THE JOKE . . . .
Two types of stochastic scenarios sets I will be discussing:

<table>
<thead>
<tr>
<th>Real World (&quot;RW&quot;)</th>
<th>Risk Neutral (&quot;RN&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Also referred to as simulation</td>
<td>Also referred to as market-consistent</td>
</tr>
<tr>
<td>Used by actuaries for financial analysis</td>
<td>Used to set capital market prices</td>
</tr>
<tr>
<td>and valuation purposes</td>
<td>Sometimes shorter-term focus</td>
</tr>
<tr>
<td>Long-term focus</td>
<td>Characteristics:</td>
</tr>
<tr>
<td>Intended to be consistent with historical</td>
<td>- Expected return = risk-free rate</td>
</tr>
<tr>
<td>market returns</td>
<td>- Implied volatility based on marketplace prices</td>
</tr>
<tr>
<td>However, often generated as forward looking, not actual historical results</td>
<td>- Combination of lower expected return and (sometimes) higher volatility results in higher expected cost than real world</td>
</tr>
<tr>
<td>Used for “real” expectations of risk/return tradeoffs</td>
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A Brief Example of What we’re Talking About

- Imagine a game that takes this form
  - A quarter and a dime are both flipped
  - For each coin, heads means the player receives 1,000 times the value of the coin
  - Tails on either coin means he pays that same amount – 1,000 times the value
  - It’s a totally fair game, right? So no up-front cost to play
  - Possible results are: -$350, -$150, +$150, and +$350, with equal probability

- Now imagine “the house” offers insurance
  - “The house,” by definition here, is “The MARKET”
  - The offer is, for a $100 premium, you can play, and the house will cover any of your losses over and above $100 for one game
  - Possible results are now: -$200, -$200, $50, and +$250, with equal probability (net of the $100 premium for the insurance)

- What scenarios make sense of the house offer of insurance (a put option)?
  - The “real world” tells you that the probabilities are all equal, so the expected outcome of the game, including the insurance, is $25 per play – this is the risk premium
  - This would indicate that the insurance is (as usual with a “house”) a bad deal
  - But what if you only brought $200 with you, and you really, REALLY . . .
  - . . . REALLY want to PLAY??
  - Then a market-consistent approach gives you the cost of adequately managing your risk
GAAP: Which Measure is Better for VA Guarantees?

- **What is FAS133 telling you?**
  - VAs contain guarantees
  - The market sets the price for guarantees
  - The actuary must evaluate the guarantees consistently with that market (use RN scenarios)

- **What is SOP 03-1 telling you?**
  - VAs contain guarantees with a mortality component
  - Only actuaries can properly value those
  - Use RW-ish scenarios, consistent with DAC economic assumptions
    - May include mean reversion
    - Probably a fairly optimistic combination of expected return and volatility

- **Any other important GAAP guidance**
  - What about valuation of hedge assets?
  - For various reasons, may also need stochastic valuation, probably RN scenarios
    - Dynamic hedging programs
    - Complex options with no closed-form solutions for values

Stat: Which Measure is Better for VA Guarantees?

- **What are regulators telling you (reserves)?**
  - Scenarios should be RW
  - Expected returns may be fairly optimistic
  - Tail returns will be somewhat conservative

- **What are rating agencies and regulators telling you (capital)?**
  - Scenarios should be RW
  - Expected returns may be fairly optimistic
  - Tail returns will be extremely conservative

- **Except . . .**
  - What if you are hedging?
  - Hedges can be included to at least some extent in asset adequacy testing
  - But can we evaluate the cost of hedging on a RW basis? Probably not
  - Testing the program may be a little of both
What Internal Conflicts can you get Into

What is the company Risk Manager telling you?
- Maybe: We need to hedge this. What will it cost? (Use RN approach)
- Maybe: We need to reinsure this. What will it cost? (Hmmmm.)
- Maybe: We need to cover out to the X%-ile of tail risk? (Use RW approach)
- Maybeeee: Are there any potential issues related to charging too much? More on that later . . . .

What is the Appointed Actuary telling you?
- Probably some of the same stuff
- Maybe: Shouldn’t we consider stochastic mortality? ($%^*&#$%^!!)
- Mostly, he wants to know that you are considering reserves and capital properly

What does the Marketing Department have to say?
- Probably: What can I get for free?
- Or better yet: If our 5 key competitors can each offer one modern rider feature for 50 basis points, . . .
- . . . why can’t our company have all 5 of those for 50 basis points?

What Potential Snags Lie Out There?

Conflict between RN and RW results?
- Let’s say you decide to charge the projected mean RN cost:
- What percentile of RW results will you cover?
- Any guesses out there?
- What would affect the answer?

Can there be statutory reporting conflicts?
- “RW-regulatory” vs. “your view on RW”?
- Even more so for “RW-rating agency”

Who will be upset if you decide to go with the higher cost?
- Marketing certainly (Should you even mention what RW %-ile you cover?)
- Anyone else?
- How about an eventual jury?
Other Conflicts

- **Name some things triggered by scenario results**
  - Lapse Rates
  - Annuitizations
  - Other Dynamic Eventualities

- **Even if RN scenarios value options correctly, are they correct for this purpose?**

- **What is different?**

GAAP for Variable Annuities: 2007

- **DAC, SOP03-1, and FAS133 liabilities (and assets?) potentially valued 3 completely different ways**

- **What are some issues that this causes for reporting of GAAP income?**
In Summary

Stochastic Modeling for Variable Annuities takes many conflicting forms, and it can be a lot of work to make sense of the overall set of results that you get . . . .

Perhaps even more thought should go into deciding how to communicate those results!!
Stochastic Modeling for Life Insurance: Stochastic Mortality

2007 SOA Annual Meeting

David J. Weinsier

October 17, 2007

Outline

- Overview
- Sources of variance in mortality
- Techniques for generating stochastic mortality scenarios
- Case study: structured settlements
- Closing thoughts
### Overview

Mortality has traditionally been viewed as a deterministic process

- Mortality assumptions have been defined by a table of death rates. This approach is simple and allows for risks to be easily replicated and compared.
- Either industry tables (e.g., SOA 1975-80, 2001VBT) or company-specific tables are utilized.
- Factors are often applied to the table to represent variations in mortality by risk class, duration, or band.

- The deterministic approach, while often sufficient for most analyses, ignores two important aspects of mortality:
  - Mortality volatility risk
  - Misestimation risk
**Stochastic mortality techniques can provide enhanced insights into financial performance**

- Increased focus on quarterly earnings leads to focus on volatility in mortality results
  - How much volatility is reasonable in one quarter?
- Stochastic mortality can also provide guidance in setting economic capital levels
- Stochastic mortality can be especially useful when:
  - There are a limited number of lives at risk
  - The economic consequences of death have a high severity but low probability of occurrence, such as the case of stop loss reinsurance
- Stochastic mortality projections are typically a required component of any life insurance securitization

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**Sources of Variance in Mortality**
Sources of mortality variance can be broken down as follows

Sources of mortality variance

<table>
<thead>
<tr>
<th>Mortality Volatility Risk</th>
<th>Mis-estimation Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard random fluctuations in mortality</td>
<td>Mis-estimation due to sampling</td>
</tr>
<tr>
<td>Fluctuation in population mortality due to various environmental factors</td>
<td>Mis-estimation of mortality slope</td>
</tr>
<tr>
<td>Fluctuation due to catastrophic events</td>
<td></td>
</tr>
</tbody>
</table>

Standard random fluctuations in mortality

- Many assume that variance in mortality due to random fluctuations is immaterial due to the law of large numbers
- However, even in a portfolio of 1,000,000 lives, actual to expected ratios in a single year can have a standard deviation of 3%
- Depends on mix of business
- There is general agreement that this can be approximated using a binomial function
Fluctuation in population mortality due to various environmental factors

- External environmental factors can have a measurable effect on general mortality levels, and in turn, insured mortality levels
- U.S. population annual mortality improvement has varied from (2.3%) to 3.1% from 1981-2001

![Graph showing US Population Mortality Improvement from 1981 to 2001](image)

Time series models can be used to incorporate this fluctuation into stochastic mortality models

Fluctuation due to catastrophic events

- A large scale catastrophic event can have a material impact on mortality
- The 1918 Influenza epidemic is most often cited as a catastrophic mortality event in the U.S.
  - The epidemic resulted in an extra 5 deaths per 1,000 in the U.S. over a single year
- Can be modeled as a one-time shock to mortality
## Mis-estimation due to sampling

- Typically, a mortality assumption is based on, or at least influenced by, the mortality experience of a finite number of lives
  - For example, the results of a 5-year mortality study
- We can never be certain of the “true” mortality rate
- Mis-estimation due to sampling may be estimated in Monte Carlo analysis using standard deviations from the underlying mortality studies used to set the mortality assumption(s)

## Mis-estimation of mortality slope

- A current issue in mortality is slope of the underlying mortality table
- The slope of SOA 1975-80 is generally flatter than SOA 2001 VBT
  - The last industry mortality table based on homogenous risk selection techniques throughout the select and ultimate periods is the SOA 1975-80 table
  - Introduction of smoker distinct rates, lower blood testing limits and preferred rates have influenced the slope of later industry tables
- Mis-estimation of mortality slope is more challenging to build into a stochastic process
  - It may be better to look at this risk on a deterministic basis, but could have “standard deviation” developed using delphi techniques
Techniques for Generating Stochastic Mortality Scenarios

Monte Carlo simulation is a common technique used to generate stochastic mortality scenarios

- Monte Carlo simulations associate a sequence of random numbers with a probability distribution to explain a real-life process, system or behavior.
- The key elements of a Monte Carlo simulation include:
  - Random number generator
  - Choice of a parameterized probability distribution
  - Real-life interpretation of the random number generation
- A graphical representation of Monte Carlo simulation follows:

  ![Monte Carlo Simulation Diagram]

  - Random Number Generator
    \[ X_i = \varepsilon [0,1] \]
  - PDF (Defined Process)
    \[ Y_i \sim \text{Dist}(\text{parm1, parm2,...}) \]
  - Solve for \( Y_i \) st
    \[ X_i = F(Y_i; \text{parm1, parm2,...}) \]
Example of Monte Carlo simulation of death rates on a cohort of N policies:
- N random numbers \( X_i \) are generated on the unit interval.
- The parameterized probability distribution \( Y_i \sim \text{Bernoulli}(q_{x+t}^i) \).
- \( Y_i = 1 \) if \( X_i > q_{x+t}^i \) (real-life interpretation: insured survives), 0 if \( X_i \leq q_{x+t}^i \) (real-life interpretation: insured dies).

The 1918 Flu Epidemic can be utilized to demonstrate how catastrophic mortality fluctuation can be incorporated into a stochastic mortality model; characteristics of the Flu Epidemic are as follows:
- It is often characterized as a once every 100 years event
- It resulted in an additional five deaths per 1,000

A catastrophic event, such as the Flu Epidemic, can be modeled using Monte Carlo simulation techniques as follows:
\[
q_{x+1} = (a_t \cdot q_x + b \cdot I_t) \cdot \prod_{k=1}^{t} (1 - i_k),
\]
where
- \( q_x \) = tabular mortality
- \( a_t \) = random deviations around \( q_x \)
- \( i_k \) = mortality improvement
- \( b = 5 \) deaths per 1,000
- \( I_t \) = Bernoulli variable with probability of 0.01
Comparisons of mortality slope can provide a range for deterministic scenario

Mortality Slope Comparison – Male age 65

SOA 1985-90 and VBT Composite multiplied by 98% and 169% respectively such that duration 1 mortality is equal for all three tables

Case Study: Structured Settlements

Practical Applications of Stochastic Mortality
<table>
<thead>
<tr>
<th>Case Study: reserves released on death for structured settlements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structured settlements contain an element of mortality risk</td>
</tr>
<tr>
<td>- Various payment structures (life annuity, certain annuity,</td>
</tr>
<tr>
<td>certain + life annuity, bullet payments, etc.)</td>
</tr>
<tr>
<td>- Value of a policy is the combination of discounted certain</td>
</tr>
<tr>
<td>payments and estimated life contingent payments in the</td>
</tr>
<tr>
<td>future</td>
</tr>
<tr>
<td>- Reserves backing the life contingent components released</td>
</tr>
<tr>
<td>when the policyholder dies</td>
</tr>
<tr>
<td>- Profits decline when fewer than expected number of</td>
</tr>
<tr>
<td>deaths occur or policies with large payments survive</td>
</tr>
<tr>
<td>longer than expected</td>
</tr>
<tr>
<td>- This demonstration focuses on reserves released on death</td>
</tr>
<tr>
<td>and the volatility of this metric</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario and methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>- A 300-policy structured settlement block was analyzed</td>
</tr>
<tr>
<td>- All policies are life contingent only</td>
</tr>
<tr>
<td>- Life contingent reserves at quarter-end have been</td>
</tr>
<tr>
<td>projected for five years based on gender, issue/rated</td>
</tr>
<tr>
<td>age, attained age and the selected mortality basis</td>
</tr>
<tr>
<td>- Large variance in size among contracts</td>
</tr>
<tr>
<td>- Methodology</td>
</tr>
<tr>
<td>- Monte Carlo simulation to project deaths each quarter for</td>
</tr>
<tr>
<td>five years</td>
</tr>
<tr>
<td>- Reserves released upon death and aggregated each</td>
</tr>
<tr>
<td>quarter</td>
</tr>
<tr>
<td>- 1,000 scenarios projected</td>
</tr>
</tbody>
</table>
Quarterly reserves released on death are extremely volatile

Scenario representing 99th percentile in one quarter is not necessarily the 99th percentile scenario in a different quarter

Cumulative reserves released on death are significantly less volatile than quarterly reserves

Scenario representing 99th percentile in one quarter is not necessarily the 99th percentile scenario in a different quarter
Observations

- Quarterly reserves released
  - Very erratic
  - Median was 0 and mean greater than 75th percentile, i.e., results were very positively skewed

- Cumulative reserves released
  - Ranged across a large band
  - Mean was materially greater than the median

Closing Thoughts

- Stochastic modeling of mortality can be a useful enhancement of life insurance modeling
  - Particularly useful in certain cases
    - Low number of lives covered
    - Impact of mortality results on financial results is discontinuous

- Stochastic mortality may be especially relevant in assessing life insurance capital market solutions

- Certain organizations may be significantly exposed to mortality risk
  - Reinsurers
  - Investors in longevity risk