Mortality, Health and Marriage:

A Study Based on Taiwan's Population Data

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Summary

Life expectancy has been increasing significantly since the start of the 20th century and mortality improvement trends are likely to continue in the 21st century. Stochastic mortality models are used frequently to predict the expansion in life expectancy. In addition to gender, age, period and cohort are three main risk factors considered in constructing mortality models. Other than these factors, it is believed marriage status is related to health and longevity and many studies have found the married have lower mortality rates than the unmarried. In this study, we use Taiwan's marital data for the whole population (married, unmarried, divorced/widowed) to evaluate if marriage status can be a preferred criteria.

In addition to checking if mortality rates vary for different marital statuses, we want to know if this preferred criteria will be valid in the future. We chose two popular mortality models, Lee-Carter and age-period-cohort, to model the mortality improvements of various marital statuses. Because of linear dependence in the parameters of the age-period-cohort model, we used computer simulation to chose the appropriate estimation method. Based on Taiwan marital data, we found the married have significantly lower mortality rates than the single and, if converting the difference into a life insurance policy, the discount amount is even larger than that for smokers/nonsmokers.

Keywords: Longevity Risk, Mortality Improvement, Age-Period-Cohort Model, Marriage Status, Simulation

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1. Introduction

The prolonging of life expectancy has been a common phenomenon in many countries since the turn of the 20th century. For example, the life expectancies of U.S. males and females were both less than 50 years in 1900; they reached about 74 and 80 years in 2000, respectively. On average, males and females gained about 30 years of life over the past 100 years (figure 1), which is equivalent to gaining about 0.3 years of life annually. Life expectancies in the United States have been steadily growing and do not show apparent signs of slowing down. The situation in Taiwan has been similar since the end of World War II. Life expectancies of males and females also have gained about 0.2 to 0.3 years annually (figure 1).
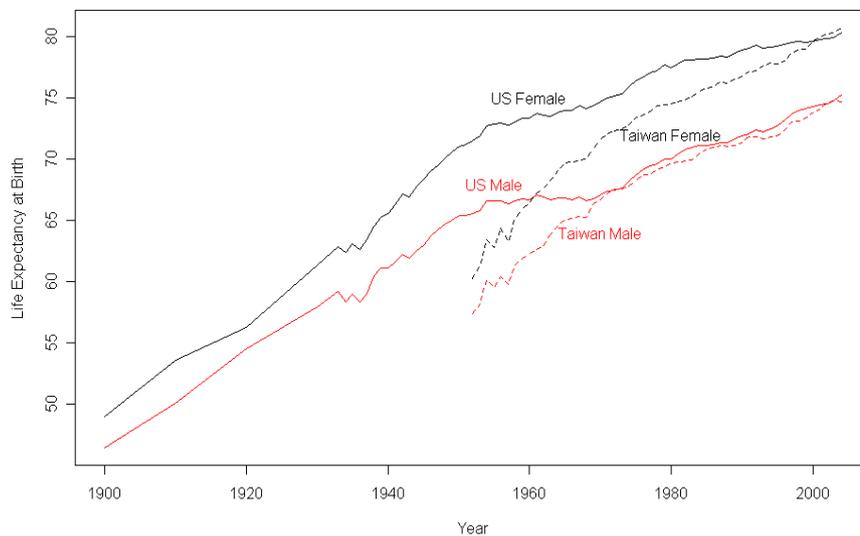

Figure 1. Life expectancy in Taiwan and the United States

Traditional life insurance products are based on fixed mortality rates, and prolonged life expectancies create problems in calculating insurance premiums. For example, if one purchases a deferred life annuity today at age 30 with benefits starting at age 60, there will be at least six years (assume that the annual increment of life expectancy at age 60 is 0.2 years; $30 \times 0.2 = 6$) of difference in life expectancy if today's mortality rates are used to compute the insurance premiums. In other words, there is a risk of underestimating insurance premiums for life annuity policies and the insurers may go into insolvency, i.e., longevity risk. Many previous studies suggest that mortality risk may cause substantial losses if handled improperly. See, for example, Huang, Yue and Yang (2008) for a detailed discussion.

In recent years, there have been intensive discussions about dealing with longevity risk. Using the dynamic life table, or, equivalently, the cohort life table, to replace the traditional life table with fixed mortality rates is one way of handling longevity risk. The search for a reliable mortality model is crucial for implementing the dynamic life table. There are several mortality models and the Lee-Carter (L.C.) model (Lee and Carter 1992) probably is the most popular choice.

If $m_{x,t}$ denotes the central death rate for a person age x at time t, then the L.C. model assumes that

$$\ln(m_{x,t}) = \alpha_x + \beta_x \, \kappa_t + \varepsilon_{x,t} \tag{1}$$

where parameter α_x denotes the average age-specific mortality, κ_t represents the general mortality level, and the decline in mortality at age x is captured by β_x. The mortality level κ_t is usually a linear function in time. The term $\varepsilon_{x,t}$ denotes the deviation of the model from the observed log-central death rates and is assumed to be white noise with 0 mean and relatively small variance (Lee 2000). The L.C. model can be treated as a model with the age effect, plus a mixed effect of age and time.

There have been several modifications to the L.C. model since it was first introduced in 1992. Many modifications consider including extra effects, in addition to the age and age-time effects. For example, Cairns et al. (2009) evaluated seven modifications of the original L.C. model, using data from England, Wales and the United States. Among these modifications, two models (M2 and M8) that include "cohort" effect have the best performances. However, unlike in the L.C. model where singular value decomposition (SVD) is suggested, Cairns et al. did not give suggestions for the estimation of the model parameters.

Adding the cohort effect to the L.C. model, there will be three effects: age, period and cohort. This is similar to the age-period-cohort (APC) model in epidemiology. Note that these three effects would produce a linear dependent structure, i.e., age = time − cohort. It has been shown that the estimation process is critical in giving reliable parameter estimates in the APC model. In fact, we found that the estimation process is also crucial in applying the modified L.C. models (such as M2 and M8).

In addition to age, period and cohort, there are factors believed to be related to human longevity. For example, smokers generally have higher mortality rates and are

charged with higher premiums for life policies than nonsmokers. Marriage status is another risk factor and many studies show that the married have lower mortality rates. However, past marriage studies used sample data and there are doubts about sampling errors. In this study, we will use Taiwan's marital data, a population data, to evaluate if marriage status is a feasible choice for a preferred criteria.

Besides checking if the mortality rates are the same for different marital statuses, we wanted to know if this preferred criteria will be valid in the future. We chose two popular mortality models, Lee-Carter and age-period-cohort, to model the mortality improvements of various marital statuses. Because of the linear dependence in parameters of the age-period-cohort model, we used computer simulation to chose an appropriate estimation method. Specifically, we examined if the estimation methods produced reliable and unbiased estimates for the model parameters.

2. The APC Model

The APC model is used in epidemiology as a preliminary tool in disease incidence and mortality. It provides a means of descriptive statistics for summarizing the information in a two-way table classified by age group and time period. In general, the APC model includes three effects: age, period and cohort.

$$Y_{ij} = \log(R_{ij}) = \log(\frac{O_{ij}}{N_{ij}}) = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ij} \tag{2}$$

where O_{ij} is the number of deaths for the ith age group ($i = 1,\ldots,m$), in the jth year ($j = 1,\ldots,n$), among N_{ij} people at risk. It is generally assumed that O_{ij} follows Poisson distribution. The model parameters α_i, β_j and γ_k each specify the effects of age, time (period) and cohort, respectively. Note that the cohort effect γ_k satisfies $k = j - i + m$ and $k = 1, 2, \ldots, m + n - 1$. To avoid the problem in computing, it is usually assumed that $\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j = \sum_{k=1}^{m+n-1} \gamma_k = 0$.

Because the three effects create linear dependency, i.e., determining two effects automatically determining the third effect, there exists an identification problem. Several methods have been proposed to solve the identification problem, such as the

sequential method, autoregressive model and individual record approach. Robertson, Gandini and Boyle (1999) gave a thorough literature review for these methods, and they found that all methods reviewed give acceptable estimates of parameters in the case of the nonlinear component. However, Robertson, Gandini and Boyle did not consider the variances of the parameter estimates.

Yang, Fu and Land (2004) introduced a new method for solving the identification problem. The method is called the intrinsic estimator (I.E.); it uses a generalized matrix inverse to acquire a unique solution for the parameters. Fu and Rohan (2004) showed the consistency and asymptotic properties of I.E. and used simulation and empirical data to demonstrate I.E. can yield satisfactory results. The parameter estimates and their variances can be derived via principal component regression.

Although it has been proven I.E. has good statistical properties, there are no studies comparing I.E. to other solutions of the identification problem. In the next section, we use computer simulation to compare I.E. to two other solutions (sequential method and autoregressive model). The simulation results can be used as a guideline in deriving the estimation method of adding cohort effect to the L.C. model, as with the L.C.-cohort model proposed by Renshaw and Haberman (2005).

3. Simulation

We choose two solutions for the identification problem, the sequential method and autoregressive model, to compare with I.E. The process of applying the sequential method is similar to that of the approximation method (Lee and Carter 1992) for the L.C. model. The estimation of the sequential method can be done in the order of age-period-cohort (apc) or age-cohort-period (acp), according to the importance of the effects. This kind of estimation can be applied to model M2 in Cairns et al. (2009),

$$\ln (m_{x,t}) = \alpha_x + \beta_x \, \kappa_t + \lambda_x \, \delta_{t\text{-}x} + \varepsilon_{x,t} \tag{3}$$

where the age, period and cohort effects (i.e., α_x, κ_t, $\delta_{t\text{-}x}$) can be estimated using the sequential method.

The period effect, κ_t, in the L.C. model is usually a linear function of time plus an autoregressive effect. Similarly, the autoregressive model of the APC model assumes

that γ_k satisfies $\gamma_k = \varphi \gamma_{k-1} + e_k$, with $e_k \overset{i.i.d}{\sim} N(0, \sigma^2)$, still keeping the constraints

$$\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j = 0 .$$ The autoregressive relationship is applied to the cohort effect

because the adjacent cohorts are likely to share similar properties and the correlation between these cohorts shall be positive, or $\varphi > 0$.

There are 10 five-age groups, five periods and 14 cohorts in the simulation. The age effect is a U-shaped curve, like the usual mortality curve; the period effect is a (decreasing) linear function of time; and the cohort effect looks like an upside-down U-shaped curve. These parameter settings mimic the three effects in Taiwan. The simulation is repeated 100 times, assuming that the variance in (2) satisfies $\varepsilon_{ij} \sim N(0,3)$. Note that computations and simulations in this study were run on an IBM-compatible PC.

We shall check the bias and coverage probability to evaluate the sequential method, autoregressive model and I.E. Figure 2 shows the average bias from 100 simulation runs.
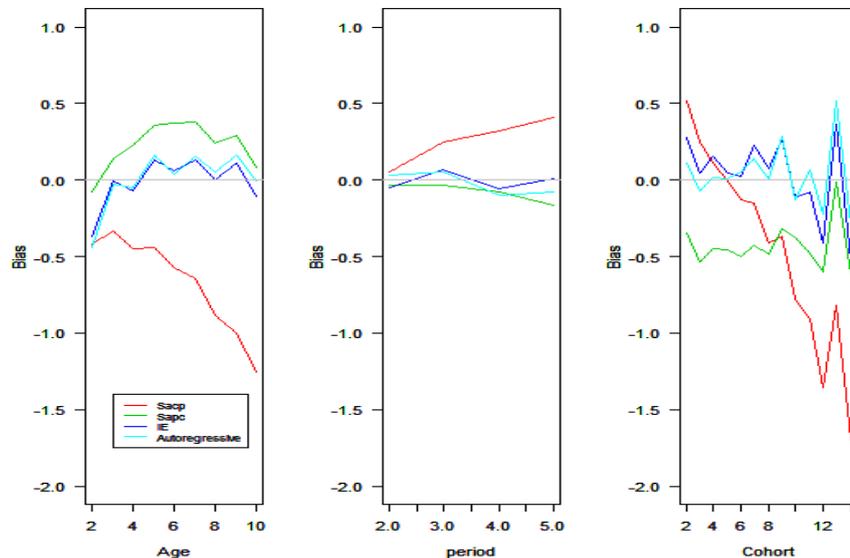

Figure 2. Bias of the parameter estimates for the APC model
Note: Sacp and Sapc are sequential approaches,
with a, p and c indicating age, period and cohort

On average, the autoregressive model and I.E. have the best performances, and the age, period and cohort effects are close to unbiased. The apc sequential method has satisfactory results, but the age effects are slightly overbiased and the cohort effects are underbiased. The acp sequential method is not as good, with the age and cohort effects

underbiased and the period effects overbiased. The results of the sequential method are similar to those in the M2 model, where the order of apc would create more satisfactory results than acp.

To see if the estimation methods produce acceptable results, we also use the coverage probability as a check (figure 3). The confidence coefficient is set at 90 percent. Since there are 100 simulation runs, the estimation results are satisfactory if the numbers of coverage are between 84 and 96. In some occasions of age, period and cohort effects, the acp sequential method has numbers of coverage smaller than expected. The other three methods have satisfactory results, and the autoregressive model and I.E. have numbers of coverage close to the nominal value. We recommend using these two methods, and only I.E. will be used in the rest of this study.
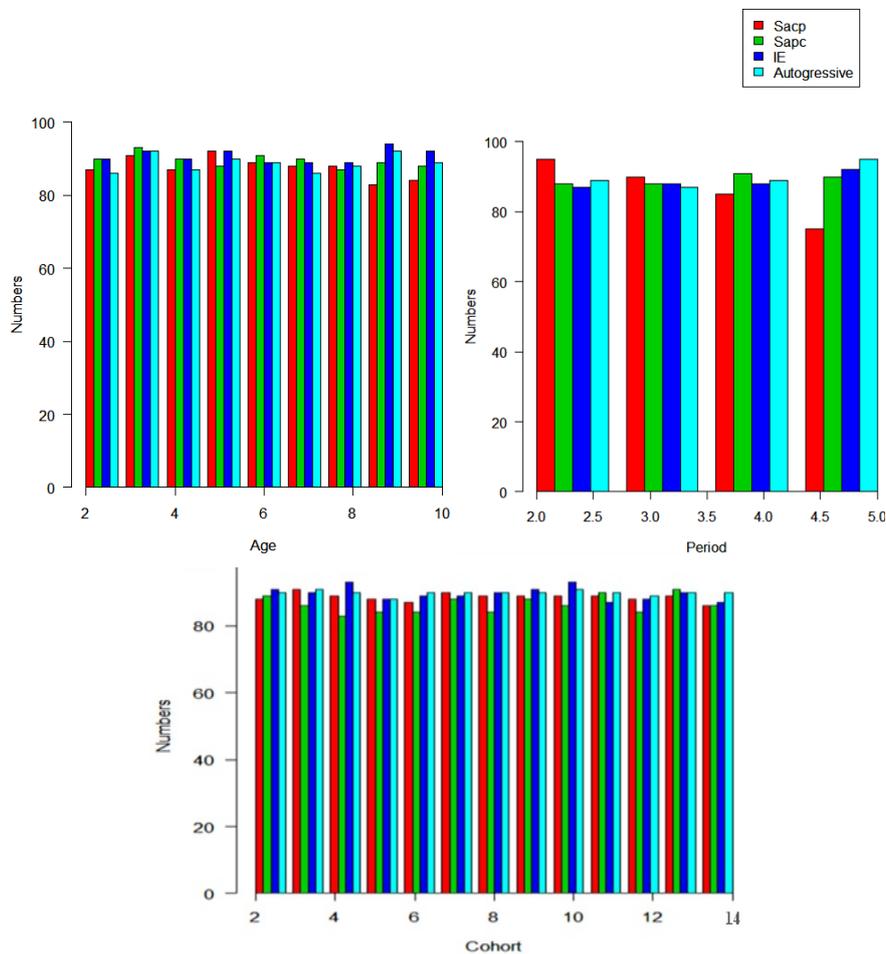

Figure 3. Numbers of coverage for the APC model parameters (100 runs, 90 percent)

4. Empirical Study

It has been recognized that mortality rates are highly correlated to marriage status. According to Trowbridge (1994), the possible explanations for the married having lower mortality include selection at marriage, responsibility, living arrangements and reciprocal care-giving, interactions and social interaction. In recent years, the percentage of married males and females in Taiwan has been decreasing significantly. For example, for the age group 25–29, the percentages of married females dropped from 90 percent in the 1970s to 40 percent in the 2000s. Overall life expectancies, however, continue to increase every year. It would be interesting to see if the married still have lower mortality rates than those of other marriage statuses.

The mortality data by marriage status in this study are from the Ministry of the Interior, Taiwan government. There are four groups of marriage status: single, married, divorced and widowed. Considering the data size, we combined divorced and widowed into one group, divorced/widowed. Table 1 lists the records of populations and deaths since 1973, the first year the data are available by marriage status.

Table 1. Taiwan mortality data by marriage status

Year	Population	Deaths
1973–74	15～50+	N/A
1975–91	15～50+	15～85+
1992–93	15～50+	15～95+
1994–97	15～100+	15～95+
1998–2006	15～100+	15～100+

Because there are detailed mortality records since 1994, we shall first compare the mortality rates between the periods 1994–96 and 2004–06. We use the results of females as a demonstration (figure 4). The mortality rates by marriage status in 1994–96 are marked with characters (m: married, s: single, d: divorced/widowed). The groups of the married and divorced/widowed have lower mortality rates in all age groups from 1994–96 to 2004–06. The single have lower mortality rates for ages smaller than 50 but have obviously higher rates for females age 60 to 80. It seems the mortality improvement does not occur at the older groups of single females in Taiwan.
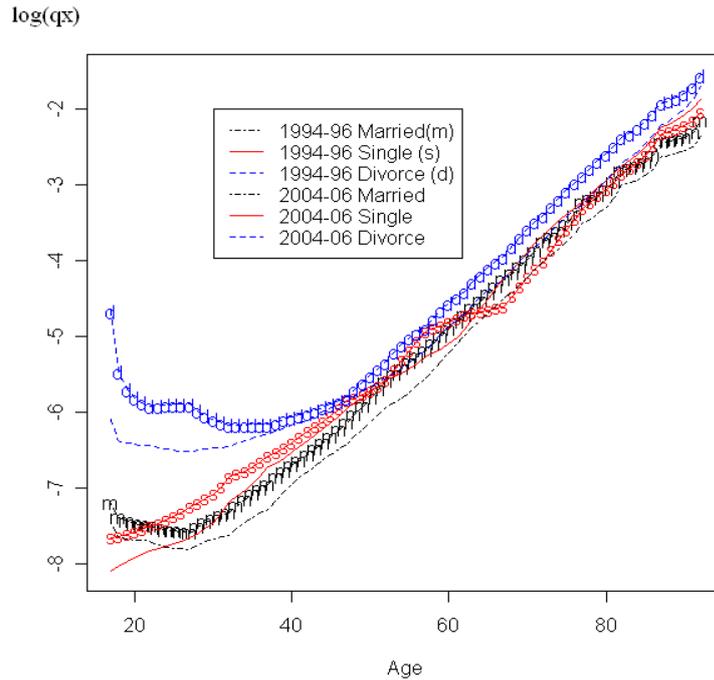
Figure 4. Female mortality rates by marriage status in 1994–96 and 2004–06

To further investigate the differences of mortality improvements by marriage status over 1994–96 and 2004–06, we compute the mortality ratio (figure 5). The mortality rates of the single are treated as the standard group. The ratios of married vs. single and divorced/widowed vs. single both reduced from 1994–96 to 2004–06, except for younger age groups (ages less than 30). The reduction is especially noticeable for ages over 40 at married vs. single, and for ages over 60 at divorced/widowed vs. single. In 2004–06, the married have the smallest mortality rates for all ages and the divorced/widowed have almost the same mortality rates compared to the single. In other words, the single have the smallest mortality improvement from 1994–96 to 2004–06. A further examination of the mortality rates for the married vs. the single over a longer period (1975–2006, figure 6) shows similar results; the married male has larger mortality improvements as well.
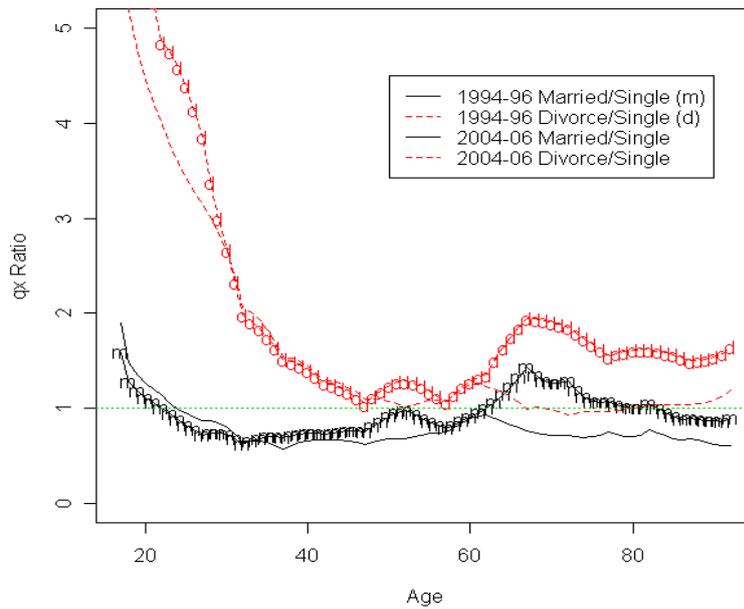
Figure 5. Female mortality ratio in 1994–96 and 2004–06 by marriage status

Figure 7 shows the gains in life expectancies by marriage status over the 10-year period (1994－96 to 2004－06). The divorced/widowed have the largest increments for the female and about the same increments as the married. Single males still have positive gains at the younger age groups, but single females experience decreases in life expectancy for all ages. This indicates that the mortality improvements are not homogeneous by marriage status. Both the married and the divorced/widowed show noticeable mortality improvements from 1994–96 to 2004–06.

In addition to the comparison of mortality rates, we also use the APC model to measure the period and cohort effects of mortality improvement by marriage status. As shown in the previous section, I.E. is chosen as the estimation method since it has better performances. Also, it requires a certain amount of observations to apply the APC model, and thus the data in the format of five-age groups (ages 15–49, seven groups) and five-year periods (year 1975–2006, seven data points) are chosen to guarantee a sufficient data size.
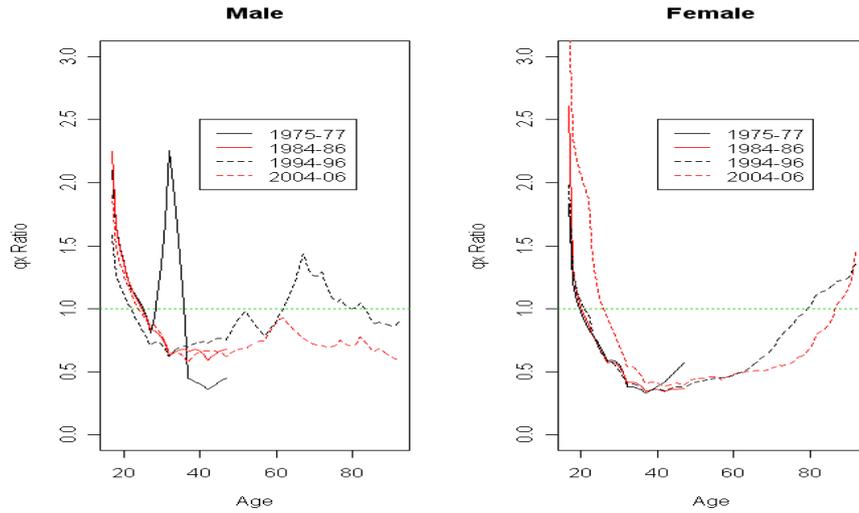
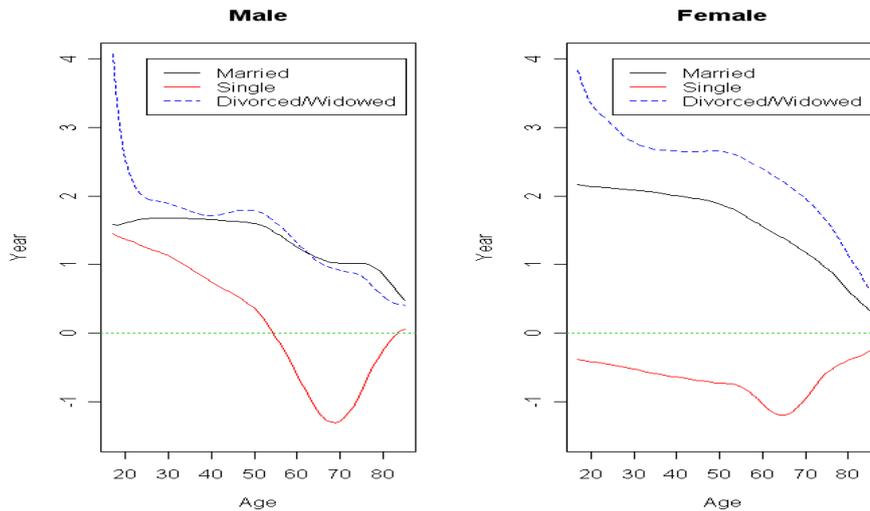Figure 6. Male and female mortality ratios (married vs. single)

Figure 7. Increments of life expectancy by marriage status (1994–96 to 2004–06)

To see the change of a certain age group, we only need to compute the sum of period and cohort effects. The exponential value of this sum is equal to the reduction rate. For example, if the sum is −0.07 for the married, then, on average, the married would have a reduction of $1 - e^{-0.07} \cong 6.76\%$ in mortality rates for all ages every five years. Table 2 lists the averages of period and cohort effects over 1975–2006 for three marital groups. The divorced/widowed have the largest reduction, which are 7.3 percent and 7.7 percent every five years for the male and female, respectively. The married male and single male have very close reduction rates, which are 2.0 percent and 2.5 percent. But the married female have larger reduction rates than the single female.

These results are slightly different than those in figure 4 (1994–96 vs. 2004–06) because more observations (1975–2006) are used and the estimates of "period + cohort" effects would be different.

Table 2. The estimate of period and cohort effects by marriage status

	Married	Single	Divorced/Widowed
Male	−0.0196	−0.0251	−0.0762
Female	−0.0697	−0.0510	−0.0798

5. Applications of Marriage Mortality Rate

The population aging has increased the burden of social insurances in Taiwan. For example, the government raised the premiums for the National Health Insurance and National Pension Insurance in early 2013, and it is likely the premiums will be raised again in the near future. It seems the social insurances cannot (and should not) take care all the people's needs. Unfortunately, about one-third of people in Taiwan do not own any insurance policies, and those who have insurance policies are underinsured (Yue and Huang 2011). To reduce the burden of public finance, Taiwan's Insurance Bureau, Financial Supervisory Commission, in 2007 started to encourage life insurance companies to develop preferred-status life insurance products, hoping the premium discount would raise demand for insurance products.

Two risk factors are usually considered in preferred-status insurance, smoking and overweight (obesity). However, these two factors in Taiwan are rarely listed in the health exam form, and the response to asking if someone smokes often is doubted by the insurers. At least 25 percent of the insured do not honestly disclose if they currently smoke, according to some insurers (Schwaiger 2005). The information if a person is married is easier to collect (it can be found in the population registration system), and thus marital status can be used in pricing preferred-status insurance. As mentioned in the previous section, Taiwan's mortality data of marital status are a record of the whole population (i.e., census data) and do not have the problem of selection bias in most sample data. Of course, the mortality rates calculated in the last section require some adjustments before they can be used to price insurance products.

The influences of marriage have been studied for a long time, and generally it is believed marriage has a role in health protection (Lillard and Panis 1996; Liu and Umberson 2008; Henretta 2007; Manzoli et al. 2007; Williams and Umberson 2004; Van den Berg and Gupta 2007) and the married lives longer than the unmarried, or have lower mortality rates (Bennett et al. 1994; Cheung 2000; Gardner and Oswald 2004; Gove 1973; Ikeda et al. 2007; Kaplan and Kronick 2006; Martikainen et al. 2005; Murphy et al. 2007; Trowbridge 1984; Hu and Goldman 1990). Therefore, it seems that using marital status as a risk factor in preferred-status insurance is a feasible choice.

We will first calculate the differences of age-specific mortality rates between the married and the single, and compare them with those between the nonsmoking and the smoking. Note that the mortality data of smoker status are based on the claim experience of Taiwan insurance companies. We shall use the mortality ratio to check the differences in mortality rates for different marital and smoking statuses (figure 8). In general, the married have smaller mortality rates than the single, and the amounts of reductions are about the same as those of the nonsmoking over the smoking. Males apparently benefit more from marriage and the mortality rates of married males are about 40 percent of those for the single male around ages 30–50. It seems marital status is a feasible choice for designing the preferred-status insurance, especially for those between 30 and 60 years old.
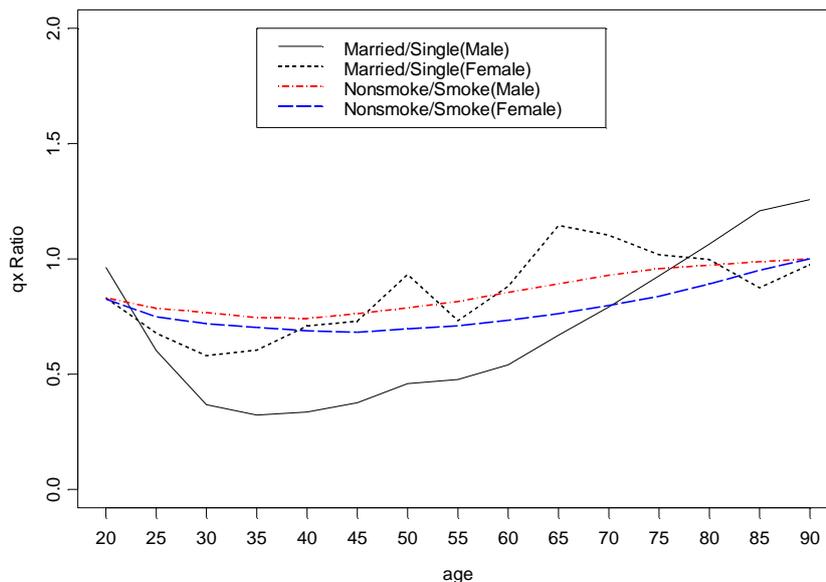

Figure 8. Mortality ratios of marital and smoking status

The impact of using marital status in designing life insurance products can be evaluated by the survival curves. Figure 9 shows the survival curves, staring from age 15, of the single and the married in 2007. The areas between survival curves (shadow) can be treated as the differences in life expectancy between the single and the married, and they match the results of life expectancy in figure 10. It seems that the area between the married and the single are larger for the male case, similar to that seen in figure 8. The advantage of marriage starts at around age 40, and it quickly accumulates. We expect that the premiums of life insurance products for the married would be significantly lower than those of the single.
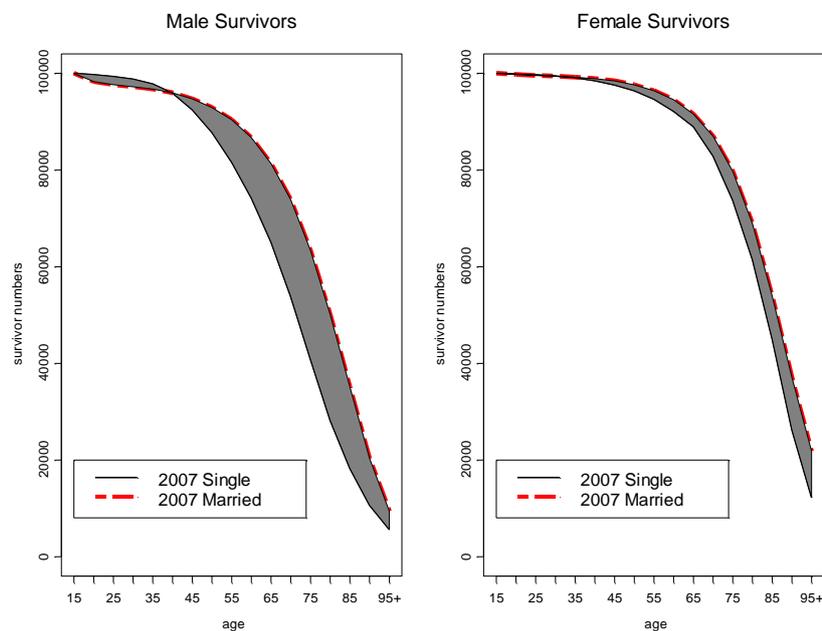

Figure 9. Number of survivors for marital status

We use whole life insurance to demonstrate the premium differences of using marital status for the preferred-status insurance. The married and single life tables are constructed using the Whittaker graduation, where the exposures of raw mortality rates are based on the assumption of 30/360, or 30 days per month and 360 days per year. The graduation process is to calculate the raw mortality rates of five-age groups first. After graduating five-age mortality rates, we use the interpolation formula (cubic spline) and the assumption uniform death distribution to obtain age-specific mortality rates for a single age. Note that the mortality rates of people 80 and older are graduated using the Gompertz law assumption. The estimation of parameters in the Gompertz law is with weighted least squares (Yue 2002).
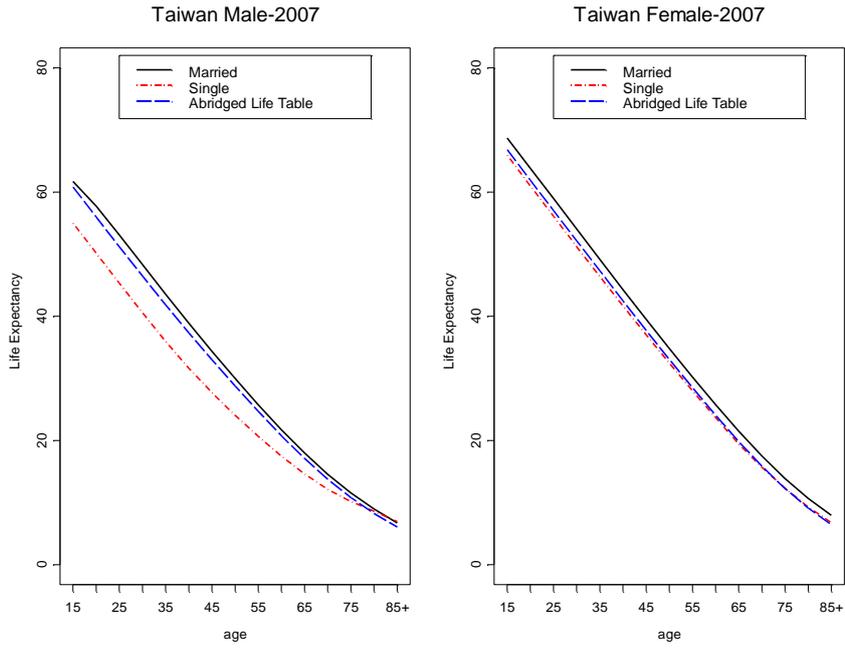
Figure 10. Life expectancy of various marriage statuses

The insured ages of whole life insurance considered are from 30 to 60, with a payment period of 20 years, and the interest rate is 2 percent or 5 percent. Tables 3 and 4 are the annual pure premiums for coverage amount per $1,000. As expected, married males have a larger discount in premiums and the discount is about 40 percent and 35 percent for 5 percent and 2 percent interest rates, respectively. We also compare the married with the whole population, with population data from the Human Mortality Database. This was to avoid marriage discrimination. In other words, we shall treat all insureds equally and only give discounts to those married. Given a 5 percent interest rate, the premium discounts for the married vs. the whole population are about 7 percent~16 percent and 6 percent~8 percent for males and females, respectively. The discounts in the case of 2 percent interest are smaller.

Table 3. Premiums of 20-year-pay whole life (interest rate: 5 percent)

Age	Male				Female			
	Married	Single	Married/ Single	Married/ HMD	Married	Single	Married/ Single	Married/ HMD
30	23.13	37.06	0.62	0.84	15.52	19.17	0.81	0.92
35	28.78	48.19	0.60	0.86	19.40	23.54	0.82	0.93
40	35.48	61.12	0.58	0.88	24.24	28.64	0.85	0.94
45	43.83	75.70	0.58	0.90	30.21	35.00	0.86	0.94
50	55.18	93.68	0.59	0.91	37.90	43.52	0.87	0.94
55	71.31	116.99	0.61	0.93	48.57	55.10	0.88	0.93
60	94.11	151.31	0.62	0.93	64.01	74.34	0.86	0.92

Table 4. Premiums of 20-year-pay whole life (interest rate: 2 percent)

Age	Male				Female			
	Married	Single	Married/ Single	Married/ HMD	Married	Single	Married/ Single	Married/ HMD
30	37.28	49.36	0.76	0.91	30.60	33.81	0.91	0.96
35	42.14	59.25	0.71	0.91	34.10	37.67	0.91	0.96
40	47.89	70.99	0.67	0.91	38.28	42.05	0.91	0.96
45	55.23	84.65	0.65	0.92	43.35	47.43	0.91	0.96
50	65.53	102.04	0.64	0.93	49.96	54.84	0.91	0.96
55	80.76	125.34	0.64	0.94	59.48	65.56	0.91	0.95
60	103.03	159.90	0.64	0.93	73.93	84.03	0.88	0.93

6. Discussions and Conclusion

Mortality improvement is a common phenomenon in many countries and people are expected to live longer. Prolonging life expectancy puts the insurers into a higher risk of insolvency if the longevity risk is underestimated. Because of rapid population aging and the growing need for life annuity products, the insurance industry needs to figure out solutions for dealing with this risk. The dynamic life table (or cohort life table) is one possible solution, and most of these tables rely on mortality models. The Lee-Carter model probably is the most popular mortality model and several modifications of the L.C. model have been proposed. Among all modifications, introducing the "cohort" effect into the model is a common variation of the L.C. model,

in addition to age and period effects. However, because the three effects would create linear dependency, the estimation of the effects needs to be careful.

The age-period-cohort (APC) model has been a frequently used tool in epidemiology and many solutions have been proposed to deal with linear dependency. In this study, we use computer simulations to evaluate three estimation methods for the APC model, including a recently proposed method, intrinsic estimator. Judging from the bias and coverage probability, we found I.E. is a feasible method; in addition, the estimates and their variances can be derived directly. The autoregressive model can be treated as an alternative method for the parameter estimation. We suggest using the approaches of I.E. and autoregressive models to deal with the problem of linear dependency when the cohort effect is introduced in the L.C. model.

The empirical study of Taiwan mortality by marriage status suggests the divorced/widowed have the largest gains in life expectancy. The marrieds' mortality rates at all age groups have declined significantly. On the other hand, the single have the smallest mortality improvement among the three marriage statuses, and the single elderly even experienced an increase in mortality rates. Applying the mortality data by marriage status to the APC model, the results are similar and the divorced/widowed have the largest mortality reductions, but the differences between the married male and single male are smaller. The results of applying the Lee-Carter model are similar and are omitted here.

The married male would receive a 24 percent to 42 percent discount in pure premium above the single male. It seems that married or not can be regarded as a mortality risk factor for the male. An advantage of the research is our use of the complete population record data in Taiwan to analysis the relationship of marriage status and mortality. We have the same results as scholars in many countries. Hence, we suggest the future social security disbursement should also take marriage forecast into consideration and provide insurance companies some references in designing life insurance products. It also indirectly encourages marriage motivation to increase the fertility rate.

Further, we use the computer simulation to evaluate the estimation methods of the APC model. The goal is to study the estimation methods and provide possible suggestions for the L.C. model, if the cohort effect is to be introduced into the mortality model. We found that the I.E. and autoregressive models outperform the sequential approach in the parameter estimation of the APC model. Since adding the cohort effect

in the Lee-Carter model usually adapts an approach similar to the sequential APC model (Renshaw and Haberman 2005; Cairns et al. 2009), there may be room for improving the estimation. Of course, we do not suggest that the APC model is a better model or adding the cohort effect is the only way to modify the L.C. model. For example, Debon et al. (2008) proposed a geostatistical modification to the L.C. model, adding a spatial autocorrelation to the residuals of the L.C. model.

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