Multiple Objective Asset Allocation for Retirees Using Simulation

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The asset portfolios of retirees' serve many purposes. Retirees may need them to provide stable cash flow to cover living costs. They may gradually sell their assets when social retirement benefits and asset cash flows are not enough to meet financial needs such as unexpected medical costs. They may also want to leave a certain amount of their estate to their children. Multiple objectives with different levels of importance lead to a complex asset allocation problem for retirees.

Multiple Objectives

Depending on the retiree's specific situation, a variety of objectives are expected for asset allocation.

1. **Current income.** With limited income after retirement, a retiree is likely to draw down his/her asset to pay for living costs. Assets that can generate stable and regular cash flow are more favorable.
2. **Liquidity.** A higher level of liquidity is needed for retirees compared to workers. A reduced amount of income leads to a higher probability that assets need to be sold to meet liquidity requirements. Liquid assets with less bid-ask spread are more favorable for retirees.
3. **Purchasing power.** Retirees are concerned with maintaining their living standard in case of hyperinflation. Assets that grow with inflation are preferred.
4. **Longevity risk.** Retirees are also concerned they may outlive their assets. Annuity products that protect retirees from longevity risk need be included in the asset allocation plan.
5. **Wealth growth.** A higher return is always better; however, it may not be the top priority.
6. **Estate.** Some retirees may want to leave an estate for their heirs. This also needs to be considered in the asset allocation plan depending on the importance of this objective to the retiree.
7. **Time horizon.** The asset allocation plan for a new retiree would be very different from that for a retiree after 15 years of retirement.
8. **Tax minimization.** Retirees would also want to take advantage of tax-efficient assets to reduce both estate tax and investment income tax.
9. **Relative importance of multiple objectives.** The final asset allocation plan needs to find an appropriate balance among multiple objectives according to their relative importance to the investor.

Current Methods

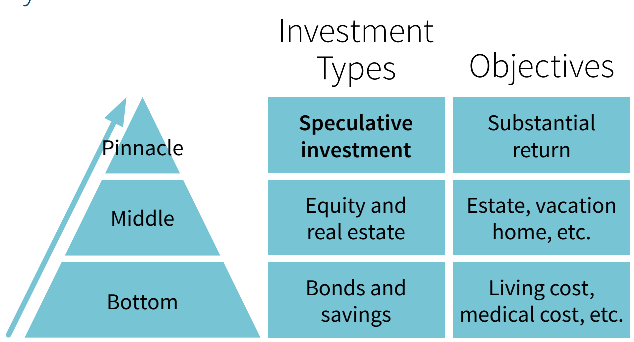
Existing asset allocation methods normally focus on a subset of the multiple objectives of retirees in an approximate way. Age-based asset allocation uses this rule of thumb to determine the allocation between equity and fixed income securities: (100 – age) percent of assets is suggested to be invested in equity. This can only provide high level guidance to limit the risk without recognizing specific situations of each retiree. Many other objectives are neglected by this method.

Asset allocation based on modern portfolio theory such as mean-variance optimization has the goal of maximizing the expected return given a specified level of risk. The risk level is determined by the investor's willingness and ability to take risk. In theory, this single objective decision-making method can lead to the maximal expected economic value for investors. However, some objectives of retirees need to be translated into a risk-aversion score and the translation could be quite ambiguous and subjective. Other objectives such as current income and sufficient liquidity conflict with the goal return maximization and are hard to be incorporated into the model. The optimal solution is also very sensitive to assumptions of the expected return and volatility of each asset class and correlation between asset classes.

Contrary to asset allocation based on modern portfolio theory, asset allocation based on the risk pyramid sets the allocation plan by meeting individual objectives sequentially. It starts from the most important objective such as paying basic living costs and uses the most conservative assets such as bank savings and government bonds to achieve the objective. It then goes up to less important objectives such as estate or vacation and uses riskier assets to support them. Retirees are willing to accept uncertainty for a higher expected return for less critical objectives.

Figure 1 shows the risk pyramid including objectives and corresponding asset classes. The pyramid structure does not consider all the objectives together, nor does it consider the diversification between asset classes. The resulting asset allocation plan is not economically optimal.

Figure 1 Asset Allocation Based on Risk Pyramid



The asset allocation method based on the analytic hierarchy process (AHP)[1] explicitly considers the multiple objectives and their priorities when choosing an allocation plan. Investors need to provide pairwise assessment of objectives regarding their importance. Asset allocation plans are ranked by the weighted performance for all the objectives where the weight is based on the priorities of the objectives. However, the resulting asset allocation is often subjective and not economically optimal.

None of the current methods discussed above has a clear way to find the optimal solution when considering all the objectives together. A more direct method is needed to make sure all objectives are incorporated in the optimization process according to their relative importance.

Simulation-Based Multiple Objective Asset Allocation

The simulation-based multiple objective asset allocation method objectively assesses each allocation plan against multiple objectives in a consistent way and provides a more holistic picture of possible outcomes. This information is critical for finding the optimal allocation plan. The optimization is based on the weighted performance relative to multiple objectives. The implementation follows several steps:

1. With a specified asset allocation plan, the retiree's future income and spending under different economic, mortality and morbidity scenarios are projected. Under each scenario, the projected result is checked against each objective in terms of whether the objective can be met and how well it is met. The weighted performance is used to measure the aggregate performance regarding the objectives. The weight is the relative importance of each objective. The return measure is the average of the weighted performance in each scenario. The risk measure could be the volatility, value at risk (VaR) or tail value at risk (TVaR) of weighted performance.
2. Repeat the exercise for all possible asset allocation plans.
3. Construct the efficient frontier using the average weighted performance as the return measure and the volatility/VaR/TVaR as the risk measure.
4. Choose the portfolio on the efficient frontier according to the investor's risk tolerance.

Figure 2 illustrates the process of simulation-based multiple objective asset allocation.

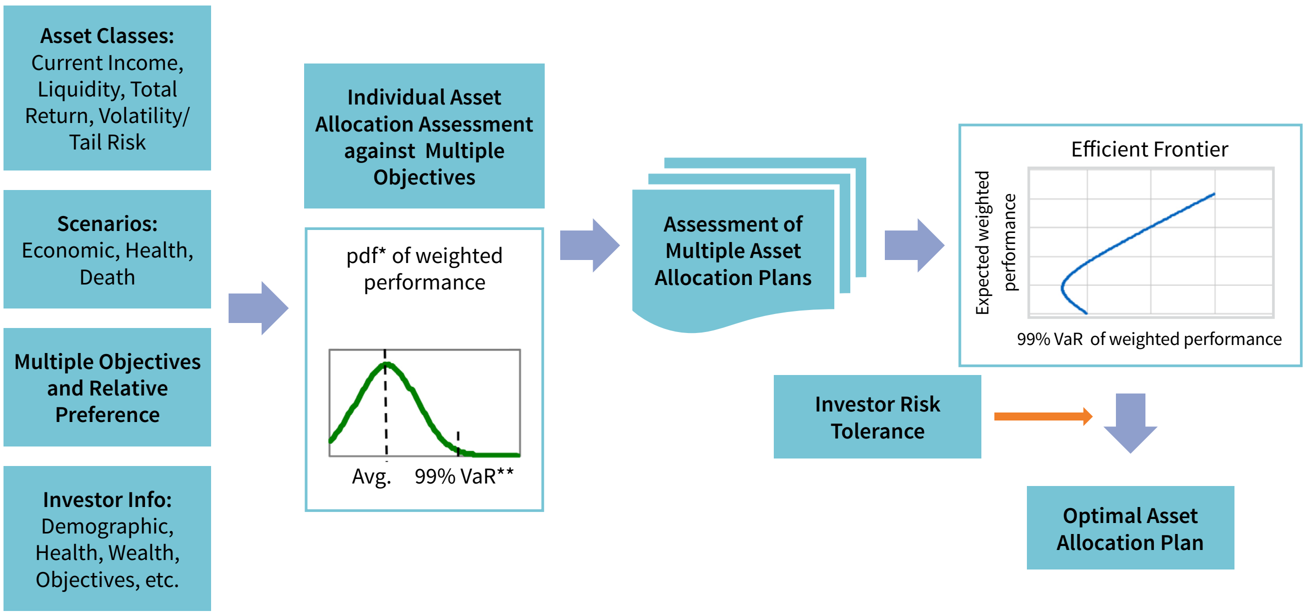
Example

A simplified example is illustrated here to show the process of simulation-based multiple objective asset allocation. Assume a male retiree at age 65 is considering his asset allocation plan. He has five objectives:

1. High current income no less than 2 percent of the asset value (CI)
2. Maintain the purchase power of the portfolio (PP)
3. Maintain sufficient liquidity to cover living costs and unexpected medical costs (AL)
4. Minimize longevity risk (LR)
5. Leave an estate of $100,000 for his children (ES)

1 Steven Le, "Asset Allocation: An Application of the Analytic Hierarchy Process," *Journal of Business & Economics Research 6,* no.9 (2008).

Figure 2 Simulation-Based Multiple Objective Asset Allocation Process



* pdf: probability density function
** 99% VaR is one of many possible risk measures and is for illustration only

Table 1 shows the retiree's relative preference of the five objectives.

The suggested scale for AHP by Hobbs and Meier (2000)[2] is used. For example, CI is moderately more important than PP. The reciprocal means that the relationship of the two objectives is switched.

a. 1: If the two attributes are judged to be equally important
b. 3: If attribute I is judged to be slightly more important than attribute II
c. 5: If attribute I is judged to be moderately more important than attribute II
d. 7: If attribute I is judged to be strongly more important than attribute II
e. 9: If attribute I is judged to be extremely more important than attribute II
f. 2,4,6,8: If intermediate values between two adjacent judgments are needed

Table 1 Relative Preference of Retirement Objectives

	CI	PP	AL	LR	ES
CI	1	5	3	1	7
PP	1/5	1	3	1/2	5
AL	1/3	1/3	1	3	5
LR	1	2	1/3	1	7
ES	1/7	1/5	1/5	1/7	1

Based on the preference matrix, the weight assigned to each objective can be calculated by dividing each entry by the sum of the column and then taking the average of the row, as in the AHP (see Table 2).

Each objective has its own measure of performance. The measurement could be performed for the entire time horizon to get the average performance or the time period with the worst performance. The measures need to be normalized before calculating the weighted

2 Benjamin F. Hobbs and Peter Meier, *Energy Decisions and the Environment: A Guide to the Use of Multicriteria Methods* (Boston: Kluwer Academic Publishers, 2000).

Table 2 Weight of Retirement Objectives

	CI	PP	AL	LR	ES
Weight	36%	18%	21%	22%	3%

Table 3 Performance Measure of Retirement Objectives

	CI	PP	AL	LR	ES
Type of measure	Average	Average	Worst	Average	Average
Performance measurement	1.5	2	0.9	0.75	−0.8

Table 4 Asset Class Profile

Asset Class	Expected Return	Risk	Liquidity	Current Income
Government bond	Low	Low	High	High
Stock index	High	High	Low	Low
Short-term savings	Very low	Very Low	High	Medium
Real estate	High	High	Very low	Very low
Life annuity	Medium	Low	Low	High

performance. In this example, normalization is omitted for simplicity.

1. **CI:** (current income rate – 2%)/2%. Current income rate is the weighted average of savings interest rate, bond coupon rate, stock dividend rate and real estate rental income rate.
2. **PP:** (investment return – inflation rate)/2%
3. **AL:** (AL – living cost – unexpected medical cost)/(living cost + unexpected medical cost)
4. **LR:** (age at which assets are outlived – age @ life expectancy)/(99^{th} percentile of the age – age @ life expectancy)
5. **ES:** (estate @ life expectancy – 100,000)/100,000

Assume under one scenario, we get the performances against the five objectives shown in Table 3.

The weighted performance using the weights derived from the preference matrix is 1.22.

The retiree only considers four asset classes and one life annuity product. Assets are assumed infinitely divisible for simplicity although constraints can be added according to the reality. See Table 4.

The retiree's financial information is summarized in Table 5.

Table 5 Example: Financial Information

Net invested asset	$200,000
Real estate (residence)	$300,000
Retirement income (social program)	$2,000/month
Current living cost	$3,500/month
Contingent medical cost	$100,000

Stochastic scenarios including interest rate, equity return, inflation rate and mortality rate are used to generate the distribution of the aggregate performance. See Table 6.

By testing multiple asset allocation plans, the relationship between the return measure (average weighted performance) and the risk measure (average weighted performance – worst 1% performance) can be established. See Figure 3.

Table 6 Assumptions of Stochastic Scenarios

Stochastic Scenarios Assumption						
Insurance Assumption						
Mortality (MR)	2008 Valuation Basic Tables (VBT) with 20% volatility					
Economic Assumption*						
Initial yield curve	**Term**			**Risk Free Rate (%)**		
	1			0.30		
	2			0.64		
	3			1.05		
	4			1.54		
	5			2.03		
	7			2.74		
	10			3.42		
	30			4.35		
Interest rate model (IR)	One-factor Hull-White model (σ = 10%, α = 0.05)					
Equity model (EQ)	Log-normal model (Risk premium= 4%, σ = 25%)					
Real estate model (RE)	Log-normal model (μ = 4%, σ = 25%)					
Inflation rate model (IN)	Log-normal model (μ = 2.3%, σ = 13%)					
Correlation among variables		**MR**	**IR**	**EQ**	**IN**	**RE**
	MR	1	0	0	0	0
	IR	0	1	0.1	0.6	0.05
	EQ	0	0.1	1	−0.1	0.7
	IN	0	0.6	−0.1	1	0.2
	RE	0	0.05	0.7	0.2	1

** The economic assumptions used are for illustration purpose. They are based on the same framework used in Kailan Shang et al., "Pension Plan Embedded Option Valuation," Society of Actuaries report (2013). Details are not listed here, as they are not the focus of this article.*

A weighted performance of zero means that the minimum requirement is met. The efficiency of an asset plan can be measured using the risk measure divided by the (return measure – 0). The investor needs to have a minimum expected weighted performance of 0.5 with less than a 1 percent chance of having a performance less than –0.1. Based on this risk tolerance, we can find the optimal asset allocation plan with the highest Sharpe ratio. See Table 7.

Implementation Challenges

Assessing the relative preference of multiple objectives is a difficult task and could be time consuming. Normally, pair comparison is used to help investors quickly choose the more important objective of the two. But the number of pairs an investor needs to compare could be large. For example, nine objectives would need 36 pairs[3] of comparisons to finish assessment. In addition, the comparisons may

Figure 3 Performance (Return vs. Risk)

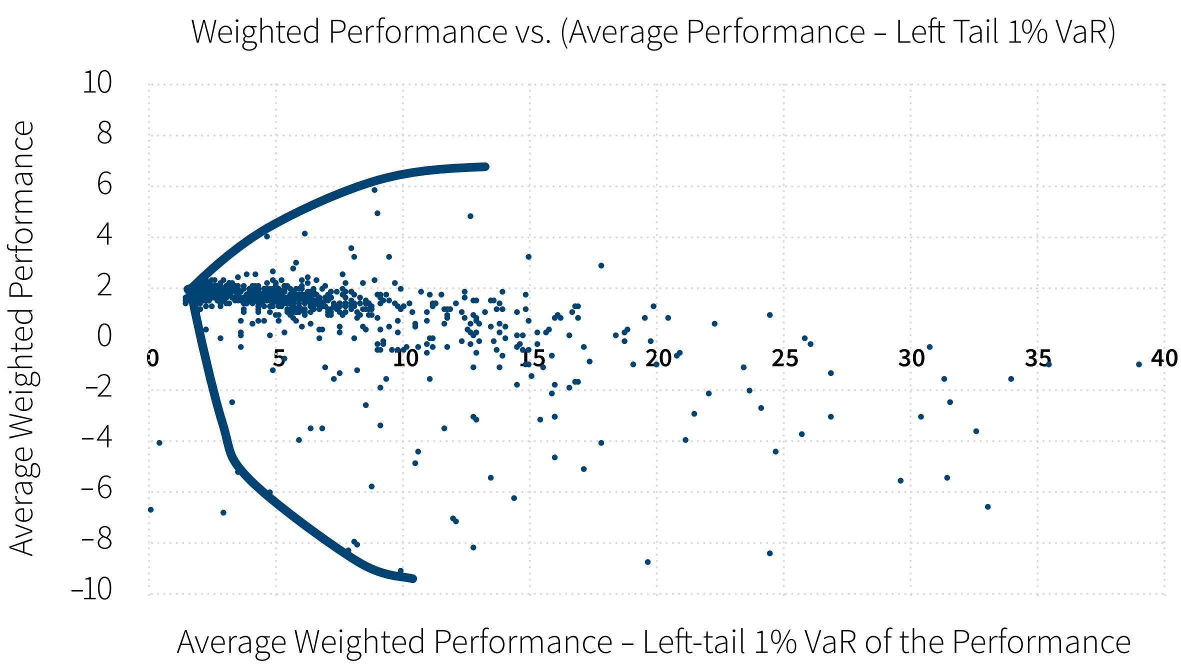


Table 7 Optimal Asset Allocation Plan

Savings	Bond	Equity	Real Estate	Annuity (Monthly Payment with 2% Annual Increase)	Weighted Performance	Risk	Sharpe Ratio
10%	90%	0%	0%	500	1.94	1.96	1.01

3 $C_9^2 = \frac{9\times8}{2} = 36$

be inconsistent. An investor may prefer objective A to B, prefer objective B to C and prefer objective C to A. Consistency of the matrix needs to be checked, as suggested by Saaty (1980, 1994).[4] Inconsistent preference inputs need to be communicated to the investor and adjusted.

For an integrated analysis using scenarios including economic and insurance risk factors, the correlation among risk variables need to be reflected. For example, an unexpected rising inflation could cause lower stock returns due to the rising input cost. Inflation may cause lower purchasing power and also higher medical costs. This would require complicated modeling using correlation matrices, copula or structured models. In addition, the result could be very sensitive to the correlation assumption. Stress testing is needed to test the robustness of the resulting optimal asset allocation plan.

Protection types of insurance products are also included in the financial planning. Unlike assets that return and risk depending on investment performance, the benefit of insurance products depend on insurance events such as death and sickness. Traditional approaches cannot be used for optimization that considers insurance products. A simulation-based multiple objective approach can consider assets and insurance products together using cash flow projection, but it significantly increases the number of asset allocation plans that need to be tested.

Time horizon is an important factor in asset allocation planning. The asset allocation plan needs to be reviewed regularly to reflect a changing time horizon.

Conclusion

A simulation-based multiple objective approach can systematically assess asset allocation plans against multiple objectives and use the aggregate performance to find the optimal plan. It is a flexible and extensible framework that can incorporate different objectives, asset classes and insurance products.

By projecting the cash flows over the time horizon, the new approach can easily measure the performance. At the same time, it requires more inputs and advanced modeling.

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4 Thomas L. Saaty, *The Analytic Hierarchy Process: Planning, Setting Priority, Resource Allocation*, 2nd ed. (Pittsburgh, Pa.: RWS Publications, 1980); Thomas L. Saaty, "How to Make a Decision: The Analytical Hierarchy Process," Interfaces 24, no. 6 (1994): 19–43.