SOCIETY OF ACTUARIES

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE SOLUTIONS

This set of sample questions includes those published on the interest theory topic for use with previous versions of this examination. In addition, the following have been added to reflect the revised syllabus beginning June 2017:

- Questions 155-158 on interest rate swaps have been added. Questions 155-157 are from the previous set of financial economics questions. Question 158 is new.
- Questions 66, 178, 187-191 relate to the study note on approximating the effect of changes in interest rates.
- Questions 185-186 and 192-195 relate to the study note on determinants of interest rates.
- Questions 196-202 on interest rate swaps were added.

March 2018 – Question 157 has been deleted.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

The following model solutions are presented for educational purposes. Alternative methods of solution are, of course, acceptable.

In these solutions, s_m is the *m*-year spot rate and ${}_m f_t$ is the *m*-year forward rate, deferred *t* years.

FM-10-17

1. Solution: C

Given the same principal invested for the same period of time yields the same accumulated value, the two measures of interest $i^{(2)} = 0.04$ and δ must be equivalent, which means: $\left(1+\frac{i^{(2)}}{2}\right)^2 = e^{\delta}$ over a one-year period. Thus,

$$e^{\delta} = \left(1+\frac{i^{(2)}}{2}\right)^2 = 1.02^2 = 1.0404$$

$$\delta = \ln(1.0404) = 0.0396.$$

2. Solution: E

From basic principles, the accumulated values after 20 and 40 years are

$$100[(1+i)^{20} + (1+i)^{16} + \cdots + (1+i)^4] = 100\frac{(1+i)^4 - (1+i)^{24}}{1-(1+i)^4}$$

$$100[(1+i)^{40} + (1+i)^{36} + \cdots + (1+i)^4] = 100\frac{(1+i)^4 - (1+i)^{44}}{1-(1+i)^4}.$$

The ratio is 5, and thus (setting $x = (1+i)^4$)

$$5 = \frac{(1+i)^4 - (1+i)^{44}}{(1+i)^4 - (1+i)^{24}} = \frac{x - x^{11}}{x - x^6}$$

$$5x - 5x^6 = x - x^{11}$$

$$5 - 5x^5 = 1 - x^{10}$$

$$x^{10} - 5x^5 + 4 = 0$$

$$(x^5 - 1)(x^5 - 4) = 0.$$

Only the second root gives a positive solution. Thus

$$x^5 = 4$$

$$x = 1.31951$$

$$X = 100\frac{1.31951 - 1.31951^{11}}{1-1.31951} = 6195.$$

Annuity symbols can also be used. Using the annual interest rate, the equation is

$$100\frac{s_{\overline{40}|}}{a_{\overline{4}|}} = 5(100)\frac{s_{\overline{20}|}}{a_{\overline{4}|}}$$

$$\frac{(1+i)^{40}-1}{i} = 5\frac{(1+i)^{20}-1}{i}$$

$$(1+i)^{40} - 5(1+i)^{20} + 4 = 0$$

$$(1+i)^{20} = 4$$

and the solution proceeds as above.

3. Solution: C

Eric's (compound) interest in the last 6 months of the 8th year is $100\left(1+\frac{i}{2}\right)^{15}\frac{i}{2}$.

Mike's (simple) interest for the same period is $200\frac{i}{2}$.

Thus,

$$100\left(1+\frac{i}{2}\right)^{15}\frac{i}{2} = 200\frac{i}{2}$$

$$\left(1+\frac{i}{2}\right)^{15} = 2$$

$$1+\frac{i}{2} = 1.047294$$

$$i = 0.09459 = 9.46\%.$$

4. Solution: A

The periodic interest is 0.10(10,000) = 1000. Thus, deposits into the sinking fund are 1627.45-1000 = 627.45.

Then, the amount in sinking fund at end of 10 years is $627.45 s_{\overline{10}|0.14} = 12{,}133$. After repaying the loan, the fund has 2,133, which rounds to 2,130.

5. Solution: E

The beginning balance combined with deposits and withdrawals is 75 + 12(10) – 5 – 25 – 80 – 35 = 50. The ending balance of 60 implies 10 in interest was earned.

The denominator is the average fund exposed to earning interest. One way to calculate it is to weight each deposit or withdrawal by the remaining time:

$$75(1)+10\left(\frac{11}{12}+\frac{10}{12}+\cdots+\frac{0}{12}\right)-5\frac{10}{12}-25\frac{6}{12}-80\frac{5}{24}-35\frac{2}{12}=90.833.$$

The rate of return is 10/90.833 = 0.11009 = 11.0%.

6. Solution: C

$$\begin{aligned}77.1 &= v\left(Ia\right)_{\overline{n}|}+\frac{nv^{n+1}}{i}\\ &= v\left[\frac{\ddot{a}_{\overline{n}|}-nv^{n}}{i}\right]+\frac{nv^{n+1}}{i}\\ &= \frac{a_{\overline{n}|}}{i}-\frac{nv^{n+1}}{i}+\frac{nv^{n+1}}{i}\\ &= \frac{a_{\overline{n}|}}{i}=\frac{1-v^{n}}{i^{2}}=\frac{1-v^{n}}{0.011025}\end{aligned}$$

$$0.85003=1-v^{n}$$

$$1.105^{-n}=0.14997$$

$$n=-\frac{\ln(0.14997)}{\ln(1.105)}=19.$$

To obtain the present value without remembering the formula for an increasing annuity, consider the payments as a perpetuity of 1 starting at time 2, a perpetuity of 1 starting at time 3, up to a perpetuity of 1 starting at time n + 1. The present value one period before the start of each perpetuity is 1/i. The total present value is $(1/i)(v+v^{2}+\cdots+v^{n})=(1/i)a_{\overline{n}|}$.

7. Solution: C

The interest earned is a decreasing annuity of 6, 5.4, etc. Combined with the annual deposits of 100, the accumulated value in fund Y is

$$\begin{aligned}
&6(Ds)_{\overline{10}|0.09} + 100 s_{\overline{10}|0.09} \\
&= 6\left(\frac{10(1.09)^{10} - s_{\overline{10}|0.09}}{0.09}\right) + 100(15.19293) \\
&= 565.38 + 1519.29 \\
&= 2084.67.
\end{aligned}$$

8. Deleted

9. Solution: D

For the first 10 years, each payment equals 150% of interest due. The lender charges 10%, therefore 5% of the principal outstanding will be used to reduce the principal.

At the end of 10 years, the amount outstanding is $1000(1-0.05)^{10} = 598.74$.

Thus, the equation of value for the last 10 years using a comparison date of the end of year 10 is

$$\begin{aligned}
598.74 &= X a_{\overline{10}|10\%} = 6.1446X \\
X &= 97.44.
\end{aligned}$$

10. Solution: B

The book value at time 6 is the present value of future payments:

$$BV_6 = 10,000v^4 + 800a_{\overline{4}|\,0.06} = 7920.94 + 2772.08 = 10,693.$$

The interest portion is 10,693(0.06) = 641.58.

11. Solution: A

The value of the perpetuity after the fifth payment is 100/0.08 = 1250. The equation to solve is:

$$\begin{aligned}
1250 &= X(v + 1.08v^2 + \cdots + 1.08^{24}v^{25}) \\
&= X(v + v + \cdots + v) = X(25)/1.08 \\
X &= 50(1.08) = 54.
\end{aligned}$$

12. Solution: C

Equation of value at end of 30 years:

$$10(1-d/4)^{-40}(1.03)^{40}+20(1.03)^{30}=100$$

$$10(1-d/4)^{-40}=[100-20(1.03)^{30}]/1.03^{40}=15.7738$$

$$1-d/4=1.57738^{-1/40}=0.98867$$

$$d=4(1-0.98867)=0.0453=4.53\%.$$

13. Solution: E

The accumulation function is $a(t)=\exp\left[\int_0^t (s^2/100)ds\right]=\exp(t^3/300).$

The accumulated value of 100 at time 3 is $100\exp(3^3/300)=109.41743.$

The amount of interest earned from time 3 to time 6 equals the accumulated value at time 6 minus the accumulated value at time 3. Thus

$$\left(109.41743+X\right)[a(6)/a(3)-1]=X$$

$$(109.41743+X)(2.0544332/1.0941743-1)=X$$

$$(109.41743+X)0.877613=X$$

$$96.026159=0.122387X$$

$$X=784.61.$$

14. Solution: A

$$167.50=10a_{\overline{5}|9.2\%}+10(1.092)^{-5}\sum_{t=1}^{\infty}\left[\frac{(1+k)}{1.092}\right]^t$$

$$167.50=38.86955+6.44001\frac{(1+k)/1.092}{1-(1+k)/1.092}$$

$$(167.50-38.86955)[1-(1+k)/1.092]=6.44001(1+k)/1.092$$

$$128.63045=135.07046(1+k)/1.092$$

$$1+k=1.0399$$

$$k=0.0399\Rightarrow K=3.99\%.$$

15. Solution: B

Option 1: $2000 = Pa_{\overline{10}|0.0807}$

$P = 299 \Rightarrow$ Total payments $= 2990$

Option 2: Interest needs to be $2990 - 2000 = 990$

$990 = i[2000 + 1800 + 1600 + \cdots + 200]$

$= 11,000i$

$i = 0.09 = 9.00\%$

16. Solution: B

Monthly payment at time t is $1000(0.98)^{t-1}$.

Because the loan amount is unknown, the outstanding balance must be calculated prospectively. The value at time 40 months is the present value of payments from time 41 to time 60:

$$OB_{40} = 1000[0.98^{40}v^1 + \cdots + 0.98^{59}v^{20}]$$

$$= 1000\frac{0.98^{40}v^1 - 0.98^{60}v^{21}}{1 - 0.98v}, \; v = 1/(1.0075)$$

$$= 1000\frac{0.44238 - 0.25434}{1 - 0.97270} = 6888.$$

17. Solution: C

The equation of value is

$$98S_{\overline{3n}|} + 98S_{\overline{2n}|} = 8000$$

$$\frac{(1+i)^{3n} - 1}{i} + \frac{(1+i)^{2n} - 1}{i} = 81.63$$

$$(1+i)^n = 2$$

$$\frac{8-1}{i} + \frac{4-1}{i} = 81.63$$

$$\frac{10}{i} = 81.63$$

$$i = 12.25\%$$

18. Solution: B

Convert 9% convertible quarterly to an effective rate of *j* per month:

$(1+j)^3 = \left(1+\frac{0.09}{4}\right)$ or $j = 0.00744$.

Then

$$2(Ia)_{\overline{60}|0.00744} = 2\frac{\ddot{a}_{\overline{60}|0.00744} - 60v^{60}}{0.00744} = 2\frac{48.6136-38.4592}{0.00744} = 2729.7.$$

19. Solution: C

For Account K, the amount of interest earned is 125 – 100 – 2X + X = 25 – X.

The average amount exposed to earning interest is 100 – (1/2)X + (1/4)2X = 100. Then

$$i = \frac{25-X}{100}.$$

For Account L, examine only intervals separated by deposits or withdrawals. Determine the interest for the year by multiplying the ratios of ending balance to beginning balance. Then

$$i = \frac{125}{100}\frac{105.8}{125-X} - 1.$$

Setting the two equations equal to each other and solving for X,

$$\frac{25-X}{100} = \frac{13,225}{100(125-X)} - 1$$

$$(25-X)(125-X) = 13,225 - 100(125-X)$$

$$3,125 - 150X + X^2 = 13,225 - 12,500 + 100X$$

$$X^2 - 250X + 2,400 = 0$$

$$X = 10.$$

Then *i* = (25 – 10)/100 = 0.15 = 15%.

20. Solution: A

Equating present values:

$$100 + 200v^n + 300v^{2n} = 600v^{10}$$
$$100 + 200(0.76) + 300(0.76)^2 = 600v^{10}$$
$$425.28 = 600v^{10}$$
$$0.7088 = v^{10}$$
$$0.96617 = v$$
$$1.03501 = 1 + i$$
$$i = 0.035 = 3.5\%.$$

21. Solution: A

The accumulation function is:

$$a(t) = e^{\int_0^t \frac{1}{8+r}dr} = e^{\ln(8+r)\big|_0^t} = \frac{8+t}{8}.$$

Using the equation of value at end of 10 years:

$$20,000 = \int_0^{10} \left(8k + tk\right) \frac{a(10)}{a(t)} dt = k\int_0^{10} (8+t) \frac{18/8}{(8+t)/8} dt = k\int_0^{10} 18dt$$
$$= 180k \Rightarrow k = \frac{20,000}{180} = 111.$$

22. Solution: D

Let C be the redemption value and $v = 1/(1+i)$. Then

$$X = 1000ra_{\overline{2n}|i} + Cv^{2n}$$
$$= 1000r\frac{1-v^{2n}}{i} + 381.50$$
$$= 1000(1.03125)(1-0.5889^2) + 381.50$$
$$= 1055.11.$$

23. Solution: D

Equate net present values:

$$-4000 + 2000v + 4000v^2 = 2000 + 4000v - Xv^2$$
$$\frac{4000 + X}{1.21} = 6000 + \frac{2000}{1.1}$$
$$X = 5460.$$

24. Solution: E

For the amortization method, the payment is determined by

$$20,000 = Xa_{\overline{20}|0.065} = 11.0185, \quad X = 1815.13.$$

For the sinking fund method, interest is 0.08(20,000) = 1600 and total payment is given as X, the same as for the amortization method. Thus the sinking fund deposit = X – 1600 = 1815.13 – 1600 = 215.13.

The sinking fund, at rate j, must accumulate to 20,000 in 20 years. Thus, $215.13s_{\overline{20}|j} = 20,000$, which yields (using calculator) j = 14.18%.

25. Solution: D

The present value of the perpetuity = X/i. Let B be the present value of Brian's payments.

$$B = Xa_{\overline{n}|} = 0.4\frac{X}{i}$$

$$a_{\overline{n}|} = \frac{0.4}{i} \Rightarrow 0.4 = 1 - v^n \Rightarrow v^n = 0.6$$

$$K = v^{2n}\frac{X}{i}$$

$$K = 0.36\frac{X}{i},$$

Thus the charity's share is 36% of the perpetuity's present value.

26. Solution: D

The given information yields the following amounts of interest paid:

$$\text{Seth} = 5000\left(\left(1+\frac{0.12}{2}\right)^{10} - 1\right) = 8954.24 - 5000 = 3954.24$$

$$\text{Janice} = 5000(0.06)(10) = 3000.00$$

$$\text{Lori} = P(10) - 5000 = 1793.40 \text{ where } P = \frac{5000}{a_{\overline{10}|6\%}} = 679.35$$

The sum is 8747.64.

27. Solution: E

For Bruce, $X = 100[(1+i)^{11} - (1+i)^{10}] = 100(1+\mathrm{i})^{10} i$. Similarly, for Robbie, $X = 50(1+i)^{16} i$ .Dividing the second equation by the first gives $1 = 0.5(1+i)^6$ which implies $i = 2^{1/6} - 1 = 0.122462$. Thus $X = 100(1.122462)^{10}(0.122462) = 38.879$.

28. Solution: D

Year t interest is $ia_{\overline{n-t+1}|i} = 1 - v^{n-t+1}$.

Year t+1 principal repaid is $1-(1-v^{n-t}) = v^{n-t}$.

$X = 1 - v^{n-t+1} + v^{n-t} = 1 + v^{n-t}(1-v) = 1 + v^{n-t} d.$

29. Solution: B

For the first perpetuity,

$32 = 10(v^3 + v^6 + \cdots) = 10v^3 / (1 - v^3)$

$32 - 32v^3 = 10v^3$

$v^3 = 32/42.$

For the second perpetuity,

$X = v^{1/3} + v^{2/3} + \cdots = v^{1/3} / (1 - v^{1/3}) = (32/42)^{1/9} / [1 - (32/42)^{1/9}] = 32.599.$

30. Solution: D

Under either scenario, the company will have 822,703(0.05) = 41,135 to invest at the end of each of the four years. Under Scenario A these payments will be invested at 4.5% and accumulate to $41{,}135 s_{\overline{4}|0.045} = 41{,}135(4.2782) = 175{,}984.$ Adding the maturity value produces 998,687 for a loss of 1,313. Note that only answer D has this value.

The Scenario B calculation is
$41{,}135 s_{\overline{4}|0.055} = 41{,}135(4.3423) = 178{,}621 + 822{,}703 - 1{,}000{,}000 = 1{,}324.$

31. Solution: D.

The present value is

$$5000[1.07v + 1.07^2 v^2 + \cdots + 1.07^{20} v^{20}]$$

$$= 5000\frac{1.07v - 1.07^{21} v^{21}}{1 - 1.07v} = 5000\frac{1.01905 - 1.48622}{1 - 1.01905} = 122{,}617.$$

32. Solution: C.

The first cash flow of 60,000 at time 3 earns 2400 in interest for a time 4 receipt of 62,400. Combined with the final payment, the investment returns 122,400 at time 4. The present value is $122,400(1.05)^{-4} = 100,699.$ The net present value is 699.

33. Solution: B.

Using spot rates, the value of the bond is:

$60/1.07 + 60/1.08^2 + 1060/1.09^3 = 926.03.$

34. Solution: E.

Using spot rates, the value of the bond is:

$60/1.07 + 60/1.08^2 + 1060/1.09^3 = 926.03.$ The annual effective rate is the solution to $926.03 = 60a_{\overline{3}|i} + 1000(1+i)^{-3}$. Using a calculator, the solution is 8.9%.

35. Solution: C.

Duration is the negative derivative of the price multiplied by one plus the interest rate and divided by the price. Hence, the duration is –(–700)(1.08)/100 = 7.56.

36. Solution: C

The size of the dividend does not matter, so assume it is 1. Then the duration is

$$\frac{\sum_{t=1}^{\infty} tv^t}{\sum_{t=1}^{\infty} v^t} = \frac{(Ia)_{\overline{\infty}|}}{a_{\overline{\infty}|}} = \frac{\ddot{a}_{\overline{\infty}|}/i}{1/i} = \frac{1/(di)}{1/i} = \frac{1}{d} = \frac{1.1}{0.1} = 11.$$

37. Solution: B

Duration = $$\frac{\sum_{t=1}^{\infty} tv^t R_t}{\sum_{t=1}^{\infty} v^t R_t} = \frac{\sum_{t=1}^{\infty} tv^t 1.02^t}{\sum_{t=1}^{\infty} v^t 1.02^t} = \frac{(Ia)_{\overline{\infty}|j}}{a_{\overline{\infty}|j}} = \frac{\ddot{a}_{\overline{\infty}|j}/j}{1/j} = \frac{1}{d}.$$

The interest rate j is such that $(1+j)^{-1} = 1.02v = 1.02/1.05 \Rightarrow j = 0.03/1.02.$ Then the duration is $1/d = (1+j)/j = (1.05/1.02)/(0.03/1.02) = 1.05/0.03 = 35.$

45. Solution: A

For the time weighted return the equation is:

$$1+0=\frac{12}{10}\frac{X}{12+X}\Rightarrow 120+10X=12X\Rightarrow 120=2X\Rightarrow X=60.$$

Then the amount of interest earned in the year is 60 – 60 – 10 = –10 and the weighted amount exposed to earning interest is 10(1) + 60(0.5) = 40. Then Y = –10/40 = –25%.

46. Solution: A

The outstanding balance is the present value of future payments. With only one future payment, that payment must be 559.12(1.08) = 603.85. The amount borrowed is $603.85a_{\overline{4}|0.08}=2000.$ The first payment has 2000(0.08) = 160 in interest, thus the principal repaid is 603.85 – 160 = 443.85.

Alternatively, observe that the principal repaid in the final payment is the outstanding loan balance at the previous payment, or 559.12. Principal repayments form a geometrically decreasing sequence, so the principal repaid in the first payment is $559.12/1.08^3=443.85.$

47. Solution: B

Because the yield rate equals the coupon rate, Bill paid 1000 for the bond. In return he receives 30 every six months, which accumulates to $30s_{\overline{20}|j}$ where j is the semi-annual interest rate. The equation of value is $1000(1.07)^{10}=30s_{\overline{20}|j}+1000\Rightarrow s_{\overline{20}|j}=32.238.$ Using a calculator to solve for the interest rate produces j = 0.0476 and so $i=1.0476^2-1=0.0975=9.75\%.$

48. Solution: A

To receive 3000 per month at age 65 the fund must accumulate to 3,000(1,000/9.65) = 310,880.83. The equation of value is $310{,}880.83=X\ddot{s}_{\overline{300}|0.08/12}=957.36657X\Rightarrow 324.72.$

49. Solution: D

(A) The left-hand side evaluates the deposits at age 0, while the right-hand side evaluates the withdrawals at age 17.

(B) The left-hand side has 16 deposits, not 17.

(C) The left-hand side has 18 deposits, not 17.

(D) The left-hand side evaluates the deposits at age 18 and the right-hand side evaluates the withdrawals at age 18.

(E) The left-hand side has 18 deposits, not 17 and 5 withdrawals, not 4.

50. Deleted

51. Solution: D

Because only Bond II provides a cash flow at time 1, it must be considered first. The bond provides 1025 at time 1 and thus 1000/1025 = 0.97561 units of this bond provides the required cash. This bond then also provides 0.97561(25) = 24.39025 at time 0.5. Thus Bond I must provide 1000 – 24.39025 = 975.60975 at time 0.5. The bond provides 1040 and thus 975.60975/1040 = 0.93809 units must be purchased.

52. Solution: C

Because only Mortgage II provides a cash flow at time two, it must be considered first. The mortgage provides $Y / a_{\overline{2}|0.07} = 0.553092Y$ at times one and two. Therefore, 0.553092Y = 1000 for Y = 1808.02. Mortgage I must provide 2000 – 1000 = 1000 at time one and thus X = 1000/1.06 = 943.40. The sum is 2751.42.

53. Solution: A

Bond I provides the cash flow at time one. Because 1000 is needed, one unit of the bond should be purchased, at a cost of 1000/1.06 = 943.40.

Bond II must provide 2000 at time three. Therefore, the amount to be reinvested at time two is 2000/1.065 = 1877.93. The purchase price of the two-year bond is $1877.93/1.07^2 = 1640.26$.

The total price is 2583.66.

54. Solution: C

Given the coupon rate is greater than the yield rate, the bond sells at a premium. Thus, the minimum yield rate for this callable bond is calculated based on a call at the earliest possible date because that is most disadvantageous to the bond holder (earliest time at which a loss occurs). Thus, X, the par value, which equals the redemption value because the bond is a par value bond, must satisfy

Price = $1722.25 = 0.04Xa_{\overline{30}|0.03} + Xv_{0.03}^{30} = 1.196X \Rightarrow X = 1440.$

55. Solution: B

Because 40/1200 is greater than 0.03, for early redemption the earliest redemption should be evaluated. If redeemed after 15 years, the price is $40a_{\overline{30}|0.03} + 1200/1.03^{30} = 1278.40$. If the bond is redeemed at maturity, the price is $40a_{\overline{40}|0.03} + 1100/1.03^{40} = 1261.80$. The smallest value should be selected, which is 1261.80.

56. Solution: E

Given the coupon rate is less than the yield rate, the bond sells at a discount. Thus, the minimum yield rate for this callable bond is calculated based on a call at the latest possible date because that is most disadvantageous to the bond holder (latest time at which a gain occurs). Thus, X, the par value, which equals the redemption value because the bond is a par value bond, must satisfy

Price = $1021.50 = 0.02Xa_{\overline{20}|0.03} + Xv_{0.03}^{20} = 0.851225X \Rightarrow X = 1200.$

57. Solution: B

Given the price is less than the amount paid for an early call, the minimum yield rate for this callable bond is calculated based on a call at the latest possible date. Thus, for an early call, the effective yield rate per coupon period, *j*, must satisfy Price = $1021.50 = 22a_{\overline{19}|j} + 1200v_j^{19}$. Using the calculator, *j* = 2.86%. We also must check the yield if the bond is redeemed at maturity. The equation is $1021.50 = 22a_{\overline{20}|j} + 1100v_j^{20}$. The solution is *j* = 2.46% Thus, the yield, expressed as a nominal annual rate of interest convertible semiannually, is twice the smaller of the two values, or 4.92%.

58. Moved to Derivatives section

59. Solution: C

First, the present value of the liability is $PV = 35,000a_{\overline{15}|6.2\%} = 335,530.30.$

The duration of the liability is:

$$\bar{d} = \frac{\sum tv^t R_t}{\sum v^t R_t} = \frac{35,000v + 2(35,000)v^2 + \cdots + 15(35,000)v^{15}}{335,530.30} = \frac{2,312,521.95}{335,530.30} = 6.89214.$$

Let X denote the amount invested in the 5 year bond.

Then, $\frac{X}{335,530.30}(5) + \left(1 - \frac{X}{335,530.30}\right)(10) = 6.89214 \Rightarrow X = 208,556.$

60. Solution: A

The present value of the first eight payments is:

$$PV = 2000v + 2000(1.03)v^2 + ... + 2000(1.03)^7 v^8 = \frac{2000v - 2000(1.03)^8 v^9}{1-1.03v} = 13{,}136.41.$$

The present value of the last eight payments is:

$$PV = 2000(1.03)^7 0.97v^9 + 2000(1.03)^7 (0.97)^2 v^{10} + \cdots + 2000(1.03)^7 (0.97^8) v^{16}$$

$$= \frac{2000(1.03)^7 0.97v^9 - 2000(1.03)^7 (0.97)^9 v^{17}}{1-0.97v} = 7{,}552.22.$$

Therefore, the total loan amount is L = 20,688.63.

61. Solution: E

$$2000 = 500\exp\left(\int_0^t \frac{\frac{r^2}{100}}{3+\frac{r^3}{150}}\,dr\right)$$

$$4 = \exp\left(0.5\int_0^t \frac{\frac{r^2}{50}}{3+\frac{r^3}{150}}\,dr\right) = \exp\left[0.5\ln\left(3+\frac{r^3}{150}\right)\Big|_0^t\right]$$

$$4 = \exp\left[0.5\ln\left(1+\frac{t^3}{450}\right)\right] = \left(1+\frac{t^3}{450}\right)^{\frac{1}{2}}$$

$$16 = \left(1+\frac{t^3}{450}\right)$$

$$t = 18.8988$$

62. Solution: E

Let F, C, r, and i have their usual interpretations. The discount is $(Ci - Fr)a_{\overline{n}|}$ and the discount in the coupon at time t is $(Ci - Fr)v^{n-t+1}$. Then,

$$194.82 = (Ci - Fr)v^{26}$$

$$306.69 = (Ci - Fr)v^{21}$$

$$0.63523 = v^5 \Rightarrow v = 0.91324 \Rightarrow i = 0.095$$

$$(Ci - Fr) = 194.82(1.095)^{26} = 2062.53$$

$$\text{Discount} = 2062.53a_{\overline{40}|0.095} = 21{,}135$$

63. Solution: A

$$699.68 = Pv^{8-5+1}$$
$$P = 842.39 \text{ (annual payment)}$$
$$P_1 = \frac{699.68}{1.0475^4} = 581.14$$
$$I_1 = 842.39 - 581.14 = 261.25$$
$$L = \frac{261.25}{0.0475} = 5500 \text{ (loan amount)}$$
$$\text{Total interest} = 842.39(8) - 5500 = 1239.12$$

64. Solution: D

$$OB_{18} = 22,000(1.007)^{18} - 450.30 s_{\overline{18}|0.007} = 16,337.10$$
$$16,337.10 = Pa_{\overline{24}|0.004}$$
$$P = 715.27$$

65. Solution: C

If the bond has no premium or discount, it was bought at par so the yield rate equals the coupon rate, 0.038.

$$d = \frac{\frac{1}{2}\left(1(190)v + 2(190)v^2 + \cdots + 14(190)v^{14} + 14(5000)v^{14}\right)}{190v + 190v^2 + \cdots + 190v^{14} + 5000v^{14}}$$
$$d = \frac{95\left(Ia\right)_{\overline{14}|} + 7(5000)v^{14}}{190a_{\overline{14}|} + 5000v^{14}}$$
$$d = 5.5554$$

Or, taking advantage of a shortcut:

$d = \ddot{a}_{\overline{14}|0.038} = 11.1107$. This is in half years, so dividing by two, $d = \frac{11.1107}{2} = 5.5554$.

66. Solution: A

$$\overline{v} = \frac{7.959}{1.072} = 7.425$$
$$P(0.08) = P(0.072)\left[1 - (\Delta i)\overline{v}\right]$$
$$P(0.08) = 1000\left[1 - (0.008)(7.425)\right] = 940.60$$

67. Solution: E

$(1+s_3)^3 = (1+s_2)^2(1+{}_1f_2)$

$0.85892 = \frac{1}{(1+s_3)^3}, s_3 = 0.052$

$0.90703 = \frac{1}{(1+s_2)^2}, s_2 = 0.050$

$1.052^3 = 1.050^2(1+{}_1f_2)$

${}_1f_2 = 0.056$

68. Solution: C

Let d_0 be the Macaulay duration at time 0.

$d_0 = \ddot{a}_{\overline{8}|0.05} = 6.7864$

$d_1 = d_0 - 1 = 5.7864$

$d_2 = \ddot{a}_{\overline{7}|0.05} = 6.0757$

$\frac{d_1}{d_2} = \frac{5.7864}{6.0757} = 0.9524$

This solution employs the fact that when a coupon bond sells at par the duration equals the present value of an annuity-due. For the duration just before the first coupon the cash flows are the same as for the original bond, but all occur one year sooner. Hence the duration is one year less.

Alternatively, note that the numerators for d_1 and d_2 are identical. That is because they differ only with respect to the coupon at time 1 (which is time 0 for this calculation) and so the payment does not add anything. The denominator for d_2 is the present value of the same bond, but with 7 years, which is 5000. The denominator for d_1 has the extra coupon of 250 and so is 5250. The desired ratio is then 5000/5250 = 0.9524.

69. Solution: A

Let *N* be the number of shares bought of the bond as indicated by the subscript.

$N_C(105) = 100, N_C = 0.9524$

$N_B(100) = 102 - 0.9524(5), N_B = 0.9724$

$N_A(107) = 99 - 0.9524(5), N_A = 0.8807$

70. Solution: B

All are true except B. Immunization requires frequent rebalancing.

71. Solution: D

Set up the following two equations in the two unknowns:

$$A(1.05)^2 + B(1.05)^{-2} = 6000$$

$$2A(1.05)^1 - 2B(1.05)^{-3} = 0.$$

Solving simultaneously gives:

$$A = 2721.09$$

$$B = 3307.50$$

$$|A - B| = 586.41.$$

72. Solution: A

Set up the following two equations in the two unknowns.

$$(1) \quad 5000(1.03)^3 + B(1.03)^{-b} = 12,000 \Rightarrow$$

$$5463.635 + B(1.03)^{-b} = 12,000 \Rightarrow B(1.03)^{-b} = 6536.365$$

$$(2) \quad 3(5000)(1.03)^3 - bB(1.03)^{-b} = 0 \Rightarrow 16,390.905 - b6536.365 = 0$$

$$b = 2.5076$$

$$B = 7039.27$$

$$\frac{B}{b} = 2807.12$$

73. Solution: D

$$P_A = A(1+i)^{-2} + B(1+i)^{-9}$$

$$P_L = 95,000(1+i)^{-5}$$

$$P_A' = -2A(1+i)^{-3} - 9B(1+i)^{-10}$$

$$P_L' = -5(95,000)(1+i)^{-6}$$

Set the present values and derivatives equal and solve simultaneously.

$$0.92456A + 0.70259B = 78,083$$

$$-1.7780A - 6.0801B = -375,400$$

$$B = \frac{78,083(1.7780/0.92456) - 375,400}{0.70259(1.7780/0.92456) - 6.0801} = 47,630$$

$$A = [78,083 - 0.70259(47,630)]/0.92456 = 48,259$$

$$\frac{A}{B} = 1.0132$$

74. Solution: D

Throughout the solution, let $j = i/2$.

For bond A, the coupon rate is $(i + 0.04)/2 = j + 0.02$.

For bond B, the coupon rate is $(i - 0.04)/2 = j - 0.02$.

The price of bond A is $P_A = 10,000(j+0.02)a_{\overline{20}|j} + 10,000(1+j)^{-20}$.

The price of bond B is $P_B = 10,000(j-0.02)a_{\overline{20}|j} + 10,000(1+j)^{-20}$.

Thus,

$$P_A - P_B = 5,341.12 = [200-(-200)]a_{\overline{20}|j} = 400a_{\overline{20}|j}$$

$$a_{\overline{20}|j} = 5,341.12/400 = 13.3528.$$

Using the financial calculator, $j = 0.042$ and $i = 2(0.042) = 0.084$.

75. Solution: D

The initial level monthly payment is

$$R = \frac{400,000}{a_{\overline{15\times12}|0.09/12}} = \frac{400,000}{a_{\overline{180}|0.0075}} = 4,057.07.$$

The outstanding loan balance after the 36th payment is

$$B_{36} = Ra_{\overline{180-36}|0.0075} = 4,057.07a_{\overline{144}|0.0075} = 4,057.07(87.8711) = 356,499.17.$$

The revised payment is 4,057.07 – 409.88 = 3,647.19.

Thus,

$$356,499.17 = 3,647.19a_{\overline{144}|j/12}$$

$$a_{\overline{144}|j/12} = 356,499.17 / 3,647.19 = 97.7463.$$

Using the financial calculator, $j/12 = 0.575\%$, for $j = 6.9\%$.

76. Solution: D

The price of the first bond is

$$1000(0.05/2)a_{\overline{30\times2}|0.05/2} + 1200(1+0.05/2)^{-30\times2} = 25a_{\overline{60}|0.025} + 1200(1.025)^{-60}$$
$$= 772.72 + 272.74 = 1,045.46.$$

The price of the second bond is also 1,045.46. The equation to solve is

$$1,045.46 = 25a_{\overline{60}|j/2} + 800(1+j/2)^{-60}.$$

The financial calculator can be used to solve for $j/2 = 2.2\%$ for $j = 4.4\%$.

77. Solution: E

Let n = years. The equation to solve is

$$1000(1.03)^{2n} = 2(1000)(1.0025)^{12n}$$
$$2n\ln 1.03 + \ln 1000 = 12n\ln 1.0025 + \ln 2000$$
$$0.029155n = 0.69315$$
$$n = 23.775.$$

This is 285.3 months. The next interest payment to Lucas is at a multiple of 6, which is 288 months.

78. Solution: B

The ending balance is 5000(1.09) + 2600sqrt(1.09) = 8164.48.

The time-weighted rate of return is (5200/5000) x [8164.08/(5200 + 2600)] – 1 = 0.0886.

79. Solution: A

Equating the accumulated values after 4 years provides an equation in *K*.

$$10\left(1+\frac{K}{25}\right)^4 = 10\exp\left(\int_0^4 \frac{1}{K+0.25t}\,dt\right)$$

$$4\ln(1+0.04K) = \int_0^4 \frac{1}{K+0.25t}\,dt = 4\ln(\mathrm{K}+0.25\,\mathrm{t})\Big|_0^4 = 4\ln(K+1) - 4\ln(K) = 4\ln\frac{K+1}{K}$$

$$1+0.04K = \frac{K+1}{K}$$

$$0.04K^2 = 1$$

$$K = 5.$$

Therefore, $X = 10(1+5/25)^4 = 20.74.$

80. Solution: C

To repay the loan, the sinking fund must accumulate to 1000. The deposit is 2(1000*i*). Therefore,

$$1000 = 2000is_{\overline{5}|0.8i}$$

$$0.5 = i\frac{(1+0.8i)^5 - 1}{0.8i}$$

$$(1+0.8i)^5 = 1.4$$

$$1+0.8i = 1.0696$$

$$i = 0.0696/0.8 = 0.087.$$

81. Solution: D

The outstanding balance at time 25 is $100(Da)_{\overline{25}|} = 100\frac{25-a_{\overline{25}|}}{i}$. The principle repaid in the 26th payment is $X = 2500 - i(100)\frac{25-a_{\overline{25}|}}{i} = 2500 - 2500 + 100a_{\overline{25}|} = 100a_{\overline{25}|}$. The amount borrowed is the present value of all 50 payments, $2500a_{\overline{25}|} + v^{25}100(Da)_{\overline{25}|}$. Interest paid in the first payment is then

$$i\left[2500a_{\overline{25}|} + v^{25}100(Da)_{\overline{25}|}\right]$$

$$= 2500(1-v^{25}) + 100v^{25}(25-a_{\overline{25}|})$$

$$= 2500 - 2500v^{25} + 2500v^{25} - v^{25}100a_{\overline{25}|}$$

$$= 2500 - Xv^{25}.$$

82. Solution: A

The exposure associated with i produces results quite close to a true effective rate of interest as long as the net amount of principal contributed at time t is small relative to the amount in the fund at the beginning of the period.

83. Solution: E

The time-weighted weight of return is

j = (120,000 / 100,000) x (130,000 / 150,000) x (100,000 / 80,000) – 1 = 30.00%.

Note that 150,000 = 120,000 + 30,000 and 80,000 = 130,000 – 50,000.

84. Solution: C

The accumulated value is $1000\ddot{s}_{\overline{20}|0.816} = 50,382.16$. This must provide a semi-annual annuity-due of 3000. Let n be the number of payments. Then solve $3000\ddot{a}_{\overline{n}|0.04} = 50,382.16$ for n = 26.47. Therefore, there will be 26 full payments plus one final, smaller, payment. The equation is $50,382.16 = 3000\ddot{a}_{\overline{26}|0.04} + X(1.04)^{-26}$ with solution X = 1430. Note that the while the final payment is the 27th payment, because this is an annuity-due, it takes place 26 periods after the annuity begins.

85. Solution: D

For the first perpetuity,

$$\frac{1}{(1+i)^2-1}+1=7.21$$

$$\frac{1}{6.21}=(1+i)^2-1$$

$$i=0.0775.$$

For the second perpetuity,

$$R\left[\frac{1}{(1.0775+0.01)^3-1}+1\right](1.0875)^{-1}=7.21$$

$$1.286139R=7.21(1.0875)(0.286139)$$

$$R=1.74.$$

86. Solution: E

$$10,000 = 100(Ia)_{\overline{5}|} + Xv^5 a_{\overline{15}|} = 100\left(\frac{\ddot{a}_{\overline{5}|} - 5v^5}{0.05}\right) + Xv^5 a_{\overline{15}|}$$

$$10,000 = 1256.64 + 8.13273X$$

$$1075 = X$$

87. Solution: C

$$5000 = Xs_{\overline{10}|0.06}(1.05)^5$$

$$X = \frac{5000}{13.1808(1.2763)} = 297.22$$

88. Solution: E

The monthly payment on the original loan is $\frac{65,000}{a_{\overline{180}|8/12\%}} = 621.17$. After 12 payments the outstanding balance is $621.17a_{\overline{168}|8/12\%} = 62,661.40$. The revised payment is $\frac{62,661.40}{a_{\overline{168}|6/12\%}} = 552.19$.

89. Solution: E

At the time of the final deposit the fund has $750s_{\overline{18}|0.07} = 25,499.27$. This is an immediate annuity because the evaluation is done at the time the last payments is made (which is the end of the final year). A tuition payment of $6000(1.05)^{17} = 13,752.11$ is made, leaving 11,747.16. It earns 7%, so a year later the fund has 11,747.16(1.07) = 12,569.46. Tuition has grown to 13,752.11(1.05) = 14,439.72. The amount needed is 14,439.72 – 12,569.46 = 1,870.26

90. Solution: B

The coupons are 1000(0.09)/2 = 45. The present value of the coupons and redemption value at 5% per semiannual period is $P = 45a_{\overline{40}|0.05} + 1200(1.05)^{-40} = 942.61$.

91. Solution: A

For a bond bought at discount, the minimum price will occur at the latest possible redemption date. $P = 50a_{\overline{20}|0.06} + 1000(1.06)^{-20} = 885.30$.

92. Solution: C

$$\frac{1.095^5}{1.090^4} - 1 = 11.5\%$$

93. Solution: D

The accumulated value of the first year of payments is $2000 s_{\overline{12}|0.005} = 24,671.12$. This amount increases at 2% per year. The effective annual interest rate is $1.005^{12} - 1 = 0.061678$. The present value is then

$$P = 24,671.12 \sum_{k=1}^{25} 1.02^{k-1}(1.061678)^{-k} = 24,671.12 \frac{1}{1.02} \sum_{k=1}^{25} \left(\frac{1.02}{1.061678} \right)^k$$

$$= 24{,}187.37 \frac{0.960743 - 0.960743^{26}}{1 - 0.960743} = 374,444.$$

This is 56 less than the lump sum amount.

94. Solution: A

The monthly interest rate is 0.072/12 = 0.006. 6500 five years from today has value $6500(1.006)^{-60} = 4539.77$. The equation of value is

$$4539.77 = 1700(1.006)^{-n} + 3400(1.006)^{-2n}.$$

Let $x = 1.006^{-n}$. Then, solve the quadratic equation

$$3400x^2 + 1700x - 4539.77 = 0$$

$$x = \frac{-1700 + \sqrt{1700^2 - 4(3400)(-4539.77)}}{2(3400)} = 0.93225.$$

Then,

$$1.006^{-n} = 0.9325 \Rightarrow -n\ln(1.006) = \ln(0.93225) \Rightarrow n = 11.73.$$

To ensure there is 6500 in five years, the deposits must be made earlier and thus the maximum integral value is 11.

95. Solution: C

$$\frac{(1-d/2)^{-4}}{(1-d/4)^{-4}}=\left(\frac{39}{38}\right)^4 \Rightarrow \frac{1-d/2}{1-d/4}=\frac{38}{39}\Rightarrow 39-39(d/2)=38-38(d/4)$$

$$d\left(39/2-38/4\right)=39-38$$

$$d=1/(19.5-9.5)=0.1$$

$$1+i=(1-d/2)^{-2}=.95^{-2}=1.108\Rightarrow i=10.8\%.$$

96. Solution: C

The monthly interest rate is 0.042/12 = 0.0035. The quarterly interest rate is $1.0035^3-1=0.0105$. The investor makes 41 quarterly deposits and the ending date is 124 months from the start. Using January 1 of year *y* as the comparison date produces the following equation:

$$X+\sum_{k=1}^{41}\frac{100}{1.0105^k}=\frac{1.9X}{1.0035^{124}}$$

Substituting $1.0105=1.0035^3$ gives answer (C).

97. Solution: D

Convert the two annual rates, 4% and 5%, to two-year rates as $1.04^2-1=0.0816$ and $1{,}05^2-1=0.1025$.

The accumulated value is

$$100\ddot{s}_{\overline{3}|0.0816}(1.05)^4+100\ddot{s}_{\overline{2}|0.1025}=100(3.51678)(1.21551)+100(2.31801)=659.269.$$

With only five payments, an alternative approach is to accumulate each one to time ten and add them up.

The two-year yield rate is the solution to $100\ddot{s}_{\overline{5}|i}=659.269$. Using the calculator, the two-year rate is 0.093637. The annual rate is $1.093637^{0.5}-1=0.04577$ which is 4.58%.

98. Solution: C

$$(1.08)^{1/12}-1=0.006434$$

$$\frac{1}{1.08^{15}}25{,}000\ddot{a}_{\overline{4}|8\%}=X\ddot{a}_{\overline{216}|0.6434\%}$$

$$X=\frac{25{,}000(3.57710)}{3.17217(117.2790)}=240.38$$

99. Solution: B

$$\text{PV}_{\text{perp.}} = \left[\frac{1}{0.1} + \frac{\frac{1}{0.08} - \frac{1}{0.1}}{1.1^{10}} \right] (15,000) + 15,000$$

$$= 164,457.87 + 15,000 = 179,457.87$$

$$X\left(\ddot{a}_{\overline{10}|0.10} + \frac{\ddot{a}_{\overline{15}|0.08}}{1.10^{10}} \right) = 179,458$$

$$X\left(6.759 + \frac{9.244}{1.10^{10}} \right) = 179,458$$

$$X = 17,384$$

100. Solution: A

$$1050.50 = (22.50 + X)a_{\overline{14}|0.03} + X\left(\frac{a_{\overline{14}|0.03} - 14(1.03)^{-14}}{0.03} \right) + 300(1.03)^{-14}$$

$$1050.50 = (22.50 + X)11.2961 + X\left(\frac{11.2961 - 9.25565}{0.03} \right) + 198.335 \Rightarrow 79.3111X = 598 \Rightarrow X = 7.54$$

101. Solution: D

The amount of the loan is the present value of the deferred increasing annuity:

$$(1.05)^{-10}\left[500\ddot{a}_{\overline{30}|0.05} + 500(I\ddot{a})_{\overline{30}|0.05} \right] = (1.05^{-10})(500)\left[\ddot{a}_{\overline{30}|0.05} + \frac{\ddot{a}_{\overline{30}|0.05} - 30(1.05)^{-30}}{0.05/1.05} \right] = 64,257.$$

102. Solution: C

$$50,000\left[\frac{(1+i)^{30} - (1.03)^{30}}{(1+i)^{30}(i - 0.03)} \right](1+i) = 5,000\left[\frac{(1+i)^{30} - (1.03)^{30}}{i - 0.03} \right]$$

$$50,000/(1+i)^{29} = 5,000$$

$$(1+i)^{29} = 10$$

$$i = 10^{1/29} - 1 = 0.082637$$

The accumulated amount is

$$50,000\left[\frac{(1.082637)^{30} - (1.03)^{30}}{(1.082637)^{30}(0.082637 - 0.03)} \right](1.082637) = 797,836.82$$

103. Solution: D

The first payment is 2,000, and the second payment of 2,010 is 1.005 times the first payment. Since we are given that the series of quarterly payments is geometric, the payments multiply by 1.005 every quarter.

Based on the quarterly interest rate, the equation of value is

$$100,000 = 2,000 + 2,000(1.005)v + 2,000(1.005)^2 v^2 + 2,000(1.005)^3 v^3 + \cdots = \frac{2,000}{1-1.005v}.$$

$$1-1.005v = 2,000/100,000 \Rightarrow v = 0.98/1.005.$$

The annual effective rate is $v^{-4} - 1 = \left(0.98/1.005\right)^{-4} - 1 = 0.10601 = 10.6\%$.

104. Solution: A

Present value for the first 10 years is $\dfrac{1-\left(1.06\right)^{-10}}{\ln\left(1.06\right)} = 7.58$

Present value of the payments after 10 years is

$$\left(1.06\right)^{-10} \int_0^\infty \left(1.03\right)^s \left(1.06\right)^{-s} ds = \frac{0.5584}{\ln\left(1.06\right) - \ln\left(1.03\right)} = 19.45$$

Total present value = 27.03

105. Solution: C

$$\left[10,000\left(1.06\right)^5 + X\left(1.06\right)^2\right] e^{\int_5^{10} \frac{1}{t+1} dt} = 75,000$$

$$\left(13,382.26 + 1.1236X\right)\frac{11}{6} = 75,000$$

$$1.1236X = 27,526.83$$

$$X = 24,498.78$$

106. Solution: D

The effective annual interest rate is $i = (1-d)^{-1} - 1 = (1-0.055)^{-1} - 1 = 5.82\%$

The balance on the loan at time 2 is $15,000,000(1.0582)^2 = 16,796,809$.

The number of payments is given by $1,200,000 a_{\overline{n}|} = 16,796,809$ which gives n = 29.795 => 29 payments of 1,200,000. The final equation of value is

$$1,200,000 a_{\overline{29}|} + X(1.0582)^{-30} = 16,796,809$$

$$X = (16,796,809 - 16,621,012)(5.45799) = 959,490.$$

107. Solution: C

$1-v^2 = 0.525(1-v^4) \Rightarrow 1 = 0.525(1+v^2) \Rightarrow v^2 = 0.90476 \Rightarrow v = 0.95119$

$1-v^2 = 0.1427(1-v^n) \Rightarrow 1-\mathrm{v}^n = (1-0.90476)/0.1427 = 0.667414 \Rightarrow v^n = 0.332596$

$n = \ln(0.332596)/\ln(0.95119) = 22$

108. Solution: B

Let *X* be the annual deposit on the sinking fund. Because the sinking fund deposits must accumulate to the loan amount, $L = Xs_{\overline{11}|0.047} = 13.9861X$. At time 7 the fund has $Xs_{\overline{7}|0.047} = 8.0681X$. This is 6241 short of the loan amount, so a second equation is *L* = 8.0681*X* + 6241. Combining the two equations gives 13.9861*X* – 8.0681*X* = 6241 with implies *X*=1055.

109. Solution: C

The monthly payment is $200,000/a_{\overline{360}|0.005} = 1199.10$. Using the equivalent annual effective rate of 6.17%, the present value (at time 0) of the five extra payments is 41,929.54 which reduces the original loan amount to 200,000 – 41,929.54 = 158,070.46. The number of months required is the solution to $158,070.46 = 1199.10a_{\overline{n}|0.005}$. Using calculator, *n* = 215.78 months are needed to pay off this amount. So there are 215 full payments plus one fractional payment at the end of the 216th month, which is December 31, 2020.

110. Solution: D

The annual effective interest rate is 0.08/(1 – 0.08) = 0.08696. The level payments are $500,000/a_{\overline{5}|0.08696} = 500,000/3.9205 = 127,535$. This rounds up to 128,000. The equation of value for *X* is

$128,000a_{\overline{4}|0.08696} + X(1.08696)^{-5} = 500,000$

$X = (500,000-417,466.36)(1.51729) = 125,227.$

111. Solution: B

The accumulated value is the reciprocal of the price. The equation is
X[(1/0.94)+(1/0.95)+(1/0.96)+(1/0.97)+(1/0.98)+(1/0.99)] = 100,000.

X= 16,078

112. Solution: D

Let P be the annual payment. The fifth line is obtained by solving a quadratic equation.

$$P(1-v^{10}) = 3600$$

$$Pv^{10-6+1} = 4871$$

$$\frac{1-v^{10}}{v^5} = \frac{3600}{4871}$$

$$1-v^{10} = 0.739068v^5$$

$$v^5 = 0.69656$$

$$v^{10} = 0.485195$$

$$i = 0.485195^{-10} - 1 = 0.075$$

$$X = P\frac{1-v^{10}}{i} = \frac{3600}{0.075} = 48,000$$

113. Solution: A

Let j = periodic yield rate, r = periodic coupon rate, F = redemption (face) value, P = price, n = number of time periods, and $v_j = \frac{1}{1+j}$. In this problem, $j = (1.0705)^{\frac{1}{2}} - 1 = 0.03465$, r = 0.035, P = 10,000, and n = 50.

The present value equation for a bond is $P = Fv_j^{\,n} + Fra_{\overline{n}|j}$; solving for the redemption value F yields

$$F = \frac{P}{v_j^{\,n} + ra_{\overline{n}|j}} = \frac{10,000}{(1.03465)^{-50} + 0.035a_{\overline{50}|0.03465}} = \frac{10,000}{0.18211 + 0.035(23.6044)} = 9,918. \; .$$

114. Solution: B

Jeff's monthly cash flows are coupons of 10,000(0.09)/12 = 75 less loan payments of 2000(0.08)/12 = 13.33 for a net income of 61.67. At the end of the ten years (in addition to the 61.67) he receives 10,000 for the bond less a 2,000 loan repayment. The equation is

$$8000 = 61.67a_{\overline{120}|i^{(12)}/12} + 8000(1+i^{(12)}/12)^{-120}$$

$$i^{(12)}/12 = 0.00770875$$

$$i = 1.00770875^{12} - 1 = 0.0965 = 9.65\%.$$

115. Solution: B

The present value equation for a par-valued annual coupon bond is $P = Fv_i^n + Fra_{\overline{n}|i}$; solving for the coupon rate r yields $r = \frac{P - Fv_i^n}{Fa_{\overline{n}|i}} = \frac{P}{a_{\overline{n}|i}}\left(\frac{1}{F}\right) - \frac{v_i^n}{a_{\overline{n}|i}}$.

All three bonds have the same values except for F. We can write $r = x(1/F) + y$. From the first two bonds:

$0.0528 = x/1000 + y$ and $0.0440 = x/1100 + y$. Then,

$0.0528 – 0.044 = x(1/1000 – 1/1100)$ for $x = 96.8$ and $y = 0.0528 – 96.8/1000 = –0.044$. For the third bond, $r = 96.8/1320 – 0.044 = 0.2933 = 2.93\%$.

116. Solution: A

The effective semi-annual yield rate is $1.04 = \left(1 + \frac{i^{(2)}}{2}\right)^2 \Rightarrow \frac{i^{(2)}}{2} = 1.9804\%$. Then,

$$582.53 = c(1.02)v + c(1.02v)^2 + \cdots + c(1.02v)^{12} + 250v^{12}$$

$$= c\frac{1.02v - (1.02v)^{13}}{1 - 1.02v} + 250v^{12} = 12.015c + 197.579 \Rightarrow c = 32.04.$$

$$582.53 = c\frac{1.02v - (1.02v)^{13}}{1 - 1.02v} + 250v^{12} = 12.015c + 197.579 \Rightarrow c = 32.04$$

117. Solution: E

Book values are linked by BV3(1 + i) – Fr = BV4. Thus 1254.87(1.06) – Fr = 1277.38. Therefore, the coupon is Fr = 52.7822. The prospective formula for the book value at time 3 is

$$1254.87 = 52.7822\frac{1 - 1.06^{-(n-3)}}{0.06} + 1890(1.06)^{-(n-3)}$$

$$375.1667 = 1010.297(1.06)^{-(n-3)}$$

$$n - 3 = \frac{\ln(375.1667/1010.297)}{-\ln(1.06)} = 17.$$

Thus, n = 20. Note that the financial calculator can be used to solve for n – 3.

118. Solution: A

Book values are linked by BV3(1 + i) – Fr = BV4. Thus BV3(1.04) – 2500(0.035) = BV3 + 8.44. Therefore, BV3 = [2500(0.035) + 8.44]/0.04 = 2398.5. The prospective formula for the book value at time 3 is, where m is the number of six-month periods.

$$2398.5 = 2500(0.035)\frac{1-1.04^{-(m-3)}}{0.04} + 2500(1.04)^{-(m-3)}$$
$$211 = 312.5(1.04)^{-(m-3)}$$
$$m-3 = \frac{\ln(211/312.5)}{-\ln(1.04)} = 10.$$

Thus, m = 13 and n = m/2 = 6.5. Note that the financial calculator can be used to solve for m – 3.

119. Solution: C

$$s_1 = {}_1f_0 = 0.04$$
$${}_1f_1 = 0.06 = \frac{(1+s_2)^2}{(1+s_1)} - 1 \;\Rightarrow\; s_2 = \sqrt{(1.06)(1.04)} - 1 = 0.04995$$
$${}_1f_2 = 0.08 = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 \;\Rightarrow\; s_3 = [(1.08)(1.04995)^2]^{1/3} - 1 = 0.05987 = 6\%.$$

120. Solution: D

Interest earned is 55,000 – 50,000 – 8,000 + 10,000 = 7,000.

Equating the two interest measures gives the equation

$$\frac{7,000}{50,000 + (16,000/3) - 10,000(1-t)} = \frac{52}{50}\frac{62}{60}\frac{55}{52} - 1 = 0.13667$$
$$7,000 = 0.13667(55,333.33 - 10,000 + 10,000t)$$
$$t = [7,000 - 0.13667(45,333.33)]/1,366.7 = 0.5885.$$

121. Solution: B

The Macaulay duration of Annuity A is $0.93 = \frac{0(1)+1(v)+2(v^2)}{1+v+v^2} = \frac{v+2v^2}{1+v+v^2}$, which leads to the quadratic equation $1.07v^2 + 0.07v - 0.93 = 0$. The unique positive solution is v = 0.9.

The Macaulay duration of Annuity B is $\frac{0(1)+1(v)+2(v^2)+3(v^3)}{1+v+v^2+v^3} = 1.369.$

122. Solution: D

With v =1/1.07,

$$D = \frac{2(40,000)v^2 + 3(25,000)v^3 + 4(100,000)v^4}{40,000v^2 + 25,000v^3 + 100,000v^4} = 3.314.$$

123. Deleted

124. Solution: C

$$30 = MacD = \frac{\sum_{n=0}^{\infty} nv^n}{\sum_{n=0}^{\infty} v^n} = \frac{Ia_{\overline{\infty}|}}{\ddot{a}_{\overline{\infty}|}} = \frac{1/(di)}{1/d} = \frac{(1+i)/i^2}{(1+i)/i} = \frac{1}{i} \text{ and so } i = 1/30.$$

Then, $ModD = \frac{MacD}{1+i} = \frac{30}{1+\frac{1}{30}} = 29.032.$

125. Solution: D

Let D be the next dividend for Stock J. The value of Stock F is $0.5D/(0.088 - g)$. The value of Stock J is $D/(0.088 + g)$. The relationship is

$$\frac{0.5D}{0.088-g} = 2\frac{D}{0.088+g}$$

$$0.5D(0.088+g) = 2D(0.088-g)$$

$$2.5g = 0.132$$

$$g = 0.0528 = 5.3\%.$$

126. Solution: B

I) False. The yield curve structure is not relevant.

II) True.

III) False. Matching the present values is not sufficient when interest rates change.

127. Solution: A

The present value function and its derivatives are

$$P(i) = X + Y(1+i)^{-3} - 500(1+i)^{-1} - 1000(1+i)^{-4}$$
$$P'(i) = -3Y(1+i)^{-4} + 500(1+i)^{-2} + 4000(1+i)^{-5}$$
$$P''(i) = 12Y(1+i)^{-5} - 1000(1+i)^{-3} - 20,000(1+i)^{-6}.$$

The equations to solve for matching present values and duration (at i = 0.10) and their solution are

$$P(0.1) = X + 0.7513Y - 1137.56 = 0$$
$$P'(0.1) = -2.0490Y + 2896.91 = 0$$
$$Y = 2896.91 / 2.0490 = 1413.82$$
$$X = 1137.56 - 0.7513(1413.82) = 75.36.$$

The second derivative is

$$P''(0.1) = 12(1413.82)(1.1)^{-5} - 1000(1.1)^{-3} - 20,000(1.1)^{-6} = -1506.34.$$

Redington immunization requires a positive value for the second derivative, so the condition is not satisfied.

128. Solution: D

This solution uses time 8 as the valuation time. The two equations to solve are

$$P(i) = 300,000(1+i)^2 + X(1+i)^{8-y} - 1,000,000 = 0$$
$$P'(i) = 600,000(1+i) + (8-y)X(1+i)^{7-y} = 0.$$

Inserting the interest rate of 4% and solving:

$$300,000(1.04)^2 + X(1.04)^{8-y} - 1,000,000 = 0$$
$$600,000(1.04) + (8-y)X(1.04)^{7-y} = 0$$
$$X(1.04)^{-y} = [1,000,000 - 300,000(1.04)^2] / 1.04^8 = 493,595.85$$
$$624,000 + (8-y)(1.04)^7(493,595.85) = 0$$
$$y = 8 + 624,000 / [493,595.85(1.04)^7] = 8.9607$$
$$\text{X} = 493,595.85(1.04)^{8.9607} = 701,459.$$

129. Solution: A

This solution uses Macaulay duration and convexity. The same conclusion would result had modified duration and convexity been used.

The liabilities have present value $573/1.07^2 + 701/1.07^5 = 1000.$ Only portfolios A, B, and E have a present value of 1000.

The duration of the liabilities is $[2(573)/1.07^2 + 5(701)/1.07^5]/1000 = 3.5.$ The duration of a zero coupon bond is its term. The portfolio duration is the weighted average of the terms. For portfolio A the duration is [500(1) + 500(6)]/1000 = 3.5. For portfolio B it is [572(1) + 428(6)]/1000 = 3.14. For portfolio E it is 3.5. This eliminates portfolio B.

The convexity of the liabilities is $[4(573)/1.07^2 + 25(701)/1.07^5]/1000 = 14.5.$ The convexity of a zero-coupon bond is the square of its term. For portfolio A the convexity is [500(1) + 500(36)]/1000 = 18.5 which is greater than the convexity of the liabilities. Hence portfolio A provides Redington immunization. As a check, the convexity of portfolio E is 12.25, which is less than the liability convexity.

130. Solution: D

The present value of the liabilities is 1000, so that requirement is met. The duration of the liabilities is $402.11[1.1^{-1} + 2(1.1)^{-2} + 3(1.1)^{-3}]/1000 = 1.9365.$ Let X be the investment in the one-year bond. The duration of a zero-coupon is its term. The duration of the two bonds is then $[X + (1000 - X)(3)]/1000 = 3 - 0.002X$. Setting this equal to 1.9365 and solving yields $X = 531.75$.

131. Solution: A

Let x, y, and z represent the amounts invested in the 5-year, 15-year, and 20-year zero-coupon bonds, respectively. Note that in this problem, one of these three variables is 0.

The present value, Macaulay duration, and Macaulay convexity of the assets are, respectively,

$$x+y+z, \frac{5x+15y+20z}{x+y+z}, \frac{5^2x+15^2y+20^2z}{x+y+z}.$$

We are given that the present value, Macaulay duration, and Macaulay convexity of the liabilities are, respectively, 9697, 15.24, and 242.47.

Since present values and Macaulay durations need to match for the assets and liabilities, we have the two equations

$$x+y+z=9697, \quad \frac{5x+15y+20z}{x+y+z}=15.24.$$

Note that 5 and 15 are both less than the desired Macaulay duration 15.24, so z cannot be zero. So try either the 5-year and 20-year bonds (i.e. y = 0), or the 15-year and 20-year bonds (i.e. x = 0).

In the former case, substituting y = 0 and solving for x and z yields

$$x=\frac{(20-15.24)9697}{20-5}=3077.18 \text{ and } z=\frac{(15.24-5)9697}{20-5}=6619.82.$$

We need to check if the Macaulay convexity of the assets exceeds that of the liabilities.

The Macaulay convexity of the assets is $\frac{5^2(3077.18)+20^2(6619.82)}{9697}=281.00$, which exceeds the Macaulay convexity of the liabilities, 242.47. The company should invest 3077 for the 5-year bond and 6620 for the 20-year bond.

Note that setting x = 0 produces y = 9231.54 and z = 465.46 and the convexity is 233.40, which is less than that of the liabilities.

132. Solution: E

The correct answer is the lowest cost portfolio that provides for $11,000 at the end of year one and provides for $12,100 at the end of year two. Let H, I, and J represent the face amount of each purchased bond. The time one payment can be exactly matched with $H + 0.12J = 11,000$. The time two payment can be matched with $I + 1.12J = 12,100$. The cost of the three bonds is $H/1.1 + I/1.2321 + J$. This function is to be minimized under the two constraints. Substituting for H and I gives $(11,000 – 0.12J)/1.1 + (12,100 – 1.12J)/1.2321 + J = 19,820 – 0.0181J$. This is minimized by purchasing the largest possible amount of J. This is 12,100/1.12 = 10,803.57. Then, H = 11,000 – 0.12(10,803.57) = 9703.57. The cost of Bond H is 9703.57/1.1 = 8,821.43.

133. Solution: C

The strategy is to use the two highest yielding assets: the one-year bond and the two-year zero-coupon bond. The cost of these bonds is $25,000/1.0675+20,000/1.05^2=41,560$.

134. Solution: E

Let P be the annual interest paid. The present value of John's payments is $Pa_{\overline{X}|0.05}$. The present value of Karen's payments is $P(1.05)^{-X}a_{\overline{\infty}|0.05}=P(1.05)^{-X}/0.05$. Then,

$$P(1.05)^{-X}/0.05=1.59Pa_{\overline{X}|0.05}$$
$$\frac{1.05^{-X}}{0.05}=1.59\frac{1-1.05^{-X}}{0.05}$$
$$1.59=2.59(1.05)^{-X}$$
$$\ln 1.59=\ln 2.59-X\ln 1.05$$
$$X=10.$$

135. Solution: A

Cheryl's force of interest at all times is ln(1.07) = 0.06766. Gomer's accumulation function is from time 3 is $1 + yt$ and the force of interest is $y/(1 + yt)$. To be equal at time 2, the equation is $0.06766 = y/(1 + 2y)$, which implies $0.06766 + 0.13532y = y$ for $y = 0.07825$. Gomer's account value is 1000(1 + 2x0.07825) = 1156.5.

136. Solution: D

One way to view these payments is as a sequence of level immediate perpetuities of 1 that are deferred n-1, n, n+1,… years. The present value is then

$$v^{n-1}/i+v^n/i+v^{n+1}/i+\cdots=(v^{n-2}/i)(v+v^2+v^3+\cdots)=v^{n-2}/i^2.$$

Noting that only answers C, D, and E have this form and all have the same numerator,

$$v^{n-2}/i^2=v^n/(vi)^2=v^n/d^2.$$

137. Solution: B

The monthly interest rate is $j=(1.08)^{1/12}-1=0.643\%$. Then,

$$20,000s_{\overline{4}|0.08}=X\ddot{s}_{\overline{252}|0.00643},\quad 90,122.24=630.99X,\quad X=142.83.$$

138. Solution: D

$\bar{a}_{\overline{20|}} = 1.5\bar{a}_{\overline{10|}}$, $\dfrac{1-e^{-20\delta}}{\delta} = 1.5\dfrac{1-e^{-10\delta}}{\delta}$, $e^{-20\delta} - 1.5e^{-10\delta} + 0.5 = 0$. Let $X = e^{-10\delta}$. We then have the quadratic equation $X^2 - 1.5X + 0.5 = 0$ with solution $X = 0.5$ for $\delta = \ln 0.5/(-10) = 0.069315$. Then, the accumulated value of a 7-year continuous annuity of 1 is

$$\bar{s}_{\overline{7|}} = \frac{e^{7(0.069315)} - 1}{0.069315} = 9.01.$$

139. Solution: B

The present value is

$$v^3 + v^{10} + v^{17} + \cdots + v^{-4+7n}$$

$$= \frac{v^3 - v^{3+7n}}{1-v^7} = \frac{(1-v^{3+7n}) - (1-v^3)}{1-v^7} = \frac{a_{\overline{3+7n|}} - a_{\overline{3|}}}{a_{\overline{7|}}}.$$

140. Solution: C

From the first annuity, $X = 21.8 s_{\overline{n|}0.109} = 21.8 \cdot \dfrac{1.109^n - 1}{0.109} = 200[1.109^n - 1]$.

From the second annuity, $X = 19,208(v^n + v^{2n} + \cdots) = 19,208\dfrac{v^n}{1-v^n} = 19,208\dfrac{1}{1.109^n - 1}$.

Hence,

$$200[1.109^n - 1] = 19,208\frac{1}{1.109^n - 1}$$

$$[1.109^n - 1]^2 = 19,208/200 = 96.04$$

$$1.109^n - 1 = 9.8$$

$$X = 200(9.8) = 1960.$$

141. Solution: C

$$2(Ia)_{\overline{60|}1\%} = 2\frac{\ddot{a}_{\overline{60|}} - 60v^{60}}{0.01} = 2\frac{45.4 - 33.03}{0.01} = 2,474.60.$$

142. Solution: E

Let j be the semi-annual interest rate. Then,

$$475{,}000 = 300 + 300a_{\overline{\infty}|j} + (1+j)^{-1}200(Ia)_{\overline{\infty}|j} = 300 + 300/j + 200/j^2$$

$$474{,}700j^2 - 300j - 200 = 0$$

$$j = \frac{300 + \sqrt{300^2 - 4(474{,}700)(-200)}}{2(474{,}700)} = 0.02084$$

$$i = (1+j)^2 - 1 = 0.04212 = 4.21\%.$$

143. Solution: B

The present value is

$$4a_{\overline{\infty}|0.06} + 2(Ia)_{\overline{\infty}|0.06} = 4/0.06 + 2(1.06)/0.06^2 = 655.56.$$

144. Solution: A

The present value of the income is $100a_{\overline{\infty}|0.1025} = 100/0.1025 = 975.61$. The present value of the investment is

$$X\left[1 + 1.05/1.1025 + (1.05/1.1025)^2 + (1.05/1.1025)^3 + (1.05/1.1025)^4 + (1.05/1.1025)^5\right]$$

$$= X[1 + 1.05^{-1} + 1.05^{-2} + 1.05^{-3} + 1.05^{-4} + 1.05^{-5}] = X\frac{1-1.05^{-6}}{1-1.05^{-1}} = 5.3295X.$$

Then 975.61=5.3295X for X = 183.06.

145. Solution: A

The present value of the ten level payments is $X\ddot{a}_{\overline{10}|0.05} = 8.10782X$. The present value of the remaining payments is

$$X(v^{10}1.015 + v^{11}1.015^2 + \cdots) = X\frac{v^{10}1.015}{1-v1.015} = X\frac{1.015/1.05^{10}}{1-1.015/1.05} = 18.69366X.$$

Then, 45,000 = 8.10782X + 18.69366X = 26.80148X for X = 1679.

146. Solution: D

The equation of value is

$$10,000 = X(v + v^2 0.996 + v^3 0.996^2 + \cdots) = X \frac{v}{1 - v0.996} = X \frac{e^{-0.06}}{1 - e^{-0.06} 0.996} = 15.189X.$$ The solution is X = 10,000/15.189 = 658.37.

147. Solution: D

Discounting at 10%, the net present values are 4.59, –2.36, and –9.54 for Projects A, B, and C respectively. Hence, only Project A should be funded. Note that Project C's net present value need not be calculated. Its cash flows are the same as Project B except being 50 less at time 2 and 50 more at time 4. This indicates Project C must have a lower net present value and therefore be negative.

148. Solution: D

The loan balance after 10 years is still 100,000. For the next 10 payments, the interest paid is 10% of the outstanding balance and therefore the principal repaid is 5% of the outstanding balance. After 10 years the oustanding balance is $100,000(0.95)^{10} = 59,874$. Then, $X = 59,874 / a_{\overline{10}|0.1} = 59,874 / 6.14457 = 9,744.$

149. Solution: B

First determine number of regular payments:
$4000 = 600v^4 a_{\overline{n}|0.06}$, $a_{\overline{n}|0.06} = (4000/600)1.06^4 = 8.4165$. Using the calculator, n = 12.07 and thus there are 11 regular payments. The equation for the balloon payment, X, is:

$$4000 = 600v^4 a_{\overline{11}|0.06} + Xv^{16} = 3748.29 + 0.39365X, \ X = 639.43.$$

150. Solution: C

$$20,000 = X\left(a_{\overline{5}|0.11} + 1.11^{-5} a_{\overline{5}|0.12}\right) = X(3.69590 + 3.60478/1.68506) = 5.83516\text{X}$$

$$\text{X} = 20,000/5.83516 = 3427.50.$$

151. Solution: A

The principal repaid in the first payment is 100 – iL. The outstanding principal is L – 100 + iL = L + 25. Hence, iL = 125. Also,

$$L = 300a_{\overline{16}|} - 200a_{\overline{8}|} = \frac{300(1-v^{16}) - 200(1-v^8)}{i}$$

$$125 = iL = 100 + 200v^8 - 300v^{16}$$

$$300v^{16} - 200v^8 + 25 = 0$$

$$v^8 = \frac{200 \pm \sqrt{200^2 - 4(300)(25)}}{600} = \frac{200 \pm 100}{600} = 0.5.$$

The larger of the two values is used due to the value being known to exceed 0.3. The outstanding valance at time eight is the present value of the remaining payments:

$$300a_{\overline{8}|} = 300\frac{1-0.5}{2^{1/8}-1} = 1657.$$

152. Solution: E

Let j be the monthly rate and X be the level monthly payment. The principal repaid in the first payment is 1400 = X – 60,000j. The principal repaid in the second payment is 1414 = X – (60,000 – 1400)j. Substituting X = 1400 + 60,000j from the first equation gives 1414 = 1400 + 60,000j – 58,600j or 14 = 1400j and thus j = 0.01 and X = 2000. Let n be the number of payments. Then $60,000 = 2000a_{\overline{n}|0.01}$ and the calculator (or algebra) gives n = 35.8455. The equation for the drop payment, P, is $60,000 = 2000a_{\overline{35}|0.01} + Pv^{36} = 58,817.16 + 0.698925P$ for P = 1692.

153. Solution: C

The accumulated value is

$$1000\left(s_{\overline{24}|0.06/12}(1+0.08/12)^{24} + s_{\overline{24}|0.08/12}\right) = 1000(25.4320(1.1729) + 25.9332) = 55,762.$$

154. Deleted

155. Solution: E

The notional amount and the future 1-year LIBOR rates (not given) do not factor into the calculation of the swap's fixed rate.

Required quantities are

(1) Zero-coupon bond prices:

$1.04^{-1} = 0.96154, 1.045^{-2} = 0.91573, 1.0525^{-3} = 0.85770, 1.0625^{-4} = 0.78466, 1.075^{-5} = 0.69656.$

(2) 1-year implied forward rates:

$0.04, 1.045^2/1.04 - 1 = 0.05002, 1.0525^3/1.045^2 = 0.06766,$

$1.0625^4/1.0525^3 - 1 = 0.09307, 1.075^5/1.0625^4 - 1 = 0.12649.$

The fixed swap rate is:

$$\frac{\begin{array}{c}0.96154(0.04) + 0.9173(0.05002) + 0.85770(0.06766) \\ + 0.78466(0.09307) + 0.69656(0.12649)\end{array}}{0.96154 + 0.91573 + 0.85770 + 0.78466 + 0.69656} = 0.07197.$$

The calculation can be done without the implied forward rates as the numerator is 1 – 0.69656 = 0.30344.

156. Solution: C

In the second year of the swap contract, Company ABC has the following interest payment outflows:

Existing debt: 2,000,000 × (LIBOR + 0.5%) = 2,000,000 × (4.0% + 0.5%) = 90,000.

Swap contract, fixed rate, to the swap counterparty: 2,000,000 × 3.0% = 60,000.

Also, in the second year of the swap contract, ABC has the following interest payment inflow:

Swap contract, variable rate, to the swap counterparty: 2,000,000 × LIBOR = 2,000,000 × 4.0% = 80,000.

Thus, the combined net payment that Company ABC makes is:

(90,000 + 60,000) – (80,000) = 70,000, which is an outflow.

157. DELETED

158. Solution: D

First, the implied forward rates are:

Year	1	2	3	4	5	6
Implied forward rate	2.5%	3.7%	4.0%	4.2%	5.62%	5.21%

PV (floating payments) = $\left(\frac{0.04}{(1.034)^3}+\frac{0.042}{(1.036)^4}+\frac{0.0562}{(1.04)^5}\right)$ = 0.03618+0.3646+0.04619=0.11883.

PV (fixed payments) = $\left(\frac{r}{(1.034)^3}+\frac{r}{(1.036)^4}+\frac{r}{(1.04)^5}\right)$ = (0.90456 + 0.86808 + 0.82193)r = 2.59457r.

Equating floating to fixed payments: 0.11883 = 2.59457r for $r = \left(\frac{0.1183}{2.59457}\right) = 4.6\%$.

159. Solution: C

Each month the principal paid increases by $1.1^{1/12}$. Thus, the amount of principal paid increases to $500(1.1^{1/12})^{30-6} = 500(1.1)^2 = 605.$

160. Solution: C

$$\text{Int}_{11} = i\cdot\left[900\cdot a_{\overline{20|}i} + 300a_{\overline{10|}i}\right] = 900(1-v^{20}) + 300(1-v^{10}) = 1200 - 300v^{10} - 900v^{20}$$

$$\text{Int}_{21} = i\left[900\cdot a_{\overline{10|}i}\right] = 900(1-v^{10})$$

$$\text{Int}_{11} = 2\text{Int}_{21} \Rightarrow 1200 - 300v^{10} - 900v^{20} = 1800 - 1800v^{10}$$

$$\Rightarrow 9v^{20} - 15v^{10} + 6 = 0 \Rightarrow v^{10} = 2/3$$

$$\text{Int}_{21} = 900(1-v^{10}) = 300$$

161. Solution: C

$$427.50s_{\overline{5|}0.115}(1.115)^5 + Xs_{\overline{5|}0.115} = 10,000$$

$$X = \frac{10,000}{s_{\overline{5|}0.115}} - 427.50(1.115)^5 = \frac{10,000}{6.2900} - 736.73 = 853.10.$$

162. Solution: C

The equation to solve is

$$(965.76-3000i)s_{\overline{4}|0.08}=3000$$
$$(965.76-3000i)4.5061=3000$$
$$965.76-3000i=665.7642$$
$$3000i=300\Rightarrow i=0.10$$

163. Solution: C

The original monthly payment is $85,000/a_{\overline{240}|0.005}=85,000/139.5808=608.97$. On July 1, 2009 there has been 4 years of payments, hence 16x12 = 192 remaining payments. The outstanding balance is $608.97a_{\overline{192}|0.005}=608.97(123.2380)=75,048.24$. The number of remaining payments after refinancing is determined as

$$75,048.24=500a_{\overline{n}|0.0045}=500\frac{1-1.0045^{-n}}{0.0045}$$
$$0.67543=1-1.0045^{-n}$$
$$n=-\ln(0.32457)/\ln(1.0045)=250.62.$$

Thus the final payment will be 251 months from June 30, 2009. This is 20 years and 11 months and so the final payment is May 31, 2030.

164. Solution: B

Just prior to the extra payment at time 5, the outstand balance is $1300a_{\overline{20}|0.07}=1300(10.5940)=13,772.20$. After the extra payment it is 11,172.20. Paying this off in 15 years requires annual payments of $11,172.20/a_{\overline{15}|0.07}=11,172.20/9.1079=1226.65$.

165. Solution: C

During the first redemption period the modified coupon rate is 1000(0.035)/1250 = 2.80% which is larger than the desired yield rate. If redeemed during this period, bond sells at a premium and so the worst case for the buyer is the earliest redemption. The price if called at that time is $35a_{\overline{20}|0.025}+1250(1.025)^{-20}=35(15.5892)+762.84=1308.46$. During the second redemption period the modified coupon rate is 1000(0.035)/1125 = 3.11% which is also larger than the desired yield rate and the worst case for the buyer is again the earliest redemption. The price if called at that time is $35a_{\overline{40}|0.025}+1125(1.025)^{-40}=35(25.1028)+418.98=1297.58$. Finally, if the bond is not called, its value is $35a_{\overline{60}|0.025}+1000(1.025)^{-60}=35(30.9087)+227.28=1309.08$.

The appropriate price is the lowest of these three, which relates to the bond being called after the 40th coupon is paid.

166. Solution: B

Because the yield is less than the coupon rate, the bond sells at a premium and the worst case for the buyer is an early call. Hence the price should be calculated based on the bond being called at time 16. The price is $100a_{\overline{16}|0.05} + 1000(1.05)^{-16} = 100(10.0378) + 458.11 = 1542$.

167. Solution: A

All calculations are in millions. For the ten-year bond, at time ten it is redeemed for $2(1.08)^{10} = 4.31785$. After being reinvested at 12% it matures at time twenty for $4.31785(1.12)^{10} = 13.4106$. The thirty-year bond has a redemption value of $4(1.08)^{30} = 40.2506$. For the buyer to earn 10%, it is sold for $40.2506(1.1)^{-10} = 15.5184$. The gain is 13.4106 + 15.5184 – 6 = 22.9290.

168. Solution: A

The book value after the third coupon is $7500(0.037)a_{\overline{37}|0.0265} + C(1.0265)^{-37} = 6493.05 + 0.379943C$ and after the fourth coupon it is $7500(0.037)a_{\overline{36}|0.0265} + C(1.0265)^{-36} = 6387.61 + 0.390012C$. Then,

$$6493.05 + 0.379943C - (6387.61 + 0.390012C) = 28.31$$
$$105.44 - 0.010069C = 28.31$$
$$C = 7660.15.$$

169. Solution: C

The semiannual yield rate is $1.1^{1/2} - 1 = 0.0488$. Assuming the bond is called for 2900 after four years, the purchase price is $150a_{\overline{8}|0.0488} + 2900(1.0488)^{-8} = 150(6.4947) + 1980.87 = 2955.08$. With a call after the first coupon, the equation to solve for the semi-annual yield rate (j) and then the annual effective rate (i) is

$$2955.08 = (150 + 2960) / (1 + j)$$
$$1 + j = 1.05242$$
$$i = 1.05242^2 - 1 = 0.10759.$$

170. Solution: C

The book value after the sixth coupon is $1000(r/2)a_{\overline{34}|0.036}+1000(1.036)^{-34}=9716.01r+300.45$. After the seventh coupon it is $1000(r/2)a_{\overline{33}|0.036}+1000(1.036)^{-33}=9565.79r+311.26$. Then,

$$4.36=9565.79r+311.26-(9716.01r+300.45)=10.81-150.22r$$
$$r=(10.81-4.36)/150.22=0.0429.$$

171. Solution: B

The two equations are:

$$P=(10,000r)a_{\overline{5}|0.04}+9,000(1.04)^{-5}=44,518.22r+7,397.34$$
$$1.2P=[10,000(r+0.01)]a_{\overline{5}|0.04}+11,000(1.04)^{-5}=44,518.22r+9,486.38.$$

Subtracting the first equation from the second gives 0.2P = 2089.04 for P = 10,445.20. Inserting this in the first equation gives r = (10,445.20 – 7,397.34)/44,518.22 = 0.0685.

172. Solution: C

When the yield is 6.8% < 8%, the bond is sold at a premium and hence an early call is most disadvantageous. Therefore, $P=40a_{\overline{10}|0.034}+1000(1.034)^{-10}=1050.15$. When the yield is 8.8% > 8%, the bond is sold at discount. Hence, Q < 1000 < P. and thus Q = 1050.15 – 123.36 = 926.79. Also, because the bond is sold at a discount, the latest call is the most disadvantageous. Thus,

$$926.79=40a_{\overline{2n}|0.044}+1000(1.044)^{-2n}=\frac{40}{0.044}+(1.044)^{-2n}\left(1000-\frac{40}{.044}\right)=909.09+90.90(1.044)^{-2n}$$
$$17.70=90.90(1.044)^{-2n}$$
$$2n=-\ln(17.70/90.90)/\ln(1.044)=38$$
$$n=19.$$

173. Solution: B

The fund will have $500(1.05)^4 - 100s_{\overline{4}|0.05} = 176.74$ after four years. After returning 75% to the insured, the insurer receives 0.25(176.74) = 44.19. So the insurer's cash flows are to pay 100 at time 0, receive 125 at time 2, and receive 44.19 at time four. The equation of value and the solution are:

$$100(1+i)^4 - 125(1+i)^2 - 44.19 = 0$$

$$(1+i)^2 = \frac{125 \pm \sqrt{(-125)^2 - 4(100)(-44.19)}}{200} = 1.5374$$

$$1+i = 1.2399$$

$$i = 24\%.$$

174. Solution: A

If the value of X increases, the 9% rate from July 1 to December 31 counts more heavily than the (4320 – 4000)/4000 = 8% rate from January 1 to June 30. So the annual effective yield rate increases.

The time-weighted rate depends only on percentage increases in each sub-period and thus it remains unchanged**.**

175. Solution: B

The amount of interest earned is 100,000 +50,000 – 30,000 – 100,000 = 20,000. The amount invested for the year is 100,000 + (1 – 5/12) 30,000 – (1 – 3/4) 50,000 = 105,000. The dollar-weighted rate of return is 20,000/105,000 = 19.05%.

176. Solution: B

The Macaulay duration of the perpetuity is $\frac{\sum_{n=1}^{\infty} nv^n}{\sum_{n=1}^{\infty} v^n} = \frac{(Ia)_{\overline{\infty}|}}{a_{\overline{\infty}|}} = \frac{(1+i)/i^2}{1/i} = \frac{1+i}{i} = 1 + 1/i = 17.6.$

This implies that i = 1/16.6. With i = 2i = 2/16.6, the duration is 1 + 16.6/2 = 9.3.

177. Solution: A

Because the interest rate is greater than zero, the Macaulay duration of each bond is greater than its modified duration. Therefore, the bond with a Macaulay duration of c must be the bond with a modified duration of a and $a = c/(1+i)$ which implies $1 + i = c/a$. The Macaulay duration of the other bond is $b(1+i) = bc/a$.

178. Solution: B

$P(0.1025) \approx P(0.10)\left(\frac{1.10}{1.1025}\right)^{11} = 0.97534P(0.10).$ Therefore, the approximate percentage price change is 100(0.97534 – 1) = –2.47%.

179. Solution: C

The present value of the dividends is:

$$\frac{2\times1.07^{10}}{1.04^{10}}\times\left(\frac{1}{1.1}+\frac{1.07}{1.04\cdot1.1^2}+\cdots\right)=\frac{2\times1.07^{10}}{1.04^{10}}\times\frac{1}{1.1}\times\frac{1}{1-1.07/(1.04\times1.1)}=37.35.$$

180. Solution: B

Cash-flow matching limits the number of investment choices available to the portfolio manager to a subset of the choices available for immunization.

181. Solution: C

Options for full immunization are:

2J (cost is 3000), K+2L (cost is 2500), and M (cost is 4000). The lowest possible cost is 2500. Another way to view this is that the prices divided by total cash flows are 0.6, 0.5, 0.5, and 0.8. The cheapest option will be to use K and L, if possible.

182. Solution: B

The present value of the assets is 15,000 + 45,000 = 60,000 which is also the present value of the liability. The modified duration of the assets is the weighted average, or 0.25(1.80) + 0.75*D*mod. The modified duration of the liability is 3/1.1 and so *D*mod = (3/1.1 – 0.45)/0.75 = 3.04.

183. Solution: C

Let *A* be the redemption value of the zero-coupon bonds purchased and *B* the number of two-year bonds purchased. The total present value is:

$$1783.76 = A/1.05 + B(100/1.06 + 1100/1.06^2) = 0.95238A + 1073.3357B.$$

To exactly match the cash flow at time one, *A* + 100*B* = 1000. Substituting *B* = 10 – 0.01*A* in the first equation gives 1783.76 = 0.95238*A* + 10733.357 – 10.733357*A* for *A* = 8949.597/9.780977 = 915. The amount invested is then 915/1.05 = 871.

184. Solution: B

The company must purchase 4000 in one-year bonds and 6000 in two-year bonds. The total purchase price is $4000/1.08 + 6000/1.11^2 = 8573$.

185. Solution: E

See Section 6.3 of the Study Note for a discussion of this issue.

186. Solution: B

The rate on borrowing between reserve balances is the federal fund rate, which is 3.25%.

187. Solution: C

The modified duration is 11/1.10 = 10. Then,
$P(0.1025) \approx P(0.10)[1-(0.1025-0.10)10] = 0.975P(0.10)$. Therefore, the approximate percentage price change is 100(0.975 – 1) = –2.50%.

188. Solution: B

$$P(0.08) \approx 1000\left(\frac{1.072}{1.08}\right)^{7.959} = 942.54.$$

189. Solution: E

Modified duration = (Macaulay duration)/(1 + i) and so Macaulay duration = 8(1.064) = 8.512.

$E_{MAC} = 112,955\left(\frac{1.064}{1.07}\right)^{8.512} = 107,676$ and $E_{MOD} = 112,955[1-(0.07-0.064)(8)] = 107,533$.

Then, $E_{MAC} - E_{MOD} = 107,676 - 107,533 = 143$.

190. Solution: C

The Macaulay duration of the portfolio is $\frac{35,000(7.28)+65,000(12.74)}{35,000+65,000} = 10.829$. Then,

$$105,000 = 100,000\left(\frac{1.0432}{1+i}\right)^{10.829} \Rightarrow \frac{1.0432}{1+i} = \left(\frac{105,000}{100,000}\right)^{1/10.829} = 1.004516 \Rightarrow i = 0.0385.$$

191. Solution: A

$121,212 = 123,000\left(\frac{1.05}{1.054}\right)^{D_{MAC}} \Rightarrow D_{MAC} = \frac{\ln(121,212/123,000)}{\ln(1.05/1.054)} = 3.8512$. Then,
$D_{MOD} = 3.8512/1.05 = 3.67$.

192. Solution: D

$QR = \frac{360}{180}\frac{100,000-95,000}{100,000} = 0.1$. The annual effective yield is the solution to

$95,000(1+j)^{180/365} = 100,000$ which implies $j = \left(\frac{100,000}{95,000}\right)^{365/180} - 1 = 0.1096$. Then,

$j - QR = 0.1096 - 0.1 = 0.0096 = 0.96\%$.

193. Solution: D

The quoted rate on the Canadian T-Bill is $\frac{365}{120}\frac{100,000-98,000}{98,000} = 0.062075$. Thus, statement A is true.

The quoted rate on the U.S. T-Bill is $\frac{360}{120}\frac{100,000-98,000}{100,000} = 0.06$. Thus, statement B is true.

For both, the annual effective yield is the solution to $(1+i)^{120/365} = \frac{100,000}{98,000}$, which is i = 0.063377, thus statement C is true.

Based on the previous calculations, statement D is false and statement E is true.

194. Solution: D

Assume that 1 is borrowed. Anderson wants to receive exp(5*0.05) = 1.28403 as compensation for deferred consumption. The actual rate charged will be $0.05+\delta$. The expected amount received, given the probability of default is
$(1-0.007)e^{5(0.05+\delta)} + 0.007(0.3)e^{5(0.05+\delta)} = 0.9951e^{0.25}e^{5\delta} = 1.27773e^{5\delta}$. Setting this equal to 1.28403 and solving produces $\delta = 0.2\ln(1.28403/1.27773) = 0.00098$.

195. Solution: C

The first year, the government pays 0.032 + 0.024 = 0.056 compounded continuously. In the next two years the rates will be 0.032 + 0.028 = 0.060 and 0.032 + 0.042 = 0.074 respectively. The amount owed after three years is $100,000e^{0.056}e^{0.060}e^{0.074} = 120,925$.

196. Solution: E

It is an amortizing, not an accreting, swap. The settlement period is one year, not three. The notional amount changes each year. Trout Bank is not a counterparty. The counterparties are Katarina and Lily.

197. Solution: C

$$R = \frac{\sum_{i=1}^{n} Q_{t_i} f^*_{[t_{i-1},t_i]} P_{t_i}}{\sum_{i=1}^{n} Q_{t_i} P_{t_i}} = \frac{Q_1 f^*_{[0,1]} P_1 + Q_2 f^*_{[1,2]} P_2 + Q_3 f^*_{[2,3]} P_3}{Q_1 P_1 + Q_2 P_2 + Q_3 P_3}$$

$$f^*_{[1,2]} = \frac{(1+r_2)^2}{(1+r_1)^1} - 1 = \frac{(1.046)^2}{1.043} - 1 = 0.049008629 \text{ and } f^*_{[2,3]} = \frac{(1+r_3)^3}{(1+r_2)^2} - 1 = \frac{(1.051)^3}{(1.046)^2} - 1 = 0.061071816$$

$$= \frac{(300,000)(0.043)(1.043)^{-1} + (200,000)(0.04901)(1.046)^{-2} + (100,000)(0.06107)(1.051)^{-3}}{(300,000)(1.043)^{-1} + (200,000)(1.046)^{-2} + (100,000)(1.051)^{-3}}$$

$$= 0.04777 = 4.78\%$$

198. Solution: D

$$R = \frac{P_{t_0} - P_{t_n}}{\sum_{i=1}^{n} P_{t_i}} = \frac{P_2 - P_5}{P_3 + P_4 + P_5} = \frac{(1.046)^{-2} - (1.056)^{-5}}{(1.051)^{-3} + (1.054)^{-4} + (1.056)^{-5}} = 0.06266 = 6.27\%$$

199. Solution: A

Miaoqi is receiving the fixed interest rate and paying the variable rate. This means that the first year she will receive the fixed rate of 4% and pay the variable rate of 3.8%. During the second year, she will receive the fixed rate of 4% and pay the variable rate of $f_{[1,2]}$.

$$f_{[1,2]} = \frac{(1.041)^2}{1.038} - 1 = 0.044008671$$

Market Value = Present Value of Expected Cash Flows

$$= \frac{(250,000)(0.04 - 0.038)}{1.038} + \frac{(250,000)(0.04 - 0.044008671)}{(1.041)^2} = -443.09$$

200. Solution: C

The net interest payment is the interest paid on the loan plus any net swap payment made by the loan holder less any net swap payment received by the loan holder. The interest paid on the laon is (500,000)(*LIBOR* + 0.012) = (500,000)(0.056 + 0.012) = 34,000.

The net swap payment received by SOA is (500,000)(*LIBOR* +0.005 – 0.0535) = 3,750.

The net interest payment is 34,000 – 3,750 = 30,250.

Note that this can also be calculated as the notional amount multiplied by the sum of the interest rate paid by the loan holder under the swap plus any spread between the loan and the swap. That produces a net interest payment of (500,000)(0.0535 + 0.012 – 0.005) = 30,250.

201. Solution: C

$$R = \frac{1 - P_1}{P_{0.25} + P_{0.5} + P_{0.75} + P_1} = \frac{1 - (1.0192)^{-1}}{(1.015)^{-0.25} + (1.0165)^{-0.5} + (1.0179)^{-0.75} + (1.0192)^{-1}}$$
$$= 0.0047612 = 48 \text{ bp}$$

Note that this is a quarterly effective interest rate.

202. Solution: A

First we need to calculate the swap rate:

$$R = \frac{1 - P_4}{P_1 + P_2 + P_3 + P_4} = \frac{1 - 0.825}{0.965 + 0.92 + 0.875 + 0.825} = 0.0488145.$$

At the end of the first year, Josh owes the fixed rate: (200,000)(0.0488145) = 9,762.90. At the end of the first year, Phillip owes the variable rate: (200,000)(1/0.965 – 1) = 7,253.89. Thus, Josh pays 9,762.90 – 7,253.89 = 2,509.01.