Mortality Comparisons and Risk Exposures in the Older Age U.S. Financial Services Market

Sponsored by
Committee on Life Insurance Research
Pension Section Research Team
Reinsurance Section
Society of Actuaries

Prepared by
William H. Bowman, FSA, MAAA
and
Roger N. Freeman, ASA, FCIA, FIA, MAAA

December 2011

The opinions expressed and conclusions reached by the authors are their own and do not represent any official position or opinion of the Society of Actuaries or its members. The Society of Actuaries makes no representation or warranty to the accuracy of the information.

© 2011 Society of Actuaries, All Rights Reserved
Table of Contents

Executive Summary ................................................................. 3
Purpose of the Study ............................................................... 5
Methods of Gathering Data ................................................... 6
Methodologies for Analysis of Data and Comparisons ............ 8
Mortality Rate Comparison ...................................................... 12
Reasons for Different Mortality Rates between Products ....... 15
Arbitrage Calculations .......................................................... 18
Arbitrage Exposure for Life Insurance and SPIAs ................. 21
Arbitrage Exposure for SPDAs .............................................. 26
Arbitrage Exposure for Pension Plans .................................... 28
Implications of Arbitrage Exposure ...................................... 29
Conclusions ........................................................................... 32
Acknowledgments ................................................................... 33
Appendix—Participating Companies ...................................... 34
Executive Summary

The Society of Actuaries (SOA) has undertaken research to uncover the differences in mortality expectations between life insurance, annuity and pension products at older issue ages, and to increase awareness of potential impacts that these differences may have on managing the risk assumed for various financial services products in the United States. The products examined include basic life insurance and annuity products, and do not include products like annuities with guaranteed minimum withdrawal benefits or the products developed as alternatives to single premium immediate annuities (SPIAs) by brokerage houses.

The research included surveying insurance companies to ask about their mortality and pricing assumptions for large amount life insurance and annuity products. Fifteen companies participated. Pension mortality for lump sum calculations was not surveyed, since it is now governed by a prescribed table. This table has been included in the analysis and examined along with the annuity and insurance mortality.

The researchers found significant differences between the mortality assumptions in different product lines, and even between different companies within a single product line. Aggregate projected mortality rates are being used by many companies in the pricing assumptions for SPIA and single premium deferred annuity (SPDA) products, compared to the select and ultimate mortality assumptions for life insurance.

There are many legitimate explanations for these mortality rate differences. For example, the markets where a company operates will certainly impact a company’s experience. In addition, the degree of underwriting (if any) has an effect on the assumed mortality rates (and on company expenses as well). Different populations of insureds will be taken into account when preparing mortality assumptions and may lead to differences between mortality rates between companies and between product lines as well.

After a review of individual companies’ mortality assumptions for older issue ages, the researchers conclude that the vectors of mortality rates for insurance and annuity products may be compared through the calculation of a whole life net single premium (NSP).

Widespread differences in mortality expectations between products may give individuals an advantage by acting on inefficiencies in the financial services market. In the most common situation, an individual may purchase an SPIA and use its annual income to pay the gross annual premiums (GAPs) of a life insurance policy. If the individual can do this at a lesser cost than paying a lump sum to the life insurance company, then the companies involved are exposed to arbitrage. Clearly the larger the case size, the more likely arbitrage may be sought and found as such cases are frequently “shopped around.”
Based on the contributing companies, in an extreme case, an 80-year-old male may expect to gain as much as 35 percent of the lump sum he might pay to the life insurance company. Since the degree of arbitrage decreases as issue ages decrease, there are lower exposures at younger ages, but the degree of arbitrage is still significant.

Life insurance companies are generally careful about classifying the risk for older prospective insureds (and especially so for large face amounts), so the potential losses from this arbitrage will likely fall on the annuity issuer. Thus, annuity pricing actuaries need to be very careful about mortality and other assumptions in their products.

Insurance companies also need to review mortality assumptions at older ages to ensure consistency. The data in this study revealed companies where life insurance and annuity mortality are inconsistent, and also where SPIA and SPDA mortality within one company are inconsistent between the two annuity products.

The degree of arbitrage exposure was at its greatest between life insurance and SPIA companies, while some SPDA companies incurred additional exposure due to inconsistent mortality assumptions.

Pension plan mortality is also of interest when considering arbitrage opportunities. For some time now, pension plan mortality for lump sum calculations has been set to a required, specific unisex mortality table for ERISA-covered plans, and that table has on average slightly higher mortality rates than SPDA and SPIA mortality assumptions. The fact that pension benefits are not subject to underwriting (and additionally they are unisex) means that individuals with an option to choose lump sum distributions or choose single life pension benefits versus joint life pension benefits can potentially create a similar arbitrage as for SPIA and SPDA purchasers.

Because a single company may not have credible mortality experience data at ages over 60, it is important to access SOA intercompany mortality studies. If the SOA can increase its focus on older age mortality in all product lines, this will serve the industry well and give both life insurers and annuity issuers access to invaluable current mortality experience data.
Purpose of the Study

The Society of Actuaries proposed a research study in the fall of 2010 to examine mortality assumptions in the older age U.S. financial services market. It questioned whether some older age individuals might gain an advantage from arbitraging one product against another. If individuals are able to select against the financial services industry, then widespread use of this practice may cause financial harm to individual companies or to the industry as a whole.

The objective of this project was threefold:

1. To educate actuaries and other interested parties on the relative differences in older age mortality expectations in insurance, annuity and pension products.

2. To identify possible reasons for the difference in mortality expectations between the financial service products at the older ages.

3. To increase awareness of implications that any differences in mortality expectations can have on managing mortality risks for older age financial services products.

The SOA proposed to examine several currently offered financial services products—life insurance, SPIAs, deferred annuities and defined-benefit pension plans. Since group insurance products are generally available prior to age 65, they have not been included in this analysis.

Different actuaries will certainly have different expectations for future mortality, based on historical experience, underwriting criteria, profit goals and risk tolerance of the company. Different expectations will lead to different prices of financial services products. The magnitude of the difference will determine if arbitrage opportunities are available for a given individual.
Methods of Gathering Data

The researchers, with the assistance of the Project Oversight Group (POG) and the SOA staff, prepared a survey to be sent to leading companies in the life insurance and annuity marketplace. Individual companies who participated will receive a copy of this report describing the overall aggregate results, as well as a report detailing their own company results.

The survey form asked companies to list mortality rates assumed for the following financial services products, if they were currently available for sale:

- **Life insurance:** Assuming the purchase of a $1 million policy for male and female; issued at quinquennial ages between 55 and 80; Best Preferred Nonsmoker class, Standard Nonsmoker class, Standard Smoker class and Substandard Nonsmoker class (assuming Table 8).

- **SPIAs:** Assuming a deposit of $500,000, for male and female; issued at quinquennial ages between 55 and 80.

- **SPDAs:** Assuming the purchase of an immediate annuity at male and female issue ages 60 and 65.

Participating companies were identified only by number, so company names were not included in the data, and anonymity could be assured for participants. Companies were also asked to list their expense assumptions and interest rate assumptions used in pricing these products.

A total of 15 companies participated in the survey. (Please see the Appendix for company names.) Thirteen companies submitted data on their life insurance plans; 13 submitted immediate annuity data; and nine submitted deferred annuity data. The distribution of companies is shown in Table 1:

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Life Insurance</th>
<th>SPIA</th>
<th>SPDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Insurance Only</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPIA Only</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>SPDA Only</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Life Insurance and SPIA</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Life Insurance and SPDA</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>SPIA and SPDA</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>All Three Products</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Total Number of Companies</td>
<td><strong>13</strong></td>
<td><strong>13</strong></td>
<td><strong>9</strong></td>
</tr>
</tbody>
</table>
Mortality rates were received on both an Age Last Birthday and an Age Near Birthday basis. The latter rates were adjusted, so that all rates could be compared on an Age Last Birthday basis.

In addition to data collected through the survey, several recent industry and population mortality tables were added to the analysis for comparison purposes, as indicated in Table 2:

Table 2

<table>
<thead>
<tr>
<th>Table</th>
<th>Source</th>
<th>Notes</th>
<th>Symbol in Charts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008 Valuation Basic Table</td>
<td>Society of Actuaries</td>
<td></td>
<td>VBT</td>
</tr>
<tr>
<td>2006 Experience under Social Security</td>
<td>Social Security Administration</td>
<td>Most recent actual mortality experience</td>
<td>SSA</td>
</tr>
<tr>
<td>2011 Unisex Table</td>
<td>Pension Protection Act of 2006</td>
<td>Used to calculate lump sum benefits</td>
<td>PPA</td>
</tr>
</tbody>
</table>

The VBT and SSA tables project no mortality improvement in future years, although there certainly have been improvements in mortality experience over the last 20 years or more. The PPA table does have between 7 and 15 years of mortality improvement included, depending on the age of the participant. Some of the participating companies may be projecting mortality improvements (and are very likely doing so for annuities), so long-term mortality rates among participating companies may fall below these reference rates for that reason.

All mortality rates were expressed as 1000 qx. 1 While some companies assumed that the rate would rise to 1000 (i.e., no one survives beyond an omega age), others had small numbers of survivors at extreme old ages. To keep all companies on a comparable basis, the omega age was set to 115, so that 55-year-olds had 60 years of assumed rates.

---

1 “qx” represents the proportion of a group aged “x” who die within one year.
Methodologies for Analysis of Data and Comparisons

Having up to 60 years of assumed mortality rates, for three products, two sexes, six issue ages, and a number of companies results in a lot of data! To make meaningful comparisons among companies and products for a given sex/age combination, the calculation of an NSP for a whole life policy is proposed. While this is a common calculation for life insurance, it is not usually discussed in the context of an annuity or pension plan. However, the concept is useful in comparing mortality from all lines of business. The NSP is the present value of the cost of providing $1,000 of life insurance from the issue age until the end of the mortality table, under each product’s mortality assumption.

Net Single Premium Calculation

Starting with rates of 1000 qx, a spreadsheet was constructed to compute 1000 dx,\(^2\) and then 5 percent interest was used in the present value calculation. Deaths were assumed to occur at the end of each year. The assumption of when deaths occur and the use of a 5 percent discount rate are rather arbitrary, but the resulting NSPs are reasonable and are to be used only for comparison purposes.

NSPs were used to review each participating company’s mortality rates for inconsistencies. A few companies were contacted a second time to verify their submissions.

The Excel function RANK was applied to the NSPs for a given product/sex/age combination to order the participating company values. While most companies ranked near the median values, some NSPs fell outside a reasonable range, so the underlying mortality rates were again examined to determine if there might be an error in coding.

The Excel function PERCENTILE was applied to each product/sex/age combination to calculate the 10\(^{th}\) percentile, median and 90\(^{th}\) percentile values of the NSPs. It was felt that the median value would be a good representation of all of the mortality data submitted by the participating companies. In addition, the 10\(^{th}\) and 90\(^{th}\) percentile values would represent the minimum and maximum values, without allowing a reader to see the exact mortality rates being assumed.

The Excel workbook accompanying this report contains a spreadsheet with a chart allowing the user to examine the NSPs for any sex and age. For example, Chart 1 shows

\(^2\) “dx” represents the number of deaths occurring at age “x”.

NSP results for a Male at age 65. Similar charts can easily be derived within the attached workbook.

The three bars of similar height on the left-hand side of the chart for the illustrated example show that most companies have similar mortality assumptions in the Best Nonsmoker class for life insurance, resulting in NSPs in a relatively narrow range around $300 for a $1,000 policy. In contrast, for the Substandard class at this age, NSPs show a wide range, from about $420 at the 10th percentile to about $550 at the 90th percentile.

Chart 1 also allows the user to look at how the mortality assumptions for SPIAs and SPDAs stack up against the assumptions for life insurance. Both annuity products seem to have NSPs falling between the NSPs for the Standard Nonsmoker and the Standard Smoker classes. The implication is that annuity companies are currently using aggregate projected mortality rates in their pricing calculations.

The Pension bar shows that its mortality expectation is similar to that of annuities, although it is slightly higher than the median annuity bars. The Social Security bar shows a level comparable to the median Substandard class, for this age. The Social Security data represent mortality for the overall U.S. population.

**Chart 1**

![Chart 1](image)

Since SPDA data were collected only for ages 60 and 65, the user will see that the SPDA bars disappear from the Excel workbook and chart at other ages.
Since the bars are very close together, in most cases, it may be instructive to look at a table showing the values being represented. For the illustrated example for the Males at age 65,

Table 3

<table>
<thead>
<tr>
<th></th>
<th>10th Percentile</th>
<th>Median</th>
<th>90th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best NS</td>
<td>287.66</td>
<td>304.04</td>
<td>314.59</td>
</tr>
<tr>
<td>Std NS</td>
<td>325.99</td>
<td>344.82</td>
<td>368.86</td>
</tr>
<tr>
<td>Std Smoker</td>
<td>402.24</td>
<td>433.04</td>
<td>463.89</td>
</tr>
<tr>
<td>Substd</td>
<td>428.45</td>
<td>480.00</td>
<td>555.26</td>
</tr>
<tr>
<td>SPIA</td>
<td>374.99</td>
<td>396.02</td>
<td>412.89</td>
</tr>
<tr>
<td>SPDA</td>
<td>352.94</td>
<td>387.79</td>
<td>410.05</td>
</tr>
<tr>
<td>Pension</td>
<td></td>
<td>404.17</td>
<td></td>
</tr>
<tr>
<td>Social Security</td>
<td></td>
<td>463.28</td>
<td></td>
</tr>
</tbody>
</table>

For this particular sex and age, there seems to be a narrow range for the Best Nonsmoker class (only $27.00), while the Substandard class has an extremely large range (of almost $130.00).

It may be easier to see the relationships between the values by comparing them to a standard, such as the population mortality from the Social Security Administration.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>10th Percentile</th>
<th>Median</th>
<th>90th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best NS</td>
<td>62%</td>
<td>66%</td>
<td>68%</td>
</tr>
<tr>
<td>Std NS</td>
<td>70%</td>
<td>74%</td>
<td>80%</td>
</tr>
<tr>
<td>Std Smoker</td>
<td>87%</td>
<td>93%</td>
<td>100%</td>
</tr>
<tr>
<td>Substd</td>
<td>92%</td>
<td>104%</td>
<td>120%</td>
</tr>
<tr>
<td>SPIA</td>
<td>81%</td>
<td>85%</td>
<td>89%</td>
</tr>
<tr>
<td>SPDA</td>
<td>76%</td>
<td>84%</td>
<td>89%</td>
</tr>
<tr>
<td>Pension</td>
<td></td>
<td>87%</td>
<td></td>
</tr>
<tr>
<td>Social Security</td>
<td></td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

In another example, the Excel workbook accompanying this report can be changed to show a chart for another age and sex, in this case Females at age 65 (Chart 2).
Here the Social Security experience is almost equal to the unisex Pension mortality, while the expected SPIA and SPDA mortality is once again midway between the four life insurance classes.
Mortality Rate Comparison

The first objective of the study was “To educate actuaries and other interested parties on the relative differences in older age mortality expectations in insurance, annuity and pension products.”

The ranking of NSPs described in the previous section also allows a comparison of the underlying mortality rates. Since companies were ranked by the size of their NSPs at a given sex and issue age, those same rankings were used in determining the mortality curves for comparison. The 10th percentile, median and 90th percentile values of NSPs are usually determined as interpolated values between the NSPs of two companies. Likewise, the 10th percentile, median and 90th percentile mortality curves use the same interpolation between the same two companies.

For example, say that companies A and B have the 11th- and 12th-ranked highest NSP values among the 13 life insurance companies at a given sex and age. According to the definition of the PERCENTILE function, the 90th percentile value can be obtained by interpolating between the value of the 11th company and of the 12th company. This technique was used to determine the height of the NSP bar, and it was also used to determine the 90th percentile values of the mortality rates.

However, all mortality rates are not created with equal impact on the NSP! Depending on the issue age, early rates (say, for policy years 1 through 5), middle rates (say, for years 10 through 20) and distant rates (say, for years over 20) may have widely varying impacts on the price of a product. Thus, in addition to looking at the values of 1000 qx, the curve of 1000 dx is also shown in the charts accompanying this report.

Sample charts are shown in Chart 3 for a Life Insurance product, Female age 60, for a Nonsmoker not in the Preferred class (i.e., Class 2). The left-hand scale shows the value of 1000 qx, and the right-hand scale shows the value of 1000 dx. The Valuation Basic Table, Social Security Administration table and pension table are added to the chart for comparison purposes.

While the left-hand chart shows 1000 qx values between attained ages 60 and 79, the right-hand chart extends the curves to attained age 100. The scale of 1000 dx is the same between the charts, while the scale of the 1000 qx curve may differ.

---

3 The PERCENTILE function uses the formula 1 + p% * (Count – 1), so for 13 companies, the 90th percentile value is 1 + 0.9 * 12, or 11.8.
The curve of 1000 dx shows the relative “importance” of the mortality rates in the distant years. After around age 95, the curve of 1000 dx begins to trail off, indicating that these long-duration mortality rates have less effect on pricing than, say, the peak ages of 85 to 95 as illustrated below.

Chart 3

A third mortality rate chart for this sex/age combination is shown to the right. Here, the 10th percentile curve and the 90th percentile curve are compared to the median. The 1000 dx curve is also shown again. An interesting aspect of these charts is that 10th percentile curves are not always less than the median, and 90th percentile curves are not always greater than the median. This occurs because, as explained above, companies are ranked on their NSPs, which discount all mortality rates back with 5 percent interest. Quite often, the 90th percentile curve can fall below the median because its early-duration mortality rates are not as
“important” as its later rates; likewise, the 10\textsuperscript{th} percentile curve can exceed the median in long durations because its long-duration rates are not as “important” as its earlier rates.

The Excel workbook accompanying this report allows a user to review the mortality curves for each product/sex/age combination. In the case of life insurance, each underwriting class is another variable. The charts shown here are merely for illustration purposes (and may be difficult to read with so many lines); a review of the accompanying Excel workbook is the best way to analyze the data for each age and sex.
Reasons for Different Mortality Rates between Products

The second objective of the study was “To identify possible reasons for the difference in mortality expectations between the financial service products at the older ages.”

There can be large differences in older age mortality expectations between financial services product lines as well as within a single product line.

For example, historically the mortality for a $5,000 burial expense product is very different from that of a $5 million individual life policy. The socioeconomic status of the insureds, the additional protective underwriting tools utilized for the higher face amount policies, and the health selection criteria for each product contribute to the mortality differences between the products.

Historically, SPIA products issued in the United States largely have not been subject to any underwriting, and a similar relationship between larger policy annuitant mortality and smaller policy annuitant mortality exists. Larger policies exhibit better mortality than smaller policies, and it seems reasonable to assume that socioeconomic status is one factor responsible for this mortality difference. Overall, individuals who purchase annuities on average have better mortality than the population at large—but companies targeting higher socioeconomic level individuals and larger annuity sizes should expect an even greater gap between annuitant mortality and population mortality.

SPDAs are not underwritten, but there is no variation by contract size in mortality levels used for payout options. The contract always specifies the mortality basis to be used and typically is similar to the SPIA mortality. Note that because an SPDA income start date may be deferred many years, the mortality is “locked in” and creates an opportunity to the policyholder who may either take the SPDA plan options that may be many years out of date or cash out and buy an SPIA.

Pension products in the United States, like annuities, are not subject to underwriting. One would expect mortality to follow a similar pattern as SPIAs, where pensioners receiving larger pension benefits would exhibit better mortality than pensioners with smaller pension benefits. Additionally, the mortality experience of pension products could vary materially from plan to plan.

Pension plans have many options “built in” that allow a person to choose cash, single life annuities or joint life annuities based on their health and financial situation. One would expect a higher probability that the healthier lives will take a life annuity option rather than cash, and thus the mortality of that group is better than it would be if no cash option were available.
A product’s target socioeconomic population is one key to explaining differences in mortality by product. Underwriting tools and selection criteria are others. Life insurance companies typically gather more medical data for the highest face amount bands (the largest policy sizes), and these bands have the most favorable mortality and consequently the lowest premium rates.

Insurance companies use medical underwriting to determine the appropriate class for proposed insureds. In this study, we inquired about four classes—Best Nonsmoker, Standard Nonsmoker, Standard Smoker and Substandard for a $1 million life insurance policy. Some companies may have additional classes available. In Chart 4, each curve represents median (typical) mortality assumptions (from our survey companies) associated with each underwriting class for a male aged 65.

The existence of medical underwriting affects mortality rate assumptions between the product lines. Pension and annuity products in the United States generally are not subject to any medical underwriting. However, larger individual life policies generally do have significant underwriting at the older ages, usually based on smoker status, face amount, and medical condition of the insured. The initial difference between the select and ultimate mortality can be very high for individual life policies at the older ages due to this reason.
If an individual purchases a life insurance policy and also purchases an SPIA, note that the SPIA mortality assumptions for this individual from Chart 4 may be quite different in relation to the life insurance mortality assumption.

A male individual at age 65 who has substantial wealth will have a need for life insurance for estate planning purposes. Perhaps his broker will search the market for the very best rates and choose one company offering a $10 million life insurance policy using “Best Nonsmoker” rates. Notice the underlying mortality assumption from Chart 4.

The broker may then suggest to the wealthy individual that he take some of that cash he has lying around in the bank and buy an SPIA to pay the premiums on that $10 million policy. Because the annuity company does not typically underwrite such policies and bases the payout for this individual on a much higher mortality assumption, the individual is able to capitalize on the underlying assumption implicit in that product. An arbitrage opportunity exists and if executed will cost the company “most wrong” on its mortality assumption. Most likely to be hurt is the SPIA company because the life insurance underwriting has generated additional mortality information on which to base its mortality expectation. Even if it is the same company issuing the SPIA and life insurance policy, the life underwriting information will not likely be used to adjust the SPIA mortality assumption.

To the extent that pricing is based on experience studies, and experience studies are performed regularly, then if companies properly take into account the impact of these large cases on their mortality rates one might argue that no one gets hurt, as each portfolio will look after itself. But, historical mortality results for annuity writers are dependent on the historic mix of large annuity policies. An annuity writer is at risk for this mix changing over time. Also in reality, internal company mortality study data (especially for annuities) is thin at higher ages. The impact of a changing mix of large cases may be very significant in the long run, but will not be reflected in the bottom line nor in the mortality studies until long after life expectancies are exceeded! With no monitoring in place for changes in mix, an annuity writer is at risk for adverse changes in mortality.

If the SPIA portfolio on the books of an annuity company is very large, perhaps the company will not notice the impact of a few arbitrated cases. But prevention is far better than a cure, so surely a better management approach is to implement rules based on size of the SPIA and to underwrite larger cases. Unfortunately, this rarely has happened.
Arbitrage Calculations

The third objective of the study was “To increase awareness of implications that any differences in mortality expectations can have on managing mortality risks for older age financial services products.”

Although the comparisons of mortality curves and NSPs give a good snapshot of the financial services industry’s current assumptions for mortality in different product lines, another purpose of the study was to determine if arbitrage exposures exist between product lines.

Arbitrage Potential

An individual has at least two methods of purchasing a life insurance policy:

1. Pay a periodic premium for the life insurance, or
2. Borrow money to purchase an SPIA to pay the premiums on the life insurance.

If method (2) is less “costly” than method (1), then an arbitrage opportunity is deemed to exist—that is, the individual may be able to take advantage of the inefficiencies in the financial services market. Note that an individual purchasing an SPIA in combination with a life insurance policy will have a poorer financial result should death occur in the early durations; however, over an entire portfolio of lives, the net death benefit will be higher.

A similar situation may occur when an individual purchasing an SPDA exercises an option to convert that annuity into a stream of income, and then uses that income to purchase a life insurance policy. Also, a person receiving a life income from a defined-benefit pension plan may gain an advantage in the purchase of life insurance. Although arbitrage exposure exists for these two situations, most of the analysis below looks at the life insurance and SPIA situation.

To begin this examination, gross premiums for the various products need to be approximated.

Life Insurance—Gross Annual Premium

For life insurance, 11 of the 13 companies submitted data on their expense assumptions used in the current pricing of their products. Expenses are expressed in terms of percentages of premium in first year and renewal years, and dollars per $1,000 of insurance in first year and renewal years. The average expenses for all 11 companies is 130 percent of premium in the first year, 8 percent of premium in renewal years, $1,000
per policy (or, $1.00 per $1,000 of insurance) in the first year, and $280 per policy (or, $0.28 per $1,000) in renewal years.

The life insurance gross premium is assumed to be an annual premium, payable at the beginning of each year, so an annuity due must be calculated. An “lx”\(^4\) table is needed, and can be derived from the table of dx values, or from the qx values. Another approach, giving the same result, is to solve for an annuity due in the relationship:

\[
\text{Net Single Premium for $1 of whole life insurance} = 1 - (\text{discount rate}) \times \frac{\text{Annuity Due}}{} 
\]

The present value of the $1 million insurance policy expenses can now be expressed as:

\[
\text{Expenses} = (1.22 + 0.08 \times \text{annuity due}) \times \text{Gross Annual Premium} + (720 + 280 \times \text{annuity due})
\]

The GAP must be sufficient to cover the cost of the insurance coverage and the expenses. The final step in determining the GAP is to rearrange terms, giving

\[
\text{GAP} = \frac{(\text{Net Single Premium} + 720 + 280 \times \text{annuity due})}{(\text{Annuity due} \times 0.92 - 1.22)}
\]

Note that this GAP is a hypothetical figure, since average company expenses are used, instead of each participating company’s own expense assumptions. The effect of possible mortality arbitrage can be examined, since mortality is the only assumption that differs between the companies surveyed.

Four life insurance classes are considered: Best Nonsmoker, Standard Nonsmoker, Standard Smoker and Substandard. Of course, the arbitrage opportunity (or exposure from the issuing company’s perspective) is greatest for the best insurance class.

\textit{Note also that no profit margin has been assumed in determining a GAP. The reader may wish to adjust results of the arbitrage calculations by the amount of an assumed profit margin.}

\textbf{SPIA Gross Premium}

Eight of the 13 companies submitting immediate annuity mortality also submitted their pricing assumptions for annuities with a $500,000 deposit. In general, these assumptions are much simpler than those for the life insurance product. Although some companies use both percentage of premium and per policy figures for expenses, the average expense of the eight companies is 6 percent of the single premium, with no addition for per policy expense.

\textit{lx} represents the number of persons living at age x; since “dx” represents the number dying at that age, “qx” is dx divided by lx.
Thus, the calculation of the gross premium for an SPIA is simply 106 percent times an annuity due, using a company’s own mortality assumptions and 5 percent interest. As was the case for the life insurance calculation, no provision is made for a profit margin.

**Equivalent SPIA Cost**

Another special calculation can be considered the “Equivalent SPIA Cost” for a life insurance policy. This is the lump sum a purchaser would have to pay to cover all the future GAPs for the insurance policy. It is simply the annuity due, calculated for the life insurance policy above, multiplied by the GAP.

Although very few single premium life insurance policies are sold in the United States today, due to taxation issues, the “Equivalent SPIA Cost” represents a single sum that a purchaser would need to pay for the life insurance policy. Perhaps this sum would be taken from a trust or be borrowed in a lump sum, if one desired to pay for life insurance in this way, and it does allow the use of a single figure for comparison to annuities in the marketplace.
Arbitrage Exposure for Life Insurance and SPIAs

In a potential arbitrage situation, the purchaser of a $1 million life insurance policy may, on the one hand, pay the “Equivalent SPIA Cost” calculated above. That single payment would pay all future life insurance GAPs under the assumptions given. On the other hand, he or she may decide to use funds to purchase an SPIA, and then take the resulting annual annuity income and use it to pay GAPs for the life insurance. If the latter transaction can be made at a smaller cost than the former, then the purchaser has gained an advantage.

The degree of arbitrage is calculated as the difference (“Equivalent SPIA Cost” less SPIA deposit) divided by “Equivalent SPIA Cost.” If the difference is zero or negative, there is no arbitrage exposure.

From a life insurance company perspective, a company may take its premiums for the Best Nonsmoker class, determine the “Equivalent SPIA Cost” figure, and then examine all available SPIA products to see if there is any arbitrage exposure.

This calculation was made for each life insurance company, for both sexes and for each issue age; and the degree of arbitrage percentages were compiled. Then, for each issue age, the degree of arbitrage was ranked from highest to lowest. Along the bottom (x axis) of the charts, companies are ranked from first (highest) to last (lowest) in terms of degree of arbitrage.

Maximum Arbitrage Exposure

A chart showing the maximum degree of arbitrage from the life insurance company perspective is shown in Chart 5. For males, the maximum arbitrage is 35 percent for issue age 80; for females, the maximum is 30 percent.
From the perspective of the 13 life insurance companies participating in this survey, the maximum arbitrage for a male at issue age 65 would range between 16 and 20 percent. This means that a purchaser at this age could maximize his opportunity at 20 percent.

Note also that no profit margin has been assumed in determining premiums. The reader may wish to adjust results of the arbitrage calculations by the amount of an assumed profit margin for the two products. For example, if the user assumes that the life insurance has a 4 percent of premium profit margin and the SPIA a 2 percent profit margin, then it would be reasonable to deem anything over 6 percent as an exposure to risk.

For this particular situation, one life insurance company would charge a hypothetical GAP for the $1 million of coverage of $22,500 for an insured in the Best Nonsmoker class. Multiplying this by the calculated annuity due, the “Equivalent SPIA Cost” would be $340,000. The annuity company with the most favorable SPIA rate at this sex and age would charge a single premium of $12,100 for an annuity paying $1,000 annually. Since the purchaser needs to have annual income of $22,500, the cost of that SPIA will be $272,000. The difference, $68,000, represents 20 percent of the “Equivalent SPIA Cost” of $340,000.

The chart shows a very clear pattern of an increasing degree of arbitrage based on issue age. Pricing actuaries for both life insurance and annuity products need to pay particular attention to the mortality assumptions at these advanced ages, and need to check their consistency between different product lines within their own companies. The actuary will
not be able to control the prices of products outside his company, but he or she should attempt to reduce opportunities within a single company.

**Arbitrage Exposure at 90\textsuperscript{th} Percentile**

The calculation of the maximum arbitrage exposure highlights the maximum risk to which a life insurance company might be exposed. However, there is a fear that one result (an “outlier”) may distort the true degree of exposure. Thus, a 90\textsuperscript{th} percentile degree of arbitrage and a median degree of arbitrage were also calculated.

**Chart 6**

This 90\textsuperscript{th} percentile chart still shows a significant degree of arbitrage for the upper issue ages, so any “outliers” in the calculations are not having too drastic an effect on the results.

**Median Arbitrage**

Since a company might be subjected to arbitrage only rarely at the maximum or 90\textsuperscript{th} percentile level, a separate chart shows the median amount of arbitrage exposure a life company might experience. For the males at age 65, the range is from 11 to 7 percent.
The Excel workbook accompanying this report allows a user to specify the two products to be examined for each sex. Each issue age will be represented by a different line on a chart. The maximum degree of arbitrage, the 90th percentile rankings, and a median degree of arbitrage can be seen.

**Changing the Company Perspective**

From the standpoint of an SPIA company, its products may be subject to arbitrage in the purchase of a life insurance product if a purchaser can gain an advantage by using the SPIA to buy life insurance. A similar calculation has been made to rank the degree of arbitrage for each of the 13 SPIA companies in the survey.

The arbitrage percentages are ranked from highest to lowest, and then displayed in a chart (for maximum, 90th percentile and median). The highest degree of arbitrage for an SPIA company is 20 percent for a male at issue age 65, as shown in Chart 8.
Chart 8

MAXIMUM ARBITRAGE - SPIA COMPANY
PERSPECTIVE - MALES
Arbitrage Exposure for SPDAs

One way that an individual may take advantage of an arbitrage opportunity is to purchase a SPDA and then immediately turn that annuity into an income stream to purchase life insurance. In other words, the SPDA purchaser may gain a marginal advantage over an SPIA purchaser if the cost of the SPDA is less than the cost of the SPIA.

In four cases of the eight companies who submitted both SPDA and SPIA data, we found that the age-65 purchaser of the SPDA gained an advantage within the same company. For males, the average additional gain was about 5 percent, and for females the additional gain was about 4 percent. For these calculations, we assumed the same expense levels for the SPDA as for the SPIA, and assumed no profit margins, so those assumptions may need modification for an individual company.

When the analysis is extended beyond the possible internal advantage within the same company, many other situations occur, leading to additional maximum arbitrage exposure as high as 15 percent.

Charts 9 and 10 show the additional arbitrage exposure for 65-year-olds, from an SPDA company perspective. Males have slightly more additional arbitrage than females.

**Chart 9**

![Maximum Arbitrage--SPDA vs. SPIA--Males at 65](image)
Chart 10

Maximum Arbitrage--SPDA vs. SPIA--Females at 65

SPDA Company Ranking

If a male at age 65 chooses an SPDA company offering a 15 percent additional arbitrage exposure, he may be able to add that figure to the 20 percent arbitrage exposure at the SPIA company, and receive a compounded arbitrage benefit of 35 percent.
Arbitrage Exposure for Pension Plans

An individual reaching retirement age may have several opportunities to take advantage of differences in mortality assumptions. If he or she is in good health, it would be possible to elect a single life annuity (instead of a joint life annuity) and use the excess income to purchase life insurance. The life insurance can then provide income to the joint annuitant at a greater level than if the joint life annuity had been elected. If healthy risks were to choose the single life annuity and poor risks are left with joint annuities, then both single life and joint life annuities might be negatively affected.

In the case of the 2011 unisex pension table, we assumed a retiree may wish to convert his or her stream of income into a lump sum, and then use that sum to purchase an SPIA. For this calculation, we assumed no expenses in the pension plan and only the 5 percent discount rate. The results showed that a male at age 65 would have an advantage at only one of the 13 SPIA companies, while a female at age 65 would not be able to gain an incremental advantage with this technique.

Chart 11

The U.S. government updates the pension lump sum mortality table regularly. To the extent companies update their SPIA mortality assumptions less frequently, this could increase potential arbitrage exposures.
Implications of Arbitrage Exposure

The pricing and marketing of a new life insurance product or a new annuity product cannot be done in a vacuum. Pricing actuaries must be alert to other products in the marketplace and the impact they might have on the new product being introduced.

Annuity products must take into account life insurance products in the marketplace; otherwise, the annuity company may unwittingly increase its exposure to arbitrage. If the annuity company, for instance, encourages its producers to write annuities with million-dollar deposits, and yet it does no underwriting and assigns Standard Smoker mortality to its annuitants, it will increase its exposure.

Insurers in the United Kingdom typically underwrite their SPIA sales, giving “enhanced” payouts to those annuitants considered substandard. The annuity market in the United Kingdom is large, since tax legislation requires a portion of one’s retirement benefit be invested in a lifetime annuity. Even with a large market and underwritten annuity sales, U.K. insurers are exposed to arbitrage, so insurers there must also stay alert to attempts to reap an advantage from mortality assumption differences. The practice of medically underwriting SPIAs has also been developing in the United States (see the 2006 SOA report “Substandard Annuities”). By implication, growth in the substandard market will leave the residual market with improving mortality, and, if ignored, will impact profit margins in the “regular” SPIA market.

As a “worst-case” scenario, we will look at males at issue age 80, since we saw above that the arbitrage exposure is at its highest for the oldest issue ages.

The company with the lowest NSP for a Best Nonsmoker male at issue age 80 has an NSP of $472.00 for $1,000 of whole life coverage. This is 92 percent of the median value of $511.00, so the figure does not seem out of line.

Chart 12

Of course, at this issue age, life expectancy is not as long as at younger ages, and Chart 12 displays only 20 years of mortality curves. Here we can see that this company assumes mortality rates less than the median rates for all years, and less than the 10th percentile rates from ages 86 to 99.
Following the calculations for this company, its assumed GAP for the $1 million life insurance policy is $53,000, and its “Equivalent SPIA Cost” is $588,000.

By contrast, one annuity company has assumed a rather short life expectancy for males at age 80, with high mortality rates. Using these mortality assumptions, its NSP is $678.00 for a $1,000 whole life policy, or about 44 percent more than the life insurer is assuming.

The mortality rates for this company are far above the median and 90th percentile values for all SPIA companies in the survey, as shown in Chart 13.

**Chart 13**

The result of this mortality curve and our assumed SPIA expenses is a gross premium for the annuity of $7,200 at this age. Since the life insurance company will charge $53,000 for its GAP, the needed deposit for the annuity in this case is about $382,000, a 35 percent discount from the “Equivalent SPIA Cost” that the life insurance company assumes.

Thus, we are looking at a situation that maximizes the arbitrage exposure for these companies, and the question is: Which company’s mortality expectations for the purchaser are more likely to be realized? Since the life insurance company has underwritten the risk to classify the insured in the Best Nonsmoker class, and the SPIA company has not likely done that, it seems that the SPIA company’s mortality assumptions will more often not be realized, and it will see losses from this transaction.

Another situation, but no less disturbing, comes when a single company prices its life insurance and annuity products in “silos,” where neither side realizes what assumptions the other is making. This can be demonstrated by looking at the calculations (again, for males at age 80) for a different company (not used in the arbitrage calculations above). In this case, the company’s NSP for whole life in the Best Nonsmoker class is $489.00 for a $1,000 policy, which is 96 percent of the median value on that measure. Its NSP for its SPIA at that age is $628.00, equal to the median value.
The mortality rates themselves show the potential problem.

Although this company’s SPIA rates are the median rates for all companies, its own life insurance mortality assumptions are far below the median life insurance company assumption. In addition, it has not recognized that its assumptions are inconsistent internally.

Chart 14

Unfortunately, this company may realize too late that its mortality assumptions in the two product lines are inconsistent, so it may suffer financial losses before it can correct the situation.
Conclusions

Given the amount of potential arbitrage exposure at older ages, pricing actuaries need to pay particular attention to mortality assumptions and risk selection at these ages. In the charts shown above, arbitrage exposure almost always increases as the issue age increases. Unfortunately, life insurance pricing at older ages may be under-emphasized because the potential for sales may be limited. This could be a mistake, as the potential for significant financial losses increases as the issue age increases.

Annuity pricing actuaries need to reassess mortality assumptions at older issue ages. In a situation with potential arbitrage, it is most likely that the product line with the least amount of selection at issue (annuities and pensions) will be the product line most at risk for losses. The losses may not be “realized” for many years, but they may be significant when they occur.

In addition to reviewing mortality assumptions between the life insurance and annuity lines, companies also need to address potential inconsistent assumptions within the annuity line, for SPDA and SPIA annuities.

The greater the age, the more potential for arbitrage, so classification of the risk at higher ages is an important consideration. As a corollary, expense assumptions at older ages may need to be re-examined, as pinning down the mortality risk may be more costly than current average assumptions for underwriting costs.

Companies should be wary of increasing life insurance issue ages much above 80, especially for larger policies. In addition, they should examine all assumptions, not just mortality, at these upper issue ages, if they wish to sell both life insurance and annuities.

Single-company mortality studies at ages over 60 may not have sufficient data to be credible, so it is important for actuaries to access SOA intercompany studies. The SOA should increase its focus on older age mortality (say, ages 60 to 100) in all product lines. Given the risk from arbitrage exposure at high issue ages, this project should be a priority for the SOA and SOA sections.
Acknowledgments

The researchers wish to acknowledge the guidance and feedback of the members of the Project Oversight Group (POG), and the staff members of the SOA, who assisted in the review of these materials and advised us on the contents of this report.

**Project Oversight Group**
- Clark Himmelberger, Chair
- Gavin Benjamin
- Ing Chian Ching
- Jean-Marc Fix
- Jeffrey C. Harper
- Edward Hui
- Sebastian Kleber
- Stephen Manley

**Society of Actuaries**
- Jack Luff, Experience Studies Actuary
- Ronora Stryker, Research Actuary
- Jan Schuh, Research Administrator
Appendix—Participating Companies

The researchers and the SOA thank the companies participating in our survey and the actuaries who sent us the details of their mortality assumptions.

**Participating Companies**

<table>
<thead>
<tr>
<th>Company</th>
<th>Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allstate Insurance</td>
<td>Nationwide Financial</td>
</tr>
<tr>
<td>Aviva</td>
<td>Northwestern Mutual Life</td>
</tr>
<tr>
<td>Federated Life</td>
<td>Penn Mutual Life</td>
</tr>
<tr>
<td>Genworth Financial</td>
<td>Principal Financial Group</td>
</tr>
<tr>
<td>Guardian Life</td>
<td>Protective Life</td>
</tr>
<tr>
<td>Hartford Life</td>
<td>Prudential Insurance</td>
</tr>
<tr>
<td>Kansas City Life</td>
<td>Western &amp; Southern Financial</td>
</tr>
<tr>
<td>Lincoln Financial Group</td>
<td></td>
</tr>
</tbody>
</table>