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Looking Beyond Hedge Effectiveness

Charles L. Gilbert
Looking Beyond Hedge Effectiveness

Equity Based Insurance Guarantees Conference
13 October 2009

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• What has the recent crisis taught us?
  • large losses, higher capital and reserves
  • breakage from basis risk
  • stress testing of dynamic hedging programs
• How should performance of hedging strategies be evaluated?
• Do hedge portfolio gains and hedge effectiveness tell the whole story?
• What should decision makers focus on?
ERM Framework And Risk Appetite Statement

- ERM strategic decision-making framework provides direction for balancing hedge objectives
- Risk appetite statement defines
  - Risk Capacity => How much risk you are able to take
  - Risk Strategy => What risks are you willing / unwilling to take
- Risk taking is then based on risk profile
  - Ensure fairly compensated for risks
  - Hedge cost vs. hedge effectiveness

Hedge Effectiveness vs Hedge Cost

- Relationship between hedge effectiveness and hedge cost
  - High effectiveness can increase hedging cost and reduce profit
- Trade-off between
  1. Higher expected profit, cost of capital and tail exposure
  2. Lower expected profit, cost of capital and tail exposure
- Dynamic hedging strategies with high hedge effectiveness can have higher losses for a given scenario
### Active Naked Strategies Used By Some VA Writers

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Execution</th>
<th>Description</th>
<th>Cost / Risk Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naked (passive)</td>
<td>Hold capital</td>
<td>• Unsophisticated&lt;br&gt; • Do nothing approach&lt;br&gt; • Company may not be compensated fairly for risk</td>
<td>• Zero hedge cost&lt;br&gt; • Highest cost of capital&lt;br&gt; • Highest downside&lt;br&gt; • Highest expected profit</td>
</tr>
<tr>
<td>Naked (active)</td>
<td>Hold capital; take view on market</td>
<td>• Sophisticated systems&lt;br&gt; • Company runs risk naked when fairly compensated&lt;br&gt; • Risk profile aligned to risk appetite and fin. objectives&lt;br&gt; • Based on market view of implied vol, interest rates&lt;br&gt; • Based on level of premiums charged</td>
<td>• Potentially low hedge cost&lt;br&gt; • High cost of capital&lt;br&gt; • Potential for most attractive risk profile if market view is correct</td>
</tr>
</tbody>
</table>

### Costs Vary Widely Between 1st And 2nd Order Strategies

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<th>Strategy</th>
<th>Execution</th>
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<tbody>
<tr>
<td>First Order Dynamic Hedging*</td>
<td>Delta, Rho, Vega* hedging with futures and swaps</td>
<td>• Replicate 1st order sensitivity of embedded option in liabilities&lt;br&gt; • No initial cash outlay&lt;br&gt; • Short volatility&lt;br&gt; • Exposure to realized vol</td>
<td>• Hedge cost not known until maturity&lt;br&gt; • Lowest expected hedge cost (next to naked)&lt;br&gt; • Highest tail exposure (next to naked)</td>
</tr>
<tr>
<td>Second Order Dynamic Hedging</td>
<td>Gamma, Vega, Vanna, Vomma hedging w/ options, vol swaps</td>
<td>• Replicate 1st and 2nd order sensitivities&lt;br&gt; • Premium paid for options&lt;br&gt; • Long volatility (hedge portfolio)&lt;br&gt; • Rollover risk = exposure to implied vol</td>
<td>• Hedge cost not known until maturity&lt;br&gt; • High expected hedge cost&lt;br&gt; • Greatest potential for losses if not executed properly</td>
</tr>
</tbody>
</table>
## “Static” Strategies Are A Subset Of Dynamic Hedging

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<th>Strategy</th>
<th>Execution</th>
<th>Description</th>
<th>Cost / Risk Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Hedging</td>
<td>OTC, exchange traded options and</td>
<td>• Match embedded option in liabilities</td>
<td>• Hedge cost known</td>
</tr>
<tr>
<td></td>
<td>exotic options</td>
<td>• Monitor / hedge basis risk</td>
<td>• Highest expected hedge cost</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Premium paid for options</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Long volatility (hedge portfolio)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• No exposure to implied vol</td>
<td></td>
</tr>
<tr>
<td>Semi-Static Hedging</td>
<td>Portfolio of options</td>
<td>• Approximate match embedded option in liabilities</td>
<td>• Hedge cost not known until maturity</td>
</tr>
<tr>
<td>(active / passive)</td>
<td></td>
<td>• Monitor / hedge basis risk</td>
<td>• High expected hedge cost</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Premium paid for options</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Long volatility</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Exposure to implied vol</td>
<td></td>
</tr>
</tbody>
</table>

## Risk-Reward Analysis

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Average Payoff</th>
<th>Tail Ranking</th>
<th>Hedge Effectiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naked</td>
<td>Best</td>
<td>4 (worst)</td>
<td>N/A</td>
</tr>
<tr>
<td>Delta only</td>
<td>Lower</td>
<td>3</td>
<td>~ 50%</td>
</tr>
<tr>
<td>Delta / Rho</td>
<td>Lowest</td>
<td>2</td>
<td>~ 70%</td>
</tr>
<tr>
<td>Delta / Rho / Vega</td>
<td>Lower (same as delta)</td>
<td>1 (best)</td>
<td>~ 90%</td>
</tr>
</tbody>
</table>
Case Study: First Order Passive Hedging Strategy

- Guarantee: GMAB, ROP
- Term to Maturity: 5 years
- Premium: 100,000
- Greeks Hedged: Delta only
- Hedging Frequency: Daily
- # of Simulations: 10,000
- Volatility: Constant
- Average Hedge Effectiveness
  \( R^2 = 59.7\% \)
- CTE(95) = 22,138

Cost to Replicate GMAB

Expected Cost = 15,780

\( R^2 = 42.7\% \)

Average Cost = 12,914

\( R^2 = 76.7\% \)

Negative correlation between hedge effectiveness and replicating cost

High realized vol results in lower hedge effectiveness under strategy
Case Study: First Order Passive Hedging Strategy

- Guarantee: GMAB, ROP
- Term to Maturity: 5 years
- Premium: 100,000
- Greeks Hedged: Delta only
- Hedging Frequency: Weekly
- # of Simulations: 10,000
- Volatility: Constant
- Average Hedge Effectiveness
  \[
  R^2 = 55.6\%
  \]
- CTE(95) = 22,515

Only slight decrease in expected cost and hedge effectiveness

\[y = -4E-05x + 1.2251\]
\[R^2 = 0.2563\]

Case Study: First Order Passive Hedging Strategy

- Guarantee: GMAB, ROP
- Term to Maturity: 5 years
- Premium: 100,000
- Greeks Hedged: Delta only
- Hedging Frequency: Weekly
- # of Simulations: 10,000
- Volatility: Constant
- Average Hedge Effectiveness
  \[
  R^2 = 55.6\%
  \]
- CTE(95) = 22,515

Lower correlation between hedge effectiveness and replicating cost
Case Study: First Order Passive Hedging Strategy

- Guarantee: GMAB, ROP
- Term to Maturity: 5 years
- Premium: 100,000
- Greeks Hedged: Delta only
- Hedging Frequency: Monthly
- # of Simulations: 10,000
- Volatility: Constant
- Average Hedge Effectiveness
  \( R^2 = 42.3\% \)
- CTE(95) = 24,025

No change in expected cost but much lower hedge effectiveness

Case Study: First Order Passive Hedging Strategy

- Guarantee: GMAB, ROP
- Term to Maturity: 5 years
- Premium: 100,000
- Greeks Hedged: Delta only
- Hedging Frequency: Monthly
- # of Simulations: 10,000
- Volatility: Constant
- Average Hedge Effectiveness
  \( R^2 = 42.3\% \)
- CTE(95) = 22,515

Negligible correlation between hedge effectiveness and replicating cost
Case Study: Comparison Of Hedging Strategies

- Guarantee: GMAB, ROP
- Term to Maturity: 5 years
- Premium: 100,000
- Greeks Hedged: Delta only
- Hedging Frequency: Weekly
- # of Simulations: 10,000
- Volatility: Constant

Higher hedge effectiveness comes at higher expected hedging cost

Hedge Effectiveness And Hedge Cost Based On S&P500

High hedge effectiveness resulted in high hedge cost due to volatility
Hedge Effectiveness And Hedge Cost Based On S&P500

How did hedge perform at market bottom?
• Hedge effectiveness was 85%
• Gain of 21,799
• Compare to naked strategy: increase in liability of 25,513

Roll forward to today
• Hedge cost = 16,962
• Naked cost = 7,440

Gain of 21,799 from Hedge Portfolio
Increase in liability of 25,513
Total Cost of Hedge 16,962
(M2M + interest + exercise)

Need to consider hedge performance over lifetime of guarantee

Hedge Effectiveness Based On S&P500

Higher rebalancing frequency resulted in higher hedge cost
Higher Order Greeks –
$1^{st}$, $2^{nd}$, $3^{rd}$ Order Sensitivities Of Option Price

<table>
<thead>
<tr>
<th>First Order</th>
<th>Delta</th>
<th>Vega</th>
<th>Rho</th>
<th>Theta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second Order</td>
<td>Gamma</td>
<td>Vomma</td>
<td>Rho Convexity</td>
<td></td>
</tr>
<tr>
<td>Third Order</td>
<td>Speed</td>
<td>Ultima</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Higher Order Greeks –
Cross Sensitivities Of Option Price

<table>
<thead>
<tr>
<th>Underlying (S)</th>
<th>Volatility (v)</th>
<th>Time to Expiry (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>Gamma</td>
<td>Vanna</td>
</tr>
<tr>
<td>Gamma</td>
<td>Speed</td>
<td>Vanna</td>
</tr>
<tr>
<td>Vega</td>
<td>Vanna</td>
<td>Vomma</td>
</tr>
<tr>
<td>Vomma</td>
<td>Zomma</td>
<td>Ultima</td>
</tr>
</tbody>
</table>
Attribution Analysis

Hedge effectiveness in terms of higher order greeks provides insight

Necessary to track risk exposure to changes in term structure
In Summary…

- Hedge effectiveness an important measure for managing risk
  - Attribution analysis with higher order greeks and cross greeks can provide insight for improving hedge effectiveness
- Hedge cost, risk distribution equally important as effectiveness
  - Higher hedge effectiveness can be less profitable and have less desirable risk profile
- Need to assess hedge portfolio “gains” or “losses” over lifetime of hedge not at a single point in time
  - May be better than naked at given point in time but not over period
- Hedging strategy should be determined within ERM framework

Thank You!

Nexus Risk Management provides a clear focus for managing risk to maximize value. As a company, we focus exclusively on risk. We deliver tools, training, expertise and execution to enable our clients needed to achieve their financial objectives.

Worldwide we work closely with insurance companies and pension plans, rating agencies and regulators, leading experts from academia and the investment industry as well as reinsurers and other counterparties to bring together deep industry knowledge, leading edge techniques and best practices from around the world. Using robust ALM and LDI frameworks that are fully integrated within ERM we deliver state of the art dynamic hedging programs, asset management within a risk framework and risk optimization.

Through our offices in Toronto and Seoul we provide highly specialized services to our clients globally.