Notation and Terminology Used on Exam LTAM
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Introduction

This notation note completely replaces similar notes used on previous examinations.

In actuarial practice there is notation and terminology that varies by country, by application, and by source. The purpose of this study note is to present notation and terminology that will be used on the LTAM examination for situations where notation or terms differ from that in the recommended resources (*Actuarial Mathematics for Life Contingent Risks* (3rd edition) (*AMLCR*)) for the examination and notation or terms that are unique to the examination. For notation and terms not discussed here, the meaning in *AMLCR* or the appropriate Study Note will apply.

The format of this note is to list common alternative notations for a given item. The specific notation(s) that will be used on the examination will then be provided.

Notation and Terminology

The **force of mortality** may be represented by μ_x or $\mu(x)$ or μ_{x+t} or $\mu(x+t)$ where x and $x+t$ are attained ages. The symbol $\mu_{[x]+t}$ indicates selection at age x and attained age $x+t$. The symbols μ_x, μ_{x+t}, and $\mu_{[x]+t}$ will be used on the examination.

Both the **survival function, salary rate function and salary scale** may be represented by *S or s*. On the examination, the symbols $\bar{s}_x$ and s_x will indicate the salary rate function and salary scale respectively. For the survival function, there are multiple symbols (all involving S) used in *AMLCR* and the Study Notes. When the symbols for the survival function are used on the examination, the definition will be clear from the context of the question or the question will define the symbol.

The **number of lives** at age x can be represented by ℓ_x or l_x. Either symbol may be used on the examination.

The **complete future lifetime of (*x*) random variable** can be represented by T_x or $T(x)$. Similarly, the symbols used for joint life status can be T_{xy} or $T(xy)$, and for last survivor status can be $T_{\overline{xy}}$ or $T\left(\overline{xy}\right)$. The symbols T_x, T_{xy} and $T_{\overline{xy}}$ will be used on the examination.

The **curtate future lifetime of (x) random variable** can be represented by K_x or $K(x)$. Similarly, the symbols used for joint life status are K_{xy} or $K(xy)$, and for last survivor status are $K_{\overline{xy}}$ or $K\left(\overline{xy}\right)$. The symbols K_x, K_{xy} and $K_{\overline{xy}}$ will be used on the examination.

The **present value of future losses random variable** may be represented by L or ${}_0L$ or L_0 for loss at issue and ${}_tL$ or L_t for loss from t years after issue. Superscripts may be included. When the symbol L is used to represent present value of future losses random variable the symbol including any subscripts or superscripts will be defined in the text of the question.

Duration subscripts can be used differently. For example, something happening in the first duration (between ages x and x+1) may be identified with a 0 or 1. The text of the question will define any notation used.

If benefits can vary continuously, the **benefit** at time t is represented by b_t. If benefits vary but as a step function, the **benefit** at the end of period k is represented by b_k. The text of the question will define the benefit either by formula or in words.

Actuarial present value and **expected present value** are terms used for the expectation of the random variable representing the present value of one or more contingent future payments. Either term may be used on the examination.

Fully discrete insurance is an insurance where both the premiums and the benefits are paid only at discrete time points. **Semi-continuous** insurance is an insurance where the premiums are paid at discrete time points and the death benefits are paid at the moment of death. **Fully continuous** insurance is an insurance where the premiums are paid continuously and the death benefits are paid at the moment of death. Unless stated otherwise in the text of the question discrete time points are the beginnings of years for premium payments and the ends of years for death benefit payments.

Special insurance is an insurance that has either non-level benefits or non-level premiums or both. The non-level aspects of the insurance will be described in the text of the question. If an insurance is not defined as “special” then premiums and benefits are assumed to be level, unless there is explicit information in the text of the question to the contrary.

Net premium is the premium determined by the equivalence principle and assuming no expenses. In the MLC exams prior to 2014 this was called benefit premium. The term benefit premium will no longer be used on the examination.

The **net premium for fully discrete insurances** will be represented by P with the appropriate symbols attached. P_x , $P^{1}_{x:\overline{n}|}$, $P_{x:\overline{n}|}^{\;\;1}$, and $P_{x:\overline{n}|}$ may be used on the exam.

The symbols are defined in terms of an insurance, A, and an annuity, $\ddot{a}$, as follows:

$$P_x = \frac{A_x}{\ddot{a}_x} \ , \ P^{1}_{x:\overline{n}|} = \frac{A^{1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \ , \ P_{x:\overline{n}|}^{\;\;1} = \frac{A_{x:\overline{n}|}^{\;\;1}}{\ddot{a}_{x:\overline{n}|}} \ , \ P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

The symbol P will be defined within the text of the question if it is not one of the symbols shown above.

Net premium reserves are reserves based on the net premium assuming no expenses, and using the same mortality and interest assumptions as the net premium calculation. In MLC exams prior to 2014 these were called benefit reserves. The term benefit reserve will no longer be used on the examination.

Gross premium reserves are reserves based on the policy's actual gross premium. The mortality, interest and expense assumptions for the reserve would not necessarily be the same as those used in that gross premium calculation. That gross premium may not be the gross premium that would be determined using the equivalence principle.

Unless stated otherwise in the text of a question all expenses are equal to zero. If expenses are specified in the text of a question then the expenses need to be considered in the solution to the question.

In a **multiple decrement model** $q_x^{(j)}$ is the probability a life age *x* fails in the next year due to decrement (*j*) and $q_x^{(\tau)}$ is the probability of failure due to all decrements. The associated single decrement probability of failure at age *x* due to decrement (*j*) is $q_x'^{(j)}$. The probability $q_x^{(j)}$ is also called the dependent probability. The probability $q_x'^{(j)}$ is also called the independent probability. A multiple decrement model is a special case of a multi-state model. Any of the multi-state model notation of the next paragraph may be used with a multiple decrement model on the examination.

In a **multi-state model** ${}_t p_x^{ij}$ is the probability that a life currently age *x* and in state *i* is in state *j* at age *x*+*t*. The symbol μ_x^{ij} is the force of transition between states *i* and *j* at age *x*. The symbol ${}_t p_x^{\overline{ii}}$ is the probability that a life currently age *x* and in state *i* remains in state *i* from age *x* to age *x*+*t*.

The **reserve** at time ***t*** may be represented by ${}_tV$ or V_t. The symbol ${}_tV$ will be used on the examination.

In practice, the financial statements of an insurance company will include a liability amount in respect of future outgo on a policy in force, and this amount is called the reserve. *AMLCR* calls this "the actual capital held in respect of a policy" and uses the term reserve only in this context. The exam will use the term reserve both for this context, and also for the expected value of a future loss random variable, even where this is not related to the provision in the financial statements. *AMLCR* calls the expected value of the future loss random variable a policy value. The term policy value will not be used on the exam. (This paragraph keeps the meaning of reserve on the examination the same as its meaning in examinations before 2014. *AMLCR* discusses its distinction between reserve and policy value on page 225 and in chapter 13.).

Reserves may be calculated based not only on mortality, but also with possible inclusion of other decrements (e.g. surrender/lapse, morbidity). Unless otherwise noted in the problem, reserves should be calculated based on the decrements given in the problem.

A **modified reserve** is a reserve computed without expenses but adjusting the valuation premiums to allow implicitly for initial expenses. A full preliminary term reserve is an example of a modified reserve. All modified reserves have the expected present value at issue of the benefits equal to the expected present value at issue of the valuation premiums; valuation premiums are typically lower in the first year or first few years than in later years. Any modified reserve questions on the examination other than full preliminary term reserves will specify the modification basis in the question.

If a **table of select and ultimate values** is presented in a question the format of the table will either follow the convention of (i) reading across the row of select rates and then down the column of ultimate rates for the values corresponding to each age at selection or it will follow the convention that (ii) all row entries indicate a current age but differ as to the age at selection. On the examination, the table method can be inferred from the table headers.

On the examinations the **transition probabilities** for a multi-state model may be presented in a matrix. For example, for a model with two states, 0 and 1, the transition probabilities would be presented in a matrix as follows:

$$\begin{bmatrix} p_x^{00} & p_x^{01} \\ p_x^{10} & p_x^{11} \end{bmatrix}$$

Euler's method can be used to numerically obtain an approximate solution to a differential equation. The method produces a piece-wise linear approximate solution. There are two versions, called the forward method and the backward method. In both methods the next line segment joins the current point in time and next point in time. In the forward version, the slope is determined at the beginning of the time interval, whereas in the backward method, the slope of the curve is evaluated at the other endpoint.

That is, suppose we have a differential equation $\frac{d}{dt}f(t) = d(t)$ for some functions f and d.

The **Forward Euler Approximation**, with step size h, gives

$$\frac{f(t+h)-f(t)}{h} = d(t) \Rightarrow f(t+h) = f(t) + hd(t) \ .$$

The **Backward Euler Approximation**, with step size h, gives

$$\frac{f(t+h)-f(t)}{h} = d(t+h) \Rightarrow f(t) = f(t+h) - hd(t+h).$$

Generally, the forward method is used when the boundary condition is an initial value (for example, the Kolmogorov Forward equations for transition probabilities), and the backward method is used when the boundary condition is a terminal value (for example, the Thiele equation for policy reserves). In the case of Thiele's equation for policy reserves in a life/death model, the forward method might also be used even when the boundary condition is a terminal value. For example, see *AMLCR*, page 326.

On the examination**, retirement benefits** based on years of service include any fractional years, unless explicitly stated otherwise. For example, a pension plan that stipulates that annual retirement benefits are calculated by the participant's final salary x 0.02 x the number of years of service would provide an annual benefit of 8200 for a participant with a final salary of 20,000 and 20.5 years of service (20,000 x 0.02 x 20.5).

As noted in Section 11.8 of *AMLCR*, the term **Normal Cost** is synonymous with the term **Normal Contribution**. Either term may be used on the examination.

Actuarial liability and **actuarial accrued liability** are synonymous terms and either term may appear on the exam.

On the examination, **pension reduction factors** may be presented in the context of benefit calculations for individuals who retire earlier than the normal retirement age. These factors are simple rates per unit time, rather than compound rates per unit time, unless explicitly stated otherwise.

For example, a pension plan may stipulate a 5% reduction for each year prior to normal retirement age, 65. For a participant retiring at age 60, this factor would result in a 25% reduction to the normal retirement benefit, rather than a reduction of $1 - (1 - .05)^5$.

Unless stated otherwise in the problem, the terms **death benefit**, **face amount**, **sum insured**, and **sum assured** are synonymous terms. Any of these four terms may be used on the exam.

The terms **certain period** and **guarantee period** are synonymous terms. Either term may be used on the exam.

<u>For a time period where all cash flows occur only at the beginning and end of the time period</u>:

Profit for the time period occurs at the end of the time period and is (a) minus (b) where:

(a) is the accumulated value of the sum of the reserve at the end of the previous period and the cash flows that occur at the beginning of the period; and
(b) is the sum of the value of the reserve at the end of the period and the cash flows that occur at the end of the period.

Expenses at the inception of a contract may be classified as negative profit at time 0 or may be part of period 1 cash flow and included in the profit calculation for period 1. Any initial expenses that are not part of period 1 cash flow will be identified in the question as **Pre-contract expenses**. If a reserve is to be established at time 0, before the first premium is received, it would be part of the time 0 profit. Any such reserve will be identified in the question.

Expected profit is the profit calculated using the gross premium, expected cash flows at the beginning and end of the period, and accumulating beginning of year values at the expected interest rate. The expected interest rate, and the assumptions used to calculate the expected cash flows may or may not be the same as those used to calculate the reserve.

Actual profit is calculated using the gross premium, actual cash flows at the beginning and end of the period, and accumulating beginning of year values using the actual rate of interest earned during the period.

Gain is the actual profit minus the expected profit for the period. **Gain by source** is the gain calculated where the effect of the difference between the observed values and the expected values in the profit calculations from one source is reflected, while the differences for the other sources are not. Examples of sources are: expenses, interest, mortality and withdrawal. Often, gains from multiple sources are calculated sequentially. For example, the gain from mortality might be calculated first, reflecting the difference between the observed mortality and the assumed mortality, and the gain from interest calculated second, reflecting the difference between the observed and assumed interest, while using only the observed mortality.

The examination will only include questions asking for Gain by source where the reserves are gross premium reserves and expected profits are based on the reserve assumptions. Under those conditions, the expected profit is 0 and the sum of the gains by source is equal to the actual profit.

Other terms and common equivalents

Terms used on the examination	Equivalent or similar terms (**not** used on the examination)
annuity due, annuity-due	due annuity
annuity immediate, annuity-immediate	immediate annuity
temporary life annuity	term annuity
temporary expectation of life	term expectation of life
premium paying period	premium paying term
net amount at risk	death strain at risk, sum at risk, amount at risk
net premium	benefit premium
gross premium	contract premium, expense-loaded premium, expense-augmented premium
net premium reserve	net premium policy value, benefit reserve
gross premium reserve	gross premium policy value
$\boldsymbol{\mu}_x^{ij}$	$\mu_{ij}(x)$, $\lambda_{ij}(x)$
${}_t\boldsymbol{p}_x^{ij}$	${}_t p_{ij}^{(x)}$
Variance, Var	V
${}_t\boldsymbol{q}_x^{(\tau)}$	${}_t p_x^{0\bullet}$
${}_t\boldsymbol{p}_x'^{(j)}$	${}_t p_x^{*(j)}$
$\boldsymbol{\mu}_x^{(\tau)}(t)$ or $\boldsymbol{\mu}_{x+t}^{(\tau)}$	$\mu_{x+t}^{0\bullet}$

Unless specified otherwise within the examination question, the following assumptions should be made:

1. The force of interest is constant.
2. Future lifetimes are independent.
3. All lives in a question follow the same mortality table.
4. Expenses are payable at the start of each period.
5. Expenses (including commissions) that are expressed as a percent of premium are payable when the corresponding premium is payable and end when the premiums are no longer payable.