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An Actuarial Formulation for the Retirement and Replacement of Fixed Physical Assets

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Abstract

Accurate determination of the appropriate time to retire and replace a physical asset is important for large ensembles of such assets when taken as a whole, and concerns the fate of tens of trillions of dollars in assets, such as airplane engines, vehicles, or any other kind of physical asset susceptible to obsolescence. A decision model to accomplish this is derived using the concept of actuarial present value applied to an economic theory of cost minimization. Since this is a theory coupling investor wealth creation with the long-term optimization of the use of scarce resources, it can also be offered as an economic model relevant to sustainability.

Introduction

In this paper we propose the creation of value through the management of a pervasive kind of uncertainty found throughout industry. This is enterprise risk management at its core, and is a straightforward application of basic actuarial principles to an important and vexing problem found outside of insurance that currently lacks a theoretical foundation.

Thought experiments and inductive reasoning do not have to be mathematically sophisticated, have the rigor of a proof, or follow a worn academic path in order to give articulation to groundbreaking theories. Albert Einstein once used only the Pythagorean Theorem to suggest an unprecedented idea about time and space. The basic actuarial education provides an ample toolkit that, if applied fundamentally in other industries, may result in tremendous social value and in new stem areas of actuarial practice in the field of enterprise risk management.

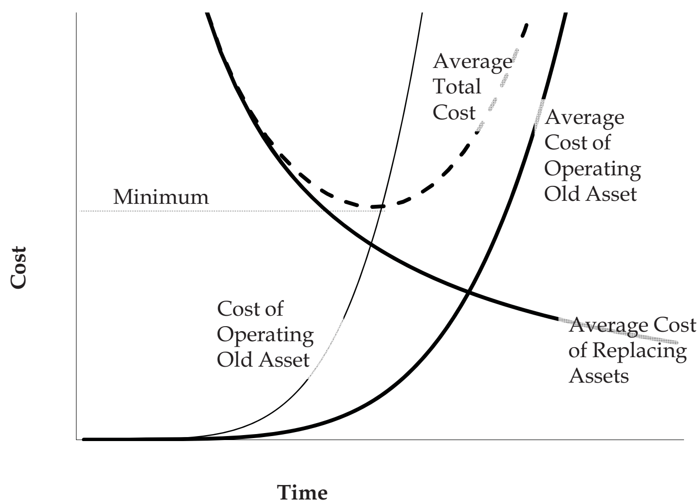
An Economic Model of Physical Asset Mortality

If we consider all the opportunity costs associated with an individual asset that are above and beyond the operating costs of having the latest, newest, like asset, and call this the Cost of Operating Old Asset, these would be costs such as unexpected repairs; potential savings in time, energy and materials inherent in more efficient technology; and the expired tax shelter of depreciation. We would find that these costs are zero by definition when the asset is new, and generally increase with the age of the asset, as is shown in the upward-sloping, thin-lined curve of the following graph.

Then let us define the Average Cost of Operating Old Asset, the bold, upward-sloping curve below, as the cumulative sum of the first curve up to a certain time, divided by that amount of time.

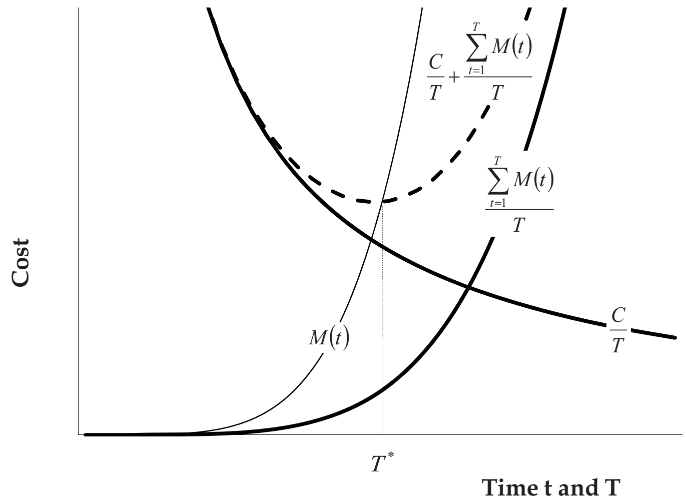
Then we define the Average Cost of Replacing Assets, the bold, downward-sloping curve below, as the cost of a new asset divided by the time between replacements.

The dotted curve with a minimum is labeled as the Average Total Cost, and is defined as the sum of the Average Cost of Operating Old Asset and the Average Cost of Replacing Assets.



This graph shows the relationship of rising Cost of Operating Old Asset, and several other curves related by the algebraic expressions in the following graph.

One can easily see that the dashed curve has a minimum at a certain time. This means that there is only one instant in time to replace an asset at which the long-term consumption of resources (cost) is minimized. The rest of this paper develops a theory for how to determine when that point in time occurs for individual assets that are part of a class or fleet of similar assets.



This graph shows the algebraic relationships of the first graph. The optimization problem is to maintain Average Total Cost at its minimum by correctly timing the replacements of individual assets.

The economic model above depicts the principle of efficient timing for the replacement of fixed assets. But, it does not provide guidance for when to replace individual assets. First of all, there is no such thing as an average asset. An individual asset's life span, or the time it is kept by one owner, can vary from the mean. Assets of the same class (locomotives, wind turbines, vehicles, etc.) do not all get replaced punctually at some average life span.

A Decision Model Using Asset Mortality Data

To render the above economic model useful for decision-making for individual assets, we first take advantage of the following equality that always is true at the minimum of the Average Total Cost:

$$M(T^*) = \frac{C}{T^*} + \frac{\sum_{t=1}^{T^*} M(t)}{T^*}$$

Where:

C = replacement cost, less salvage or trade-in value.

t = asset's age.

*T** = the optimum average inter-replacement time, at the minimum of Average Total Cost.

M(t) = Cost of Operating Old Asset at age, *t*.

M(T)* = Cost of Operating Old Asset at optimum average inter-replacement time, *T**.

This equality simply means that *M(t)* always intersects the Average Total Cost at its minimum.

But we are really concerned with timing a decision with the rising Cost of Operating Old Asset, so anytime *M(t)* is greater than (or equal to) the right side of the equation is when to replace the asset. When the following inequality is satisfied, we must replace the asset in order to minimize Average Total Costs:

$$M(t) \geq \frac{C}{T^*} + \frac{\sum_{t=1}^{T^*} M(t)}{T^*}$$

The above equation does not yet take into account the time value of money, which must be done for any costs distributed over time, such as these. Minimizing the present value of future costs, rather than the Average Total Costs as defined in the graphs, also translates more directly into shareholder wealth creation. A proof of the adjustment to this equation to include the time value of money has been derived in Wendling (2011), and to save time, we will not repeat the entire derivation, but simply show that result here:

$$M(t) \geq iC + \frac{C}{s_{\overline{T^*|}}} + \frac{(1+i)^{T^*} \sum_{t=1}^{T^*} M(t) v^t}{s_{\overline{T^*|}}}$$

Where:

i = owner's real cost of capital

and

$$v^t = \frac{1}{(1+i)^t}$$

$$s_{\overline{T|}} = \frac{(1+i)^T - 1}{i}$$

Then, since the time to replacement is random, we take the expected value of the right side. We want to do this since we are interested in minimizing the expected value of the present value (the actuarial present value) of all future costs associated with each individual asset, and its successors. The phenomenon of *M(t)* intersecting the minimum is also preserved for the actuarial present value as it was for the Average Total Costs in the previous graphs:

$$M(t) \geq E\left[iC + \frac{C}{s_{\overline{T^*|}}} + \frac{(1+i)^{T^*} \sum_{t=1}^{T^*} M(t) v^t}{s_{\overline{T^*|}}} \right]$$

$$M(t) \geq iC + E\left[\frac{C}{s_{\overline{T^*|}}} + \frac{(1+i)^{T^*} \sum_{t=1}^{T^*} M(t) v^t}{s_{\overline{T^*|}}} \right]$$

Approximation 1:

The right-most term inside the brackets is then assumed to equal zero.

$$M(t) \geq iC + E\left[\frac{C}{s_{\overline{T^*|}}} + 0 \right]$$

We do this only to get rid of that term, since it cannot be easily evaluated using objective data, and since its value is very small compared to the other terms in the equation. *M(t)* does not follow a deterministic process as depicted in the above graphs; rather it follows a stochastic process, and the eliminated term would require considerably more historic data to evaluate than can be reasonably obtained in practice. A possible range of values of this term can be obtained through scenario testing, and since the values are usually very small compared to the other terms, this approximation slightly understates the right side of the inequality.

Then dividing both sides by C:

$$\frac{M(t)}{C} \geq i + E\left[\frac{1}{s_{\overline{T|}}} \right]$$

Approximation 2:

$$E\left[\frac{1}{s_{\overline{T|}}} \right] = \sum_{i=1}^{N} \left[\frac{1}{s_{\overline{T_i|}}} \right] \Big/ N$$

Where T_1, T_2,…T_N, are empirical ages at the time of replacement (asset mortality data) from a homogeneous class of assets.

This assumes that the expected value of the above term is simply equal to the average of the term evaluated over N empirical observations of time to replacement of similar assets. Although a close approximation, it is really the inter-replacement time at each future replacement which is random, not an identical value of inter-replacement time for all future replacements. This approximation tends to slightly overstate the right side of the inequality.

The inequality now looks like this:

$$\frac{M(t)}{C} \geq i + \sum_{i=1}^{N} \left[\frac{1}{s_{\overline{T_i|}}} \right] \Big/ N$$

Considering the two approximations we have made so far:

Approximation 1 (slightly understates the right side)
\+ Approximation 2 (slightly overstates the right side)

= β

The errors due to Approximations 1 and 2 are small, and occur in opposite directions. They approximately cancel each other out, but we will call their sum β, and include it in the model. This value is a small error term that can be evaluated through simulation and judgment. The appropriateness and sensitivity of these two approximations will require further analysis from the perspectives of general research and specific application.

The model now looks like this, and can be broken down into these parts:

$$\overbrace{\overbrace{\frac{M(t)}{C}}^{\text{Individual Asset}} \geq i + \overbrace{\sum_{i=1}^{N} \left[\frac{1}{s_{\overline{T_i|}}} \right] \Big/ N}^{\text{Entire Class}}}^{\text{Objective (Empirical)}} + \overbrace{\overbrace{\beta}^{\text{Error}}}^{\text{Subjective}}$$

Where T_1, T_2, T_3,…T_N, are empirical ages at the time of replacement of assets from a class (asset mortality data), such as locomotives, gas turbines, or, even better, a homogeneous ensemble of similar vehicles in a fleet owned by the same enterprise. As soon as this inequality is satisfied for an individual asset, one must replace the asset in order to minimize costs for the entire class of assets.

It is important to note here that, since the sampled empirical ages affect the calculation of the threshold value, they also affect the values of newly generated empirical ages. Asset mortality data generated by the model is also fed back into the model, since it newly characterizes the mortality of the class. Therefore, this model is iterative. The model will converge to a steady state, but the selection of seed values for T_1, T_2, T_3,…T_N should be done with the goal of shortening the calibration period.

Model Interpretation

Because of its iterative nature using feedback, this model is a process control algorithm, and the process being controlled is the mortality within entire ensembles, fleets, or homogeneous classes of physical assets. Decisions on individual assets depend on aggregate information (asset mortality data) from the fleet to optimize the performance of the entire fleet. One can imagine that the random life distribution defining mortality of a class of assets will not remain stationary, but will migrate in shape and form over time as distributional changes caused by environmental factors, such as advancing technology or energy costs, work their way through an asset population of mixed ages. As these changes occur the model follows them with the singular goal of minimizing the actuarial present value of all future costs.

The structure of this model shows that even if the accuracy on the right side of the inequality is not perfect, the left side always provides a way to prioritize the assets for replacement in order to achieve the minimum of Average Total Cost.

Most of this model is based on objective data that can be easily obtained in practice. $M(t)$ is simply the engineering data that ordinarily goes into this type of analysis, such as energy costs, repair costs, expired depreciation, potential loss costs, potential downtime costs, etc. The model also separates the data obtained from the individual asset from the asset mortality data obtained from the entire class of assets. There is a clear role for the art of classification of assets into homogeneous groups with similar characteristics that affect their mortality.

It is beyond the scope of this paper to describe what happens when the optimized asset replacement timing indicated by this model is not met in practice; however, simple simulation experiments (Wendling, 2012) show that the systematic delay of asset replacements past an optimum time can create a material negative impact on the net worth of a company that owns a large portfolio of physical assets. Without a theoretical foundation to define exactly when to replace assets, industries are probably either spending too much on replacements or tolerating inefficient operating costs of obsolete assets for too long.

Conclusion

This decision model transcends short-term earnings and cash flow management, and allows minimization of the consumption of time, energy and materials (measured as cost) over the long term. This is arguably a major goal of sustainability. This theory is a risk management concept that combines a variety of unpredictable factors (contributing to an asset's obsolescence) to take advantage of repeatable historical patterns, much in the way that insurance works. It is an example of the actuarial toolkit applied in a new setting to manage risk and potentially recover vast operational efficiencies in the industrial management of physical assets.

Bibliography

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