefit at age 60 regardless of the number of years of service.⁶
- The pension benefit is a single-life annuity, with fixed cost-of-living increases of 2 percent annually throughout retirement.⁷

Discount rate. As many traditional state and local pension plans do, we will assume that the discount rate is 7 percent (see, e.g., National Association of State Retirement Administrators, 2017; CalPERS, 2016). That means that for the purposes of calculating liabilities and funding requirements, investments held in the pension plan are assumed to earn a 7-percent rate-of-return.⁸

Mortality rates. For simplicity, we choose a reasonable and static mortality table (see Appendix Table A1). Specifically, we use the Society of Actuaries (SOA) RP-2014 mortality table with the MP-2016 projection scale (Society of Actuaries, 2018a; Society of Actuaries, 2018b). That table with the projection scale is generational, meaning mortality rates change over time. We make the table static by fixing the projection to the year 2038, so that the resulting table reflects mortality rates as they are projected to be in 2038, which is 20 years forward from the current year of 2018.⁹

For further simplicity, we make the table gender-neutral by averaging the mortality rates of males and females. This streamlines the discussion, because we do not have to present separate results for males and females.¹⁰

Annuity factor. When an employee retires, the actuarial liability for the pension is the starting amount of the pension times an annuity factor. The annuity factor is the expected present discounted value of the employee's pension, adjusted to an initial pension amount of \$1. Calculating the annuity factor is a standard exercise (which we explain in the Appendix), and Table 1 shows the resulting annuity factors for our model pension for individuals of various ages. For example, for an employee retiring at age 60 with an initial pension amount of \$100,000, the

⁶ In the real world, 5-year vesting periods are common, and employees who terminate before vesting only get their own contributions back, so our model plan is more generous in that regard.

⁷ In the real world, joint-life annuities are common, with a surviving spouse receiving pension payments after the death of the retiree, so our model plan is less generous in that regard.

⁸ Much has been written about the topic of discount rate, and it is not our intent to wade into that discussion here. That said, we do point out that in the case of CalPERS, actual investment return has historically outperformed assumed discount rates (Sabin, 2016), and the current underfunded status is due to inadequate contributions rather than to underperforming investments (Sabin, 2015). Accordingly, our motivation here is to address the contribution side of pension plans, not the investment side, so we choose a discount rate that is historically appropriate. Choosing a different discount rate would not change our analysis and conclusions.

⁹ Our choice of a static, forward-looking mortality table mimics what CalPERS and many other pension plans do. CalPERS produces its own mortality table drawn from its own experience studies (see, e.g., CalPERS, Actuarial Office, 2014). The CalPERS table is static and projects mortality rates 20 years forward from the date of the experience study. Unfortunately, CalPERS does not publish the complete table, only excerpts. That makes it hard to use here, so instead, we use an industry-standard table.

¹⁰ We are not advocating that a gender-neutral table be used in practice; we do it here merely for convenience.

actuarial liability at the start of her retirement will be \$1,424,000 ($\$1,424,000 = 14.24 \text{ annuity factor} \times \$100,000 \text{ initial pension amount}$).

Table 1. Annuity Factors

Age	Annuity Factor
60	14.24
61	14.00
62	13.75
63	13.50
64	13.24
65	12.97
66	12.69
67	12.41
68	12.12
69	11.83
70	11.52
71	11.22
72	10.90
73	10.58
74	10.25

2.2 Normal Cost Funding

As already mentioned, state and local government pension plans typically use the entry-age-normal-cost method to determine employer funding and contributions, so that is where we start with our model pension plan. The method is well known, so here we will describe it only briefly, the intent being to specify how it is implemented in our model plan.

In the entry-age-normal-cost method, the projected cost of an employee’s pension is spread over the projected working life of the employee and expressed as a fixed percentage of salary. This percentage is called the *normal rate*. With a static mortality table, as we use here, the normal rate is the same for every employee who enters service at a given age, regardless of current age—hence, the name “entry-age-normal-cost.”

Salary growth rate. To calculate the normal rate, we need to make assumptions about how an employee’s salary grows during her working years. For the model pension plan, we assume that each employee’s salary grows at a 4-percent annual rate. That is, we adopt a year-over-year 4-percent salary growth rate assumption. For example, if the salary of an employee in the current year is \$100,000, we assume that it will be \$104,000 next year, \$108,160 the following year, and so on.¹¹

Assumed retirement age. For the purposes of calculating the normal rate, we assume that all employees age 59 or less will choose to retire at age 60. Normal rates are not calculated for employees who are already eligible to retire, which for the model pension plan means employees

¹¹ CalPERS makes more complicated salary growth rate assumptions that vary depending upon the worker’s category, entry age, and duration of service (CalPERS, Actuarial Office, 2014, pp. 62–65).

who are age 60 or older;¹² it is assumed that the pension liability for these employees has already been funded.¹³

2.2.1 Normal Rate

Calculating the normal rate is a standard exercise (which we explain in the Appendix), and Table 2 shows the *benefit factors* and normal rates that we calculated for several entry ages in our model pension plan. For example, the benefit factor at age 60 for an employee that starts working at age 30 and retires at age 60 is 60 percent ($60\% = 2\% \times 30 \text{ yrs}$). Her normal rate of contribution would be 17.68 percent, meaning that during each of her 30 years of service, 17.68 percent of her salary should be contributed to her pension plan.¹⁴ The *normal cost* for an employee for a given year is the dollar value of the amount contributed, meaning the normal rate times salary for that year. For example, an employee whose entry age is 30 and who makes \$100,000 in the current year, the normal cost would be \$17,680 ($\$17,680 = 17.68 \times \$100,000$).

Table 2. Benefit Factors and Normal Rates

Entry Age	Benefit Factor At Age 60	Normal Rate
20	80%	14.99%
30	60%	17.68%
40	40%	20.73%
50	20%	24.14%

2.2.2 Plan-Normal Rate

The *plan-normal rate* is the ratio of *A* over *B* where: *A* is the total dollar value of the normal costs of all active employees younger than age 60, and *B* is the total payroll, meaning the total dollar value of the salaries of all active employees regardless of age.

The plan-normal rate is calculated by the plan actuary as part of the yearly valuation of the pension. It is calculated by looking backward, at salary data for the past year. To illustrate, suppose that the valuation is being prepared for an effective date of December 31, 2016.¹⁵ The actuary would calculate the normal rate for each employee based on each employee's entry age and salary during 2016, and from these the actuary would calculate the plan-normal rate. The

¹² CalPERS makes more complicated assumptions about when a worker retires, assumptions that vary depending on the worker's category and years of service (CalPERS, Actuarial Office, 2014, pp. 43–54).

¹³ An equivalent view is the normal rate is zero for employees who are already eligible to retire.

¹⁴ In the body of this paper we say that the normal rate is 17.68 percent; however, in our underlying calculations, figures, and tables, the normal rate we actually use is the actual number that our model generated, 0.1768470541031830.

¹⁵ In our model plan, the fiscal year matches the calendar year, for simplicity. In the real world many plans use a different fiscal year; for example, CalPERS uses a fiscal year of July 1 through June 30 (see, e.g., CalPERS, 2016).

calculation would take place sometime during the year 2017, after the necessary data from 2016 becomes available.

The plan-normal rate calculated in the 2016 valuation would then be used to determine the annual required contribution for the year 2018. The lag is necessary because of the time needed by the actuary to obtain the data and prepare the valuation. The *plan-normal cost* for 2018 will be the plan-normal rate as calculated in the 2016 valuation times the actual payroll in 2018.

The plan-normal rate depends on the distribution of entry ages and salaries among the employees younger than 60, and on the salaries of employees 60 or older. In practice, the plan-normal rate changes little from year to year, because changes in the makeup of the workforce are gradual. Indeed, that stability is a primary motivation for selecting the entry-age-normal-cost method: a consistent plan-normal rate provides the employer with a predictable normal contribution. Here, we simply assume the plan-normal rate is 18 percent every year, which is plausible if the average entry age is around age 30 (see Table 2).

2.2.3 Accrued Liability

The *accrued liability* for an active employee is the total amount of past contributions, plus interest, that are assumed to have been made based on the employee's current salary. Basically, we can think of the accrued liability as the amount that should be in the pension to fully fund the portion of the future pension benefit that an employee has already earned with her past service, even though that benefit will not be paid to her until she retires.

The accrued liability is calculated by the plan actuary for each active employee as part of the yearly valuation of the pension. The accrued liability figures into the calculation of the funded status of the pension, as well as into the calculation of any amortization payment that might be required if there is a funding shortfall. To be sure, in this paper, we do not focus on the funded status or on the required amortization payments needed for an underfunded pension to become fully funded. However, we do make use of accrued liability in the entry-age-service-cost method that we propose, so we do need formulas for how to calculate the accrued pension liability. Here we derive the accrued liability formulas for employees who have not yet reached the retirement age of 60.

Analysis. For example, consider an employee who entered service at age 30 (entry age = 30). Let AL_x be the accrued liability at the start of the year of age x . At the start of service, the employee has not yet earned any pension benefits and no contributions have yet been made on her behalf, and thus the accrued pension liability is 0 ($AL_{30} = 0$).

During her first year of service, the amount contributed is $NR \times SAL_{30}$, where NR is the employee's normal rate (17.68% in this example, given her entry age of 30 [row 2 of column 3 of Table 2]), and SAL_{30} is her salary during the first year. At the completion of her first year, that contribution, plus interest, tells us the value of the accrued liability for age 31:

$$AL_{31} = NR \times SAL_{30} \times 1.07^{1/2}$$

The factor $1.07^{1/2}$ is included in this formula because salary is paid in installments throughout the year (e.g., monthly paychecks), which we model as earning a half-year's interest (at the 7 percent discount rate).

During the second year of service, the amount contributed is $NR \times SAL_{31}$. It is assumed that the amount contributed during the preceding year is $NR \times SAL_{31} / 1.04$; that is, it is assumed that $SAL_{31} / SAL_{30} = 1.04$, meaning the actual year-over-year salary growth follows the pension's 4-percent salary growth rate assumption. At completion of the second year, those contributions, plus interest, tell us the accrued liability for age 32:

$$\begin{aligned} AL_{32} &= (NR \times SAL_{31} / 1.04 \times 1.07^{1/2}) \times 1.07 + NR \times SAL_{31} \times 1.07^{1/2} \\ &= NR \times SAL_{31} \times 1.07^{1/2} \times (1.07 / 1.04 + 1) \end{aligned}$$

More generally, for the year of age x , where x is no greater than the retirement age of 60, it is assumed that salaries in prior years are related to the current salary by the 4-percent salary growth rate assumption, and that the amount contributed in prior years is the employee's normal rate times salary. This works out to the following formula for accrued liability at the start of the year:

$$AL_x = NR \times SAL_{x-1} \times 1.07^{1/2} \times \sum_{k=e}^{x-1} (1.07/1.04)^{k-e}$$

where e is the employee's entry age ($e = 30$ in this example). The sum is a geometric series; substituting the formula for it¹⁶ works out to:

$$AL_x = NR \times SAL_{x-1} \times 1.07^{1/2} \times \frac{(1.07/1.04)^{x-e} - 1}{(1.07/1.04) - 1}, e \leq x \leq r$$

where r is the retirement age ($r = 60$ for our model pension).

For example, consider a typical employee that we call "Alice." We created Alice because we wanted an employee whose salary and pension are expressed in easy-to-work-with round numbers. Therefore, at the outset, we assumed that Alice would have final pay (fp) of \$100,000 at age 59 after 30 years of service; therefore, she would be entitled to an initial pension of \$60,000 at age 60 ($\$60,000 = 60\% \times \$100,000 \text{ } fp = 2\% \times 30 \text{ } yos \times \$100,000 \text{ } fp$). We then worked backward from age 60 to age 30 to determine the values of all the other parameters, and Table 3 shows the results. More specifically, Table 3 shows how Alice's annual salary, and the pension's accrued liability will grow from age 30 to age 60 when she is expected to retire.

¹⁶ The formula is $\sum_{k=0}^{n-1} x^k = \frac{x^n - 1}{x - 1}$.

Table 3. Accrued Liability and Salary for Alice, a Typical Employee

Age (x)	Salary (SAL _x)	Normal Cost (NC _x = ~ 17.68% × SAL _x)	Accrued Liability (AL _x)
30	\$32,065.14	\$5,670.63	\$0.00
31	\$33,347.75	\$5,897.45	\$5,865.74
32	\$34,681.66	\$6,133.35	\$12,376.71
33	\$36,068.92	\$6,378.68	\$19,587.47
34	\$37,511.68	\$6,633.83	\$27,556.75
35	\$39,012.15	\$6,899.18	\$36,347.81
36	\$40,572.63	\$7,175.15	\$46,028.73
37	\$42,195.54	\$7,462.16	\$56,672.77
38	\$43,883.36	\$7,760.64	\$68,358.78
39	\$45,638.69	\$8,071.07	\$81,171.57
40	\$47,464.24	\$8,393.91	\$95,202.36
41	\$49,362.81	\$8,729.67	\$110,549.25
42	\$51,337.32	\$9,078.85	\$127,317.74
43	\$53,390.82	\$9,442.01	\$145,621.22
44	\$55,526.45	\$9,819.69	\$165,581.60
45	\$57,747.51	\$10,212.48	\$187,329.87
46	\$60,057.41	\$10,620.98	\$211,006.83
47	\$62,459.70	\$11,045.81	\$236,763.73
48	\$64,958.09	\$11,487.65	\$264,763.08
49	\$67,556.42	\$11,947.15	\$295,179.41
50	\$70,258.67	\$12,425.04	\$328,200.20
51	\$73,069.02	\$12,922.04	\$364,026.77
52	\$75,991.78	\$13,438.92	\$402,875.31
53	\$79,031.45	\$13,976.48	\$444,977.91
54	\$82,192.71	\$14,535.54	\$490,583.74
55	\$85,480.42	\$15,116.96	\$539,960.28
56	\$88,899.64	\$15,721.64	\$593,394.61
57	\$92,455.62	\$16,350.50	\$651,194.82
58	\$96,153.85	\$17,004.52	\$713,691.55
59	\$100,000.00	\$17,684.71	\$781,239.58
60			\$854,219.55

At the outset, columns 1 and 2 of Table 3 show how Alice’s salary would grow from a starting salary of around \$32,100 at age 30 to a final salary of \$100,000 at age 59, exactly following the 4 percent salary growth rate assumption. Under the model pension plan, that means that Alice can retire at age 60 with an initial pension of \$60,000 ($\$60,000 = 60\% \times \$100,000 = 2\% \times 30 \text{ yrs} \times \$100,000 \text{ fp}$) and fixed cost-of-living increases of 2 percent annually throughout her retirement. Figure 1 also illustrates how Alice’s salary would grow over the course of her 30-year career.

Figure 1. Accrued Liability and Salary for a Typical Employee

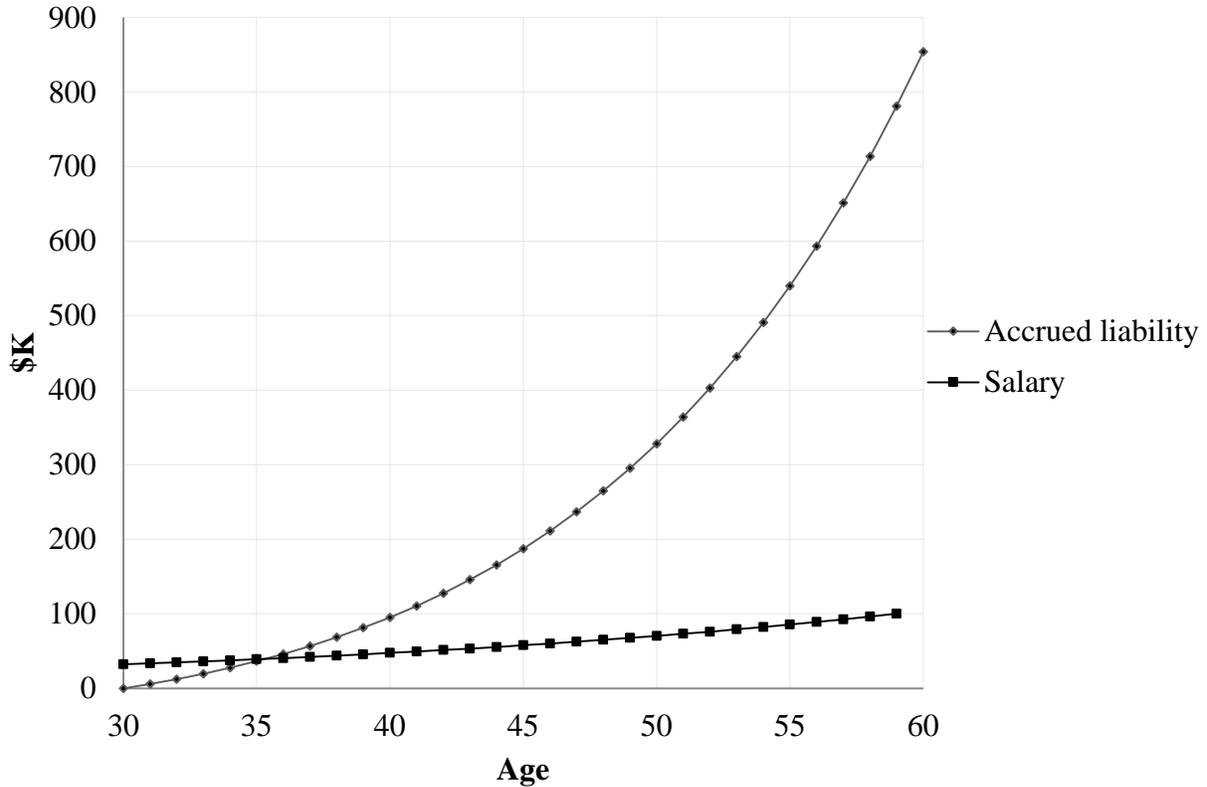


Table 3 and Figure 1 also show the accrued liability for Alice’s pension. Recall that we determined that the applicable annuity factor for a 60-year-old retiree like Alice is 14.24 (row 1 of column 2 of Table 1). Therefore, the accrued pension liability for Alice at the start of her 60th year is around \$854,000 ($\$854,400 = 14.24 \times \$60,000$), calculated as:

$$AL_{60} = 17.68\% \times 100K \times 1.07^{1/2} \times \frac{(1.07/1.04)^{60-30} - 1}{(1.07/1.04) - 1} = \$854K$$

More specifically, column 3 of Table 3 shows that the accrued liability for Alice’s pension starts at zero at her entry age of 30 and increases rapidly over the course of her 30-year career until it reaches \$854,000 at her retirement age of 60.

In summary, our example shows that if Alice starts work at age 30 with a \$32,100 salary and her salary increases by 4 percent each year thereafter, her salary will reach \$100,000 in her 30th year of work (at age 59). The example also shows that if her employer contributes 17.68 percent of her salary each year to the pension, and if those contributions earn the assumed 7 percent rate-of-return each year, then the contributions plus investment earnings would accumulate to around \$854,000 when Alice completes 30 years of service at age 60 and becomes eligible for her

pension. Accordingly, at the start of her retirement at age 60, her pension would be fully funded (i.e., the pension plan will have accumulated an amount that matches the expected present discounted value of a pension that pays her an initial amount of \$60,000 with fixed cost-of-living increases of 2 percent annually throughout her retirement).

3. The Problem with Normal-Cost Funding

In a typical final-average-pay pension plan using the entry-age-normal-cost method, the employer makes an annual required contribution (ARC) determined by the plan actuary (see, e.g., CalPERS, 2018). The ARC consists of two parts: the plan-normal cost, which is specified as the plan-normal rate times payroll; and an amortization payment, which is often specified as a dollar amount that does not depend on payroll. The plan-normal cost is intended to cover the increase in accrued liability that arises from the current year's service by active employees. That is, the plan-normal cost is intended to be an accurate representation of the *service cost*, which we define as the contribution amount that would exactly offset the year-over-year increase in the accrued pension liability of active employees. The amortization payment is a partial pay-down of unfunded liability arising from past service of active and retired employees; it is not something we focus on in this paper.

Unfortunately, plan-normal cost is not a reliably accurate measure of service cost. The problem is that accrued liability is very sensitive to changes in salary (e.g., pay raises), while plan-normal cost is not. A small change in salary can produce a big change in accrued liability, for better or worse, but it never produces a big change in plan-normal cost. In other words, the plan-normal cost masks the sensitivity to salary that is inherent in the true service cost.

This masking effect can best be understood with a simple example. Consider another typical employee whom we call “Bob.”¹⁷ Like Alice, Bob entered service at age 30, and his normal rate is also 17.68 percent (row 2 of column 3 of Table 2). Bob is currently age 55, and his salary during age 54 was \$100,000 (column 1 of Table 4). At the start of age 55, Bob's accrued pension liability (AL_{55}) would be \$656,944, based on his entry age (30) and his salary during age 54 (\$100,000) (column 2 of Table 4). The accrued liability at the start of the next year—when Bob is age 56—will depend on what his salary is during age 55, and Table 4 shows the impact of three possible age 55 salaries on the accrued liability.

¹⁷ We created Bob for this example instead of using Alice so that we could choose the salary to be easy-to-work-with round numbers at the ages of interest here.

Table 4. The Impact of Salary Increases on Accrued Pension Liability

SAL_{54}	AL_{55}	SAL_{55}	Plan-Normal Cost	AL_{56}	Bob's Normal Cost	Overfunding (Underfunding)
\$100,000	\$656,944	\$104,000	\$18,720	\$721,955	\$18,392	\$339
		\$110,000	\$19,800	\$763,607	\$19,453	(\$40,195)
		\$100,000	\$18,000	\$694,188	\$17,685	\$27,362

Case 1: A 4 Percent Pay Raise

In the first case, Bob's salary for age 55 will be \$104,000 (row 1 of column 3 of Table 4). That is, Bob's salary follows the 4 percent salary growth rate assumption ($\$104,000 SAL_{55} = 1.04 \times \$100,000 SAL_{54}$). Given that we assumed a plan-normal rate of 18 percent, the plan-normal cost for Bob is \$18,720 (row 1 of column 4 of Table 4; $\$18,720 = 18\% \text{ plan-normal rate} \times \$104,000 SAL_{55}$), and the accrued liability at the start of age 56 (AL_{56}) is \$721,955 (Row 1 of column 5 of Table 4). In this case, because the 4 percent pay raise matches the assumed 4 percent salary growth rate that was used to determine Bob's 17.68 percent normal rate, Bob's normal cost would be the correct amount to contribute to offset the year-over-year growth in accrued liability, and Bob's normal cost is just \$18,392 (row 1 of column 6 of Table 4; $\$18,392 = 17.68\% \text{ NR} \times \$104,000 SAL_{55}$). Since the plan-normal rate (18%) is slightly larger than Bob's normal rate (17.68%), the plan ends up being slightly overfunded, by \$339 (row 1 of column 7 of Table 4; $\$339 = (\$18,720 \text{ plan-normal cost} - \$18,392 \text{ Bob's normal cost}) \times 1.07^{1/2}$).

In this case, the plan-normal cost is a very good match to Bob's true service cost, because Bob's 17.68 percent normal rate is a close match to the 18 percent plan-normal rate, and the 4 percent pay raise matches the 4 percent salary growth rate assumption.

The next two cases illustrate what happens when the pay raise deviates from the 4 percent salary growth rate assumption.

Case 2: A 10 Percent Pay Raise

In the second case, Bob's salary for age 55 (SAL_{55}) is \$110,000 (row 2 of column 3 of Table 4). That is, Bob's salary at age 55 is 10 percent higher than his salary was at age 54 ($\$110,000 SAL_{55} = 1.10 \times \$100,000 SAL_{54}$). This 10 percent pay raise (\$10,000) is two-and-a-half times greater than the 4 percent salary growth rate assumption ($2.5 = 10\% / 4\% = \$10,000 / \$4,000$). This kind of 10 percent pay raise might happen, for example, if Bob were promoted to a higher-level position.

In this case, that the accrued liability at the start of age 56 (AL_{56}) is now \$763,607 (row 2 of column 5 of Table 4).¹⁸ Of course, the amount contributed is just \$19,800 (row 2 of column 4 of

¹⁸ Since accrued liability is proportional to salary, we can calculate AL_{56} as follows: $\$763,607 AL_{56}(\$10,000 \text{ pay raise}) = \$721,955 AL_{56}(\$4,000 \text{ pay raise}) \times (\$110,000 / \$104,000)$.

Table 4; $\$19,800 = 18\% \text{ plan-normal rate} \times \$110,000 \text{ SAL}_{55}$). Since that $\$19,800$ contribution is nowhere near enough to offset the growth in accrued liability, the plan ends up being underfunded, by $\$40,195$ (row 2 of column 7 of Table 4; $\$40,195 = (\$763,607 - \$721,955) - ((\$19,800 \text{ plan-normal cost} - \$18,392 \text{ normal cost}) \times 1.07^{1/2})$).

In this case, the plan-normal cost ($\$19,800$) now seriously understates Bob's true service cost, resulting in significant underfunding. For the accrued liability to be fully funded, the employer should have contributed $\$58,659$ ($\$58,659 = \$763,607 / 1.07^{1/2} - \$656,944 \times 1.07^{1/2}$), but only $\$19,800$ was contributed. This underfunding occurred even though Bob's normal rate (17.68%) was a good match to the plan-normal rate (18%). The problem is that Bob's salary grew by 10 percent ($\$10,000$) instead of 4 percent ($\$4,000$); that is, the pay raise was $\$6,000$ larger than what was assumed, and that additional $\$6,000$ pay raise increased the accrued liability by $\$41,652$, which was not funded ($\$41,652 = \$763,607 - \$721,955$). The additional $\$6,000$ pay raise did result in an increased contribution of $\$1,080$ ($\$1,080 = \$19,800 \text{ plan-normal cost} - \$18,720 \text{ normal cost} = 18\% \times \$6,000 = 18\% \times (\$110,000 - \$104,000)$), but that $\$1,080$ contribution increase was quite small compared to the actual $\$41,652$ increase in accrued liability attributable to the extra $\$6,000$ pay raise.

Case 3: No Pay Raise

In the third case, Bob's salary for age 55 (SAL_{55}) is $\$100,000$ (row 3 of column 3 of Table 4). That is, Bob's salary at age 55 is exactly the same as it was for age 54. This 0 percent pay raise is less than the 4 percent salary growth rate assumption. This could happen, for example, if the employer freezes salaries due to a recession or due to a shortfall in revenue collections.

In this case, the accrued liability at the start of age 56 (AL_{56}) is now $\$694,188$ (row 3 of column 5 of Table 4).¹⁹ The amount contributed is $\$18,000$ (row 3 of column 4 of Table 4; $\$18,000 = 18\% \text{ plan-normal rate} \times \$100,000 \text{ SAL}_{55}$). Since that $\$18,000$ contribution is more than what was needed to offset the growth in accrued liability, the plan ends up being overfunded, by $\$27,362$ (row 3 of column 7 of Table 4; $\$27,362 = (\$721,955 - \$694,188) - ((\$18,392 - \$18,000) \times 1.07^{1/2})$).

In this case, the plan-normal cost seriously overstates Bob's true service cost, resulting in a large overfunding, again despite the good match between Bob's normal rate (17.68%) and the plan-normal rate (18%). The problem is that salary grew by $\$4,000$ less than what was assumed, which reduced the accrued liability by $\$27,767$ ($\$27,767 = \$721,955 - \$694,188$) but only reduced the contribution by $\$720$ ($\$720 = \$18,720 - \$18,000 = 18\% \times \$4,000 = 18\% \times (\$104,000 - \$100,000)$).

Summary

Cases 2 and 3 illustrate that *accrued liability is very sensitive to salary changes*. The salary variation between the two cases is just $\$10,000$ ($\$10,000 = \$110,000 - \$100,000$), but that $\$10,000$ variation in year-over-year salary growth produced a difference in accrued liability of $\$67,557$ ($\$67,557 = \$40,195 \text{ up} + \$27,362 \text{ down}$). Meanwhile the $\$10,000$ range of pay raises only changed contributions by a small fraction (18%; $\$1,800 = 18\% \times \$10,000$; $\$10,000 =$

¹⁹ $\$694,188 = \$721,955 \times (\$100,000 / \$104,000)$. See note 18.

\$6,000 up + \$4,000 down). In other words, the variation in accrued liability was almost seven times larger than the variation in pay raises ($6.7557 = \$67,557 / \$10,000$).

Because we believe that an employer should fully fund each employee's accrued pension liability each year, it seems clear to us that the entry-age-normal cost method is not the proper method to use to promote full funding. The next part of this paper explains how traditional final-average-pay pension plans could begin to use individual salary, paywall and pay-raise data to determine contributions and promote full funding.

4. Service Cost

Part 3 of this paper showed by example that the true service cost of an employee can be very different from the plan-normal cost. In this part, we elaborate on that observation by providing a more detailed analysis of the service cost.

The service cost for an employee of age x is the amount that must be contributed to offset the change in accrued liability that occurs during age x . That is, the service cost (SVC_x) is the value that satisfies the formula:

$$AL_{x+1} = AL_x \times 1.07 + SVC_x \times 1.07^{1/2}$$

The starting liability AL_x represents assumed past contributions plus interest; during age x , those past contributions accrue an additional year's interest. The service cost SVC_x is contributed in installments throughout the year, as a percentage of salary; during age x it is modeled as earning a half-year's interest. Solving for the service cost (SVC_x) yields the following formula:

$$SVC_x = AL_{x+1} \times 1.07^{-1/2} - AL_x \times 1.07^{1/2}$$

In short, the service cost for a given year is the difference between the final and starting accrued liabilities, each valued at midyear to account for a half-year's interest.

4.1 Prior to Retirement Age

An example would help, and here we reconsider Bob from part 3. Recall that Bob entered service at age 30. During his first year of service, we apply the formulas for accrued liability in section 2.2.3 to get:

$$\begin{aligned} SVC_{30} &= AL_{31} \times 1.07^{-1/2} - AL_{30} \times 1.07^{1/2} \\ &= NR \times SAL_{30} \end{aligned}$$

In other words, during the first year, service cost equals normal cost.

During the second year of service, we apply the formulas for accrued liability to get:

$$\begin{aligned} SVC_{31} &= AL_{32} \times 1.07^{-1/2} - AL_{31} \times 1.07^{1/2} \\ &= NR \times SAL_{31} \times (1.07/1.04 + 1) - NR \times SAL_{30} \times 1.04 \end{aligned}$$

With a little manipulation, we rearrange this into the more intuitive formula:

$$SVC_{31} = NR \times SAL_{30} \times 1.04 + NR \times (1.07/1.04 + 1) \times (SAL_{31} - SAL_{30} \times 1.04)$$

The intuition behind it is as follows: If the year-over-year actual salary growth matches the 4 percent salary growth rate assumption that was used to calculate the normal rate—meaning, if

$SAL_{31} = SAL_{30} \times 1.04$, then the second term in the formula is 0, and the first term is $NR \times SAL_{31}$; that is, service cost equals normal cost. But if actual salary growth is different from the 4 percent salary growth rate assumption, then the second term is nonzero, and that changes the service cost. The rate of that change in service cost, which we define as the *marginal service rate* (MR_x), is the coefficient of SAL_{31} in the second term; that is, the marginal service rate is:

$$MR_{31} = NR \times (1.07/1.04 + 1)$$

With that, we can write the formula for service cost more succinctly as:

$$SVC_{31} = NR \times SAL_{30} \times 1.04 + MR_{31} \times (SAL_{31} - SAL_{30} \times 1.04)$$

More generally, for the year of age x , where x is less than the retirement age of 60, we use the formula for AL_x in part 2.2.3 to get the following formulas for service cost and marginal service rate:

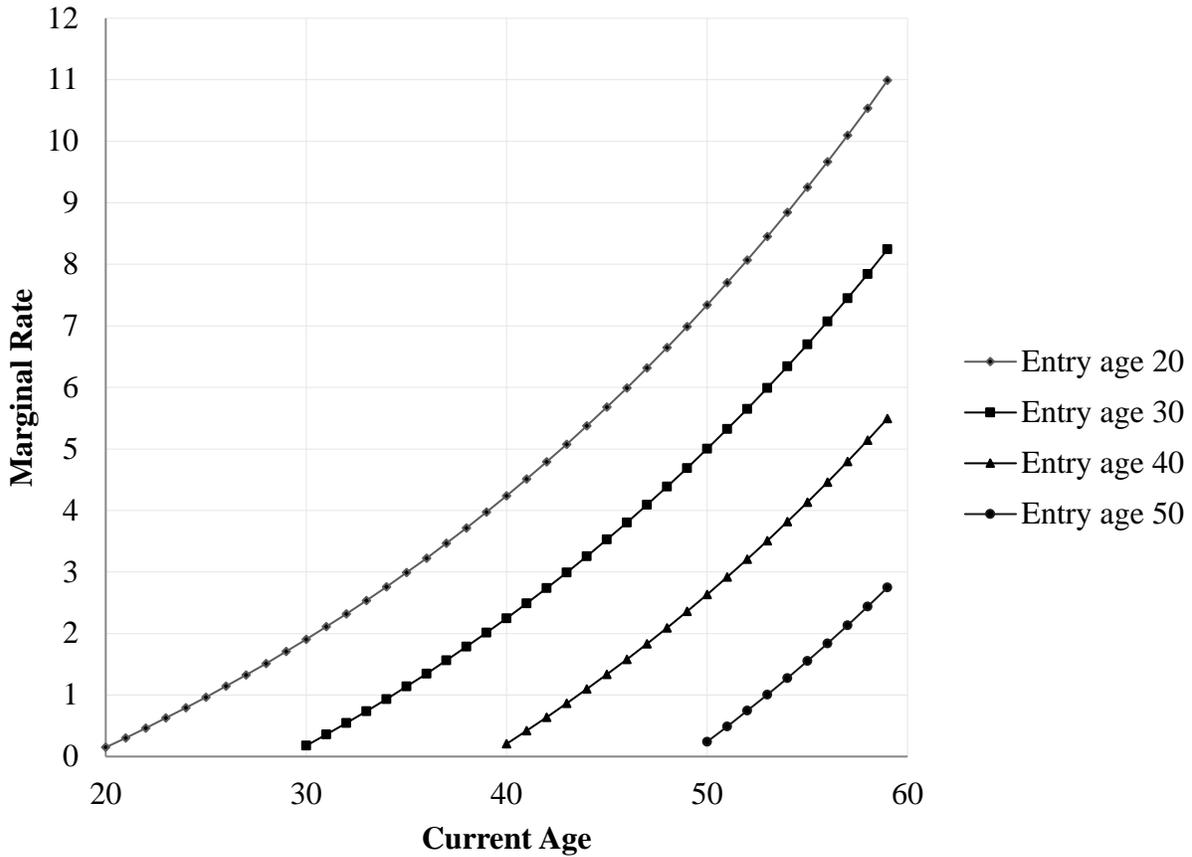
$$SVC_x = NR \times SAL_{x-1} \times 1.04 + MR_x \times (SAL_x - SAL_{x-1} \times 1.04)$$

$$MR_x = NR \times \sum_{k=e}^x (1.07/1.04)^{k-e} = NR \times \frac{(1.07/1.04)^{x+1-e} - 1}{(1.07/1.04) - 1}, e \leq x < r$$

where e is the entry age ($e = 30$ in this example) and r is the retirement age ($r = 60$ for our model pension plan). To apply the formula to the case $x = e$, meaning the entry age, we use the convention that $SAL_{e-1} = 0$, meaning the salary prior to entry age is 0.

The marginal rate MR_x is a critical parameter because it measures the sensitivity of service cost to a change in the current salary SAL_x . Figure 2 plots the marginal rate for employees with various entry ages.

Figure 2. Preretirement Marginal Rate for Several Entry Ages

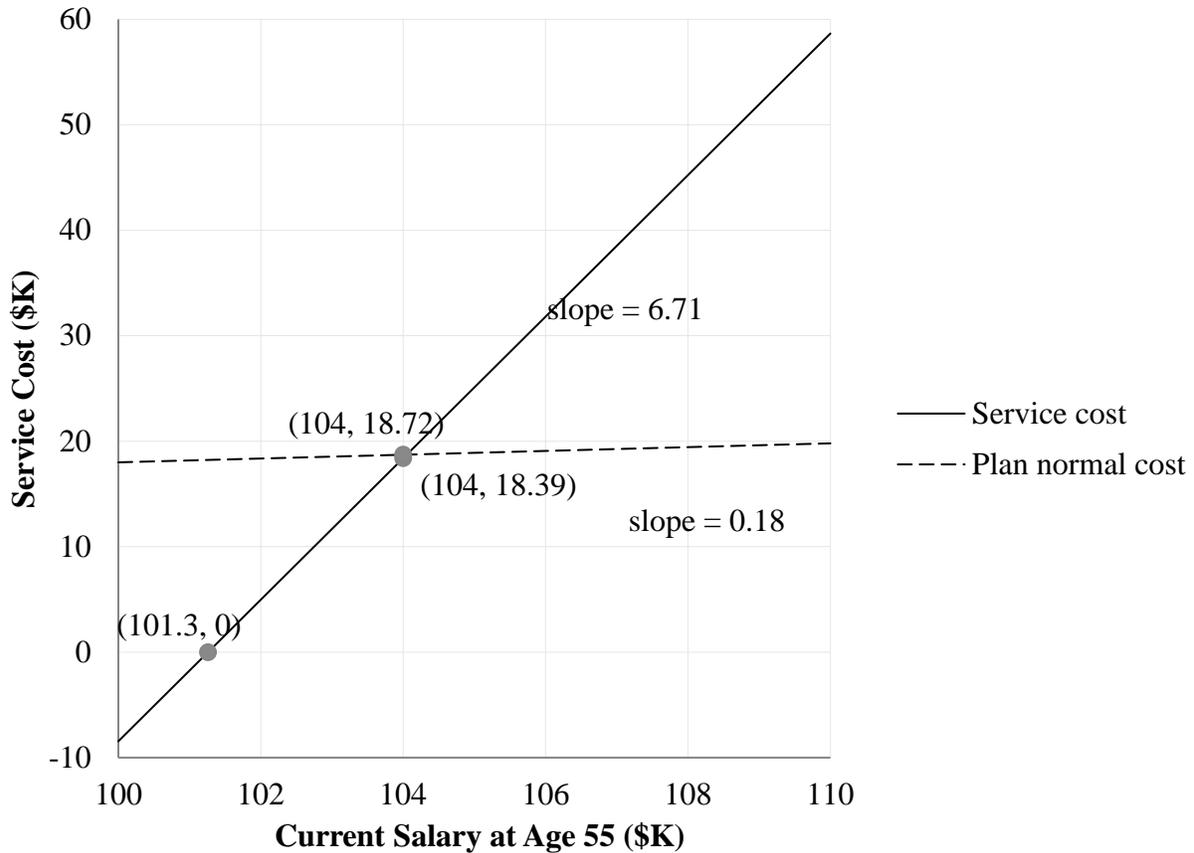


For each entry age, the marginal rate during the first year matches the normal rate. It increases every year thereafter. What is striking about the plots is how quickly the marginal rate grows. After just a few years beyond entry, the marginal rate exceeds unity, meaning a \$1 change in salary produces a greater-than-\$1 change in service cost. As the employee approaches retirement age, the marginal rate becomes quite large, meaning that a small change in salary produces a very large change in service cost. The lower the entry age, the larger the marginal rate is at any subsequent age. For an entry age of 20, the marginal rate at age 59 reaches roughly 11 (i.e., 1,100 percent)—a value that is more than 60 times larger than the 18 percent plan-normal rate ($61.111111 = 1100\% / 18\%$).

Example 1

For an example, we can reconsider Bob who entered service at age 30, is currently 55, and whose salary last year (SAL_{54}) was \$100,000. Figure 3 plots Bob’s current service cost as a function of his salary at age 55; that is, it plots SVC_{55} versus possible values of his salary at age 55 (SAL_{55}), which salary values will themselves depend upon the size of the pay raise that he gets, if any.

Figure 3. Service Cost for an Employee With Entry Age 30, Current Age 55, \$100,000 Salary at Age 54



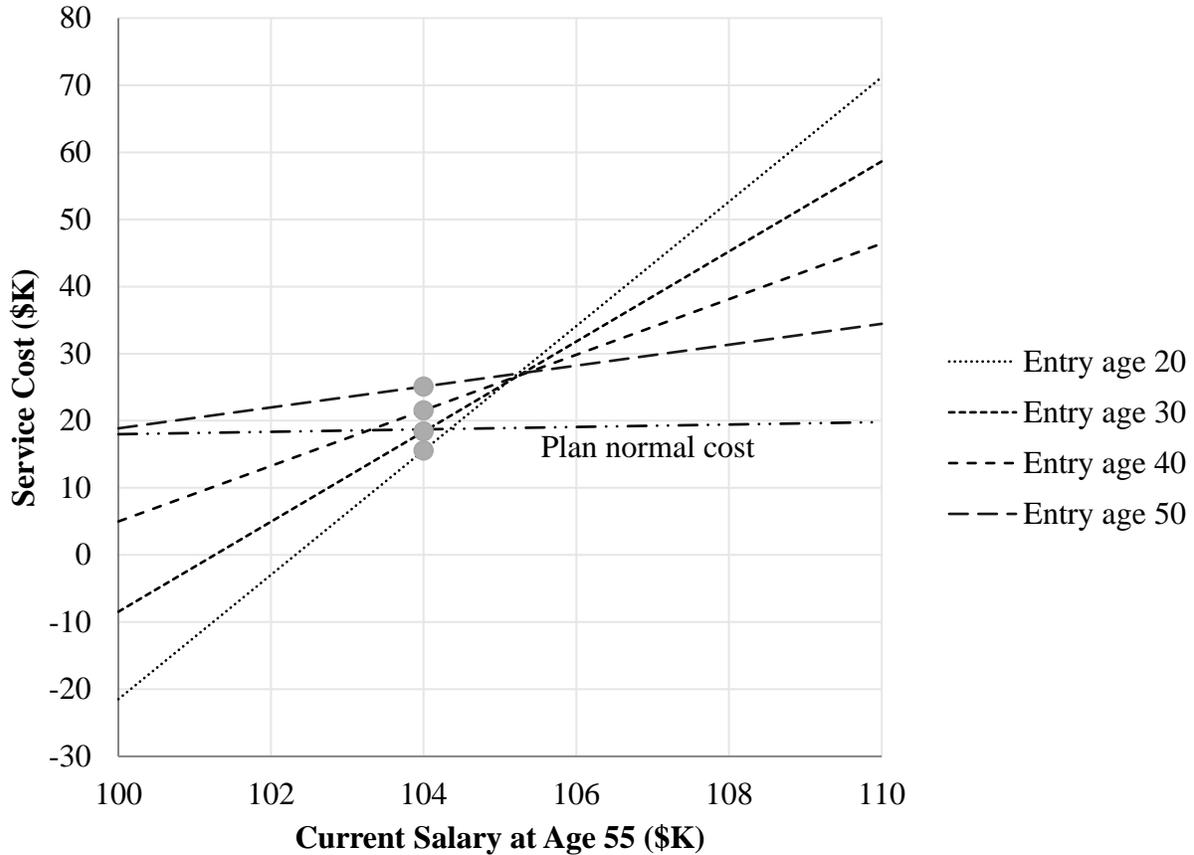
The plot is a straight line with slope $MR_{55} = 6.71$. The line passes through the point (\$104,000, \$18,392): this is the case where the pay raise matches the 4 percent salary growth rate assumption, so service cost equals Bob's normal cost ($\$18,392 = \sim 17.68\% \times \$104,000$). The line intersects the horizontal axis when salary equals \$101,259; thus service cost is positive when salary is larger than this value and negative when it is smaller. Figure 3 also plots the plan-normal cost versus salary; this is a straight line with a slope of just 0.18 (i.e., 18%, the plan-normal rate) passing through the point (\$104,000, \$18,720).

Figure 3 illustrates that plan-normal cost is generally not a very accurate measure of service cost. For Bob, whose normal rate (17.68%) is a close match to the plan-normal rate (18%), the plan-normal cost is close to the service cost when his pay raise equals the 4 percent salary growth rate assumption (and his salary goes from \$100,000 at age 54 to \$104,000 at age 55). But if he gets a larger or smaller pay raise, the plan-normal rate quickly becomes inaccurate as his salary departs from projected value of \$104,000, with each \$1 change in salary producing a \$6.71 change in service cost.

Example 2

Figure 4 is similar to Figure 3 but includes plots for employees with differing entry ages. Each plot is a line whose slope is the marginal rate for that entry age; the younger the entry age, the steeper the slope. Each line passes through the point (\$104,000, NR × \$104,000), where NR is the normal rate for an employee of that entry age.

Figure 4. Service Cost for Various Employees, Current Age 55, \$100,000 Salary at Age 54, Various Entry Ages



Example 3

Table 5 applies the service-rate calculation to a tiny model pension plan consisting of three employees of different entry ages and current ages. Table 5 also shows that the three employees also had different salaries last year, but each gets a \$5,000 pay raise this year.

Table 5. A Three-Employer Pension, Each Employee Gets a \$5,000 Pay Raise

Name	e	x	SAL _x	SAL _{x-1}	NR	MR _x	SVC _x
Charlie	20	40	\$95,000	\$90,000	0.1499	4.2454	\$19,973
Diane	30	45	\$105,000	\$100,000	0.1768	3.5324	\$21,925
Edna	40	50	\$115,000	\$110,000	0.2073	2.6394	\$25,299
Total			\$315,000	\$300,000			\$67,197

Table 5 shows the name, entry age (*e*), and current age (*x*) of each employee, along with the current salary (SAL_{*x*}) and prior year’s salary (SAL_{*x-1*}). Column 6 of Table 5 also shows the normal rate (NR) for each employee (from column 3 of Table 2).

Table 5 also shows the marginal rate MR_{*x*} and service cost SVC_{*x*}, calculated using the formulas above. For example, for Charlie, the calculations are:

$$MR_{40} = 0.1499 \times \frac{(1.07/1.04)^{40+1-20} - 1}{(1.07/1.04) - 1} = 4.2454$$

$$SVC_x = 0.1499 \times 90,000 \times 1.04 + 4.2454 \times (95,000 - 90,000 \times 1.04) = 19,973^{20}$$

The calculations for the other employees are similar.

The total service cost for the three employees is \$67,197 (row 4 of column 8 of Table 5). For comparison, the plan-normal cost is just \$56,700 (\$56,700 = 18% plan-normal rate × \$315,000 total payroll). Thus, in this example, the total service cost is considerably higher than the plan-normal cost, and contributing only the 18 percent plan-normal cost would result in an underfunding of \$10,858 (\$10,858 = (\$67,197 – \$56,700) × 1.07^{1/2}). This is not surprising, because the actual year-over-year payroll growth in this example is 5 percent (1.05 = \$315,000 / \$300,000), which is larger than the 4 percent salary growth rate assumption that was built into our calculation of the plan-normal rate.

That is only one example, of course, and Table 6 shows that by giving smaller pay raises to these employees (\$3,000 each instead of \$5,000 each), we can produce an example where service cost is less than the plan-normal cost. The ages and prior salaries in this example are the same as before, but the current salaries have been reduced by \$2,000 each, reducing the current payroll by \$6,000 (\$6,000 = \$315,000 – \$309,000). The actual year-over-year payroll growth is now just 3 percent (1.03 = \$309,000 / \$300,000), which is smaller than the 4 percent salary growth rate assumption.

²⁰ Slight discrepancies are due to rounding of table entries.

Table 6. A Three-Employer Pension, Each Employee Gets a \$3,000 Pay Raise

Name	e	x	SAL _x	SAL _{x-1}	NR	MR _x	SVC _x
Charlie	20	40	\$93,000	\$90,000	0.1499	4.2454	\$11,482
Diane	30	45	\$103,000	\$100,000	0.1768	3.5324	\$14,860
Edna	40	50	\$113,000	\$110,000	0.2073	2.6394	\$20,02
Total			\$309,000	\$300,000			\$46,362

The total service cost drops to just \$46,362 (row 4 of column 6 of Table 6). That is, the total service cost drops by \$20,835 (\$20,835 = \$67,197 – \$46,362), and that \$20,835 drop is more than three times larger than the decrease in payroll (3.47 = \$20,835 / \$6,000). The plan-normal cost drops only slightly, to \$55,620 (\$55,620 = 18% plan-normal rate × \$309,000 total payroll); that is a decrease of just \$1,980 (\$1,980 = \$57,600 – \$55,620), which is only a fraction of the decrease in payroll. Now, however, the total service cost (\$46,362) is significantly less than the plan-normal cost (\$55,620), and contributing the plan-normal cost would result in the plan being overfunded, by \$9,577 (\$9,577 = (\$55,620 – \$46,362) × 1.07^{1/2}).

4.2 Beyond Retirement Age

4.2.1 Accrued Liability

So far we have discussed employees who are younger than the minimum retirement age of 60. For each such employee, we defined the accrued liability as the accumulation of past normal contributions that we assume were made, based on the employee’s current salary. These past normal contributions, along with future normal contributions that we assume will be made, are projected to accumulate to exactly the right amount to match the actuarial liability for the employee’s pension once the retirement age of 60 is reached.

Once the retirement age of 60 is reached, however, the employee might decide to continue working rather than retire. In this case, we will need to define the accrued liability differently, since no more normal contributions will be made. We define it as the actuarial liability of the pension should the employee choose to retire at that time. That is, the accrued liability at the start of age x , where x is 60 or greater, is given by the formula:

$$AL_x = AF_x \times BF_x \times SAL_{x-1}, x \geq r$$

where: AF_x is the annuity factor when retiring at age x ; BF_x is the benefit factor when retiring at age x (i.e., 2% times the number of years of service); SAL_{x-1} is the salary during the preceding year; and $r = 60$ is the minimum retirement age.

Example

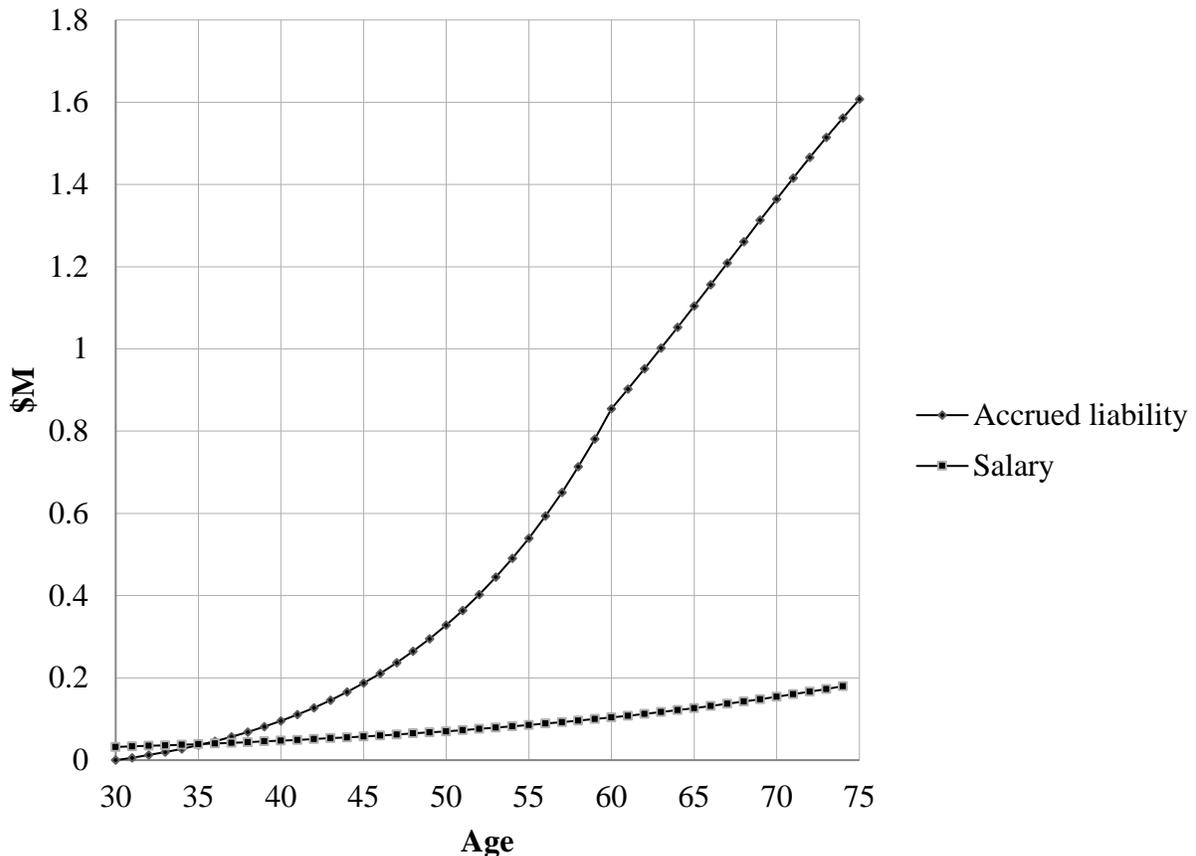
Figure 5 plots accrued liability versus age for Alice from part 2.2.3, both prior to age 60 as previously plotted in Figure 1 and after age 60 using the formula above. Figure 5 includes a plot of Alice’s yearly salary, for comparison. Alice’s salary grows by the 4 percent salary growth rate

assumption, both before and after age 60. The accrued liability at the start of age 60 is \$854K, calculated as:

$$AL_{60} = 14.24 \times 60\% \times 100K = \$854K$$

where: 14.24 is her annuity factor (row 1 of column 2 of Table 1); 60% is her benefit factor (row 2 of column 2 of Table 2); and \$100,000 is her salary during age 59. Note that this value for accrued liability matches the value obtained in part 2.2.3, which was calculated as the accumulation of past normal contributions. Figure 5 shows that the accrued liability for Alice continues to rise if she continues to work past age 59, but at a slower rate than for ages younger than 60.

Figure 5. Accrued Liability and Salary for Example Employee Alice



4.2.2 The Problem with Normal Cost Funding

In part 3, we showed that the plan-normal cost (generated by the entry-age-normal cost method) was not a reliably accurate measure of service cost for employees under the normal age of retirement (here age 60). The plan-normal cost is also not a reliably accurate measure of service cost for employees after the normal retirement age.

Again, this inaccuracy can best be understood with a simple example. Reconsider Alice from Table 3 in part 2.2.3. Assume that Alice is now age 59, but instead of retiring at age 60 she has

decided to work for one more year. At the start of age 60, Alice’s accrued pension liability would be \$854,220 (column 2 of Table 7), based on her entry age (30) and her salary during age 59 (\$100,000) (column 1 of Table 7). The accrued liability at the start of the next year when—when Alice is age 61—will depend on what her salary is during age 60, and Table 7 shows the impact of three possible age 60 salaries on the accrued liability.

Table 7. The Impact of Deferred Retirement on Accrued Pension Liability

SAL_{59}	AL_{60}	SAL_{60}	Plan-Normal Cost	AL_{61}	Overfunding (Underfunding)
\$100,000	\$854,220	\$104,000	\$18,720	\$902,561	\$30,818
		\$110,000	\$19,800	\$954,631	(\$20,135)
		\$100,000	\$18,000	\$867,847	\$64,788

Case 1: A 4 Percent Pay Raise

In the first case, Alice’s salary for age 60 will be \$104,000 (row 1 of column 3 of Table 7). That is, Alice’s salary follows the 4-percent salary growth rate assumption ($\$104,000 SAL_{60} = 1.04 \times \$100,000 SAL_{59}$). Given that we assumed a plan-normal rate of 18 percent, the plan-normal cost for Alice is \$18,720 (row 1 of column 4 of Table 7; $\$18,720 = 18\% \text{ plan-normal rate} \times \$104,000 SAL_{60}$), and the accrued liability at the start of age 61 (AL_{61}) is \$902,561 (Row 1 of column 5 of Table 7). In this case, the plan will end up being significantly overfunded, by \$30,818 (row 1 of column 6 of Table 7) ($\$30,818 = (\$854,220 AL_{60} \times 1.07 + \$18,720 \text{ plan-normal cost} \times 1.07^{1/2}) - \$902,561$).

The next two cases illustrate what happens when the pay raise deviates from the 4 percent salary growth rate assumption.

Case 2: A 10 Percent Pay Raise

In the second case, Alice’s salary for age 60 (SAL_{60}) is \$110,000 (row 2 of column 3 of Table 7). That is, Alice’s salary at age 60 is 10 percent higher than her salary was at age 59 ($\$110,000 SAL_{60} = 1.10 \times \$100,000 SAL_{59}$). In this case, the accrued liability at the start of age 61 (AL_{61}) is now \$954,631 (row 2 of column 5 of Table 7). Of course, the amount contributed is just \$19,800 (row 2 of column 4 of Table 7; $\$19,800 = 18\% \text{ plan-normal rate} \times \$110,000 SAL_{60}$). Since that \$19,800 contribution is not enough to offset the growth in accrued liability, the plan ends up being significantly underfunded, by \$20,135 (row 2 of column 6 of Table 7; $\$20,135 = \$954,631 - (\$854,220 AL_{60} \times 1.07 + \$19,800 \text{ plan-normal cost} \times 1.07^{1/2})$).

Case 3: No Pay Raise

In the third case, Alice’s salary for age 60 (SAL_{60}) is \$100,000 (row 3 of column 3 of Table 7). That is, Alice’s salary at age 60 is exactly the same as it was for age 59. This 0 percent pay raise

is less than the 4 percent salary growth rate assumption. In this case, the accrued liability at the start of age 61 (AL_{61}) is now just \$867,847 (row 3 of column 5 of Table 7). The amount contributed is \$18,000 (row 3 of column 4 of Table 7; $\$18,000 = 18\% \text{ plan-normal rate} \times \$100,000 \text{ SAL}_{60}$). Since that \$18,000 contribution is more than what was needed to offset the growth in accrued liability, the plan ends up being significantly overfunded, by \$64,788 (row 3 of column 6 of Table 7; $\$64,788 = (\$854,220 \text{ AL}_{60} \times 1.07 + \$18,000 \text{ plan-normal cost} \times 1.07^{1/2}) - \$867,847$).

Summary

Cases 2 and 3 illustrate that accrued liability is even more sensitive to salary changes after the normal retirement age. The salary variation between the two cases is just \$10,000 ($\$10,000 = \$110,000 - \$100,000$), but that \$10,000 variation in year-over-year salary growth produced a difference in accrued liability of \$84,923 ($\$67,557 = \$20,135 \text{ up} + \$64,788 \text{ down}$). Meanwhile the \$10,000 range of pay raises only changed contributions by a small fraction (18%; $\$1800 = 18\% \times \$10,000$; $\$10,000 = \$6,000 \text{ up} + \$4,000 \text{ down}$). In other words, the variation in accrued liability was eight-and-a-half times larger than the variation in pay raises ($8.4923 = \$84,923 / \$10,000$).

4.2.3 Service Cost

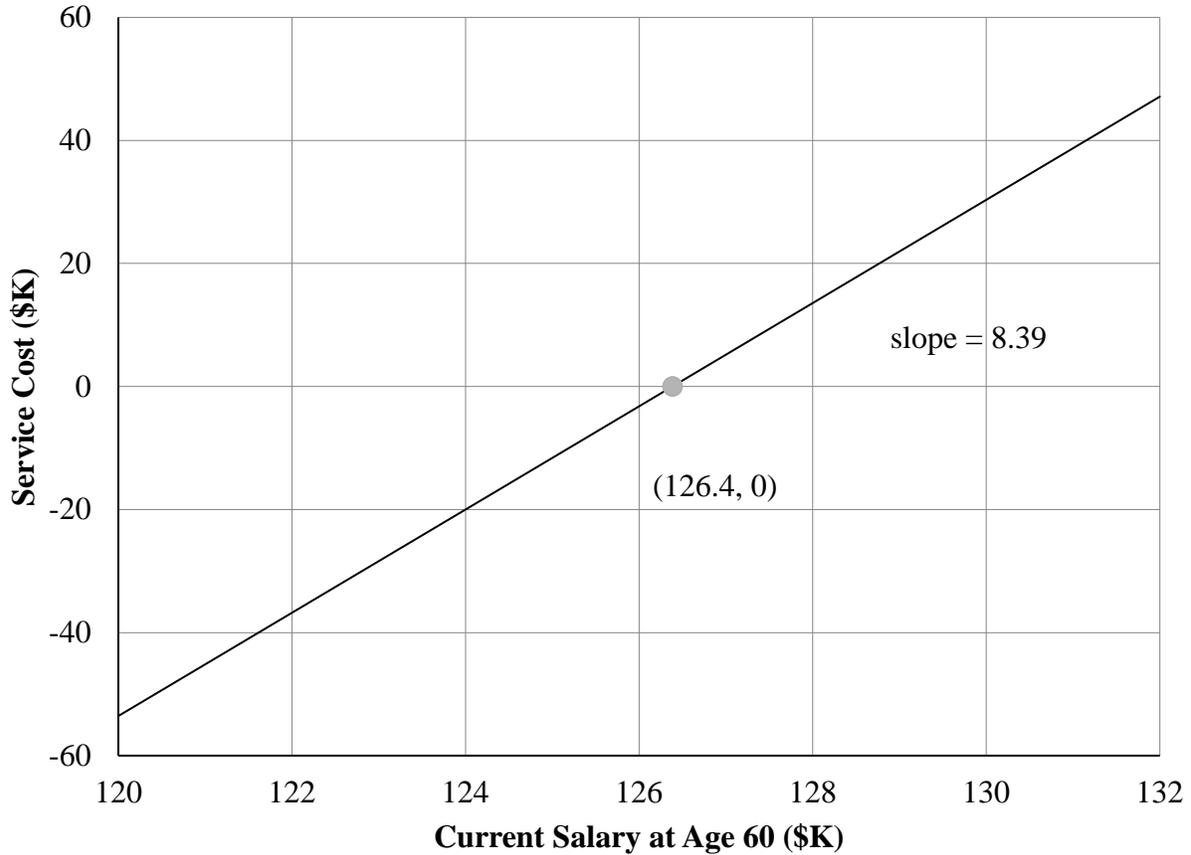
Consider an employee whose age is 60. Should the employee choose to continue to work rather than retire, then there is a service cost during age 60 for the change in accrued liability that occurs. Using the formula in part 4 for service cost and the formula in part 4.2.1 for accrued liability, we can write the formula for the service cost during age 60 as

$$\begin{aligned} \text{SVC}_{60} &= AL_{61} \times 1.07^{-1/2} - AL_{60} \times 1.07^{1/2} \\ &= \text{BF}_{61} \times \text{AF}_{61} \times \text{SAL}_{60} \times 1.07^{-1/2} - \text{BF}_{60} \times \text{AF}_{60} \times \text{SAL}_{59} \times 1.07^{1/2} \end{aligned}$$

Example 1

For an example, we can reconsider Bob from part 3 and part 4.1. Bob entered service at age 30. His benefit factors are $\text{BF}_{60} = 60$ percent and $\text{BF}_{61} = 62$ percent, based on Bob retiring with 30 or 31 years of service, respectively. The annuity factors are $\text{AF}_{60} = 14.24$ and $\text{AF}_{61} = 14.00$ (rows 1 and 2 of column 2 of Table 1). To simplify matters, we assume that Bob's salary at age 59 (SAL_{59}) was the easy-to-work-with round number \$120,000. With these numbers we can calculate Bob's service cost (SVC_{60}) versus salary (SAL_{60}), and Figure 6 shows the resulting plot—a straight line.

Figure 6. Service Cost for Bob, with Entry Age 30, Current Age 60, \$120,000 Salary at Age 59



The slope of the line is the marginal service rate. Its value in this example is 8.39, meaning a \$1 change in salary produces a \$8.39 change in accrued liability. The formula for the slope is the coefficient of SAL_{60} in the service-cost formula above; that is,

$$MR_{60} = BF_{61} \times AF_{61} \times 1.07^{-1/2}$$

The line intersects the horizontal axis when the salary has value \$126,384. We express this value as $ZR_{60} \times SAL_{59}$, where $ZR_{60} = 1.053$ is the *zero-cost rate*. The formula for the zero-cost rate is found by solving the service-cost formula for the ratio SAL_{60}/SAL_{59} with $SVC_{60} = 0$:

$$ZR_{60} = \frac{BF_{60}}{BF_{61}} \times \frac{AF_{60}}{AF_{61}} \times 1.07$$

If the actual year-over-year salary growth is less than the zero-cost rate, meaning $SAL_{60} / SAL_{59} < ZR_{60}$, then the service cost is negative; if the actual year-over-year salary growth is greater than the zero-cost rate, the service cost is positive.

This analysis generalizes to ages other than 60. For an employee who is still working at age x , where x is 60 or greater, the service cost for the year of age x and related formulas are:

$$SVC_x = MR_x \times (SAL_x - ZR_x \times SAL_{x-1})$$

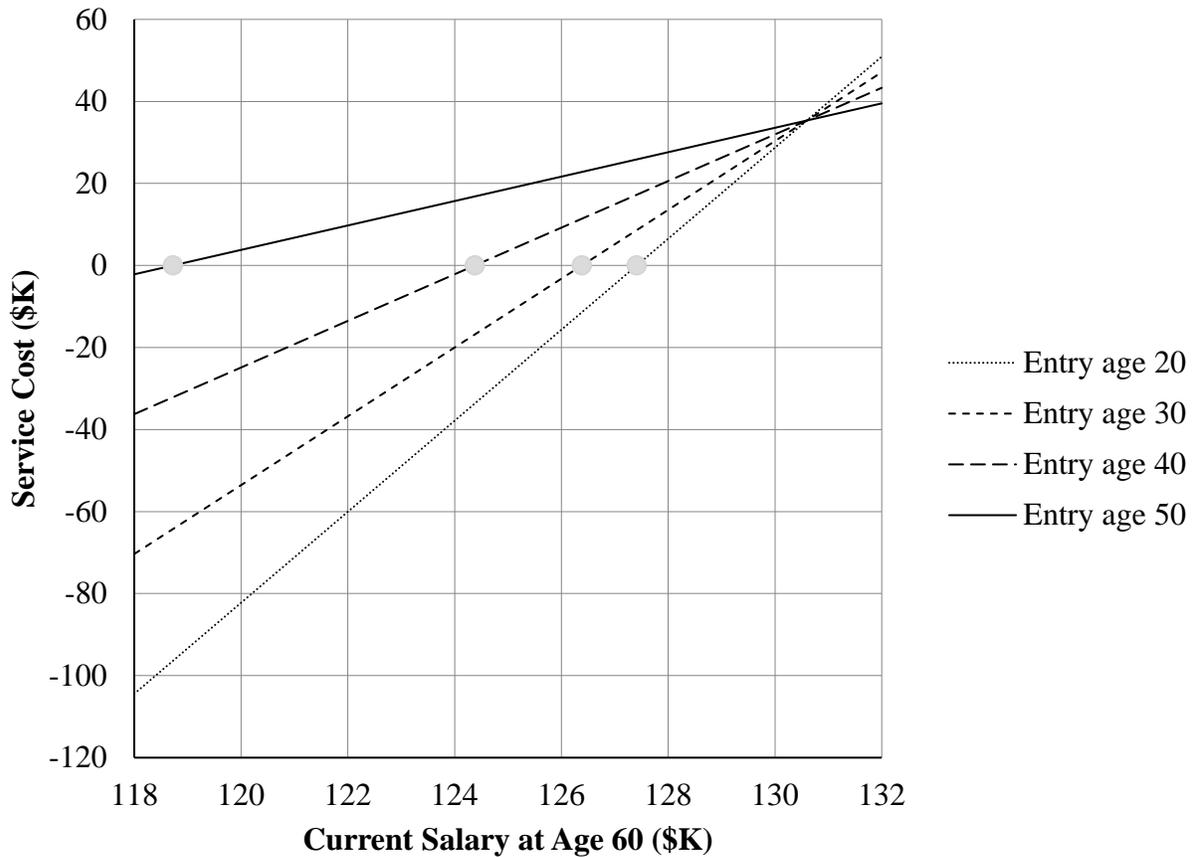
$$MR_x = BF_{x+1} \times AF_{x+1} \times 1.07^{-1/2}$$

$$ZR_x = \frac{BF_x}{BF_{x+1}} \times \frac{AF_x}{AF_{x+1}} \times 1.07$$

Example 2

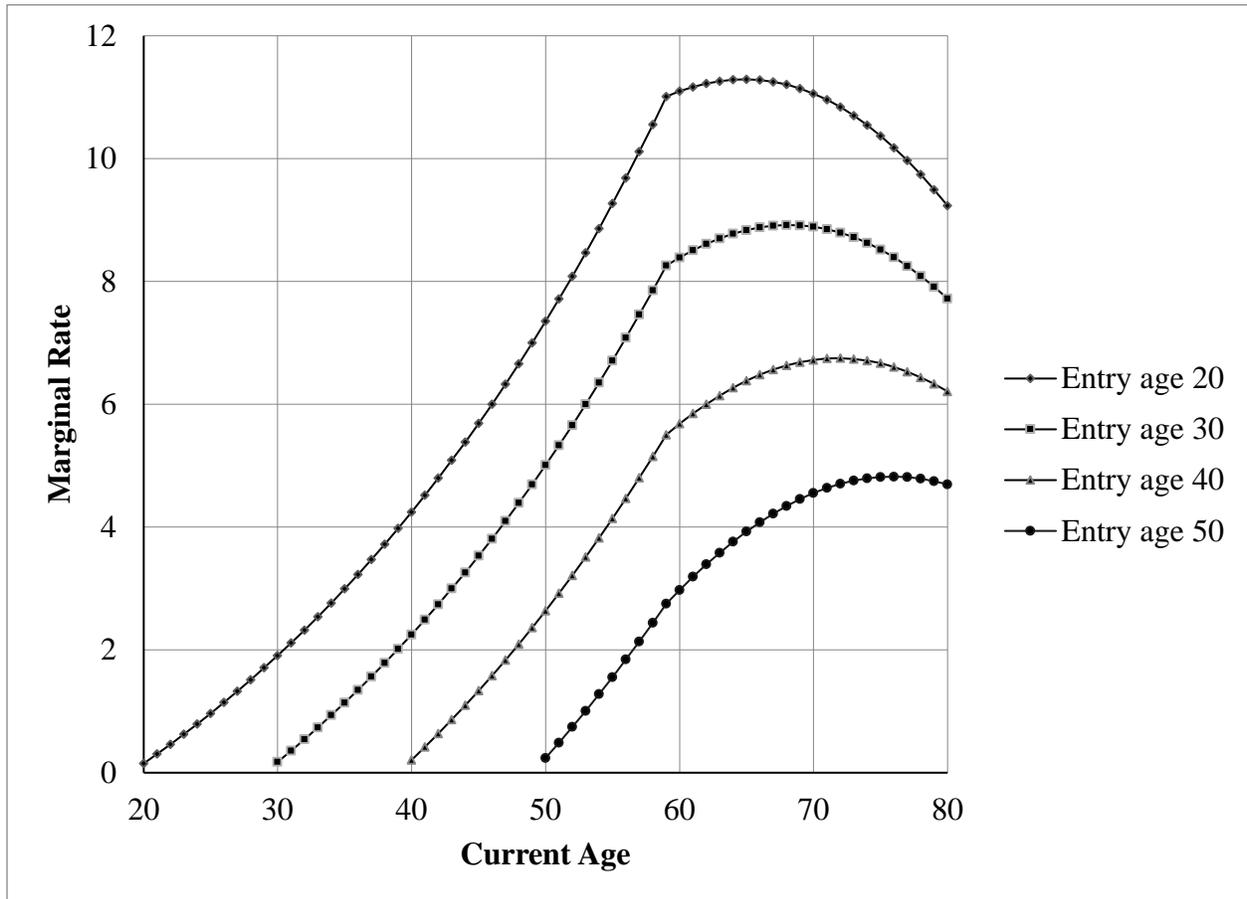
Figure 7 is similar to Figure 6 but includes plots for different entry ages. Each plot is a line whose slope is the marginal rate for that entry age; the younger the entry age, the steeper the slope. Each line passes through the point $(ZR_{60} \times \$100,000, 0)$, where ZR_{60} is the zero-cost rate for an employee of that entry age.

Figure 7. Service Cost for Various Employees, Current Age 60, \$120,000 Salary at Age 59, Various Entry Ages



The marginal rate MR_x remains a critical parameter beyond age 60 because of the high sensitivity of service cost to a change in current salary. Figure 8 plots the marginal rate for employees of several entry ages. The plot includes the previously plotted values for ages less than 60, for comparison.

Figure 8. Marginal Rates for Several Entry Ages



The marginal rate continues to grow for ages older than 60, although at a lower rate, and eventually it begins to decline for ages well past 60. But it remains strikingly high well beyond any likely working age. Thus, small changes in salary still result in much larger changes in accrued liability.

5. Discussion

5.1 Budgeting and Pay Raises

5.1.1 Budgeting for Normal Pay Raises and Promotions

Our model pension plan can be used to illustrate how employers can use pay raises as a lever to control pension costs. Basically, if employers had to currently budget the full pension cost of giving generous pay raises, employers would be much more careful about giving them. Conversely, if employers could currently budget the savings in pension costs that would result from restraining pay raises, employers would be more inclined to restrain them.

Reconsider the example of Bob from Table 4. Recall that Bob earned a \$100,000 salary at age 54. In part 3 case 1, Bob got a \$4000 pay raise for age 55 exactly in accordance with the 4 percent salary growth assumption used by the model pension plan. Not surprisingly, the impact of that pay raise on the accrued pension liability was negligible (overfunding the pension by just \$339 [row 1 of column 7 of Table 4]). Basically, to give Bob this \$4,000 pay raise, the employer just needs to come up with another \$4720 out of its annual budget: \$4,000 for the pay raise, and \$720 for the increase in the plan-normal cost ($\$720 = 18\% \times \$4,000$).

If Bob's employer instead gives him a \$10,000 (10 percent) pay raise, the employer must come up with another \$11,800 out of its annual budget: \$10,000 for the pay raise, and \$1,800 for the increase in plan-normal cost ($\$1,800 = 18\% \times 10,000$). However, that would result in an underfunding of more than \$40,000 (\$40,195 from row 2 of column 7 of Table 4). Under the current entry-age-normal-cost method, that \$40,000 underfunding would not show up in the employer's current budget but instead would simply be tacked on to the pension plan's unfunded liability. That \$40,000 in additional unfunded liability would be paid off by amortization payments, probably over the subsequent 20 to 30 years, payments that would total approximately \$80,000.²¹ We suspect, however, that few employers are cognizant of the fact that the \$10,000 pay raise would result in an \$80,000 burden to future budgets; all they notice is the \$11,800 hit to the current budget.

We believe that the employer should, instead, have to *immediately* budget for and fund both the \$10,000 pay raise and the \$40,000 increase in unfunded liability that results from it. Under the entry-age-service-cost funding method that we propose, the employer would be required to do just that. In the case of Bob's pay raise, that means Bob's employer would need to contribute another \$40,000 to the pension plan this year to offset the \$40,000 increase in unfunded liability that would otherwise result from giving Bob that \$10,000 pay raise. That is, the employer would need to come up with another \$52,000 out of its annual budget: \$10,000 for the pay raise, \$1,800 for the plan-normal cost, and \$40,000 for the increase in unfunded liability.²² The employer could still give Bob a \$10,000 pay raise, but it would have to take a budget hit of \$52,000 to do so. We suspect that most employers would be hesitant to grant a \$10,000 pay raise if it would cost them \$52,000 in the current annual budget.

That is the stick, but here is the carrot. Suppose the employer does not give Bob a pay raise at all, as might happen during a recession. Under the current entry-age-normal-cost method, the employer need not budget any amount more than what was budgeted last year, and that is likely what is motivating the employer to not give a pay raise. But the employer's savings is actually much greater than that. By not giving a pay raise, and by contributing the plan-normal cost as is

²¹ The total amount of amortization payments depends on the amortization policy of the plan. An aggressive amortization policy would be to pay off the \$40,000 underfunding with 20 annual payments of equal amounts, which with interest results in total amortization payments of \$78,113 (Appendix). A lax amortization policy would be to pay it off with 30 annual payments that increase by 3 percent each year, the intent being to make the payment a fixed percentage of payroll that is projected to grow by 3 percent annually, which with interest results in total amortization payments of \$115,601. Here we use the round number that amortization payments total to \$80,000, which is close to what it would be with an aggressive amortization policy. More generally, if the amount of underfunding is x , we will say that the total of amortization payments to pay it off is $2x$.

²² More precisely, the current budget hit should be \$51,975 ($\$51,975 = \$10,000 + \$1,800 + \$40,195$) (see row 2 of column 7 of Table 4).

required, the employer would *overfund* the pension plan by more than \$27,000 (\$27,362 from row 3 of column 7 of Table 4). That \$27,000 would not show up anywhere in the current year's annual budget; instead, it would reduce the pension plan's unfunded liability by that amount, and that \$27,000 in reduced unfunded liability would show up as an actuarial gain and be received back by the employer in the form of amortization-payment reductions, probably over the next 20 to 30 years, reductions that would total approximately \$54,000.²³ We suspect that few employers realize that by not giving Bob a pay raise, they would ease the burden on future budgets by \$54,000; all they notice is that this year's budget line for Bob is the same as last year's.

We believe that the employer should, instead, get an immediate credit in this year's annual budget for the \$27,000 decrease in unfunded liability that would result from not giving Bob a pay raise. Under the entry-age-service-cost method that we propose, the employer would get that immediate credit. That is, if Bob is not given a pay raise, Bob's employer would be permitted to reduce its contribution to the pension plan by \$27,000. In short, by freezing Bob's salary, the employer has reduced the budget line for Bob's total compensation by \$27,000 compared to the prior year.

We believe that the entry-age-service-cost method that we propose would give employers (such as California cities and other public agencies) some real incentives for fiscal discipline. In short, the entry-age-service-cost method would enable employers to use salary growth as a lever by which they could gain control over their pension funding.

5.1.2 Pension Spiking

Our model pension plan can also illustrate how to deal with the very serious problem of pension spiking. Pension spiking refers to the practice of hiking an employee's final compensation through a promotion or salary hike, right before the employee retires, thereby dramatically increasing that employee's pension for life. Spiking is difficult to prevent and ends up costing large pensions like CalPERS millions of dollars (Lifsher, 2014).

For a particularly egregious example, consider how a police department could reward a long-serving police officer with a career-end promotion to sergeant, big pay raise, and perhaps even a no-show desk job. To be sure, the department would have to budget and pay for that big, final pay raise and the plan-normal cost associated with it, but the hidden cost (i.e., the big jump in unfunded liability that would result from the spiked pension) would be paid for by future budgets in the form of amortization payments that would last for decades after the employee retired. In effect, an employer can give large rewards to a few favored employees and shift the bulk of the costs for those rewards to future budgets (and future taxpayers).

For an example, we can reconsider Alice from Table 3 in part 2.2.3. Recall that Alice's salary increased steadily by 4 percent a year from \$32,065.12 at age 30 to \$100,000 at age 59 (columns 1 and 2 of Table 3). Now suppose that instead of getting a 4 percent pay raise at age 59, her employer spiked her salary by, say, 20 percent—from \$96,153.85 at age 58 to \$115,384.62 at age 59 ($\$115,384.62 = 1.20 \times \$96,153.85 = \$96,153.85 + 20\% \times \$96,153.85 = \$96,153.85 + \$19,230.77$). With a final salary of \$115,384.62, Alice would now be entitled to a pension starting at around \$69,000 a year instead of just \$60,000 a year ($\$69,230.77 = 60\% \times$

²³ See note 21.

$\$115,384.62 = 2\% \times 30 \text{ yrs} \times \$115,384.62$). To fund Alice's larger pension, the plan would need to have $\$986,000$ instead of just $\$854,000$.²⁴ But the plan would only have around $\$857,000$ ($\$857,033.90 = \$781,239.58 AL_{59} \times 1.07 + \$17,684.71 NC_{59} \times \$115,384.62 / \$100,000 \times 1.07^{1/2}$). Thus the 20 percent salary spike would result in increasing the pension plan's unfunded liability by about $\$129,000$ ($\$128,604.08 = \$985,637.98 - \$857,033.90$), and that $\$129,000$ would not show up in the current annual budget. Instead, that $\$129,000$ would be paid off by amortization payments, probably over the next 20 to 30 years after Alice retires, payments that with interest would total to approximately $\$258,000$.²⁵ In short, that salary spike of about $\$19,000$ ($\$19,230.77 = 20\% \times \$96,153.85$) would cost future budgets (and, therefore, future taxpayers) about $\$258,000$.

We suspect that if the current annual budget had to take an immediate $\$129,000$ hit to pay for the pension spike, the spike would not happen. Under the entry-age-service-cost method, Alice's employer could still give her a 20 percent ($\$19,000$) pay raise; however, the employer would immediately have to come up with that extra $\$19,000$ for the pay raise *and* another $\$129,000$ to keep the pension plan fully funded.

5.1.3 Budgeting and Later Retirements

Our model pension plan can also be used to illustrate how employers can reduce their pension costs by encouraging employees to work beyond the normal retirement age. Basically, when an employee chooses to work beyond the normal retirement age, the employee forgoes the pension payments that he or she is otherwise eligible to receive. Those foregone payments offset some of the employer's pension cost. If employers could currently budget the savings in pension cost that result from the forgone payments, employers would be motivated to encourage employees to defer retirement.

Reconsider the example of Alice from Table 3 in part 2.2.3. Recall that Alice earned a salary of $\$100,000$ during age 59. At age 60, Alice is eligible to retire. Suppose that Alice's employer offers her a 4 percent raise if she agrees to defer retirement and work during age 60, and Alice accepts. The employer needs to budget $\$122,720$ in total compensation for Alice: $\$104,000$ for the salary ($\$104,000 = 1.04 \times 100,000$) and $\$18,720$ for the plan-normal cost ($\$18,720 = 18\% \times 104,000$); but the cost to the employer is actually much lower than that. Because Alice is working beyond the normal retirement age, the plan-normal cost that the employer is required to contribute on her behalf results in an overfunding of her pension by about $\$31,000$ ($\$30,818$ from row 1 of column 6 of Table 7). Under the entry-age-normal-cost method, that $\$31,000$ in savings would not show up anywhere in the employer's current annual budget; instead, it would reduce the pension plan's unfunded liability by that amount, and that $\$31,000$ in reduced unfunded liability would show up as an actuarial gain and be received back by the employer in the form of amortization-payment reductions, probably over the next 20 to 30 years, reductions that would total approximately $\$62,000$.²⁶ We suspect that the employer does not realize that by

²⁴ $\$985,637.98 = \$854,219.55 AL_{60} \times \$115,384.62 / \$100,000$. See note 18.

²⁵ See note 21.

²⁶ See note 21.

encouraging Alice to work another year, it would ease the burden on future budgets by \$62,000; all the employer notices is that this year's budget line for Alice is \$122,720.

5.2 Paying All Compensation Costs as They Currently Accrue: The Entry-Age-Service-Cost Method

5.2.1 General Considerations

We believe that current employers should pay the full cost of their employees as those employees perform their services. For example, if an employee is performing services that are worth \$100,000, then the employer should provide that employee with \$100,000 of compensation, no more and no less. Basically, the employer has \$100,000 to spend. If part of the compensation is paid out in fringe benefits, then less should be paid in cash salary. For example, if the employer provides its employees with a \$10,000 health insurance policy, then cash salary should be just \$90,000.²⁷

Similarly, if an employer provides a retirement benefit for its employees, then this year's cost for that benefit should be paid for out of the employer's current budget.²⁸ For example, if an employee is worth \$100,000 to an employer and they agree that the employee is to receive a \$15,000 of her compensation in the form of a contribution to her supplemental defined contribution plan, then the employer and employee would enter into a salary-reduction agreement. As a result, she would receive \$85,000 in cash compensation ($\$85,000 = \$100,000 - \$15,000$) and the employer would contribute \$15,000 to her defined contribution plan.²⁹

5.2.2 The Entry-Age-Service-Cost Method

The same pay-all-compensation-costs-currently approach should be applied to pension plans and, particularly, to traditional final-average-pay plans. In short, we believe that the plan actuary for a traditional final-average-pay pension should use the entry-age-service-cost method to fund the plan, instead of the entry-age-normal-cost method. In effect, each year, the plan actuary should (1) determine the service cost for each employee each year based on that employee's actual year-over-year salary change, (2) add the employee service costs together to get a total service cost, and (3) require that the employer contribute that amount to the pension plan for the year (instead of contributing the plan-normal cost determined under the entry-age-normal-cost method).

To see how this entry-age-service-cost method would work, we can use the example employees in the three-person model pension plan in part 4.1. As column 5 of Table 5 shows, in the prior year ($x - 1$), Charlie had a salary of \$90,000, Diane had a salary of \$100,000, and Edna had a salary of \$110,000. Then all three got \$5,000 pay raises, and we showed that the total service cost for the three employees was \$67,197 (row 4 of column 8 of Table 5), as opposed to the plan-

²⁷ Here, we ignore any tax considerations, although we acknowledge that tax-favored fringe benefits can often be more valuable than cash compensation. For example, because employer-provided health insurance is excluded from the income of employees (26 United States Code § 106), most employees find that \$10,000 of tax-free health insurance is actually more valuable to them than another \$10,000 of taxable cash compensation.

²⁸ See, e.g., American Academy of Actuaries, 2004, p. 7 (noting that "Prefunding attempts to equitably allocate to each year the cost of the pension benefits.").

²⁹ Again, we ignore any tax considerations, although we acknowledge that this \$15,000 in tax-deferred compensation is usually more valuable to the employee than \$15,000 in taxable salary.

normal cost of just \$56,700 ($\$56,700 = 18\% \text{ plan-normal rate} \times \$315,000 \text{ total payroll}$). In effect, the entry-age-service-cost method would require the employer to contribute \$10,497 *more* than the entry-age-normal-cost method ($\$10,497 = \$67,197 - \$56,700$). That additional \$10,497 is needed to ensure that the full cost of the pension benefit accrued by the three employees this year is paid for by the employer this year. The additional \$10,497 is due to the fact that the actual year-over-year pay raises were, on the average, greater than the 4 percent that was assumed by the entry-age-normal-cost method, and, as a result, the employer would have to contribute more than the plan-normal cost to fully fund the pension benefits that its employees accrued this year.

On the other hand, when each employee, instead, gets a pay raise of just \$3,000, then part 4.1 showed that the total service cost for the three employees was just \$46,362 (row 4 of column 8 of Table 6), which is lower than the plan-normal cost of \$55,620 ($\$55,620 = 18\% \text{ plan-normal rate} \times \$309,000 \text{ total payroll in year } x$). On these facts, the entry-age-service-cost method that we propose would allow the employer to contribute \$9,258 *less* than the entry-age-normal-cost method would require ($\$9,258 = \$55,620 - \$46,362$). That reduction of \$9,258 is due to the fact that the actual year-over-year pay raises were, on average, lower than the 4 percent that was assumed by the entry-age-normal-cost method, and as a result, the employer could contribute less than the plan-normal cost and still fully fund the pension benefits that its employees accrued this year.

The entry-age-service-cost method that we propose would ensure that employers make the correct contributions to keep their plans 100 percent funded at all times under the assumed discount rate and mortality assumptions. In fact, as service cost is tied closely to projected final compensation, there is a good chance that contributions made by the employer will somewhat overfund the plan. In particular, we acknowledge that (1) some employees will terminate employment before reaching their projected final compensation and so will receive lower-than-projected pensions, and (2) others may die and never receive any pension benefits at all. The entry-age-service-cost method could easily account for these events *when they occur* by reducing that year's required pension contributions. For example, suppose that Alice from part 2.2.3 is one of an employer's 101 employees, that she dies this year at age 40, and that she is not entitled to any death benefit.³⁰ Then of course, no service-cost contribution would be needed with respect to Alice. Moreover, if at her death, the employer's total service cost for the remaining 100 employees was, say, \$8 million, then her employer could be allowed to reduce its annual required contribution that year by \$95,202.36, which is the amount of Alice's accrued liability at age 40 (AL_{40} from column 4 of Table 3).

While the entry-age-service-cost method sacrifices the simplicity of the entry-age-normal-cost method, it more than makes up for it by imposing fiscal discipline on employers which would lead employers to budget responsively—by properly accounting for the *actual* cost of employee services as those services are provided to the employer.

³⁰ This is actually pretty unlikely as her death probability is quite low. For example, the death probability for a 40-year-old female employee in 2014 was just 0.0396 percent according to the RP-2014 table (Society of Actuaries, 2018b).

5.2.3 Applying the Entry-Age-Service-Cost Method to Single-Employer Pensions

In addition to applying the entry-age-service-cost method to multiple-employer pension systems like CalPERS, we believe that the same method could and should be applied by single-employer, final-average-pay pension plans. These employers would also benefit from the fiscal discipline that would come from fully funding both pay raises and the accompanying increases (or decreases) in accrued pension liabilities.

5.3 Switching Employers

On occasion, a talented employee who works for one employer is recruited away by another employer (Stecklein, 2016; Winkley, 2014). For example, reconsider Diane from Example 3 of part 4.1. Assume that Diane is an urban planner for the City of Appleton, with a \$100,000 salary last year when she was age 44 (row 2 of column 5 of Table 5). Diane was planning to stay with Appleton until she retired, but she got a great job offer from the City of Baydell, which she accepted. Diane now makes \$150,000 as Baydell's chief urban planner. Both Appleton and Baydell are (fictional) cities that have identical model pension plans in a pension system that is similar to CalPERS, and as such, Diane continues to accrue pension benefits with Baydell under the same terms that she did with Appleton. When she retires, her pension will be based on her combined years of service with the two cities.

Under our proposed entry-age-service cost method, the amount that Baydell must contribute to its plan for Diane during her first year in Baydell would be the service cost based on her increase in salary from \$100,000 at Appleton to \$150,000 at Baydell.³¹ That service cost works out to be about \$181,000:

$$\$180,878 = 3.5324 MR_{45} \times (\$150,000 - 1.04 \times \$100,000) + 0.1768 NR \times 1.04 \times \$100,000$$

In other words, as a result of the generous job offer, Baydell would have to budget an additional \$181,000 during Diane's first year with Baydell to cover the accrued pension liability increase that results, and this \$181,000 is in addition to the \$150,000 that Baydell must budget for her salary. The combined cost, \$331,000 ($\$331,000 = \$150,000 + \$181,000$), might cause Baydell to rethink its rather generous offer.³²

Suppose that Baydell decides the cost is acceptable and does indeed make the offer, and that Diane accepts. Suppose that Diane receives a 4 percent pay raise in each of her remaining years at Baydell. During age 59, her salary would be around \$260,000 ($\$259,751.47 = \$150,000 \times 1.04^{59-45}$), giving her an initial pension of around \$156,000 ($\$155,850.88 = 60\% \times \$259,751.47$). The total liability for her pension would then be $AL_{60} = \$2,218,848$ ($\$2,218,847.82 = \$155,850.88 \times 14.2369925 AF_{60}$). That total liability equals the combined service-cost contributions that were made by the two cities, along with interest. That total liability would be allocated between the two cities according to the service cost that each had contributed, along with interest, as follows.

³¹ This is the same amount that Appleton would have had to contribute had Appleton matched Baydell's offer and succeeded in keeping her.

³² For each \$10,000 by which Baydell reduces the offer, it would reduce the amount that must be budgeted by about \$45,000: \$10,000 for the reduction in salary, and \$35,000 for the reduction in service cost ($\$35,324 = 3.5324MR_{45} \times \$10,000$).

For Appleton, the final salary of Diane was \$100,000 at age 44. The accrued value of Appleton's service-cost contributions at the end of her employment there was $AL_{45} = \$337,370$ ($\$337,370 = 0.1768 \text{ NR} \times 100,000 \times 1.07^{1/2} \times [(1.07/1.04)^{45-30} - 1] / [1.07/1.04 - 1]$). With interest, that would grow to \$930,816 when Diane retires at age 60 ($\$930,816 = \$337,370 \times 1.07^{60-45}$). This \$930,816 would be Appleton's share of the total liability. Baydell would be liable for the remaining \$1,288,032 ($\$1,288,032 = \$2,218,848 - \$930,816$). More specifically, throughout the years of Diane's retirement, the two cities would be liable for her pension in proportion to these amounts, meaning Appleton would pay 41.95% of her pension each year ($41.95\% = \$930,816 / \$2,218,848$), and Baydell would pay 59.05% of her pension each year ($59.05\% = 100\% - 41.95\%$).

Note that Appleton would only be liable for the pension amount due to the salary that it paid to Diane *while she was an Appleton employee*. Baydell recruited her with a big pay raise, and that caused a big increase in total pension liability, *but that big pay raise would (and should) have no effect on Appleton's liability*. This is important, because it shields Appleton from Baydell's largesse. For its part, Baydell has control over its liability according to whatever pay raises it grants to Diane over the years that she works there. In short, what happens in Appleton stays in Appleton, and what happens in Baydell stays in Baydell.³³

6. Conclusion

In this paper, we showed how the failure of actuaries to properly account for the impact of individual pay raises of employees in traditional defined benefit pensions can (and we believe has) led to inadequate contributions to, and therefore significant underfunding of, many state and local pensions. We then showed how properly accounting for individual pay raises would help ensure full funding of these pensions. Because we believe that employers should fully fund each employee's accrued pension liability each year, it seems clear to us that the entry-age-normal cost method is not the proper method to use to promote full funding. Instead, this paper explained how adopting the service-cost method would promote full funding of state and local government pensions.

³³ We remark that this method of allocating the liability between the two cities is undoubtedly different from what CalPERS does now. We are not exactly sure what CalPERS does now, because we have not found it described in any publicly available document. However, since the entry-age-service-cost method is not the method that CalPERS currently uses, their allocation method is undoubtedly different, and we speculate that their method puts Appleton at risk for liability increases that result from Baydell's largesse.

Appendix

Mortality Table

Table 1 is the mortality table used with our model pension. The table is based on the Society of Actuaries (SOA) RP-2014 mortality table with the MP-2016 projection scale (Society of Actuaries, 2018a; Society of Actuaries, 2018b). We make the table static by fixing the projection to correspond to the year 2038. For further simplicity, we make the table gender-neutral by averaging the mortality rates of males and females. Entries are the probability of death q_x during age x .

Table A1. Mortality Table, 2038
(RP-2014, made static by using fixed projection to the year 2038 using the MP-2016 scale)

Age (x)	Death Probability (q_x)	Age (x)	Death Probability (q_x)	Age (x)	Death Probability (q_x)
60	0.005293	80	0.031065	100	0.252075
61	0.005714	81	0.034603	101	0.272181
62	0.006177	82	0.038638	102	0.292896
63	0.006682	83	0.043206	103	0.313662
64	0.007237	84	0.048396	104	0.33462
65	0.007843	85	0.05428	105	0.355497
66	0.008511	86	0.061014	106	0.376523
67	0.009245	87	0.068682	107	0.396936
68	0.010052	88	0.077234	108	0.416876
69	0.010943	89	0.086915	109	0.436401
70	0.011934	90	0.097691	110	0.455735
71	0.013032	91	0.109455	111	0.470323
72	0.014241	92	0.121891	112	0.482026
73	0.015593	93	0.135036	113	0.490095
74	0.017095	94	0.148697	114	0.495148
75	0.01878	95	0.163029	115	0.5
76	0.020675	96	0.179083	116	0.5
77	0.022803	97	0.195964	117	0.5
78	0.025223	98	0.213901	118	0.5
79	0.027956	99	0.232594	119	0.5

Note: RP-2014, made static by using fixed projection to the year 2038 using the MP-2016 scale

Annuity Factor

The annuity factor AF_x is the expected present value at the start of age x of all future payments, conditioned on being alive at the start of age x . The first payment, during age x , has value 1. Subsequent payments, during age $x + 1$ and beyond, increase by the assumed inflation rate c .

That is, the payment during age $x + k$ has the value $(1 + c)^k$. Each payment is modelled as being made at midyear if the retiree is alive; if the retiree dies prior to midyear, the payment is not made.

First consider the case when the retiree dies during age x . We will say that the (conditional) probability that the retiree dies prior to the midyear payment is $1/2$; this would be the case if the death is uniformly likely to occur at any time during the year. If the payment is made, it occurs at midyear, and its value discounted to the start of the year is $v^{1/2}$, where $v = 1/(1 + i)$ and i is the assumed interest rate. Thus at the start of age x , the (conditional) expected present value of the payment is

$$\frac{v^{1/2}}{2}.$$

Now consider the case that the retiree survives age x . The retiree receives the midyear payment for age x and goes on to receive an annuity beginning next year, at age $x + 1$, with starting payment $1 + c$. Thus, at the start of age x , the (conditional) expected present value of all future payments is

$$v^{1/2} + v(1 + c) \times AF_{x+1}.$$

Let q_x be the probability of dying during age x given that the retiree is alive at the start of age x . Then the expected present value of future payments is

$$AF_x = \frac{q_x v^{1/2}}{2} + (1 - q_x)(v^{1/2} + v(1 + c) \times AF_{x+1}).$$

In order to apply this formula to calculate annuity factors, we need a boundary condition. For the mortality table used here, there is a terminal age t such that $q_t = 1$, and $q_x < 1$ for $x < t$.³⁴ That is, the retiree may survive to age t , but not to any subsequent age. Thus,

$$AF_t = \frac{v^{1/2}}{2},$$

Because, if the retiree is alive at the start of age t , the only possible payment is the one for age t , and the probability of surviving to midyear to receive it is $1/2$. With this boundary condition, the formula can be applied recursively to calculate AF_{t-1} , AF_{t-2} , ..., AF_x .

Normal Rate

Let r denote the retirement age. The accrued liability at the start of the year of retirement is

$$AF_r \times BF_r \times SAL_{r-1}$$

where AF_r is the annuity factor, SAL_{r-1} is the salary during the year preceding retirement, and $BF_r \times SAL_{r-1}$ is the initial pension amount. For simplicity we will take $SAL_{r-1} = 1$. We assume that year-over-year salary growth occurs at a fixed rate g , meaning the salary during age x is

34 For the RP-2014 table, $t = 120$.

$$SAL_x = (1 + g)^{x-r+1}$$

Let NR denote the normal rate. The normal contribution made during age x is $NR \times SAL_x$. The contribution is modelled as being made at midyear, and thus it earns a half-year's interest during age x . After age x , there are $r - x - 1$ years until retirement begins, and the contribution made during age x earns a full year's interest in each of those subsequent years. Thus, at the start of retirement, the accrued value of the contribution made during age x is

$$NR \times SAL_x \times (1 + i)^{1/2}(1 + i)^{r-x-1}$$

where i is the assumed interest rate.

Now let e denote the entry age. The value of NR we seek is the value that results in a match between the accrued liability at the start of retirement and the accrued value of all normal contributions made during years $e, e + 1, \dots, r - 1$. That is, we seek NR that satisfies

$$AF_r \times BF_r \times SAL_{r-1} = \sum_{x=e}^{r-1} NR \times SAL_x \times (1 + i)^{1/2}(1 + i)^{r-x-1}$$

Substituting for SAL_{r-1} and SAL_x and rearranging the sum yields

$$AF_r \times BF_r = NR \times (1 + i)^{1/2} \sum_{k=0}^{r-e-1} \left(\frac{1 + i}{1 + g} \right)^k$$

Solving for NR yields

$$NR = AF_r \times BF_r \times v^{1/2} \times \left(\sum_{k=0}^{r-e-1} \left(\frac{1 + i}{1 + g} \right)^k \right)^{-1}$$

where $v = 1 / (1 + i)$. Applying the formula for a geometric sum yields

$$NR = AF_r \times BF_r \times v^{1/2} \times \frac{\left(\frac{1 + i}{1 + g} \right) - 1}{\left(\frac{1 + i}{1 + g} \right)^{r-e} - 1}$$

Amortization

Level Payments

For simplicity take the valuation year to be 2018, with valuation date December 31, 2018. Let u be the loss that occurs during 2018 as measured on the valuation date. Suppose that u is to be amortized by n consecutive annual payments, each payment of value x and made midyear, the first payment occurring in 2020 (the one-year lag being necessary to allow time to prepare the valuation). To amortize the loss u , the present value of those n annual payments on the valuation date must equal u . The present value of the k th payment is $xv^{k+1/2}$, where $v = 1/(1 + i)$ and i is the assumed interest rate. Thus the present value of the n payments is

$$u = x \sum_{k=1}^n v^{k+1/2} = xv^{1/2} \frac{1 - v^n}{i}$$

Solving for the payment value x :

$$x = \frac{i(1+i)^{1/2}}{1-v^n} u$$

The total T of the annual payments is nx .

For a numerical example, let the loss $u = \$40,000$, the number of payments $n = 20$, and the interest rate $i = 0.07$ (i.e., 7%). Then the payment value $x = \$3,905.63$, and the total of the payments $T = \$78,113$.

Level Percentage of Projected Payroll

Now suppose that u is to be amortized by n consecutive annual payments that increase by 3% annually, meaning the payments have values $x, 1.03x, 1.03^2x, \dots, 1.03^{n-1}x$. The motivation is that payroll is projected to grow by 3% annually, and thus each payment represents a fixed percentage of the projected payroll. To amortize the loss u , the present value of those n annual payments on the valuation date must equal u ; that is

$$u = x \sum_{k=0}^{n-1} 1.03^k v^{k+3/2} = xv^{3/2} \frac{1 - 1.03^n v^n}{1 - 1.03v}$$

Solving for the initial payment value x :

$$x = \frac{(1+i)^{3/2}(1-1.03v)}{1-1.03^n v^n} u$$

The total T of the annual payments is

$$T = x \sum_{k=0}^{n-1} 1.03^k = \frac{1.03^n - 1}{0.03} x$$

For a numerical example, let the loss $u = \$40,000$, the number of payments $n = 30$, and the interest rate $i = 0.07$ (i.e., 7%). Then the initial payment value $x = \$2,429.84$, and the total of the payments $T = \$115,601$.

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