

1. Solution: E  
Option E is equal to  $a_{\overline{n}|}(1+i)$
  
2. Solution: C  
Annual interest payment =  $10,000(0.09) = 900$   
Annual sinking fund deposit  $\Rightarrow$  level payment ( $s_{\overline{10}|0.08}$ ) = 10,000  

$$\text{payment} = \frac{10,000}{s_{\overline{10}|0.08}} = \frac{10,000}{14.4866} = 690.29$$
Total annual payment =  $900 + 690.29 = 1590.29$ .  
Total of all payments =  $1590.29(10) = 15,903$ .
  
3. Solution: B  
Current value =  $\frac{100}{1.20} + \frac{100}{(1.20)^2} + \frac{1100}{(1.20)^3} = 789.35$   

$$\text{Duration} = \frac{1 \cdot \left[ \frac{100}{1.20} \right] + 2 \cdot \left[ \frac{100}{(1.20)^2} \right] + 3 \cdot \left[ \frac{1100}{(1.20)^3} \right]}{789.35} = 2.70$$
  
4. Solution: A  
Present value of Seth's payments =  $X \cdot a_{\overline{n}|}$   
Present value of Susan's payments =  $X \cdot v^n \cdot a_{\overline{m}|}$   
Difference =  $X(a_{\overline{n}|} - v^n \cdot a_{\overline{m}|})$

5. Solution: B

First Bond:

Zero Coupon bond with

$P = 624.60$ ,  $F = 1000$ ,  $r = 0\%$ ,  $Fr = 0$ ,  $n = 12$ ,  $C = 1000$ , find  $i$ .

Using  $P = Fra_{\overline{n}|} + Cv^n$ .  $624.60 = 1000v^{12}$ .  $(624.60/1000) = v^{12}$ .

$$v = \frac{1}{1+i} = (0.6246)^{1/12} = 0.961539. \quad i = 0.04.$$

Now use  $i = 0.04$  to determine the value of the second bond.

Since this is an annual effective interest rate and the bond has semi-annual coupon payment, the interest rate should be converted to a semi-annual effective rate:

$$(1.04)^{1/2} - 1 = 1.98\% \text{ per 6 month.}$$

Using your financial calculator, the price of the bond is calculated as follows:

number of periods = 20

interest rate = 1.98%

payment = 30

future value = 1000

Compute the present value  $\rightarrow X = 1167.04$

6. Solution: D

The duration of the portfolio is the average of the durations weighted by the purchase price.

$$\frac{(21.46 * 980) + (12.35 * 1015) + (16.67 * 1000)}{980 + 1015 + 1000} = \frac{50236.05}{2996} = 16.77$$

7. Solution: A

$$100 + 200v + 100v^2 = 364.46$$

$$200v + 100v^2 = 264.46$$

$$100v^2 + 200v - 264.46 = 0$$

$$v = \frac{-200 \pm \sqrt{200^2 - 4(100)(-264.46)}}{2(100)} = \frac{-200 \pm 381.82}{200} = 0.9091 = \frac{1}{1+i}$$

$$i = 0.10$$

8. Solution: C

The balance after 10 payments is

$$300a_{\overline{15}|0.08} = 300(8.5595) = 2567.85$$

The balance after an additional payment of 1000 is

$$2567.85 - 1000 = 1567.85$$

The new payment over 10 years is

$$\frac{1567.85}{a_{\overline{10}|0.08}} = \frac{1567.85}{6.7101} = 233.66$$

9. Solution: D

$$A = P a_{\overline{n}|} + Q \frac{a_{\overline{n}|} - nv^n}{i}$$

$$n = 50, \quad i = 9\%, \quad P = 100, \quad Q = 1, \quad a_{\overline{n}|} = 10.9617$$

$$A = (100)(10.9617) + (1) \left[ \frac{10.9617 - 50v^{50}}{0.09} \right] = 1096.17 + 114.32 = 1210$$

10. Solution: C\*

$$i = \frac{(1.095)^2}{1.085} - 1 = 10.5\%$$

\*Due to some ambiguity in the wording of the question, answer E was also scored as correct.

11. Solution: C

The two calculated prices represent the ends of the spectrum of possible prices based on when the bond is called. If the bond is held to maturity and the investor pays anything more than 897, then she will not earn her desired yield rate. The price of 897 also guarantees that she will earn her desired rate if the bond is called any time before maturity. Thus 897 is the price she pays.

When the bond is called for 1050 at the end of 20 years, the yield rate earned is calculated using a financial calculator:

Present value = 897, future value = 1050, payment = 80, and number of periods = 20.

$i = 9.24\%$ .

12. Solution: B

I. Counter example: perpetuity-due or non-level payments

II. True

III. Counter example: non-level payments

13. Solution: D

$$j = \frac{i}{2}$$

$$1000(1+j)^4 + 1500(1+j)^2 = 2600$$

$$X = (1+j)^2$$

$$X^2 + 1.5X - 2.6 = 0$$

$$X = 1.02834$$

$$j = 0.01407$$

$$i = 2.81\%$$

14. Solution: E

$$X = 20 \cdot a_{\overline{10}|0.06} + v_{0.06}^{10} (Da)_{\overline{19}|0.06} = 147.20 + (0.5584)130.70 = 220$$

15. Solution: D

$F_1, F_2$  : face amounts of 1- and 2-year bonds.

At the end of the second year,

$$F_2 + 0.06F_2 = 10,000$$

$$F_2 = 9433.96$$

At the end of the first year,

$$(0.06)(9433.96) + F_1 + 0.04F_1 = 10,000$$

$$F_1 = 9071.12$$

The price of the 1-year bond is  $9071.12 \left( \frac{1.04}{1.05} \right) = 8984.73$

The price of the 2-year bond is  $9433.96 \left( \frac{0.06}{1.05} + \frac{1.06}{(1.05)^2} \right) = 9609.38$

The total price is  $8984.73 + 9609.38 = 18,594.11$

16. Solution: A

$$i = \frac{260}{1000 + 1000 \left( 1 - \frac{1}{3} \right) - 200 \left( 1 - \frac{1}{2} \right) - 500 \left( 1 - \frac{2}{3} \right)} = \frac{260}{1400} = 18.57\%$$

17. Solution: B

$$46530 = \frac{200}{i} + \frac{50}{i^2}$$

$$46530i^2 = 200i + 50$$

$$4653i^2 - 20i - 5 = 0$$

$$i = \frac{20 \pm \sqrt{(-20)^2 - 4(4653)(-5)}}{2(4653)} = \frac{20 \pm 305.71}{9306} = 0.035$$

18. Solution: E

If we assume purchase price of 1, the present value of 90% of purchase price is  $0.90v^n$ . Since effective annual interest rate = 8% and  $n = 2$  months out of 12,

$$0.90v^n = 0.90 \left( \frac{1}{1.08} \right)^{2/12}$$

The present value should be set equal to  $1 - \frac{X}{100}$  since  $X$  is the percentage off purchase price paid on date of sale and we assume purchase price of 1.

$$1 - \frac{X}{100} = 0.90 \left( \frac{1}{1.08} \right)^{2/12}$$

Dividing both sides by  $v^n$ , thereby converting the reference time point to 2 months from date of sale, we get  $1 - \frac{X}{100} \cdot (1.08)^{1/6} = 0.90$ .

19. Solution: C

18.6%

Derived using the simplified formula.

$$12 \left[ 1 - \left( 1 + \frac{0.189}{12} \right)^{-1} \right] = 0.186.$$

20. Solution: A

$$\text{Future value} = NX + Xi(Is)_{\overline{n}|i}$$

$$10,000 = 10X + X(0.12)(Is)_{\overline{10}|0.08}$$

$$X = 541.47$$

21. Solution: D  
Assume total cost =  $TC$ . Then the monthly interest rate is  $i$ , where

$$TC = \left(\frac{TC}{10}\right) \ddot{a}_{\overline{12}|i}$$

$$10 = \ddot{a}_{\overline{12}|i}$$

$$i = 3.503\%$$

$$\text{Annual effective rate} = 1.03503^{12} - 1 = 0.512, \text{ or } 51.2\%$$

22. Solution: B

$$\frac{(50 - X + 4 - 2)}{40} = 20\% \Rightarrow 52 - X = 8 \Rightarrow X = 44$$

23. Solution: D

$$75 = \frac{6}{i - 0.03} \Rightarrow i = 11\%$$

24. Solution: E

$$s_{\overline{n}|}(1+i) = 13.776$$

$$\left[ \frac{(1+i)^n - 1}{i} \right] (1+i) = \frac{2.476 - 1}{d} = 13.776$$

$$d = 0.10714 = \frac{i}{1+i}$$

$$i = 12\%$$

$$s_{\overline{n}|} = 12.3$$

$$n = 8$$

25. Solution: A

The quarterly interest rate is 4%. The quarterly interest is  $0.04(500) = 20$ . Since each payment is interest-only, the principal portion of every payment is 0.