SOA 09 Annual Meeting & Exhibit
October 25-28, 2009

Session 16 PD, Stochastic Modeling - IAA
Monograph on Stochastic Processes & Modeling
in Financial Reporting & Capital Assessment

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Presenters:
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Chad Michael Schuster, FRM
Stochastic Modeling:
IAA Monograph on Stochastic Processes and Modeling

Presented by:
Andrew H. Dalton

October 26, 2009

Agenda

- Overview of the monograph
- Motivation for development
- Expected usage of the monograph
- Review of Life EC Case Study
Overview of the Monograph

- Sponsored by the International Actuarial Association (IAA)
- Funded by actuarial organizations and societies around the world, including:
  - Society of Actuaries
  - Casualty Actuarial Society
  - Several non-US organizations

Overview of the Monograph

- Covers all insurance products
  - Contributing authors from
    - Life,
    - Health and
    - Property/Casualty
Overview of the Monograph: Contributing Authors

Contributing authors have credentials from the following actuarial societies worldwide:
- Society of Actuaries (North America)
- Casualty Actuarial Society (North America)
- Institute of Actuaries (United Kingdom)
- Institute of Actuaries of Australia (Australia)
- Institute of Actuaries of Japan (Japan)
- Actuarieel Genootschap (Dutch Society of Actuaries)
- Schweizerische Aktuarvereinigung (Swiss Association of Actuaries)

Overview of the Monograph: Contributing Authors

Contributing authors hold the following advanced degrees or other professional credentials:
- PhD in Chemical Physics
- PhD in Financial Mathematics
- Master of Business Administration (MBA)
- Chartered Financial Analyst Charterholder (CFA)
- Global Association of Risk Professionals (FRM)
- Chartered Enterprise Risk Analyst (CERA)
Overview of the Monograph: General Outline

Divided into Five Sections:
1. General Methodology & Techniques
2. Applications of Stochastic Modeling
3. Evaluation and Communication of Stochastic Results
4. Case Studies
5. Technical Appendix & References

Overview of the Monograph: Structure

- Each section builds on the previous one
  - Section I...
    - provides fundamental technical background material
  - Section II ...
    - applies the technical material developed in Section I to insurance models
Overview of the Monograph: Structure

- Each section builds on the previous one
  - Section III …
    - discusses practical considerations related to the models presented in Section II
  - Section IV…
    - Illustrates the real-world application of models developed in Sections I - III

Overview of the Monograph: Section I

Section I (General Methodology & Techniques):
- Risk-Neutral vs. Real World Scenarios
- Modeling Techniques
  - Stochastic vs. Non-Stochastic Methods
  - Monte Carlo Simulation
  - Lattice Models
  - Regime Switching Models
- Distributions and Fitting
- Random Number Generation
- Risk Measures
Overview of the Monograph: Section II

Section II (Applications):
- Economic scenario generation
- Capital testing
- Deflators
- Life/Health models
- Casualty Claim/Financial models
- Country/Region Specific Issues

Overview of the Monograph: Section III

Section III (Evaluation and Communication):
- Calibration
- Validation
- Auditing results
- Peer reviewing results
- Methods to communicate results
Overview of the Monograph: Section IV

Section IV (Case Studies):
- Development and management of a variable-annuity product
- Economic capital for a multi-line life insurance company
- Development of Embedded Value for a multi-line life insurance company
- Unpaid claim variability for a multi-line non-life insurance company
- Economic Capital for a multi-line non-life insurance company
- Combining Economic Capital results for life and non-life companies
- Stochastic reserve and capital calculations

Overview of the Monograph: The Case Studies

- Case studies presented in Section IV:
  - Touch on a wide variety of actuarial specialties
  - Address most of the risk factors discussed in Sections I-III
  - Illustrates use of the risk measures developed in Sections I-III
Motivation for Development

- Provide technical background in stochastic modeling to actuaries around the world
- Serve as technical reference
- Demonstrate application of stochastic modeling techniques to insurance
- Illustrate, through cases studies, real world examples of stochastic modeling in insurance

Motivation for Development

- Create stochastic modeling resource applicable to all countries
- Elevate awareness regarding importance of stochastic modeling in insurance
- Compare best practices across countries
Expected Uses

- Actuarial education
  - Exam syllabus for actuarial organizations
  - On-the-job learning resource
- Technical resource
  - For stochastic modeling concepts/techniques
  - For insurance application of stochastic modeling

Expected Uses

- By senior actuaries and senior management
  - Through case studies, to understand how stochastic modeling supports strategic and tactical decision making
  - To understand the ways in which stochastic modeling has been/is used around the world
Expected Uses

- As a starting point for future editions
  - Regional actuarial organizations may tailor and/or add case studies to fit unique needs
  - Expect to add case studies as actuarial practice evolves and expands

Review of Case Study: Economic Capital for Multi-Line Life Insurance Company

- Goals:
  - Introduce one of the case studies developed in the monograph
  - Illustrate the level of material covered in the monograph
  - Demonstrate how stochastic modeling concepts are applied to real-world settings
Background on Case Study Company

- Case study company, XYZ Life Insurance, is a large, multi-line life and health insurance company
  - Company writes primarily par individual life, variable annuity and group health business
- XYZ Life is headquartered in the U.S. However, EC analysis is not country-specific.

Economic Capital – Fundamental Concepts

- What confidence level would company like to target?
  - EC analysis intended to examine catastrophic events
  - Typical confidence levels are 99.5%, 99.9%, 99.95%

- CASE STUDY: Uses CTE-99, in which capital is set at the average of the worst 1% of losses
Economic Capital – Fundamental Concepts

- What risk metric will company use?
  - Commonly used metrics are Present Value of Future Profits (PVFP) and Greatest Present Value of Accumulated Loss (GPVL)

- **CASE STUDY**: Uses both PVFP and GPVL

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Economic Capital – Fundamental Concepts

- With what time horizon is the company concerned?
  - Short-term shocks versus long-term risk exposure

- **CASE STUDY**: Uses 30-year projection horizon; XYZ Life is concerned with long-term solvency of business.
Economic Capital – Fundamental Concepts

- What projection techniques will be used?
  - Stochastic modeling is generally required
  - Stress testing or scenario analysis are generally inadequate for EC

- **CASE STUDY:** Uses stochastic modeling. Started with XYZ’s Cash Flow Testing (or Embedded Value models, with certain adjustments).

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**XYZ Life – Key Risk Analysis**

The following table illustrates the key risk factors identified and modeled for XYZ Life:

**Table 1 - Risks Modeled for XYZ Life Insurance Company Economic Capital**

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Individual Life</th>
<th>Variable Life</th>
<th>Fixed Annuity</th>
<th>Variable Annuity</th>
<th>Fixed Income</th>
<th>Asset Management</th>
<th>Individual Disability</th>
<th>Group Health</th>
<th>Capital</th>
<th>Surplus</th>
<th>Consolidated Corporate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortality</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lapse</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market/Equity Returns</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Mortality</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Currency Exchange Rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Credit</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Strategic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Operational</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
Scenario Generation

- Based on the Key Risk Analysis, we determined that we needed 1,000 stochastic scenarios for each of the following risk factors:
  - Economic scenarios, including:
    - Interest rates
    - Equity returns
    - Spot currency exchange rates
  - Credit/default
  - Mortality
  - Morbidity
  - Lapses
  - Operational Failure and Strategic Risk

Economic Scenarios

- Equity Model
  - Uses Heston’s model (1993)
  - Has certain advantages over the traditional lognormal model, including the ability to make results consistent with certain stylized facts of the historical time series of S&P 500 index.
- Interest rate model
  - Views yield curve as having three components: level, slope and curvature.
  - Each component modeled stochastically
- Spot exchange rate model
  - Lognormal model
  - Maintains interest rate parity
Credit Risk Model

  - Borrower’s end of period state (default or no default) is driven by a normally distributed variable.

Mortality Scenarios

- Mortality generated as described in the mortality chapter in the monograph
- Scenarios rely on three models, which roll up into a single set of scenarios:
  - Baseline model
    • Reflects expected mortality and fluctuations during "normal" times.
  - Disease model
    • Reflects additional mortality that could occur during certain pandemic events.
  - Terrorism model
    • Reflects additional mortality that could occur as a result of voluntary acts of human violence.
**Morbidity Scenarios**

- Probability Distribution for New Claim Costs  
  - Developed using Milliman’s Risk Simulation software  
  - Monte Carlo analytical tool  
- Probability Distribution of Claim Runoff  
  - Also used Milliman’s Risk Simulation software

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**Lapse Scenarios**

- Developed based on two key assumptions:  
  - Historical company experience represents best-estimate for future experience.  
  - Deviation from expected rates will follow a normal distribution, with standard deviation based on company’s historical experience.
Operational Risk and Strategic Risks

- Relied on two key pieces of analysis:
  - Data gathered from profit centers related to potential operational losses which were both high impact and low frequency.
    - Business experts contributing to this exercise drew on company’s historical experience as well as knowledge of similar events at other organizations.
  - Use of STRATrisk
    - Cutting-edge methodology to determine key sources of risk that a business faces.
    - Methodology is a result of research program led by Neil Allan of Bath University in the UK.

Presentation of Results – Present Value of Future Profits

<table>
<thead>
<tr>
<th>Summary of Indicated Economic Capital</th>
<th>Baseline</th>
<th>2,500 Worst</th>
<th>Worst 1,000 for each risk factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance Risk in Lines of Business</td>
<td>4,114</td>
<td>3,106</td>
<td>1,605</td>
</tr>
<tr>
<td>Credit Defaults</td>
<td>(380)</td>
<td>(668)</td>
<td>(1,170)</td>
</tr>
<tr>
<td>Strategic and Operational Risk</td>
<td>(306)</td>
<td>(306)</td>
<td>(306)</td>
</tr>
<tr>
<td>Total Present Value of Profits</td>
<td>3,428</td>
<td>2,132</td>
<td>130</td>
</tr>
<tr>
<td>Indicated Economic Capital</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
### Presentation of Results – Greatest Present Value of Accumulated Loss

#### Summary of Indicated Economic Capital (GPVL Risk Metric)

<table>
<thead>
<tr>
<th>Risk Category</th>
<th>Baseline</th>
<th>2,500 Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance Risk in Lines of Business</td>
<td>4,114</td>
<td>(113)</td>
</tr>
<tr>
<td>Credit Defaults</td>
<td>(380)</td>
<td>(960)</td>
</tr>
<tr>
<td>Strategic and Operational Risk</td>
<td>(306)</td>
<td>(306)</td>
</tr>
<tr>
<td>Total Present Value of Profits</td>
<td>3,428</td>
<td>(1,379)</td>
</tr>
<tr>
<td>Indicated Economic Capital</td>
<td>-</td>
<td>1,379</td>
</tr>
</tbody>
</table>
Overview

- Introduction

- Policyholder Behavior Modeling

- Pricing and Valuation of Variable Annuities (VA’s) using Stochastic Modeling
Policyholder Behavior Modeling

Policyholder Behavior

- Guaranteed Minimum Benefit (GMB) Behavior Considerations:
  - Utilization
  - Persistency
  - Asset Allocation
Policyholder Behavior

- Issues Predicting GMB Policyholder
  - Most products are only several years old
  - Product evolution influences policyholder behavior
  - Utilization is company specific
  - Most sales have occurred over a steady market environment

Guarantee Utilization

- GMWB: Withdrawal
  - Dynamic or cohort

- GMIB: Annuitzation
  - Dynamic

- What if there are multiple guarantees present?
Persistency

- Dynamic Lapse Assumption
  - Typically a factor applied to base lapse rate

![Dynamic Lapse Model](image)

Asset Allocation

- Majority Of GMBs Require Asset Allocation
  - Balanced allocation 55-65% equity
  - "Lifestyle" models
    - Equity content varies considerably by company:
      - Aggressive : 75-90%
      - Moderately Aggressive : 65-85%
      - Moderate : 50-70%
      - Moderately Conservative : 40-55%
      - Conservative : 20-40%
  - Typically a single charge which introduces pricing risk
Asset Allocation

Experience on “Lifestyle” models shows policyholder’s are not necessarily maximizing economic value of guarantee
  - Focused on maintaining asset value

Predicting Policyholder Behavior

Key Considerations:
  - Monitor experience
  - Sensitivity analysis on key assumptions
  - Add additional margin to reflect uncertainty in assumptions
  - Design products to reduce policyholder behavior risk
Pricing and Valuation of VA’s using Stochastic Modeling

Pricing and Valuation Considerations

- Guaranteed Minimum Benefits (GMBs)
- Base product
- Capital
- Reserves
- Hedging
Guarantee Valuation

- The value of GMB’s depend on market movements, therefore they are types of financial derivatives
- Risk-Neutral Valuation
  - The value of a derivative is its expected payoff in a risk-neutral world, discounted at the risk-free rate
  - Consistent with Option Pricing Theory

Option Value of Guarantee

<table>
<thead>
<tr>
<th>Expected Present Value of Future Guarantee Claims</th>
<th>minus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Present Value of Future Revenues</td>
<td></td>
</tr>
<tr>
<td>over a set of Risk Neutral Scenarios</td>
<td></td>
</tr>
</tbody>
</table>

Illustrative Case Study

- Pricing analysis of a hypothetical lifetime GMWB
- Guarantee Design
  - Lifetime Guaranteed Minimum Withdrawal Benefit with a 5% withdrawal rate (issue ages 60-85)
  - Annual ratchet
  - 60% Equity / 40% Bonds
  - Rider Charge of 60bps of Benefit Base
- Modeling
  - Risk-neutral valuation of embedded guarantee
    • Theoretical cost of hedging
    • 5000 Monte Carlo simulations
  - Equity volatility of 20%
  - Interest rate of 5%
Case Study – Sample Assumptions

- **Mortality**
  - Annuity 2000 with mortality improvement based on Scale G

- **Dynamic Lapse**

  ![Dynamic Lapse Model](image)

- **Withdrawal Utilization**

<table>
<thead>
<tr>
<th>Withdrawal Delay</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50%</td>
</tr>
<tr>
<td>5</td>
<td>40%</td>
</tr>
<tr>
<td>10</td>
<td>10%</td>
</tr>
</tbody>
</table>

- **Issue Age distribution**

<table>
<thead>
<tr>
<th>Issue Age</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>10%</td>
</tr>
<tr>
<td>60</td>
<td>20%</td>
</tr>
<tr>
<td>65</td>
<td>35%</td>
</tr>
<tr>
<td>70</td>
<td>20%</td>
</tr>
<tr>
<td>75</td>
<td>10%</td>
</tr>
<tr>
<td>80</td>
<td>5%</td>
</tr>
</tbody>
</table>

Illustrative Costs

<table>
<thead>
<tr>
<th>Withdrawal Delay</th>
<th>Issue Age</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>0</td>
<td>0.78%</td>
<td>0.61%</td>
</tr>
<tr>
<td>5</td>
<td>0.51%</td>
<td>0.38%</td>
</tr>
<tr>
<td>10</td>
<td>0.29%</td>
<td>0.20%</td>
</tr>
<tr>
<td></td>
<td>0.61%</td>
<td>0.46%</td>
</tr>
</tbody>
</table>

- Decrease in cost with older ages
- Costs are also very sensitive to utilization assumption
Risk Management in Product Design

- Revised design (Design 2)
  - 5% simple bonus for each withdrawals are delayed
  - Likely results in more people waiting to withdraw

<table>
<thead>
<tr>
<th>WithdrawalDelay</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.78%</td>
<td>0.61%</td>
<td>0.44%</td>
<td>0.29%</td>
<td>0.17%</td>
<td>0.08%</td>
<td>0.45%</td>
</tr>
<tr>
<td>5</td>
<td>0.75%</td>
<td>0.58%</td>
<td>0.41%</td>
<td>0.26%</td>
<td>0.14%</td>
<td>0.07%</td>
<td>0.42%</td>
</tr>
<tr>
<td>10</td>
<td>0.61%</td>
<td>0.45%</td>
<td>0.30%</td>
<td>0.18%</td>
<td>0.09%</td>
<td>0.03%</td>
<td>0.32%</td>
</tr>
<tr>
<td>Total</td>
<td>0.72%</td>
<td>0.55%</td>
<td>0.39%</td>
<td>0.25%</td>
<td>0.14%</td>
<td>0.06%</td>
<td>0.40%</td>
</tr>
</tbody>
</table>

- Flatter costs by withdrawal delay
- Overall cost increases due to richer benefit
  - Some offset from delaying withdrawals

Sensitivity Tests – Dynamic Lapse

<table>
<thead>
<tr>
<th>Dynamic Lapse Function</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Hedge Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
</tr>
<tr>
<td>Dynamic Lapse Sensitivity</td>
</tr>
</tbody>
</table>
### Asset Allocation Sensitivity

<table>
<thead>
<tr>
<th>Asset Allocation Model</th>
<th>Hedge Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>80% Equity</td>
<td>0.60%</td>
</tr>
<tr>
<td>60% Equity</td>
<td>0.40%</td>
</tr>
<tr>
<td>40% Equity</td>
<td>0.24%</td>
</tr>
</tbody>
</table>

- Higher equity allocation results in proportionally higher cost
  - Driven by higher volatility in the funds
- Note that risk-neutral pricing means higher equity allocation does not mean higher expected growth rates

### Sensitivity Tests – Static Assumptions

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>Hedge Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (design 2)</td>
<td>40</td>
</tr>
<tr>
<td>Lapse Sensitivity - 0.75 of base lapse table</td>
<td>43</td>
</tr>
<tr>
<td>Mortality Sensitivity - 0.7 mortality multiplier</td>
<td>51</td>
</tr>
<tr>
<td>Mortality Improvement - Double rate of improvement</td>
<td>48</td>
</tr>
<tr>
<td>Mortality Improvement - Triple rate of improvement</td>
<td>55</td>
</tr>
</tbody>
</table>

- Lower persistency means higher potential claims but also higher fees
  - Sensitivity does not consider offset from base product
- Longevity risk is a joint risk with equity markets
  - In up scenarios higher longevity means more fees are collected (lower average cost)
  - In down scenarios higher longevity means higher claims (higher average cost)
Risk Management in Product Design

- Revised design
  - Withdrawal rates varying by age at first withdrawal

<table>
<thead>
<tr>
<th>Age at First Withdrawal</th>
<th>Withdrawal Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>55-59</td>
<td>4%</td>
</tr>
<tr>
<td>60-69</td>
<td>5%</td>
</tr>
<tr>
<td>70+</td>
<td>6%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design 3</th>
<th>Issue Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>55</td>
</tr>
<tr>
<td>Withdrawal Delay</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.35%</td>
</tr>
<tr>
<td>5</td>
<td>0.75%</td>
</tr>
<tr>
<td>10</td>
<td>0.61%</td>
</tr>
<tr>
<td>Total</td>
<td>0.61%</td>
</tr>
</tbody>
</table>

- Flatter costs by issue age
- Overall cost increases due to richer benefit
  - Some offset from delaying withdrawals

Nested Stochastic Projections

- Stochastic on Stochastic
  - Additional layer of economic scenario dimensionality
  - Within each “outer scenario”, one or more sets of nested “inner scenarios” are embedded
  - Projection of capital and/or reserves in one or more future projection periods
  - Simulation of hedging program
Financial Projections

- Hedge strategy simulation
  - Stochastic on Stochastic
  - Short time-step – accurately reflect actual rebalancing approach
  - Full projection of liability evolution over time as well as hedge transactions
Strategy Testing

Quarterly Profit & Loss Over 1998-2005 Market Path

- Unhedged
- Hedged
- S&P 500

Quarterly P&L Volatility Unhedged

Quarterly P&L Volatility Delta-Vega Hedged

Quarterly P&L Volatility Delta-Vega-Rho Hedged
Catastrophic Mortality Modeling

Ghalid Bagus, FSA MAAA CFA
Principal and Consulting Actuary
October 2009

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Contents

1. Model structure
2. Baseline model
3. Pandemic model
4. Terrorism model
5. Combined results

Section 1

Model structure
Model overview

**Baseline Model**
- Expected mortality
- Expected volatility
- Country specific models using historic data

**Disease Model**
- Additional mortality due to potential pandemics
- Same model for each country

**Terrorism Model**
- Additional mortality due to potential non-disease events
- Country specific model using US State Department data

**Combined Model**
- Combines baseline scenarios, pandemic scenarios and terrorism scenarios for each country
- Combines the additional deaths for each model to determine the increase in mortality

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**Non-modelled items**

- Natural disasters
- Industrial accidents
- Traditional war
- Nuclear war

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**Natural Disasters—Recent History**

<table>
<thead>
<tr>
<th>Event</th>
<th>Country</th>
<th>Year</th>
<th>Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td>Japan</td>
<td>1923</td>
<td>143,000</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Italy</td>
<td>1908</td>
<td>75,000</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Italy</td>
<td>1915</td>
<td>30,000</td>
</tr>
<tr>
<td>Hurricane</td>
<td>United States</td>
<td>1900</td>
<td>6,000</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Japan</td>
<td>1948</td>
<td>5,131</td>
</tr>
<tr>
<td>Wind Storm</td>
<td>Japan</td>
<td>1919</td>
<td>5,098</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Italy</td>
<td>1980</td>
<td>4,689</td>
</tr>
<tr>
<td>Wind Storm</td>
<td>United Kingdom</td>
<td>1912</td>
<td>4,000</td>
</tr>
<tr>
<td>Wind Storm</td>
<td>Japan</td>
<td>1917</td>
<td>4,000</td>
</tr>
<tr>
<td>Wind Storm</td>
<td>Japan</td>
<td>1945</td>
<td>1,746</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Japan</td>
<td>1933</td>
<td>3,008</td>
</tr>
<tr>
<td>Wind Storm</td>
<td>Japan</td>
<td>1934</td>
<td>3,006</td>
</tr>
<tr>
<td>Wind Storm</td>
<td>Japan</td>
<td>1933</td>
<td>3,000</td>
</tr>
<tr>
<td>Wave / Surge</td>
<td>Japan</td>
<td>1913</td>
<td>3,000</td>
</tr>
</tbody>
</table>

**More Recent (1980-2005)**

<table>
<thead>
<tr>
<th>Event</th>
<th>Country</th>
<th>Year</th>
<th>Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat Wave</td>
<td>Europe</td>
<td>2003</td>
<td>19,000</td>
</tr>
<tr>
<td>Earthquake</td>
<td>Japan</td>
<td>1995</td>
<td>5,297</td>
</tr>
<tr>
<td>Hurricane (Katrina)</td>
<td>United States</td>
<td>2005</td>
<td>1,836</td>
</tr>
<tr>
<td>Heat Wave</td>
<td>United States</td>
<td>1995</td>
<td>670</td>
</tr>
<tr>
<td>Tornado</td>
<td>United States</td>
<td>1984</td>
<td>600</td>
</tr>
<tr>
<td>Winter Storm</td>
<td>United States</td>
<td>1983</td>
<td>500</td>
</tr>
<tr>
<td>Flood</td>
<td>Japan</td>
<td>1982</td>
<td>345</td>
</tr>
<tr>
<td>Winter Storm</td>
<td>United States</td>
<td>1982</td>
<td>270</td>
</tr>
</tbody>
</table>
Section 2

Baseline model

Baseline model: modeling approach

- The baseline model uses a time series model to project base mortality rates into the future
- The approach is to test different modeling methods and to consider the goodness of fit for various algebraic models using different parameters
- 35 years of mortality data (1969 to 2003) are used for the analysis under the baseline model
Index calculation formulae

Mortality rates are calculated as:

\[ q_t = \sum_x a_x b_x q_{m,x,t} + a_x b_x q_{f,x,t} \]

Where:
- \( a_x b_x \) = the gender weightings for males and females respectively
- \( b_x \) = the weight for age group \( x \)
- \( q_{m,x,t}, q_{f,x,t} \) = the mortality rates for age group \( x \) for year \( t \) for males and females respectively

<table>
<thead>
<tr>
<th>Age and Gender Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Range</td>
</tr>
<tr>
<td>&lt; 1</td>
</tr>
<tr>
<td>25 to 34</td>
</tr>
<tr>
<td>35 to 44</td>
</tr>
<tr>
<td>45 to 54</td>
</tr>
<tr>
<td>55 to 64</td>
</tr>
<tr>
<td>65 to 74</td>
</tr>
<tr>
<td>&gt; 74</td>
</tr>
</tbody>
</table>

Baseline mortality projection

The auto-regressive formulation for the change in mortality rates is:

\[ x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \ldots + a_n x_{t-n} + \varepsilon \]

Where:
- \( x_t \) = the rate of change in the mortality rate from time \( t - 1 \) to time \( t \)
- \( \varepsilon \) = the error term

---

1995-2003 U.S. Rates per 100,000

<table>
<thead>
<tr>
<th>Year</th>
<th>Mortality Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>601.2</td>
</tr>
<tr>
<td>1996</td>
<td>583.3</td>
</tr>
<tr>
<td>1997</td>
<td>563.7</td>
</tr>
<tr>
<td>1998</td>
<td>552</td>
</tr>
<tr>
<td>1999</td>
<td>550.3</td>
</tr>
<tr>
<td>2000</td>
<td>537.4</td>
</tr>
<tr>
<td>2001</td>
<td>532.3</td>
</tr>
<tr>
<td>2002</td>
<td>527.7</td>
</tr>
<tr>
<td>2003</td>
<td>522.9</td>
</tr>
</tbody>
</table>
Results: baseline model

- The baseline model does not project large increases in mortality rates.
- Even when the effect of the projected mortality improvements is removed, the increases in mortality projected are not large enough to be considered of a catastrophic nature.

<table>
<thead>
<tr>
<th>Mortality Increase Range</th>
<th>With Mortality Improvements</th>
<th>Without Mortality Improvements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Percentage Upper Percentage</td>
<td>0.00%</td>
<td>36.19%</td>
</tr>
<tr>
<td>0.0 0.5</td>
<td>0.00%</td>
<td>38.37%</td>
</tr>
<tr>
<td>0.5 1.0</td>
<td>0.00%</td>
<td>25.45%</td>
</tr>
<tr>
<td>1.0 1.5+</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Section 3

Pandemic model
Pandemic model: modeling approach

- The pandemic model projects the additional deaths that are due to potential disease pandemics.

- The approach taken in modeling a severe pandemic is to assume a somewhat regularly occurring spike in deaths that is due to an infectious disease outbreak.

- Historic influenza pandemics were used as a guide in developing the methods and assumptions for the pandemic model.

Frequency and severity calibration

- Actuarial model projecting frequency and severity

  - Frequency
    - Modeled frequency of 7.4% per year
    - Based on 31 epidemics over the last 420 years, resulting in on average 1 event every 14 years

  - Severity
    - The severity curve used is fitted by attaching a probability of exceedance to the percentage of excess mortality as evidenced in a set of historical epidemic events
    - The 1918 severity data is based on US population experience
Modelling severity

- The severity curve is fitted using exponential and tangent functions
- The 1918 data point is placed at the 3.2 percentile level
- Other data points are attached at higher percentile levels corresponding to events of lower severity

<table>
<thead>
<tr>
<th>Pandemic</th>
<th>Actual Percentile</th>
<th>Actual Excess Mortality Percentage</th>
<th>Fitted Excess Mortality Percentage</th>
<th>Fitted Excess Mortality Rate (per 100,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted 1918</td>
<td>0.00%</td>
<td>47.00%</td>
<td>47.00%</td>
<td>200.86</td>
</tr>
<tr>
<td>1918</td>
<td>3.20%</td>
<td>25.50%</td>
<td>22.72%</td>
<td>121.07</td>
</tr>
<tr>
<td>1957</td>
<td>27.80%</td>
<td>1.30%</td>
<td>5.85%</td>
<td>20.45</td>
</tr>
<tr>
<td>2003 SARS</td>
<td>51.60%</td>
<td>0.63%</td>
<td>2.68%</td>
<td>13.64</td>
</tr>
<tr>
<td>1968</td>
<td>75.80%</td>
<td>0.73%</td>
<td>1.45%</td>
<td>7.37</td>
</tr>
<tr>
<td>1977</td>
<td>100.00%</td>
<td>0.60%</td>
<td>0.87%</td>
<td>4.51</td>
</tr>
</tbody>
</table>

Results: pandemic model

- The pandemic model projects larger increases in mortality rates
- However, the probability of extremely large increases is still remote
The objective of the model is to determine whether a potential terrorist attack would cause catastrophic increases in mortality rates.

A lack of historical data for the probability of such an event poses a serious challenge to constructing the model.

As a result, a multi-level trinomial lattice model was used.
Multi-level logic tree

- Multi-level logic tree approach using a trinomial lattice structure with 20 ‘levels’ in total
- Each “level” within the lattice structure is associated with three possible outcomes:
  - ‘Success’ assumes a random number of deaths within a pre-determined range
  - ‘Failure’ indicates no deaths have occurred
  - ‘Escalate’ to the next higher level
- Higher ‘levels’ represent increased severity in terms of deaths with a corresponding increase in the difficulty of achieving a ‘Success’

Frequency of events

- Frequency of terrorist events is chosen from a normal distribution, with an expected number of 27.2 events per quarter and standard deviation of 8.6
- This is based on a total of 730 recorded terrorist occurrences world-wide between 1999 and 2003 from U.S. State Department data

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**Terrorism Model: Number of Reported Terrorist Events**

- Frequency distribution of terrorist events per quarter from 1999 to 2003.

**Terrorism Model: Reported Deaths from Terrorist Events**

- Frequency distribution of terrorist deaths per quarter from 1999 to 2003.
Severity: defining the levels

- The model’s highest level reflects terrorist events causing between 393,217 and 786,432 deaths

- Model parameters were selected by minimizing the sum of the squares of the difference between model and actual number of deaths

<table>
<thead>
<tr>
<th>Level</th>
<th>Lower Range</th>
<th>Upper Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>96</td>
</tr>
<tr>
<td>8</td>
<td>97</td>
<td>192</td>
</tr>
<tr>
<td>9</td>
<td>193</td>
<td>384</td>
</tr>
<tr>
<td>10</td>
<td>385</td>
<td>768</td>
</tr>
<tr>
<td>11</td>
<td>769</td>
<td>1,536</td>
</tr>
<tr>
<td>12</td>
<td>1,537</td>
<td>3,072</td>
</tr>
<tr>
<td>13</td>
<td>3,073</td>
<td>6,144</td>
</tr>
<tr>
<td>14</td>
<td>6,145</td>
<td>12,288</td>
</tr>
<tr>
<td>15</td>
<td>12,289</td>
<td>24,576</td>
</tr>
<tr>
<td>16</td>
<td>24,577</td>
<td>49,152</td>
</tr>
<tr>
<td>17</td>
<td>49,153</td>
<td>98,304</td>
</tr>
<tr>
<td>18</td>
<td>98,305</td>
<td>196,608</td>
</tr>
<tr>
<td>19</td>
<td>196,609</td>
<td>393,216</td>
</tr>
<tr>
<td>20</td>
<td>393,217</td>
<td>786,432</td>
</tr>
</tbody>
</table>

Results: terrorism model

- The terrorism model does not project large enough increases in mortality to be considered catastrophic

<table>
<thead>
<tr>
<th>Mortality Increase Range</th>
<th>Proportion of Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>79.377%</td>
</tr>
<tr>
<td>0.5</td>
<td>17.495%</td>
</tr>
<tr>
<td>1.0</td>
<td>2.238%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.574%</td>
</tr>
<tr>
<td>2.0</td>
<td>0.150%</td>
</tr>
<tr>
<td>2.5</td>
<td>0.100%</td>
</tr>
<tr>
<td>3.0</td>
<td>0.029%</td>
</tr>
<tr>
<td>3.5</td>
<td>0.032%</td>
</tr>
<tr>
<td>4.0+</td>
<td>0.000%</td>
</tr>
</tbody>
</table>
Section 5

Combined results

- Combined results benefit from projected mortality improvements in the baseline model
- Catastrophic mortality increases are largely due to the pandemic model