

Applying Fuzzy Logic to Risk Assessment and Decision-Making

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Executive Summary

Complex models have long been used in risk management to assess uncertainty. With the growing availability of computing resources, advanced methods such as stochastic modeling, stress testing or even stochastic on stochastic modeling used for hedging programs are increasingly prevalent. While risk professionals strive for a better understanding of risk and employ complex models for risk assessment, many risks are still not well understood. Some remain unknown, and new risks have emerged. Many risk types still cannot be analyzed sufficiently using classical probability models. The lack of experience data and entangled cause-and-effect relationships make it difficult to assess the degree of exposure to certain risk types.

Traditional risk models are based on probability and classical set theory. They are widely used for assessing market, credit, insurance and trading risk. In contrast, fuzzy logic models are built upon fuzzy set theory and fuzzy logic, and they are useful for analyzing risks with insufficient knowledge or imprecise data. These latter types of risk typically fall into the operational risk or emerging risk category.

The fundamental difference between traditional set theory and fuzzy set theory is the nature of inclusion of the elements in the set. In traditional sets, an element is either included in the set or is not. In a fuzzy set, an element is included with a degree of truth normally ranging from 0 to 1. Fuzzy logic models allow an object to be categorized in more than one exclusive set with different levels of truth or confidence. Fuzzy logic recognizes the lack of knowledge or absence of precise data, and it explicitly considers the cause-and-effect chain among variables. Most variables are described in linguistic terms, which makes fuzzy logic models more intuitively similar to human reasoning. These fuzzy models are helpful for demystifying, assessing and learning about risks that are not well understood.

Fuzzy logic systems help simplify large-scale risk management frameworks. For risks that do not have a proper quantitative probability model, a fuzzy logic system can help model the cause-and-effect relationships, assess the degree of risk exposure and rank the key risks in a consistent way, considering both the available data and experts' opinions. For companies with diversified business, broad risk exposure and operations in multiple geographic regions, the long list of risks that need to be monitored makes in-depth risk analysis unaffordable, especially when there are entangled relationships among risk factors. Such an analysis could be costly and extremely tedious without the use of a fuzzy logic system. In addition, fuzzy logic systems include rules that explicitly explain the linkage, dependence and relationships among modeled factors. It is helpful for identifying risk mitigation solutions. Resources can then be used to mitigate the risks with the highest level of exposure and relatively low hedging cost.

Fuzzy set theory and fuzzy logic models can also be used with other types of pattern recognition and decision models. These include Bayesian and artificial neural networks, and hidden Markov and decision tree models. These extended models have the potential to solve

difficult risk assessment problems.

This paper explores areas where fuzzy logic models may be applied to improve risk assessment and risk decision-making. It discusses the methodology, framework and process of using fuzzy logic systems for risk management. With the help of practical examples, it is hoped that it will encourage wise application of fuzzy logic models to risk modeling.

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1. Introduction

Probability models are prevalent in risk quantification and assessment. They have become the fundamental basis for informed decision-making related to risk in many areas. However, a probability model built upon classic set theory may not be able to describe some risks in a meaningful and practical way. Lack of experience data, entangled cause-and-effect relationships and imprecise data make it difficult to assess the degree of exposure to certain risk types using only traditional probability models. Sometimes, even with a credible quantitative risk model calibrated to experience data, the cause of the risk and its characteristics may be incompletely understood. Other models, such as fuzzy logic, hidden Markov and decision tree models, and artificial neural and Bayesian networks, explicitly consider the underlying cause-and-effect relationships and recognize the unknown complexity. These newer models might do a better job in understanding and assessing certain risks, such as operational risk.

Interestingly, while well-accepted and complex quantitative models are available for market, credit and insurance risk, these risks are normally outside the control of business managers. On the other hand, with appropriate risk identification and risk control in place, operational risk can be significantly mitigated, despite the lack of consensus concerning which quantitative models should be used. Therefore, it may be beneficial to build and implement more appropriate operational risk models using a newer approach such as fuzzy logic.

This report focuses on the application of fuzzy logic and fuzzy set theory, introduced by mathematician Lotfi A. Zadeh in 1965, to risk management. Unlike probability theory, fuzzy logic theory admits the uncertainty of truth in an explicit way; it also can easily incorporate information described in linguistic terms. Fuzzy logic models are more convenient for incorporating different expert opinions and more adapted to cases with insufficient and imprecise data. They provide a framework in which experts' input and experience data can jointly assess the uncertainty and identify major issues. Using approximation and making inferences from ambiguous knowledge and data, fuzzy logic models may be used for modeling risks that are not fully understood. Some operational and emerging risks evolve quickly. Risk managers may not have enough knowledge or data for a full-blown assessment using models based on probability theory. Fuzzy logic models can be instrumental in assessing a business enterprise's exposure to these risks.

The remainder of the paper proceeds as follows:

- Section 2 (Fuzzy Logic and Fuzzy Set Theory) introduces the theoretical background of the fuzzy logic model and compares it to other models.
- Section 3 (Application of Fuzzy Logic) discusses the potential application of fuzzy logic to risk management.
- Section 4 (Risk Assessment Framework Based on Fuzzy Logic) discusses using a

fuzzy logic model for the identification, assessment and quantification of risks.

- Section 5 (Key Considerations) touches on some key factors for a practical risk management framework built on a fuzzy logic model.
- Section 6 (Case Studies) illustrates the risk identification, risk assessment and decision-making process at a micro level for a certain risk type and at an aggregate level for all enterprise risks.
- Section 7 summarizes the key points of this research and concludes the main body of the report.

2. Fuzzy Logic and Fuzzy Set Theory

This section introduces some basic concepts in fuzzy set theory and a comparison with other methods used for risk assessment and decision-making. It may be skipped by readers with a background in artificial intelligence or control engineering.

2.1 Basics of Fuzzy Set Theory and Fuzzy Logic

Fuzzy Sets

In classical set theory, an individual object is either a member or a nonmember of a set. However, in reality, due to insufficient knowledge or imprecise data, it is not always clear whether an object belongs to a set or not. In contrast, fuzzy sets interpret uncertainty in an approximate way. Conceptually, fuzzy set theory allows an object belonging to multiple exclusive sets in the reasoning framework. For each set, there is a degree of truth that an object belongs to a fuzzy set. Take credit scores as an example. Assume there are three levels of the score: low, average and high, which can be considered as three sets. Based on classical set theory, the full set is composed of these three exclusive sets. Once the credit score is known, the level of the score is determined. Figure 1 shows an example of classical sets for credit scores. With a credit score of 3.5, it is 100 percent true that the credit score is high.

Figure 1. Classical Set Example: Credit Score

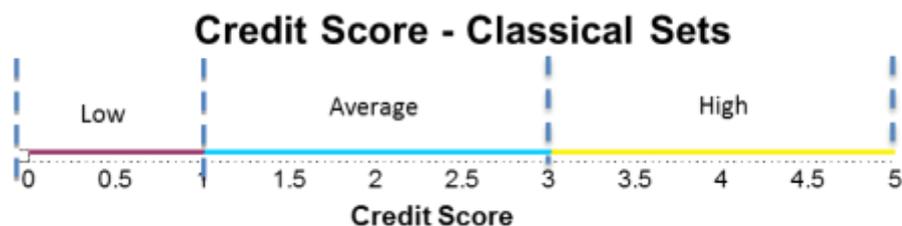
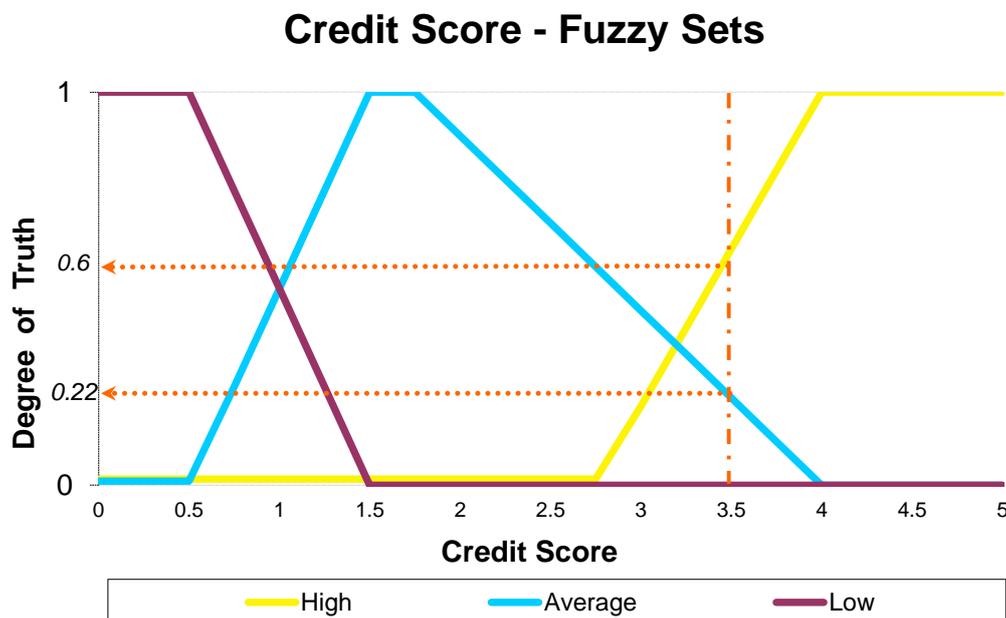


Figure 2 shows an example of fuzzy sets for credit scores. Each set has its own **membership function**, which determines the degree of truth that an element belongs to the set. For example, with a credit score of 3.5, it is 60 percent true that the score is high and 22 percent true that it is average. It is false that the score is low. In fuzzy logic theory, the degrees of truth for all sets do not necessarily add up to one for a specific object.

Figure 2. Fuzzy Set Example: Credit Score



In this example, the membership functions for the three sets are specified as below.

$$\mu^{High}(x) = \begin{cases} 0 & x \leq 2.75 \\ \frac{(x-2.75)}{1.25} & 2.75 < x \leq 4 \\ 1 & x > 4 \end{cases}$$

$$\mu^{Average}(x) = \begin{cases} 0 & x \leq 0.5 \\ \frac{(x-0.5)}{1} & 0.5 < x \leq 1.5 \\ 1 & 1.5 < x \leq 1.75 \\ \frac{(4-x)}{2.25} & 1.75 < x \leq 4 \\ 0 & x > 4 \end{cases}$$

$$\mu^{Low}(x) = \begin{cases} 1 & x \leq 0.5 \\ \frac{(1.5-x)}{1} & 0.5 < x \leq 1.5 \\ 0 & x > 1.5 \end{cases}$$

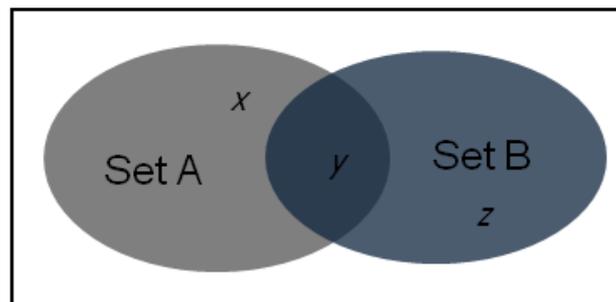
A key feature of fuzzy sets is that there are no hard rules about how their membership functions are defined. Both the mathematical form of the function and the parameters depend on the input from the experts. As long as the membership functions are consistent, on a comparative basis, the conclusion based on fuzzy sets is still meaningful. For example, the degree of truth for a credit score of 4 belonging to fuzzy set “High” should be no less than that for a credit score of 3. And only one of the membership functions may be strictly increasing for a certain range of credit score. It may be conflicting if the degree of truth for a credit score of 4 belonging to fuzzy set “High” is greater than that for a credit score of 3 while the degree of truth for a credit score of 4 belonging to fuzzy set “Average” is greater than that for a credit score of 3 at the same time.

Membership functions are typically simple for fuzzy sets. They are frequently linear and often take the shape of a triangle, trapezoid, L or r. They may also be Gaussian or gamma. Different people may have their own membership functions for a fuzzy set due to different levels of knowledge and experience. However, in general, they may still mean similar things when they make reference to a fuzzy set. For example, people may have the same opinion that a loan applicant with a high credit score is likely to get the application approved with a relatively low mortgage loan rate. Here “high credit score” fits naturally to the description of a fuzzy set. But different mortgage loan risk assessors may have different membership functions for fuzzy set “high credit score.” Fuzzy sets allow us to set up a system using our everyday language and reasoning methods.

Fuzzy Sets Operation

As in classical set theory, fuzzy sets have their own operations such as union, intersection and complement. Different from the operation on classical sets, the operations on fuzzy sets are based on the membership function. Figure 3 shows the operation on classical sets. Figure 4 shows one possible type of operation on fuzzy sets.³

Figure 3. Operation on Classical Sets



$$\begin{array}{l}
 A \cup B \quad A \cap B \quad \bar{A} \\
 = \{x, y, z\} \quad \{y\} \quad \{z\}
 \end{array}$$

$$\begin{array}{l}
 x \quad 1 \cup 0 = 1 \quad 1 \cap 0 = 0 \quad 1 - 1 = 0 \\
 y \quad 1 \cup 1 = 1 \quad 1 \cap 1 = 1 \quad 1 - 1 = 0 \\
 z \quad 0 \cup 1 = 1 \quad 0 \cap 1 = 0 \quad 1 - 0 = 1
 \end{array}$$

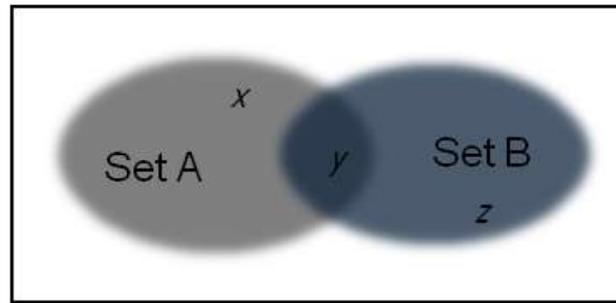
Where

$$1 : \in \quad \text{e.g. } x \in A$$

$$0 : \notin \quad \text{e.g. } x \notin A$$

³ The type of operation on fuzzy sets given in Figure 4 was invented by Zadeh (1965). There are many other types of operation such as mean, bounded sum and product. A list of commonly used operation types can be found in *The Fuzzy Systems Handbook: A Practitioner's Guide to Building, Using and Maintaining Fuzzy Systems* (Cox 1994, 133).

Figure 4. Operation on Fuzzy Sets



$$\mu_A(x) = 0.5 \quad \mu_B(x) = 0.1$$

$$\mu_A(y) = 0.6 \quad \mu_B(y) = 0.4$$

$$\mu_A(z) = 0.1 \quad \mu_B(z) = 0.7$$

	$A \cup B = \max(\mu_A, \mu_B)$	$A \cap B = \min(\mu_A, \mu_B)$	$\bar{A} = 1 - \mu_A$
x	0.5	0.1	0.5
y	0.6	0.4	0.4
z	0.7	0.1	0.9

In this example, a max-min rule is used. The degree of truth that an element belongs to the union of some fuzzy sets is the maximum of the degrees of truth that the element belongs to each of the fuzzy sets. The degree of truth that an element belongs to the intersection of some fuzzy sets is the minimum of the degrees of truth that the element belongs to each of the fuzzy sets. The degree of truth that an element belongs to the complement of a fuzzy set is one deducted by the degree of truth that the element belongs to the fuzzy set.

Inference Rules and Fuzzy Hedges

With logical operations on fuzzy sets, inference rules can be built to establish the relationship among different variables. One type of fuzzy inference rule is called the max-min inference rule.⁴ It is the max-min rule shown in Figure 4 applied to inference.

1. *If A and B, then C.*

The maximum degree of truth for C is the lesser of the degree of truth for A and that for B.

2. *If A or B, then C.*

The maximum degree of truth for C is the greater of the degree of truth for A and that for B.

3. *If not A, then C.*

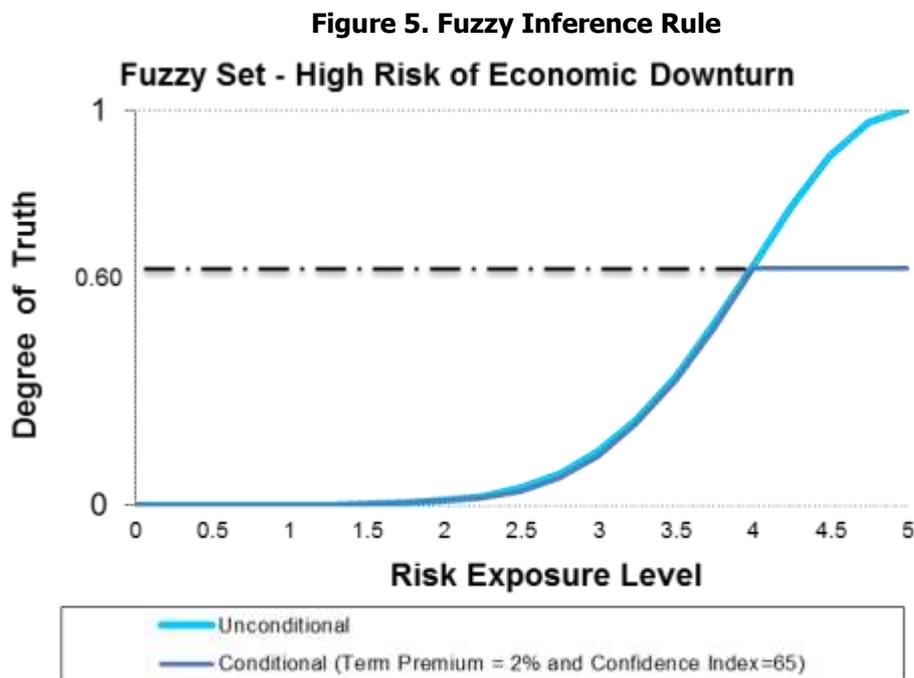
The maximum degree of truth for C is one deducted by the degree of truth for A.

⁴ In addition to the max-min inference rule, there are many other fuzzy inference rules available, such as monotonic reasoning, fuzzy additive rule, correlation minimum and correlation product.

For example, when assessing the risk of an economic downturn, term premium⁵ and investors' confidence level are the two key indicators. A possible inference rule is given below.

If the term premium is small and investors' confidence level is low, the risk of economic downturn in the near future is high.

The term premium is 2 percent with a degree of truth $\mu_{\text{small}}(2 \text{ percent})$ of 0.6. The investor's confidence index value is 65 with a degree of truth $\mu_{\text{low}}(65)$ of 0.72. Using the intersection operation on fuzzy sets as the minimum of the two degrees of truth $\mu_{\text{small}}(2 \text{ percent})$ and $\mu_{\text{low}}(65)$, the maximum degree of truth that there is a high risk of economic downturn is 0.6. The resulting fuzzy set membership function is truncated at the true value of 0.6 from the top, as shown in Figure 5.



Notes:

1. Unconditional membership function for fuzzy set “High Risk of Economic Downturn”:

$$\mu_{\text{unconditional}}^{\text{High}}(x) = \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-5)^2}{2 \times 1^2}\right)} \text{ where } x \text{ is the risk exposure level}$$

2. Conditional membership function for fuzzy set “High Risk of Economic Downturn”:

$$\mu_{\text{conditional}}^{\text{High}}(x) = \text{Min} \left(0.6, \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-5)^2}{2 \times 1^2}\right)} \right) \text{ where } x \text{ is the risk exposure level}$$

⁵ Term premium can be calculated as the difference between long-term bond yield and short-term bond yield.

Sometimes, a refinement of the membership function is necessary to reflect the inference rules with a different description. This process is called the fuzzy hedge.⁶ For example, the following similar inference rules are different in the intensity of a “high” credit score.

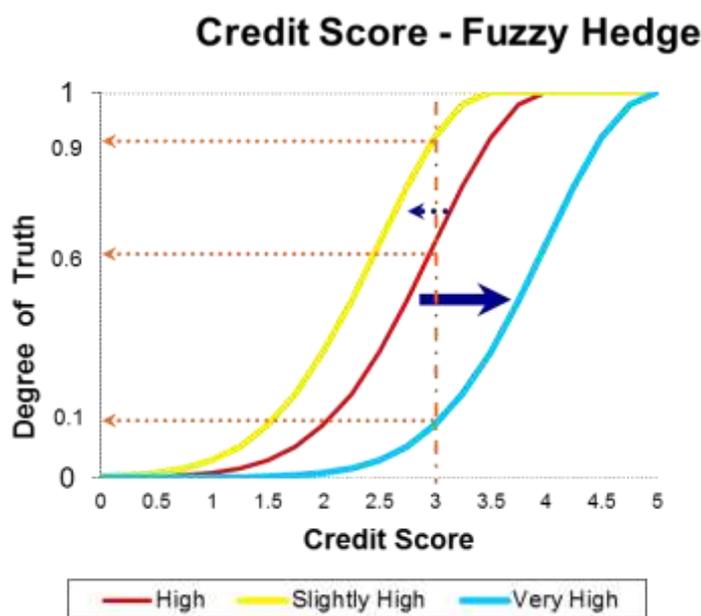
Rule 1. If a credit score is slightly high, the chance of getting a mortgage rate discount is high.

Rule 2. If a credit score is high, the chance of getting a mortgage rate discount is high.

Rule 3. If a credit score is very high, the chance of getting a mortgage rate discount is high.

To reflect the difference, the membership function of fuzzy set “High Credit Score” can be transformed to fuzzy set “Slightly High Credit Score” and “Very High Credit Score,” as shown in Figure 6. The membership function is shifted around to reflect the impact of adjectives “slightly” and “very.” In this example, for a credit score of 3, it is 60 percent true that it is high, 10 percent true it is very high, and 90 percent true it is slightly high.

Figure 6. Fuzzy Hedge



Notes:

1. Membership function for fuzzy set “High Credit Score”:

$$\mu^{High}(x) = \begin{cases} \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-4)^2}{2 \times 1^2}\right)} & 0 \leq x \leq 4 \\ 1 & 4 < x \leq 5 \end{cases} \quad \text{where } x \text{ is the credit score}$$

2. Membership function for fuzzy set “Slightly High Credit Score”:

⁶ There are many kinds of fuzzy hedges to reflect the impact of different descriptions in the inference rules. A list of fuzzy hedges can be found in Cox (1994, 162).

$$\mu^{High}(x) = \begin{cases} \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-3.5)^2}{2 \times 1^2}\right)} & 0 \leq x \leq 3.5 \\ 1 & 3.5 < x \leq 5 \end{cases} \quad \text{where } x \text{ is the credit score}$$

3. Membership function for fuzzy set “Very High Credit Score”:

$$\mu^{High}(x) = \begin{cases} \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-5)^2}{2 \times 1^2}\right)} & 0 \leq x \leq 5 \end{cases} \quad \text{where } x \text{ is the credit score}$$

Defuzzification

Defuzzification is the process of estimating the value of the dependent variable based on the resulting fuzzy set after applying the fuzzy inference rule. Three typical defuzzification methods are described below.

1. Average method: The average numerical value of the dependent variable in the output fuzzy set.
2. Average of maximum method: The average numerical value of the dependent variable with the maximum degree of truth in the output fuzzy set.
3. Centroid method: The weighted average numerical value of the dependent variable in the output fuzzy set. The weight is the degree of truth.

Different methods are appropriate in different situations. Continuing with the inference rule example from Figure 5, the output fuzzy set is the area between the conditional membership function and the x-axis.

Conditional membership function:

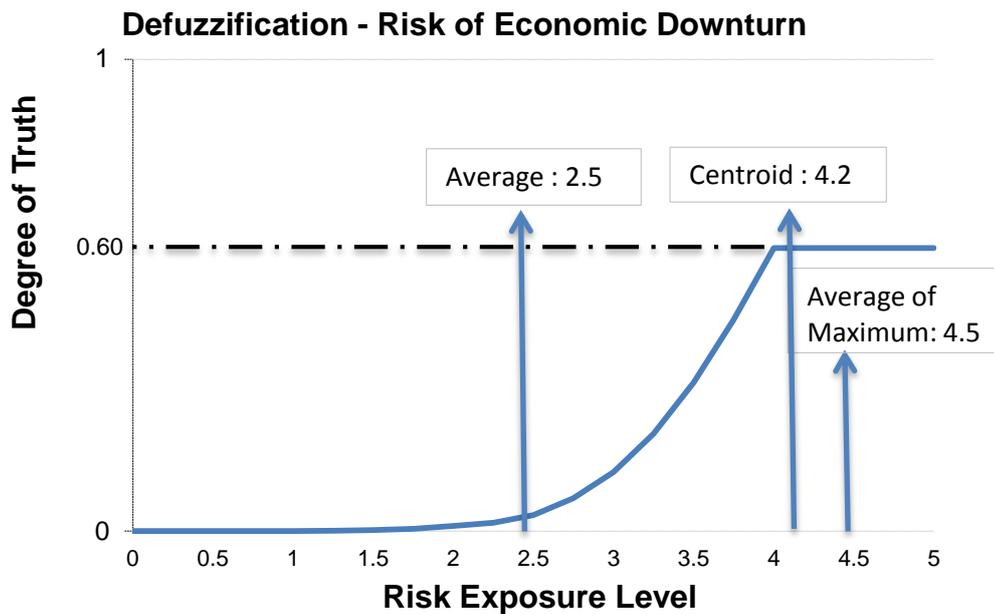
$$\mu_{conditional}^{High}(x) = \text{Min} \left(0.6, \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-5)^2}{2 \times 1^2}\right)} \right) \quad \text{where } x \in [0,5] \text{ is the risk exposure level}$$

1. Average method: The range of the risk exposure level in the output fuzzy set is [0,5]. Therefore, the result of defuzzification is 2.5, the average of 0 and 5.
2. Average of maximum method: When x is greater than 4, the value of the conditional membership function is 0.6, the maximum degree of truth in the output fuzzy set. Therefore, the result of defuzzification is 4.5, the average of 4 and 5.
3. Centroid method: The value of defuzzification is calculated as

$$\int_0^5 x \cdot \mu_{conditional}^{High}(x) = \int_0^5 x \cdot \text{Min} \left(0.6, \frac{2.5}{\sqrt{2\pi} \times 1} e^{-\left(\frac{(x-5)^2}{2 \times 1^2}\right)} \right) \approx 4.2$$

The result of defuzzification is given in Figure 7. As the risk of economic downturn is not low, the average method is not a good choice in this example.

Figure 7. Defuzzification Example

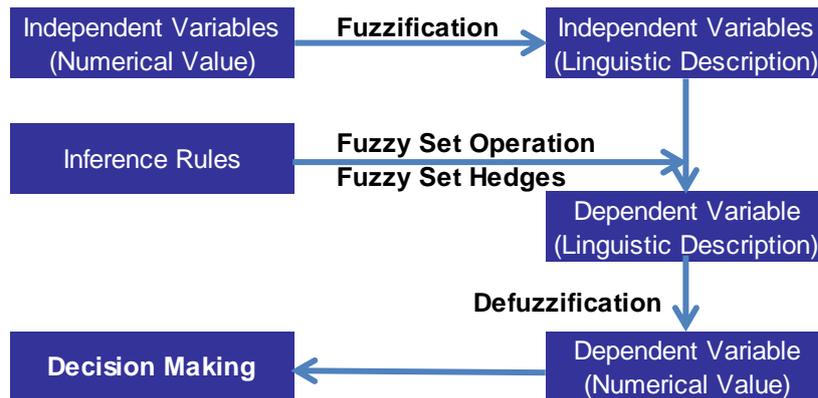


Fuzzy Logic System

With all the components, a fuzzy logic system can be built in the following steps.

- Step 1. Independent variables are selected as the key determinants or indicators of the dependent variable.
- Step 2. Fuzzy sets are created for both independent and dependent variables. Instead of using the numerical value, fuzzy sets in terms of human language are used to describe a variable. The degree of truth that each variable belongs to a certain fuzzy set is specified by the membership function.
- Step 3. Inference rules are built in the system. A fuzzy hedge may be used to tweak the membership function according to the description of the inference rules.
- Step 4. The output fuzzy set of the dependent variable is generated based on the independent variables and the inference rules. After defuzzification, a numerical value may be used to represent the output fuzzy set.
- Step 5. The result is then used for informed decision-making.

Figure 8. Fuzzy Logic System



2.2 A Numerical Example

A simple fuzzy logic system⁷ used to assess advisers' misconduct risk is illustrated in this section. Due to the incentive of high sales commission, financial advisers may be tempted to hide information about the risks of the product, provide misleading information or even advertise the product deceptively. Three key risk indicators are used to monitor this important component of an enterprise's reputation risk:

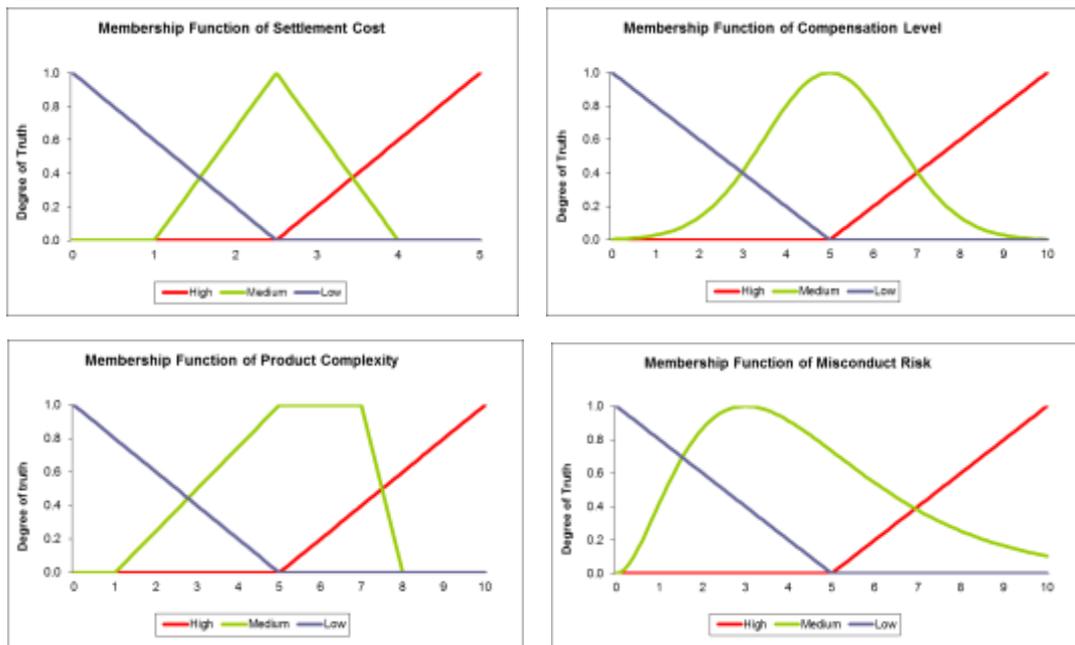
1. Settlement cost over the past year due to misleading or deceptive advertising
2. Product complexity, which measures how difficult it is for clients or advisers to understand the product being sold
3. Compensation level of advisers

Graphs of their membership functions are given below.

Figure 9. Membership Functions of Settlement Cost, Product Complexity,

⁷ There is an accompanying file, "Fuzzy Logic Examples.xls," that illustrates the calculation process and details. It can be used for some simple fuzzy logic calculation.

Compensation Level and Misconduct Risk



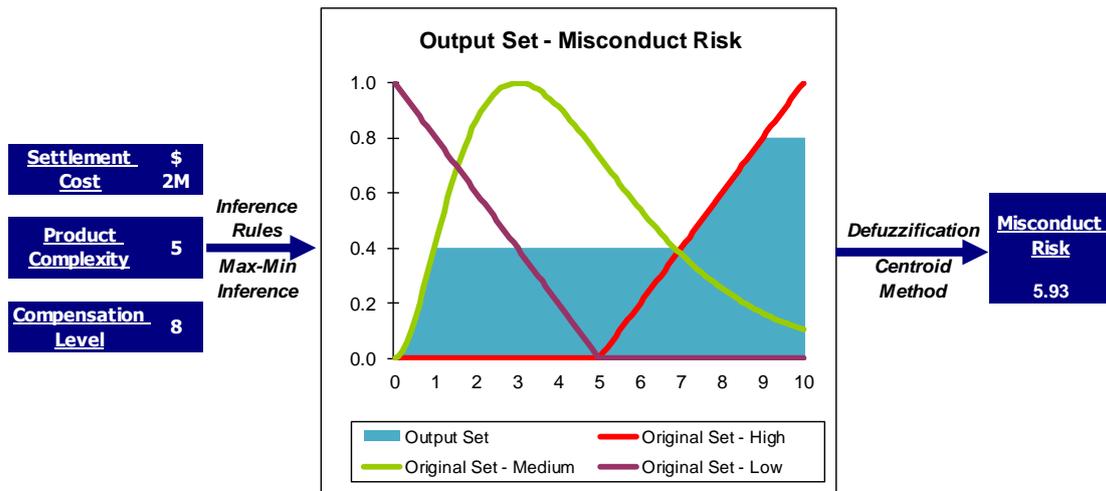
Assume that three inference rules have been specified based on the comments from subject matter experts.

1. If (product complexity is not low or compensation level is very⁸ high) and settlement cost is not low, then misconduct risk is high.
2. If (product complexity is high or settlement cost is high) and compensation level is high, then misconduct risk is high.
3. If (product complexity is not high and settlement cost is not high) and compensation level is not high, then misconduct risk is medium.

Figure 10 shows an example of calculating the value of misconduct risk given the values of the various input variables using a fuzzy logic system. The fuzzy logic system includes the membership functions, the inference rules and the chosen defuzzification method. For each product in the business portfolio, the level of misconduct risk can be assessed, given the product complexity, compensation level and historical settlement cost.

⁸ Fuzzy set “very high” is transformed from the basic fuzzy set “high.” The degree of truth being very high is smaller than that for being high for the same value. It is a fuzzy hedge as discussed on pages 11–12.

Figure 10. An Example of Calculation Using Fuzzy Logic System

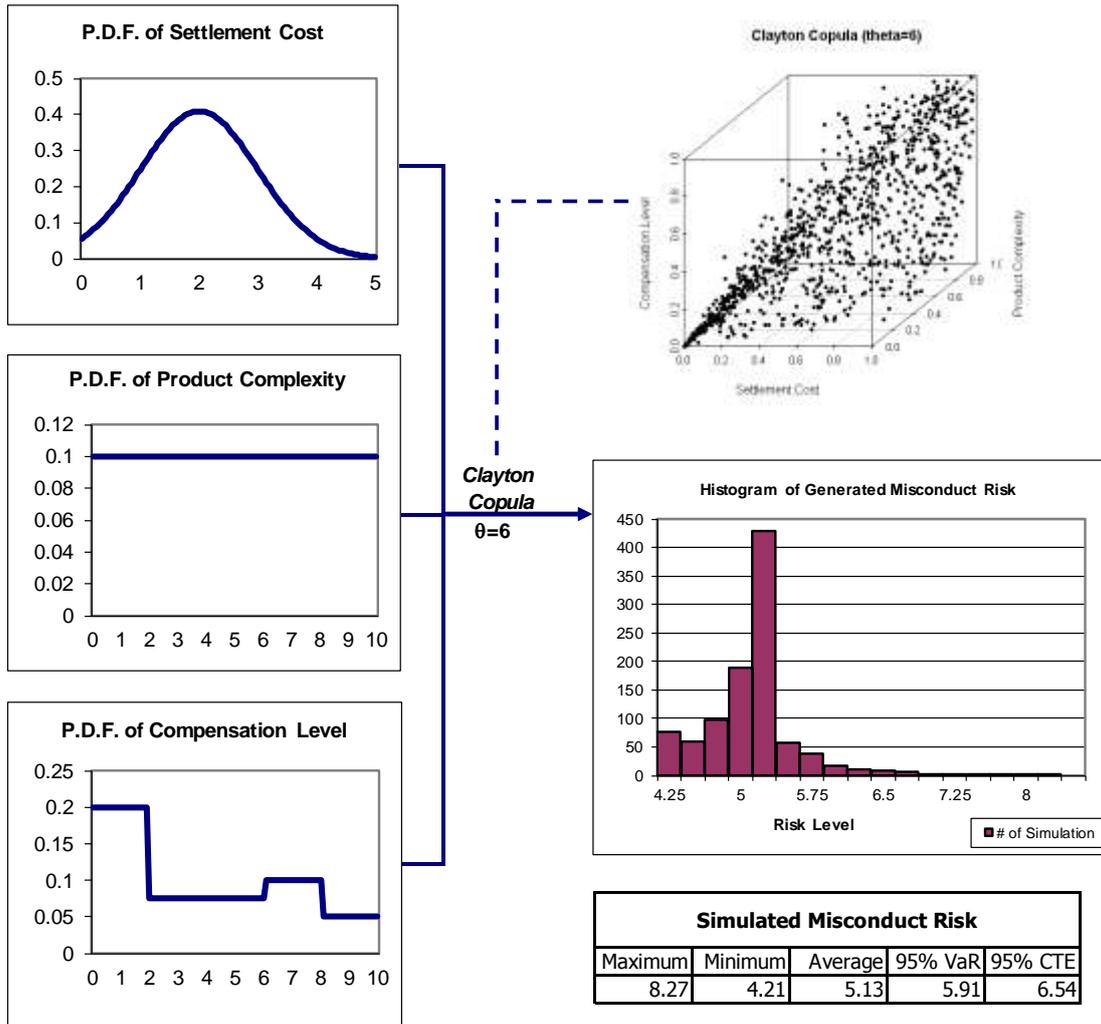


If the joint distribution of the three input variables is known, the distribution of the misconduct risk can be derived by simulation. In the example below, the marginal distributions of the input variables and their dependence are given. It is also assumed that the input variables are highly correlated. The dependence is modeled using the Clayton copula⁹ with $\theta = 6$ in this example, which indicates a strong and positive correlation. The distribution of the misconduct risk is simulated, and some descriptive statistics are calculated.

⁹ Clayton copula: $C_{\theta}^n(u) = (u_1^{-\theta} + u_2^{-\theta} + \dots + u_n^{-\theta} - n + 1)^{-1/\theta} \quad \theta > 0$

Details can be found in Nelsen (2006, 153).

Figure 11. An Example of Output Variable Simulation



P.D.F.: Probability density function.

Using this fuzzy logic model, the level of misconduct risk for each product can be calculated. The risk exposure for each product may be measured as the product of its risk level and expected new business volume. Table 1 lists five products that have different levels of complexity and compensation. The risk exposure of product B is the highest. The company may want to reduce the misconduct risk level of product B. It may consider replacing the current product with a simplified version, reducing the compensation for advisers without losing competitiveness, providing more training to advisers or improving the communication with potential clients about the risks of the product.

Table 1. Sample Misconduct Risk Monitoring

Product	Misconduct Risk Level (1)	New Business Volume (\$M) (2)	Misconduct Risk Exposure = (1) × (2)	Rank
A	8.3	2	16.6	4
B	7.6	13	98.8	1
C	5.9	5	29.5	3
D	5.1	7	35.7	2
E	3.5	3	10.5	5

2.3 Alternative Models

The most widely used quantitative model in risk assessment is the classical probability model. Objects are measured according to their numerical values. When the probability space is divided into exclusive sets, each object belongs to only one set.

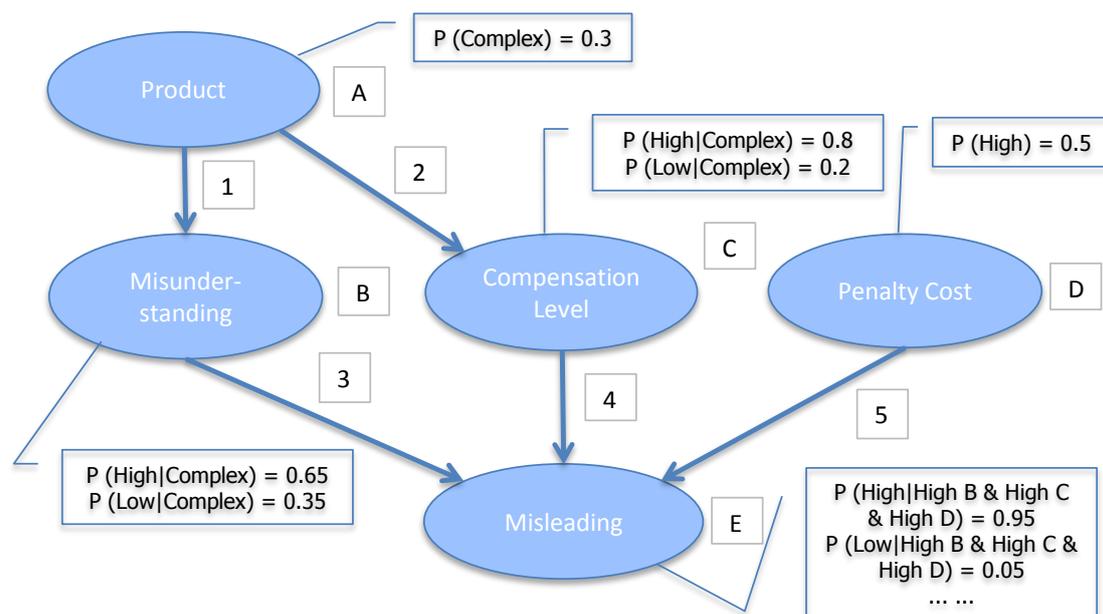
Fuzzy logic models are an alternative to the probability models grounded in traditional set theory. Fuzzy sets allow the overlapping of those traditional “exclusive sets” described in linguistic terms. For example, a fuzzy set approach can be used to gain insight into an annual loss of \$1 million due to operation error. Membership functions can be established that indicate it is 70 percent true the operational risk exposure is medium, 40 percent true it is high and 10 percent true it is low. Fuzzy logic models are able to describe risks in an imprecise way without requiring an abundance of experience data. Not every risk can be perfectly understood. As human beings continue to explore the unknown world and gain knowledge, with every step they take forward, more things are discovered. Sometimes the illusion of precision (false precision) may be another source of model risk. In addition, fuzzy logic models focus on exploring the various cause-and-effect relationships that underlie risks. They can also more readily incorporate data or opinions more easily expressed by natural language rather than mathematical. Therefore, fuzzy logic models may be useful for analyzing risks that are not well understood.

In addition to fuzzy logic models, there are alternative models that can be used for risk assessment and pattern recognition, such as Bayesian and artificial neural networks, and the hidden Markov and decision tree models. Some models may be more appropriate in solving certain problems given certain knowledge and data. They can be used with fuzzy set theory as well. To implement fuzzy logic models wisely, it is important to understand other available options and use fuzzy logic models only when they are appropriate.

The Bayesian Network

The Bayesian network, also known as the Bayesian belief network, is a directed acyclic graph composed of vertices, edges and conditional probability distribution. Using the misconduct risk example introduced earlier, let us apply it to a Bayesian network. As shown in Figure 12, there are five vertices (or variables) denoted by letters (A to E) and five edges (conditional dependences) denoted by numbers (1 to 5). The distribution of each variable is also given, either conditionally or unconditionally.

Figure 12. Bayesian Network Example: Misconduct Risk



Bayesian network models use Bayes' rules and conditional probability to describe the joint probability of the network. The relationship is embedded in the system as conditionally dependent on the parent(s) and conditionally independent of nondecedents given the value of the parent(s). Therefore, it considers both the distribution of the variables and their dependence. Bayesian network models can be used to calculate conditional probabilities, such as the probability of misleading advertisement if the product is not complex and the penalty cost is high.

Considering the number of relationships and conditional probabilities that need to be specified in a Bayesian network, however, it can be highly time consuming to build. The inference in a big network is expensive as well. Expertise about the cause-and-effect relationships is required to build the system, whether it is learned from human reasoning or from data. There is also a demand for data that specifies the conditional probability. All those features make Bayesian network models suitable for small-sized problems for which we have sufficient knowledge of the relationships.

On the other hand, fuzzy logic systems are constrained only slightly by the size of the system. They also allow for an incomplete set of rules or relationships specified in the inference system. Therefore, the fuzzy logic model is more suitable for analyzing issues with insufficient knowledge.

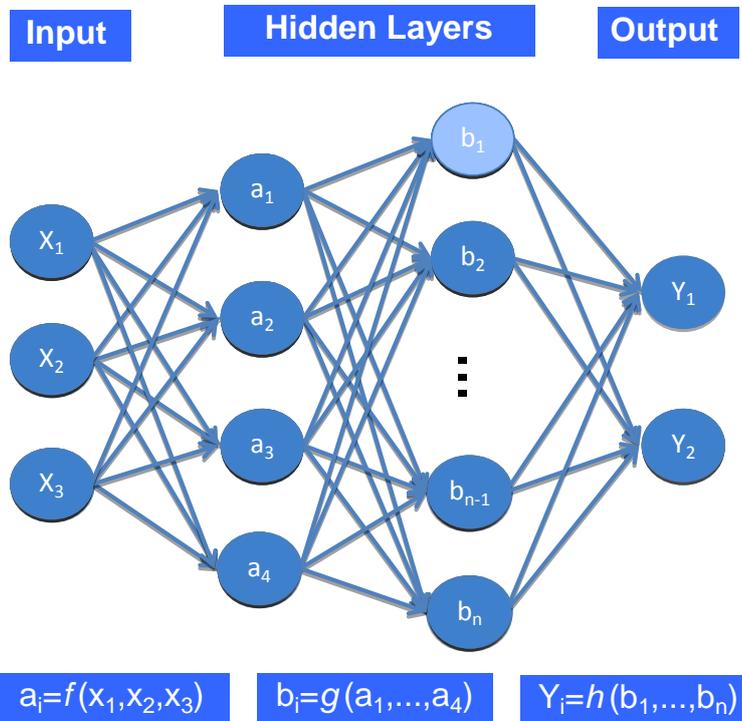
There have been some efforts to incorporate fuzzy set theory and fuzzy logic into Bayesian network models so that the variables can have both discrete and continuous values. Fuzzy sets were tested to improve the inference system in the general Bayesian network.

Those extended Bayesian networks are usually referred to as fuzzy Bayesian networks.¹⁰ But the incorporation of fuzzy sets into Bayesian networks does not necessarily reduce the need for: a) knowledge of the cause-and-effect relationships, and b) the data for calibrating the conditional probability.

Artificial Neural Networks

Artificial neural network models are used to learn the relationship among variables in a way similar to that of biological neural networks. There are many neurons in the network, and those neurons are connected in certain ways, as shown in Figure 13. Multiple hidden layers can exist between the input and the output set. Sufficient training data are required to specify the relationships represented as functions f , g and h using methods like maximum likelihood estimation, maximum a posteriori or back propagation. Artificial neural network models can be used in many areas such as pattern recognition, prediction and classification. One possible application is to detect fraud claims for auto insurance. The input information may include gender, occupation, car model, salary, location, claim amount, claim history such as frequency and severity, and cause of accident. The output may be the probability of insurance fraud and the estimated investigation cost. Based on the experience data of auto insurance fraud, this artificial neural network can be trained to determine a reasonable relationship between the input and the output through the hidden layers. It can then help to identify claims likely to be false and their expected investigation cost. The company may want to allocate resources to the most dubious claims with an affordable investigation cost.

Figure 13. Artificial Neural Network Example



¹⁰ Examples of fuzzy Bayesian network can be found in Pan (1988, 6–22).

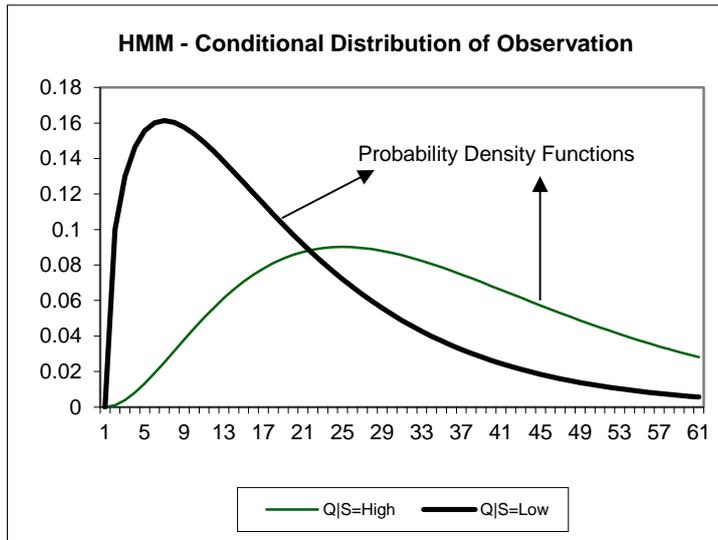
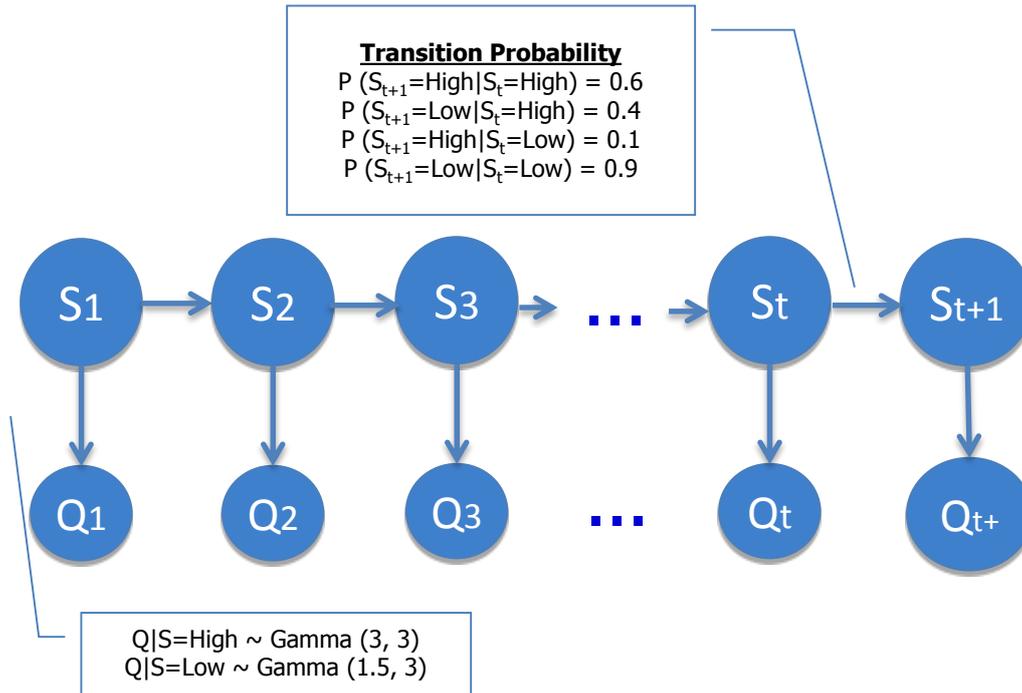
Artificial neural network models rely on large sets of training data to produce a good estimate of the relationship. The required computation is resource demanding. It is more appropriate for complex systems with sufficient observation data but vague or unknown relationships. This is quite different from fuzzy logic systems where sparse or imprecise data are often the case but there are some known relationships between the input and the output.

Neural network models have been used in some fuzzy logic systems where the inference rules are expressed in some form of the functions used in artificial neural network models, such as $Output = f(Input)$. Jang (1993) introduced the adaptive neuro fuzzy inference system (ANFIS) in which the neural networks are used to model and refine the membership function of fuzzy sets. ANFIS may be useful when some training data for the fuzzy logic system exist. The inference rules or membership functions can be trained to better fit the experience. However, it is more complicated and difficult to implement than a pure fuzzy logic system.

Hidden Markov Models

A hidden Markov model studies the Markov process of a hidden state with observations that highly depend on the hidden state. The next hidden state depends on the current hidden state but not the history of the hidden state. In most cases, a transition matrix is used to define the probability of the next state given the current one. The distribution of the observation changes with the hidden state. Based on the actual observation, an inference system built on Bayes' rule can be used to predict future hidden states. Figure 14 illustrates a hidden Markov model.

Figure 14. Hidden Markov Model Example



Notes:

S: Hidden state. It has two values, high and low. Transition probability is given as well. For example, if the current state is low, the chance of having a value of high next time is 10 percent.

Q: Observation. Given a certain state, it has a conditional distribution. For example, if the state is low, the output follows a gamma distribution with parameters $\alpha=1.5$ and $\beta=3$.

For example, when predicting damages from earthquakes, assume S is the state of crust movement, having either a high frequency or a low frequency. Q is the number of natural disasters such as earthquakes that happened in the past year. Historical data are used to estimate the transition probability of S and the conditional distribution of Q. The risk manager can use this hidden Markov model to estimate the probability of the future hidden state given current observation, such as $P(S_{t+1} = \text{High Frequency} | Q_t=30)$. If the probability of a high

frequency state is high, the company may consider increasing the rate of its insurance products or raising extra capital to be able to survive more severe losses due to natural disasters.

Similar to Bayesian networks, hidden Markov models need training data to set the appropriate transition probability and the conditional distribution of the observation based on a certain hidden state. These models also emphasize the randomness of transition from one state to another. In reality, the transition from state to state may be deterministic given exogenous factors. But those factors may be ignored in hidden Markov models due to their complexity or a lack of understanding of the cause-and-effect relationships that drive the transitions from one hidden state to the next.

Different from fuzzy logic models, hidden Markov models require a clear specification of the relationship between observations and the hidden state using conditional probability rather than possibly incomplete inference rules. It focuses on the prediction of future states in a trained but uncertain way. It is more appropriate for modeling a system where there is enough knowledge of the current situation but the evolution of the system is uncertain.

Fuzzy set theory has been used in hidden Markov models as well. The possibility of the hidden state can be described by fuzzy sets instead of classical sets,¹¹ except that fuzzy hidden Markov models have the same features as hidden Markov models based on classical set theory.

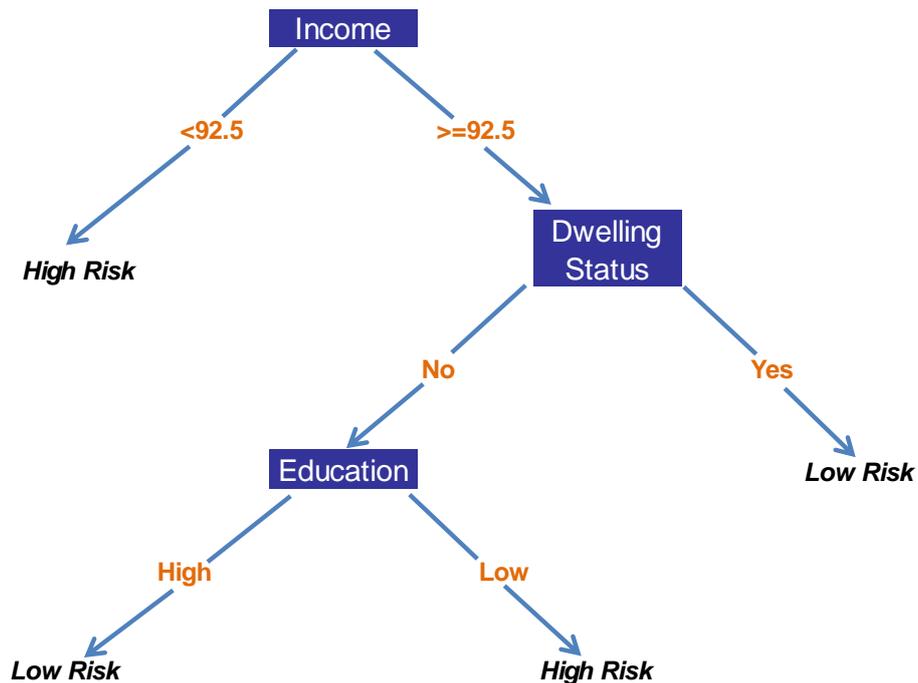
Decision Tree

A decision tree model is used to facilitate decision-making based on a set of rules presented as a tree. It uses the attributes of objects for classification and decision. For example, one can build a tree that classifies credit risk based on the person's income, age and other factors. Unlike most of the black-box modeling techniques where the internal logic can be difficult to work out, the reasoning process behind the model is clearly shown in the tree. Figure 15 describes a decision tree for classifying bank customers regarding the level of credit risk based on their income, education level and dwelling status. It can easily be translated into a set of rules used to classify a customer to make a loan decision. An example is given below.

If (income ≥ 92.5) and (dwelling status = no) and (education = high),
then (low level of credit risk = loan will be granted).

¹¹ An example of fuzzy hidden Markov model can be found in Zhang and Naghdy (2005, 3–8).

Figure 15. Decision Tree Example



To develop a decision tree model, data are split into training and validation sets. Training data are used to identify appropriate rules and find the best partition for certain attributes using techniques such as recursive partitioning. Validation data are used to validate the decision tree and make necessary adjustments to the tree. For example, unnecessary rules may be pruned based on validation data.

Decision tree models are easy to understand and useful for classification. Like fuzzy logic systems, they can work well with insufficient data if all the inference rules can be defined based on expertise. However, in a conventional decision tree model, the partition of attribute values is based on classical set theory. Due to the discreteness of the partition, sometimes a small change in the value of an attribute could lead to a different conclusion. Moreover, when the scale of a decision tree becomes large, it may no longer be easy to understand and more data will be required to identify and validate the rules. Decision tree models appear to be weak at identifying linear relationships as well, due to the discreteness. It is more appropriate for modeling relationships among discrete variables in a small system.

Realizing the shortage of partition based on classical set theory, fuzzy decision tree models are proposed¹² to avoid having a sudden change in conclusion due to a small change in the attribute value.

Summary

The strengths, weaknesses and possible application of the four alternative models

¹² Examples can be found in Janikow (1996, 12–26).

mentioned above are summarized in Table 2.

Table 2. Summary of Alternative Models

Model Type	Strengths	Weaknesses	Applications
Bayesian Network	<p>a) It presents the relationships of variables and is easy to understand.</p> <p>b) It estimates the conditional probability and distribution. Specific conditions are taken into account, and a range of values is provided for better informed decision-making.</p>	<p>a) It is not suitable for complex issues involving many variables. It may be too expensive to determine the relationships and conditional probability functions.</p> <p>b) It may be difficult to determine conditional probability without experience data.</p>	<p>Modeling and decision-making for noncomplex issues. The cause-and-effect relationships are known.</p> <p><u>Examples</u></p> <p>a) Loan lending decision-making</p> <p>b) Underwriting decision-making</p>
Artificial Neural Network	<p>a) It is complex enough to handle sophisticated pattern recognition, prediction and classification.</p> <p>b) It is full of possibility using intelligent learning algorithms.</p>	<p>a) It requires large amounts of data for credible calibration.</p> <p>b) It is very complicated and difficult for people to understand.</p> <p>c) It is highly data driven. The relationships found based purely on data may not make sense intuitively.</p> <p>d) It requires a long computing time.</p>	<p>Modeling complex issues where the cause-and-effect relationships are not well known but there are sufficient experience data. Many possible explanatory factors may exist.</p> <p><u>Examples</u></p> <p>a) Insurance fraud detection</p> <p>b) Auto insurance rate adjustment based on driving habits</p>
Hidden Markov Model	<p>a) It is suitable for issues that have structural change from time to time.</p> <p>b) It can infer the current state of the issue based on the observations of the outcome. This is helpful when the underlying state is difficult to predict but the outcome is easily</p>	<p>a) It models the transition from one state to another as a Markov process. Except for the current state, information from the past states, which might be valuable, is not used..</p> <p>b) It avoids studying the causes of the underlying state but predicts it as a random</p>	<p>Modeling complex issues where the causes are not well known but the outcome is observable. The outcome may exhibit a cycle with different levels of magnitude and volatility in different phases.</p> <p><u>Examples</u></p> <p>a) Modeling underwriting cycle</p> <p>b) Modeling the frequency</p>

Model Type	Strengths	Weaknesses	Applications
	observable.	event. c) It needs experience data to calibrate model parameters.	and severity of natural disasters
Decision Tree	a) It is easy to understand. b) It is good at dealing with discrete variables. c) It is straightforward for decision-making with limited choices.	a) It is not suitable for complex issues that require many factors and relationships. b) It is weak at identifying linear relationships due to its discreteness. c) It is designed for decision-making but not risk assessment and quantification.	Decision-making for noncomplex issues. The choices that decision-makers have are limited, normally binary (yes or no). The issues are likely to be at the individual level. <u>Examples</u> a) Loan-lending decision-making b) Underwriting decision-making

3 Application of Fuzzy Set Theory and Fuzzy Logic: A Literature Review

As discussed in Section 2, fuzzy logic can be executed in three major stages, namely fuzzification, inference and/or defuzzification. Ever since Zadeh’s (1965) contribution to this new field, there has been much literature, covering both academic research and practical implementation in almost every area, from physical to social science. The literature review here focuses on areas related—directly or indirectly—to risk management. The application is quite diversified. It might be a replacement of classical sets with fuzzy sets, a full-blown implementation of a fuzzy logic system or a hybrid model that includes a fuzzy logic model. The objective is to introduce a wide range of possible applications of fuzzy logic and is by no means intended to be exhaustive.

Risk Management

To attain faster decisions and reduce human error in the credit evaluation process, automated credit risk assessment systems play an important role. Lahsasna (2009) built and investigated the accuracy (to enable correct assessment) and transparency (to understand the decision process) of a credit-scoring model using German and Australian credit data sets and two fuzzy model types.¹³ The proposed modeling approaches allow users to perform

¹³ The two types are Takagi-Sugeno (TS) and Mamdani. Refer to Hao et al. (1998) for further details.

additional analysis such as defining customer attributes that influence the credit underwriting decision and quantifying the approximate values of these attributes.

Recognizing that the data reported in financial statements may not be exactly comparable due to differences in accounting practices and may include inaccuracy in reported numbers, Cheng et al. (2006) claimed that the observed value may be better considered as a fuzzy phenomenon but not a random one. They thereby used an interval instead of a single value for financial variables. They constructed an early-warning model for financial distress using fuzzy regression as an alternative to well-known methods, namely discriminant, logit and artificial neural network analysis.

Matsatsinis et al. (2003) found that oftentimes the analytic dependencies among the variables of a process or system are unknown or difficult to construct. Therefore, they used fuzzy rules to formulate the dependencies between the variables in the context of classification analysis for a business failures model. They used these rules in the data mining phase to predict corporate bankruptcy.

Leveraging the findings on classification problems with respect to financial and credit risk analysis, Li et al. (2011) used a fuzzy linear programming classification method with soft constraints to analyze credit cardholders' behavior.

Cherubini and Lungu (2001) observed that in pricing contingent claims, the probability measure used may not be precisely known, and therefore used a class of fuzzy measures to account for this uncertainty. They used this approach to quantify liquidity risk for pricing an asset in the presence of illiquid markets, and they further extended this to construct a fuzzified version of the seminal Merton's credit risk model.

Yu et al. (2009) proposed a multicriteria decision analysis tool for credit risk evaluation using fuzzy set theory. The tool is developed to initially allocate results obtained from alternative competing credit evaluation techniques in the form of fuzzy opinions, then aggregated into a group consensus and, lastly, defuzzified into a discrete numerical value to support an ultimate credit decision. Human reasoning, expert knowledge and imprecise information are considered valuable inputs in the estimation of operational risk.

Revez and Leon (2009) studied operational risk using the fuzzy logic inference system (FLIS) to account for the complex interaction as well as nonlinearity in these inputs. The choice of FLIS allows one to utilize the qualitative and quantitative inputs in a sound and convenient way, as well as to evaluate risk mitigation efforts ex ante.

Asset Liability Management (ALM) and Insurance

Brotons and Terceno (2011) used fuzzy logic to study immunization strategies to mitigate the risk of interest rate movements within an ALM framework where the combination of expected return and risk, chosen to achieve higher liquidity, are obtained from the midpoint

and width of relevant fuzzy numbers respectively. A risk-return map is created using this approach to account for the investor's risk aversion, which allows the investor to track differences in return of the adopted strategy for a given level of duration.

Huang et al. (2009) studied probability of ultimate ruin in an insurance risk framework where the individual claim amount is modeled as an exponentially distributed fuzzy random variable and the claim process is characterized by a Poisson process.

Lai (2006) conducted an empirical study of the underwriting profit margin of a Taiwanese property/liability (P/L) insurance company in an intertemporal capital asset pricing model (ICAPM)¹⁴ framework. He found that the best fitting parameters of the models can be expressed as an asymmetric triangular fuzzy number. He also showed how the derived skew factors could be used to forecast the underwriting profit margin. Lai (2008) extended the above study to investigate transportation underwriting of systematic risk made by the insurances related to major lines of transportation, ranging from automobile to aviation.

De Andres Sanchez and Gomez (2003) applied fuzzy regression techniques to analyze the term structure of interest rates. They focused on the quantification of interest rates and discussed applications to the pricing of life insurance contracts and P/L insurance policies.

Lazzari and Moulia (2012) studied certain parameters describing cardiovascular risk by developing a diagnosis model formulated within a fuzzy framework, and they proposed a framework for a health insurance company's expansion strategy.

Derrig and Ostaszewski (1996) studied the tax burden of a property-liability insurance company in an option theoretic framework where the appropriately priced insurance liabilities are used as a hedging instrument. The relevant parameters were modeled using fuzzy numbers to account for uncertainty in the tax rate, rate of return and the hedge liability.

Economics and Finance

Horgby (1999) provided an introduction to techniques of fuzzy inference for applications in economics. Using a set of examples, he showed the way to internalize information that is, by nature, fuzzy, and infer conclusions from a set of fuzzy "if-then" rules. Caleiro (2003) conducted an interesting study analyzing how subjective measures like consumer confidence can be approximated by objective economic measures such as the unemployment rate using fuzzy logic. Blavatsky (2011) studied risk aversion when outcomes may not be measurable in monetary terms and people have fuzzy preferences over lotteries, i.e., preferences over lotteries are expressed in a probabilistic manner.

Ng et al. (2002) established a fuzzy membership function of procurement selection criteria through an empirical study in Australia recognizing that numerous selection criteria—

¹⁴ ICAPM is a hybrid model where the algebraic insurance model is linked to CAPM to price insurance products.

such as speed, complexity, flexibility, responsibility, quality level, risk allocation and price competition—are fuzzy in nature. Xu et al. (2011) extended this approach by developing a practical risk evaluation model for public-private partnership procurement projects where the underlying risk factors are established using the Delphi survey technique and fuzzy set theory. The risk evaluation model is developed using a fuzzy synthetic evaluation approach.

Oliveira and Silva (2004) studied environmental regulation where the imperfect link between regulations and pollution-generating processes are modeled using a fuzzy logic approach. To aid effective decision-making, this study aims to provide a reasonable understanding of the complexity in interactions, which may lead to costly regulation, corruption and excessive pollution, as well as rent-seeking behavior of legislators such as providing monopoly privileges.

Sun and van Kooten (2005) applied fuzzy logic to contingent valuation of environmental amenities and public goods using a fuzzy random utility maximization (FRUM) framework. They conducted an empirical study to measure the elicited residents' willingness to pay for enhanced forest conservation using Swedish data.

Cai et al. (2009) developed a fuzzy-random interval programming (FRIP) model to identify optimal strategies in the planning of energy management systems under multiple uncertainties caused by economic, environmental and political factors. Their FRIP model was constructed by integrating interval linear programming, fuzzy-stochastic programming and mixed integer linear programming to deal with uncertainties presented as interval values.

Tucha and Brem (2006) proposed a quantitative approach to analyze functions and risk patterns in international transfer prices using the fuzzy framework. Dow and Ghosh (2004) studied the speculative demand for money using a fuzzy logic framework. They incorporate different opinions and recognize that expectations may differ when the nature of the problem prevents a precise and definitive description of the underlying variables.

Lin et al. (2008) presented a hybrid model for predicting the occurrence of currency crises by using the neuro fuzzy modeling approach. They integrate the learning ability of neural networks with the inference mechanism of fuzzy logic to uncover the causal relationships among the variables. Gulick (2010) studied the allocation problem using a fuzzy game-theoretic framework. Gulick discussed several applications ranging from cooperative investment decisions to risk capital allocation for banks and insurance companies. Leon and Machado (2011) proposed an index built using a fuzzy-logic-based inference system to conduct a comprehensive relative assessment of a financial institution's systematic importance. The proposed index uses some key importance indicators of the institution's size, its connectedness and substitutability. Expert knowledge is used for combining those indicators.

Caetano and Caleiro (2005) studied how corruption influences decisions concerning direct, foreign investment with a fuzzy logic approach recognizing that a certain level of

perceived corruption can be subject to different subjective evaluations by investors. Brochado and Martins (2005) studied cross-country variation in political indicators and their association with the level of economic, human and gender-specific development indicators using a fuzzy k-means classification algorithm. The aim was to enhance the understanding of the heterogeneity of behaviors with respect to political indicators. Sveshnikov and Bocharnikov (2009) developed a model to study the international politico-economic risk where contradictory and opposing views of countries concerning decisions on political, economic, internal and international issues are combined together using fuzzy measures and integrals. They conducted an empirical study to estimate the politico-economic risk of Ukraine.

Magni et al. (2006) studied an alternative method of firm valuation based on fuzzy logic and expert systems. In this study, the discounted cash flow analysis accounted for quantitative and qualitative variables, e.g., financial, strategic and business aspects, as well as their mutual integration via “if-then” rules used to rate and rank firms, as well as to assess the impact of managers’ decisions on value-creation and the quality of corporate governance. Smimou (2006) conducted an empirical study for the Canadian commodities futures market within the capital asset pricing model (CAPM) framework using a fuzzy regression method. Smimou provided a comparative analysis to show the superiority of the application of a fuzzy approach to capturing the risk premium in commodity futures over other competing approaches. Giovanis (2009) extended the fuzzy regression framework to generalized autoregressive conditional heteroskedasticity (GARCH) modeling and studied the day-of-the-week effect on four major stock exchanges. The principal motivation was to incorporate nonlinearities in finance and human behavior and avoid the use of binary classification in this context. Su and Fen (2011) constructed a trading strategy using a risk-controllable fuzzy inference system built on structural equation modeling, and they confirmed that it outperforms the buy-and-hold strategy.

Option Pricing

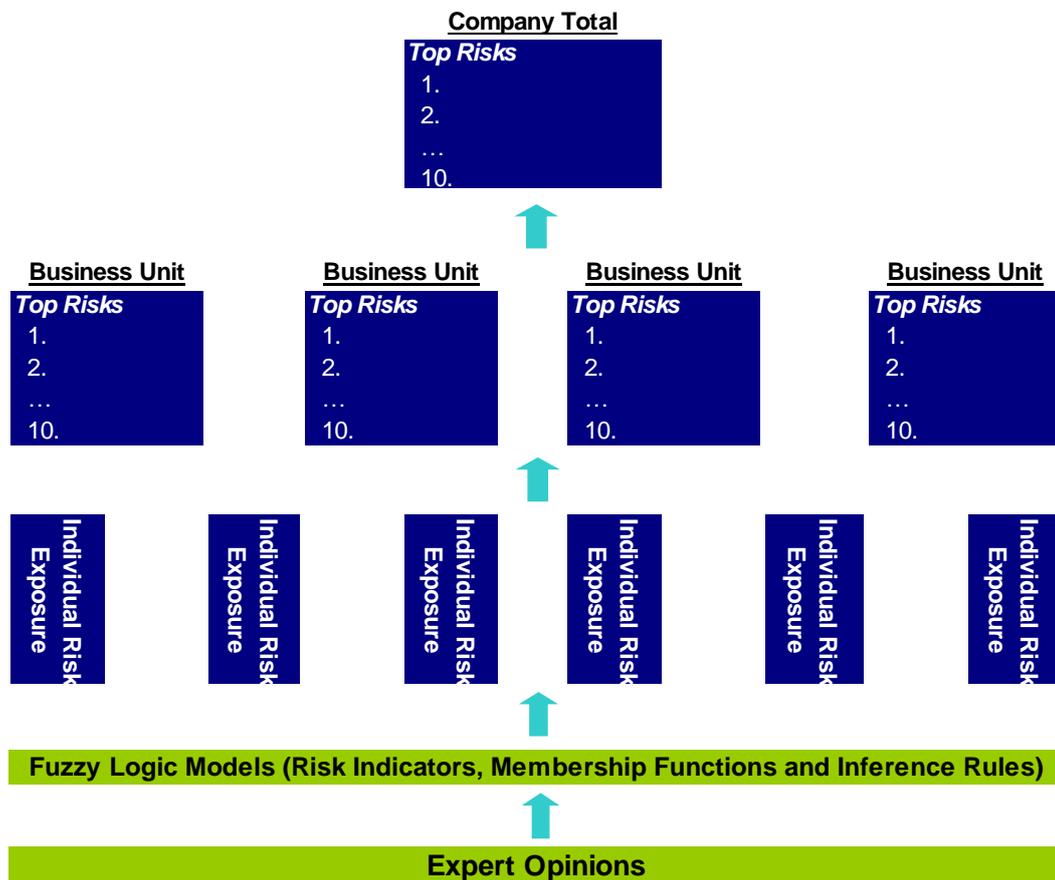
Muzzioli and Torricelli (2001) proposed a one-period binomial option pricing model (OPM) based on a risk-neutral valuation technique. They incorporated different levels of market information while modeling the option payoff by means of triangular fuzzy numbers. Lee et al. (2005) applied fuzzy set theory to the Cox, Ross and Rubinstein (CRR) interest rate model to develop a fuzzy binomial OPM that allows investors to update their portfolio strategy based on their individual risk preferences. The proposed model provides reasonable ranges of option prices allowing investors to use it for arbitrage or hedging. An empirical study using S&P 500 index options is also conducted to support their theoretical results. In the context of a real option valuation model, Zmeskal (2010) observed that the required input data often lack quality and therefore identified two types of input data uncertainty: risk and vagueness. Since risk is stochastic in nature and vagueness results from inherent fuzziness in the reported input, he proposed a fuzzy-stochastic American real option model where the inputs are used in the form of fuzzy numbers and the option value is determined as a fuzzy set.

4 Risk Assessment Framework Based on Fuzzy Logic

4.1 Risk Assessment and Decision-Making

A risk assessment and decision-making platform built on a fuzzy logic system can provide consistency when analyzing risks with limited data and knowledge. It allows people to focus on the foundation of risk assessment, which involves the cause-and-effect relationship between key factors as well as the exposure for each individual risk. Rather than a direct input for the likelihood and potential severity of a risk event, it encourages human reasoning from the facts and knowledge to the conclusion in a consistent and well-documented way. The graph below shows a sample risk assessment process based on the fuzzy logic system. It is a bottom-up structure that starts from each individual risk. The risk exposure is then aggregated at the business unit and company levels to identify the top risks.

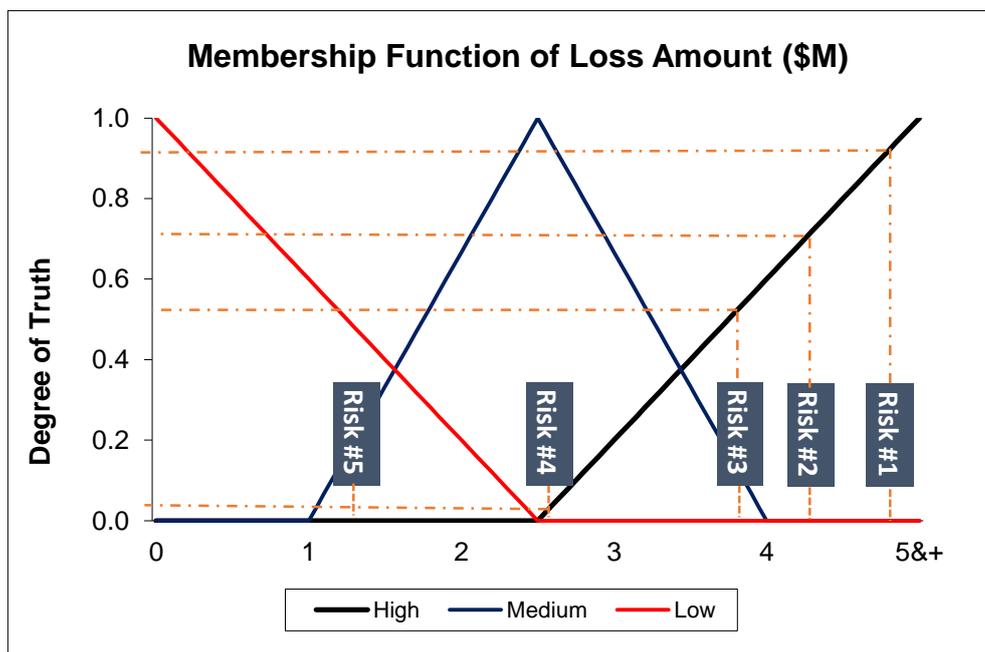
Figure 16. Risk Assessment Hierarchy Structure



To make it comparable among all kinds of risks, the same measure needs to be adopted when assessing the exposure to each risk. One possible candidate is the estimated amount of loss under extreme events. If the distribution of the loss can be simulated using a fuzzy logic model given the distribution of the independent variables, the measure could be something like the 99.5th percentile of the loss distribution (1-in-200-year event). By using the loss

amount as the output variable, risks may be ranked based on the result of defuzzification, a numerical value that measures the level of risk exposure. It is equivalent to ranking based on the degree of truth that the risk exposure is high. Figure 17 illustrates the ranking of risks based on the estimated amount of loss under extreme events. The loss amount may be estimated based on the result of defuzzification using a fuzzy logic model. The fuzzy logic model may have the loss amount as the output variable. The value of input variables under the extreme event is fed into the model to get the estimated loss amount for a certain risk. An alternative approach is the use of simulation as illustrated in Figure 11. The distribution of the loss amount may be simulated, and the value at the specified percentile can be used to represent the risk exposure.

Figure 17. Risk Ranking Based on Loss Amount



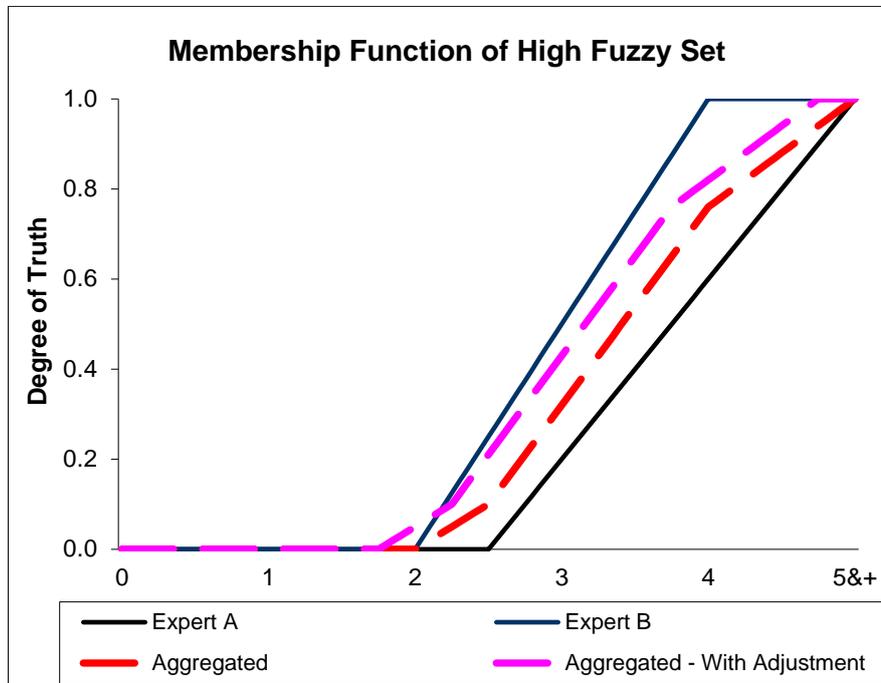
In addition to helping identify the top risks, fuzzy logic models may include information about the causes of risk exposure, or factors that have a significant impact on it. This may provide clues that lead in the direction of potential risk mitigation methods. The cost of risk-hedging or mitigation can be added as an extra output variable in the fuzzy logic model. This will help management decide which risks should be mitigated and the most cost-efficient approach of doing so.

The discussion so far assumes that all the experts share the same view of the risks. Given the experts' different levels of understanding and experience, this is unlikely to be true in actual practice. Hence, it is necessary to aggregate differing opinions. There are several approaches to aggregation.

1. Adjust the membership functions and inference rules to aggregate different opinions. In the example shown in Figure 18, the weighted average of the membership functions provided by Expert A and Expert B may be used as the aggregated

membership function for the high fuzzy set. Weights can be determined based on each expert's experience, knowledge of the investigated issue, confidence in his/her opinion and the accuracy of past estimation.

Figure 18. Aggregation of Membership Functions



It is also possible there are different opinions about the inference rules themselves. If the difference is not too large, an adjustment to the membership function may be able to incorporate that difference. Assume there are two inference rules as given below.

Expert A: If X is high, then Y is high.

Expert B: If X is not low (medium or high), then Y is high.

The aggregated membership function of the high fuzzy set can be shifted to the left, as shown in Figure 18. By changing the membership function of the high fuzzy set, it partially reflects the inference rule that *If X is medium, then Y is high*. Meanwhile, only one inference rule needs to be included in the fuzzy logic model.

If X is high, then Y is high.

However, if there are opposite opinions about the inference rules, it is necessary to understand the reasoning behind each opinion. Experts may revise their opinions after learning from the opposite side. In the event there are still opposite opinions at the end of the discussion, both inference rules may be removed entirely from the model since the disagreement may indicate a lack of knowledge and a low level of credibility.

2. Each expert may have his/her own fuzzy logic model with unique membership functions and inference rules. The aggregated risk assessment result is simply the weighted average of the results generated from the different individual models. Unlike the first approach that adjusts the model inputs, the second adjusts the model outputs by melding them all together

3. A specific case of the second approach is to assign an equal weight to all opinions, which is prevalent in the literature about fuzzy logic models. This is normally used when there are quite a few experts and the goal is to rank based on the level of risk. For example, there are n experts that provide the view of the risk level of A and B. If more than $n/2$ experts vote for A as the riskier one, A will be considered riskier than B. It may be appropriate for identifying the riskiest cases for a specific individual risk. But it does not fit well for the aggregation at the level of the business unit and the total company.

4.2 Required Economic Capital Model

It is a challenge to determine the required economic capital (REC) for risks with insufficient experience data. The lack of relevant historical loss data and the wide scope and potential range of losses resulting from these risks make it hard to quantify the exposure to them. Some companies use regulatory models or rating agency models. Others estimate exposure by making reference to peers' REC for operational risk. However, these methods are normally high-level factor-based approaches, such as x percent of revenue/premium, earnings, assets or the required capital for another risk type. Such factors may not always be able to take a full account of the difference in actual risk exposure and risk management practices among different companies.

Even without sufficient loss data for quantification, fuzzy logic systems may help estimate the REC for certain risks using a bottom-up approach, given sufficient inputs from subject matter experts. As shown in [Section 2.2 A Numerical Example](#), the output variable can be simulated to get the distribution, the value at risk (VaR) and the conditional tail expectation (CTE). If the output variable is the annual loss of the risk, the REC can be determined as, for example, the 99.5th percentile¹⁵ of the simulated loss distribution less the— average loss. An alternative is to take the difference between the estimated loss under an extreme event and the expected loss. The REC at an aggregated level may be estimated by two methods.

1. Aggregate the REC for each individual risk using a correlation matrix, as shown in an example of three risk factors given below.

¹⁵ The 99.5th percentile is equivalent to a 1-in-200-year event. The percentile may be chosen based on the risk appetite of the company.

$$\mathbf{REC}_{\text{Total}} = \sqrt{\begin{pmatrix} REC_1 & REC_2 & REC_3 \end{pmatrix} \begin{pmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{pmatrix} \begin{pmatrix} REC_1 \\ REC_2 \\ REC_3 \end{pmatrix}}$$

Notations

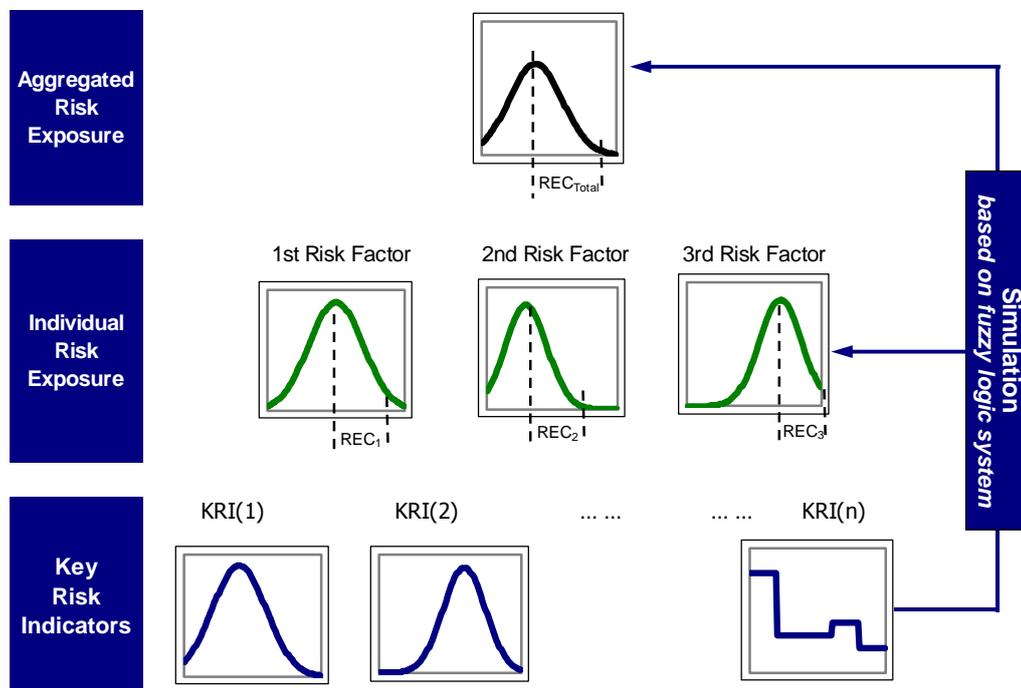
REC_{Total}: Aggregated required economic capital

REC_i: Required economic capital for risk factor i

ρ_{ij}: Correlation Coefficient of risk factors i and j

2. Generate correlated values for all input variables and run the simulation to get the distribution of the aggregated loss and then the aggregated REC, as illustrated below.

Figure 19. Aggregated REC Using Simulation



Notations

KRI: Key risk indicators

REC_{Total}: Aggregated required economic capital

REC_i: Required economic capital for risk factor i

It is not an easy task to determine the correlation of the REC for individual risks. In many cases where the fuzzy logic model is used, there is a lack of experience data. It will be even harder to figure out the appropriate correlation among different risks as it normally requires a time series of panel data for calibration. In addition, the correlation among the REC for different risks is not the same as the correlation among the risks themselves. A theoretically more reasonable approach is to build in the correlation or dependency at the root of the fuzzy logic system, which is at the level of input variables. The distribution of loss at different levels can be simulated, and the REC can be calculated based on the simulated distribution. For this reason, the second method is more appropriate for REC calculation in a fuzzy logic

system. An example of REC calculation and aggregation is given in [Section 6 Case Studies](#).

5 Key Considerations

In the application of fuzzy logic systems to risk assessment and risk decision-making, many practical issues and challenges will be encountered. Even with a solid theoretical foundation, the success of a system depends on many factors such as the quality of the experts' opinions, the system's own credibility and its linkage to management decisions. This section covers some key factors to be considered in the development and application of a practical fuzzy logic system.

5.1 Expert Opinions: Collection and Analysis

Opinions of subject matter experts or business managers are the main information source of a fuzzy logic system. It is not a one-time effort but an iterative process. A sample process is given below.

Step 1. The request for opinions about the issue or risk is sent out. It may include questions about the key factors that may cause any risk event, the value of each factor for existing business, any known cause-and-effect relationship, any risk measures that could be used and any relationship with other risk types. It can be done electronically, via interview with each expert or by means of a group discussion. If the issue is new and complicated, an introduction and discussion in a conference is more effective.

Step 2. Collected opinions are aggregated and analyzed. If there are conflicting opinions, further explanation from the experts may be needed to understand the thinking behind their opinions. After that, a proposed fuzzy logic model with specified variables, membership functions and inference rules will be communicated back to experts to get their comments and agreement.

Step 3. Feedback about the proposed model is digested and reflected in the final model specification. This may require several rounds of communication.

Step 4. After the model is finalized, relevant data collection and a risk-monitoring process need to be set up. Regular reports about the current risk exposure are prepared based on the fuzzy logic model. They are distributed to the experts for comments and information. Based on the model results, past experience, changing environment or improved understanding, experts may revise their opinions. This requires a regular review and update of the model.

To encourage the contribution of the experts to the fuzzy logic system, it is essential to present the final product and report to them so that they understand the outcome of their efforts. Normally the experts are business managers. If the model can provide them

information about the risk exposure of their existing business or future potential business strategies, they are likely to devote more time to it because it will be beneficial to their business decisions.

5.2 Selection of Membership Functions

The membership function is a critically important input for the fuzzy logic system. It may be easy to come up with the inference rules; it is not so easy to devise the membership function because it requires translating the qualitative description into a quantitative measure. There are several approaches that may be used.

1. Ask the subject-matter experts to provide inputs. Fuzzy logic models rely heavily on human reasoning. When the experts' opinions are collected, it is important to ask them to define what they mean when they say something is high, medium or low. Take the credit score, for example, as discussed earlier. Statements like "any score greater than 1.5 is not low" or "any score less than 0.5 is absolutely low" are useful for developing the membership function. A reasonable membership function for the low credit score fuzzy set could be as follows.

$$\mu^{Low}(x) = \begin{cases} 1 & x \leq 0.5 \\ \frac{1.5-x}{1} & 0.5 < x \leq 1.5 \\ 0 & x > 1.5 \end{cases}$$

The chosen membership function needs to be communicated back to those experts to get their agreement. It is a time-consuming process and requires training for people who will provide their inputs to the fuzzy logic system. Different experts may have different opinions about the membership function. It is necessary to consolidate different opinions and find a way to aggregate them. The simplest way is to add a weight to each person's opinion and use the weighted average membership function. But how the weight is determined and how to keep it simple is an art rather than a science.

2. If experience data are available, sometimes the membership function can be partially calibrated. This is usually done after using the fuzzy logic system for a certain time period. There might be some information available about whether the model worked well compared to what actually happened. Perhaps the membership functions can be refined based on the experience. The weight of each expert's opinion may be adjusted as well.
3. Other models may be used in combination with the fuzzy logic model so that the membership functions can be calibrated. The adaptive neuro fuzzy inference system (ANFIS) that combines the artificial neural network with the fuzzy logic model is an example. However, it requires a large set of training data that normally is not applicable for operational risks.

5.3 The Role of Experience Data

For most risks dealt with using fuzzy logic models, there may not be sufficient data. The reasonableness of the model is primarily in the hands of the experts or business managers. The comments on the inference rules or on the membership functions may have a material impact on the result of risk assessment. However, back testing based on experience data, if available, may be used to validate or improve the models. Comparing the actual experience with the model is an option that may be used after implementing the fuzzy logic system. Based on the experience data, the membership functions may be adjusted or calibrated to better predict the output variable. Tracking the inputs from each expert may also tell us how well they fit the experience data; the weight on each expert's opinions may be adjusted accordingly. In addition, when enough data have been collected, it may also have an impact on the experts' understanding of the subject and may change their inputs as well, including the inference rules and membership functions. In the end, with sufficient data, fuzzy logic models may be migrated to models based on probability theory, but not necessarily.¹⁶

Unlike some data-driven models, the weight put on experience data when specifying a fuzzy logic model is not heavy in most cases.

1. Experience data collected may not be statistically credible for revising the existing model parameters and inference rules. It is likely that only after the fuzzy logic model is implemented will relevant data be collected in a meaningful way.
2. For risk management, the most useful piece of information is about tail events. It will be even harder to collect data for tail events.
3. The explicit cause-and-effect relationships built in the fuzzy logic model prevent the model from changing solely based on experience data, contrary to some data-mining models. Unless the experience is analyzed and fully understood, it may not cause a change in the model.

Analyzing experience data provides opportunities to enhance our knowledge of the risks and improve the accuracy of the fuzzy logic model. The data may have information contrary to the assumed inference rules. By analyzing the data, people may be able to correct misunderstanding, discover new underlying factors and revise the inference rules.

5.4 Fuzzy Logic System Review

Like other risk management tools such as risk appetite, fuzzy logic systems need to be reviewed and updated from time to time.

1. There may be new risks to be included in the system due to new business or a change in the business environment.

¹⁶ A simple example of using experience data is included in the Appendix.

2. There may be better understanding of the issue, based on recent academic research or recently emerged loss experience.
3. There may be changes in the company's strategy. In this case, the exposure to each of the company's risks needs to be updated.

Depending on the scope of the fuzzy logic system, it may be a difficult task to produce a full update. A balance is needed to make the best use of the experts' time. It is important to make the updating process as easy and rewarding as possible. A user-friendly interface for the experts to update their opinions may be helpful. Regular reporting about the risk exposure and the implications of potential risk strategies help maintain people's interest and their willingness to engage in the ongoing process.

5.5 Linkage to Decision-Making

The ultimate goal of any risk-assessment system is to help decision-makers make informed risk decisions. Although fuzzy logic systems can be used to estimate the risk exposure quantitatively, what is really meaningful is the ranking of the risks. This enables decision-makers to identify the major risks and provides them with a better understanding of the relative magnitude of the risks. As long as the assumptions and approaches used for assessing risks are consistent, the ranking based on the fuzzy logic system will be meaningful. In addition, fuzzy logic systems can be used to estimate the cost of risk mitigation.

1. At the individual risk level: For each individual risk, the major contributors to the risk exposure may be identified by the fuzzy logic model. For example, the misconduct risk of each product a company offers can be assessed and ranked. The products exposed to high misconduct risk may then be monitored and even revised if there is a sustained period of bad risk experience. Actions may include more training for the advisers, an adjusted adviser compensation scheme that will penalize misleading advertisement or a simplification of the product. Another example is identifying the key events that may deteriorate a company's reputation. A list of events with potential risks may be collected either from the company's own risk assessment or from analyzing public opinions. A fuzzy logic model can be used to rank those events and necessary actions can be taken to manage the risk.
2. At the business unit level: The top risks can be identified by the fuzzy logic system and necessary risk monitoring and management action can then be taken at the level of the business unit. In addition to existing risks, exposure to emerging risks and new risks that may be caused by new business strategies can be assessed by the fuzzy logic system. This in turn can be fed into the business-decision process so that future business strategies include adequate consideration of the potential risks.
3. At the overall company level: At the top level, in addition to risk identification and assessment, fuzzy logic systems may play an important role in strategic planning, and may affect new business plans and strategic capital management. Decision-makers can obtain a more holistic view of the company's risks when planning its future.

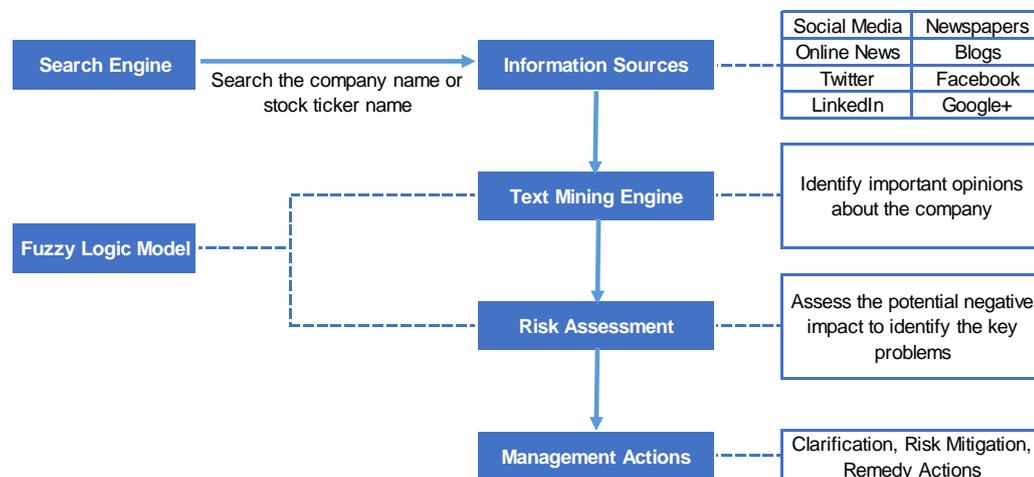
6 Case Studies

This section includes two examples of applying fuzzy logic models to risk management, one at the micro level and the other at the macro level.

6.1 Negative Public Opinion Identification and Assessment

Public opinion about a company's performance or its contribution to society are important for reputation risk management. Opinion may also have a direct impact on short-term stock performance. Therefore, collecting and analyzing public opinion is useful. It is sometimes called opinion mining or sentiment analysis. Many companies monitor public opinion by looking at social media, newspapers, online news, blogs, Twitter, etc. Normally this is done on an ad hoc basis or manually, which may be insufficient (too little, too late), especially for big and global companies. A framework capable of identifying and assessing public opinion on a more frequent, even a daily basis, is ideal since the speed at which information spreads is fast, and the impact of public negativity on stock price can be immediate. Using such information, the management team can take early actions to mitigate or even avoid the negative impact. Fuzzy logic models may be appropriate because the source materials about public opinion are generally expressed in linguistic terms. The flow chart of an identification and assessment framework based on a fuzzy logic model is given below.

Figure 20. Flow Chart of Public Opinion Monitoring



Fuzzy logic models may be used in identifying and assessing the negative public opinions that will be brought to the attention of senior management. After analyzing the collected information using a text-mining engine to get a list of potential problems, the fuzzy logic model can help identify the most problematic opinions. Instead of having a numerical value for each independent variable, inference rules based on the linguistic description of the independent variables may be used in this step. This is similar to the process of human reasoning where the input in this case is the opinions described in qualitative terms. When assessing the risk of identified problems, a different and more sophisticated fuzzy logic model

focusing on the potential impact on franchise value may be applied. As an alternative, a combined fuzzy logic model may be used to consolidate the identification and assessment into one step, but this could involve a longer processing time since a more sophisticated fuzzy logic model must process all the collected information.

A simplified example is given below to illustrate the process of building and using such a framework for reputation risk management.

Step 1. Gather the public opinions about Company XYZ from different sources, including social media, newspapers, blogs and social networks such as Twitter, Facebook, LinkedIn and Google+.

Step 2. Use advanced text-mining techniques to summarize each opinion in one sentence or a short paragraph. A sample list is given below. In a real world situation, the list may be much longer.

Table 3. A Sample List of Public Opinions

No.	Description of the "opinion"	Info Source	No. of Comments	Category	Opposite Opinions
1	It is NOT good for XYZ to buy ABC due to the large capital requirement.	Twitter	4	Business Strategy	50%
2	XYZ keeps increasing the premium rate, which makes the sales more difficult.	Blogs	5	Product Competitiveness	0%
3	XYZ helped local communities to improve children's health and education.	Newspapers	10	Social Responsibility	0%
4	XYZ's next quarterly earnings are likely to be lower than expected.	Blogs	17	Stock Performance	60%
5	Investors plan to sue XYZ for loss due to inappropriate management.	Twitter	14	Stock Performance	0%
6	The hedging program XYZ is implementing seems too conservative to get the investor the expected return on equity.	Blogs and Twitter	18	Stock Performance	40%
7	XYZ's cross selling makes some customers worried about the leakage of their private, personal information.	TV news	18	Privacy Protection	0%
8	XYZ plans to expand its business in Asia in the next five years.	Newspaper	1	Business Strategy	0%
9	I was tricked into buying a product of XYZ with a very low guaranteed	Twitter	3	False Advertisement	0%

	value.				
10	There is a chance XYZ will sell its retirement service unit in the next quarter.	Twitter	5	Inside Information	0%
... ..					

Step 3. Use the fuzzy logic model to identify important opinions about the company. An example is given below.

Table 4. Fuzzy Logic Model Example: Negative Publicity

<u>Negative Publicity</u>	
Key risk indicators	
1.	Source of information: The types of sources can be ranked in terms of importance. For example, a news item on TV is likely to be more important than a comment on Twitter.
2.	Topic popularity: It may be measured by the number of times the topic was discussed.
3.	Degree of uniformity of the opinions: This could include whether there are opposite opinions and which opinion is mostly supported.
4.	Subject of the opinion: Stock performance, product competitiveness, privacy protection, false advertisement, inside information, etc. The company may have its own priority list based on business type and risk-management strategy.
Inference rules	
1.	If [(the degree of information is important) or (the topic is popular)] and (the subject is in the priority list), then the risk is high.
2.	If (the degree of uniformity is not high) and (the subject is not in the priority list), then the risk is low.
3.	If (the degree of uniformity is high) and (the topic is popular), then the risk is not low.
... ..	
Output	
A list of potential issues regarding the public perception of the company	

The list in step 2 may be shortened by selecting the negative opinions with high risk. It may include poor sentiment about the company’s value, social responsibility, competitiveness, reputation, etc.

Table 5. A Sample List of Public Opinions with a High Risk Level

No.	Description of the “opinion”	Info Source	No. of Comments	Category	Opposite Opinions
2	XYZ keeps increasing the premium	Blogs	5	Product	0%

	rate, which makes sales more difficult.			Competitiveness	
4	XYZ's next quarterly earnings are likely to be lower than expected.	Blogs	17	Stock Performance	60%
5	Investors plan to sue XYZ for loss due to inappropriate management.	Twitter	14	Stock Performance	0%
6	The hedging program XYZ is implementing seems too conservative to get the investor the expected return on equity.	Blogs and Twitter	18	Stock Performance	40%
7	XYZ's cross selling makes some customers worried about the leakage of their private, personal information.	TV news	18	Privacy Protection	0%
10	There is a chance XYZ will sell its retirement service unit in the next quarter.	Twitter	5	Inside Information	0%
... ..					

Step 4. Use the fuzzy logic model to assess the impact on franchise value and the cost of risk-mitigation actions. The fuzzy logic model may be an extended version of the model used in step 3. Additional input variables may include whether the opinion about the company is negative and whether the impact is negative if the opinion is wrong. The output variables may include the estimated loss amount and the cost of risk mitigation.

a. Loss Amount = Base Amount × Negative Publicity Risk Factor

1. A negative publicity risk factor can be chosen to reflect a confidence level consistent with the company's risk appetite. For example, a 1-in-200-year event may be chosen over a 1-in-100-year event for a very conservative company.
2. The base amount may be determined using past experience, either company specific or industry average. The base amount may be hard to determine. Historical losses due to negative publicity may give a rough idea of a reasonable range of the base amount, which can be considered as the cost of a risk event with a risk factor of 1. It is conceptually similar to the way some companies determine operational risk required economic capital, such as using the product of a risk level factor and the Generally Accepted Accounting Principles (GAAP) earnings or the required capital for other risks. The GAAP earnings or required capital for other risks may be considered as a corresponding item to the "base amount" and the factor is something like the "negative publicity risk factor." If no experience exists, estimation from the experts is needed.

- b. The cost of risk mitigation may be estimated based on the source of information and the subject of the opinion.

Step 5. Verify the selected opinions against the fact.

Step 6. Make decisions about managing the identified key issues. Possible actions include clarifying the fact by a press release or an information session.

Table 6. A Sample List of Public Opinions and Risk-Mitigation Plans

No.	Description of the “opinion”	Possible risk-mitigation plan
2	XYZ keeps increasing the premium rate, which makes the sales more difficult.	Make the products more attractive by changing the product features, reducing the premium, increasing adviser’s compensation or educating the public about the reasons for the rate increases.
5	Investors plan to sue XYZ for loss due to inappropriate management.	Make the decision-making process and business strategy more transparent and keep the public updated about the lawsuit.
6	The hedging program XYZ is implementing seems too conservative to get the investor the expected return on equity.	Add more information in financial reports about the benefits of hedging and communicate clearly with investors.
7	XYZ’s cross selling makes some customers worried about the leakage of their private, personal information.	Stop cross selling or notify new clients about it and get their agreement.
10	There is a chance XYZ will sell its retirement service unit in the next quarter.	Investigate the source of the information and report it to regulators if insider trading is involved.

Instead of using the fuzzy logic model, this framework could be based on a model such as a decision tree. As the model grows more complex with the increasing number of independent variables, however, the fuzzy logic model may be a better choice. If the selected list is short enough, instead of using the fuzzy logic model in step 4, a full-blown analysis of the listed opinions one-by-one is also a feasible approach. However, if the amount of information to be analyzed is large, fuzzy logic models can make the process more efficient and consistent.

6.2 Risk Aggregation and Budgeting

Risks modeled using the fuzzy logic framework can be aggregated at different levels to have a holistic view of the company’s risk profile. The aggregation might be done at the business line level, business unit level or overall company level. When the potential loss amount is used as the output of the fuzzy logic model, it may be integrated into the economic

capital model where other risks are measured using probability models. The example given below illustrates the process of aggregation and the implication on risk-taking strategy. It is made as simple as possible, and the real world case may look much more complicated.

Company XYZ is a property insurance company operating in two regions, North America and the Middle East. Its main business includes auto insurance and homeowner insurance. XYZ was established 50 years ago and has plenty of internal experience data about insurance risk. It uses an economic capital framework to measure its risk exposure. For risk types with sufficient actual experience data, such as market, credit and insurance risk, quantitative models based on probability theory are used. For other risks, however, for which XYZ does not have sufficient experience or does not have enough resources to build quantitative models, the company uses fuzzy logic models to measure its risk exposure. The major risks handled by the fuzzy logic framework are climate change, cyber security, negative publicity, regional instability and terrorism. Sample fuzzy logic models are listed below for those risks, except the negative publicity, which was covered in [Section 6.1](#).

Table 7. Fuzzy Logic Model Example: Climate Change

<p><u>Climate Change</u></p> <p>Key risk indicators</p> <ol style="list-style-type: none"> 1. Frequency: Trend of increase in recent flood events 2. Severity: Trend of increase in recent flood events 3. The location of the written business <p>Inference rules</p> <ol style="list-style-type: none"> 1. If (the trend of severity increase is high) and (the insured property is located in the high-risk area), then the risk is high. 2. If (the trend of severity increase is not high) and (the trend of frequency increase is high) and (the insured property is located in the high-risk area), then the risk is medium. 3. If (the insured property is not located in the high-risk area), then the risk is low. <p>... ..</p> <p>Loss Amount = Sum Insured × Climate Change Risk Factor</p> <p>Notes</p> <ol style="list-style-type: none"> 1. The climate change risk factor can be chosen to reflect a confidence level consistent with the company's risk appetite. 2. Risk assessment may be conducted across the entire in-force business or expected new business case-by-case and then summed up.
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Table 8. Fuzzy Logic Model Example: Cyber Security

<p><u>Cyber Security</u></p> <p>Key risk indicators</p> <ol style="list-style-type: none">1. Cyber security technology2. Cyber security standards3. The scope of collected private information4. Impact of past incidents <p>Inference rules</p> <ol style="list-style-type: none">1. If (the technology is advanced) and (the standard is high), then the risk is not high.2. If (the impact of past incidents is high), then the risk is not low.3. If (the scope is not narrow), then the risk is not low.... .. <p>Loss Amount = Base Amount × Cyber Security Risk Factor</p> <p>Notes</p> <ol style="list-style-type: none">1. The cyber security risk factor can be chosen to reflect a confidence level consistent with the company's risk appetite.2. The base amount may be determined using past experience or input from experts.
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Table 9. Fuzzy Logic Model Example: Regional Instability

<p><u>Regional Instability</u></p> <p>Key risk indicators</p> <ol style="list-style-type: none">1. The location of the written business2. Loss experience due to wars3. Intervention from outside <p>Inference rules</p> <ol style="list-style-type: none">1. If (the insured property is located in a high-risk area) and (the impact of intervention from outside is high), then the risk is high.2. If (the loss due to wars was high), then the risk is not low.3. If (the insured property is not located in the high-risk area), then the risk is low.... .. <p>Loss Amount = Covered Sum Insured × Regional Instability Risk Factor</p> <p>Notes</p> <ol style="list-style-type: none">1. The regional instability risk factor can be chosen to reflect a confidence level consistent with the company's risk appetite.2. Covered sum insured: The sum insured covered in the event of wars, turmoil, chaos, etc.

3. Risk assessment may be conducted across the entire in-force business or expected new business case-by-case and then summed up.

Table 10. Fuzzy Logic Model Example: Terrorism

<p><u>Terrorism</u></p> <p>Key risk indicators</p> <ol style="list-style-type: none"> 1. The location of the written business 2. Loss experience due to terrorism 3. Security system <p>Inference rules</p> <ol style="list-style-type: none"> 1. If (the insured property is located in a high-risk area), then the risk is high. 2. If (the loss due to terrorism was high), then the risk is not low. 3. If (the insured property is not located in the high-risk area) and (the security system is good), then the risk is low. <p>Loss Amount = Covered Sum Insured × Terrorism Risk Factor</p> <p>Notes</p> <ol style="list-style-type: none"> 1. The terrorism risk factor can be chosen to reflect a confidence level consistent with the company's risk appetite. 2. Covered sum insured: The sum insured covered in the event of terrorism. 3. Risk assessment may be conducted across the entire in-force business or expected new business case-by-case and then summed up.

Using the fuzzy logic model, the estimated required economic capital at the confidence level of 99.5 percent (1-in-200-year event) for each individual risk is given in Table 11. It may be calculated as

- a. The 99.5th percentile of the simulated loss distribution less the average loss; or
- b. The estimated loss in an extreme event (historical or hypothetical) less the expected loss.

Table 11. Sample Required Economic Capital for Individual Risks

<i>US\$ Million</i>	North America	Middle East
Climate Change	100	20
Cyber Security	40	35
Negative Publicity	60	50
Regional Instability	15	120
Terrorism	50	100

As discussed in [Section 4.2](#), there are two approaches to aggregation.

1. Use the correlation matrix. Regional instability and terrorism are likely to have a high positive correlation because terrorism is normally prevalent in an instable region. Other risks are likely to have low correlation due to their distinct features and causes. Table 12 is a possible correlation matrix for those risks. In most cases, it may be determined based on subjective judgment. For risks that are new or not well understood, an assumption based on judgment with a certain degree of conservatism might be the best we can do.

Table 12 Sample Correlation Matrix for Individual Risks

Correlation	Climate Change	Cyber Security	Negative Publicity	Regional Instability	Terrorism
Climate Change	1	0.1	0.1	0.1	0.1
Cyber Security	0.1	1	0.1	0.1	0.1
Negative Publicity	0.1	0.1	1	0.1	0.1
Regional Instability	0.1	0.1	0.1	1	0.95
Terrorism	0.1	0.1	0.1	0.95	1

Applying the correlation matrix to the required economic capital for all risks will generate required economic capital (REC) of \$156 million for a North America business and an REC of \$238 million for a Middle East business.

To get the REC at the company total level, a regional correlation factor is needed as well. Considering the degree of globalization, a 90 percent correlation is assumed, which leads to a total REC of \$385 million.¹⁷

2. Model the dependency among the independent variables. Instead of quantifying the diversification in the last step, correlation can be built in among the independent variables. In this case, global scenarios that include all the independent variables in the fuzzy logic framework for every region can be designed and generated so that:
 - a. The loss caused by regional instability and by terrorism is highly correlated; and
 - b. The high risk areas for regional instability and for terrorism are almost the same.

Other independent variables can be generated separately.

Running through the stochastic or stress scenarios can generate the REC at the individual, business unit and overall company level in one step. Theoretically this approach will give us a more accurate estimate because it is not necessary to create the correlation matrix, which is likely to be based on subjective judgment.

Risks measured by fuzzy logic models can be aggregated with risks measured by probability models. Using the correlation matrix is a more feasible approach because the global scenarios required by the second approach may require the use of too many variables at the same time. Since most of the risks handled by fuzzy logic models may be operational or

¹⁷ The calculation can be found in tab “6.2 Risk Aggregation” in the accompanying EXCEL file “Fuzzy Logic Examples.xls.”

emerging risks, the assumption of correlation between operational risk and other risk types may be used. This assumption may exist in the company's current economic capital framework, regulatory capital models such as the European Union's Solvency II Directive, rating agency capital models and industry-level research reports.

Using required economic capital, for example, risks analyzed using fuzzy logic models can be compared to risks analyzed using data-driven quantitative models on the same basis. In the past, those operational and emerging risks were normally ignored during capital management and the risk-budgeting process. But now they can fit in the existing framework and become part of a more comprehensive risk profile. Most of the risks modeled by fuzzy logic models are not tradable, and therefore it is difficult to get a high return for taking the risks. The focus during the risk-budgeting and -review process is more on risk avoidance and risk mitigation to reduce exposure. For example, XYZ is highly exposed to regional instability and terrorism risk in the Middle East market. The company may consider hedging these two risks by reinsurance or changing the product features to reduce the insured amount for losses caused by wars, turmoil, terrorism, etc.

7 Conclusion

As a complement to probability models, fuzzy logic models can be applied to assess risks for which there is insufficient data and incomplete knowledge. Fuzzy logic provides a framework where human reasoning and imprecise data can contribute to risk analysis. The scope of possible applications is wide for fuzzy logic systems. Many risks are beyond control, not well understood or even unknown, as evidenced by the growing list of emerging risks.

Using an appropriate fuzzy logic system, it is possible to consistently analyze multiple risks that are not well understood. The exposure to each risk can be assessed and ranked. Key risks can be identified and managed. Resources may be used to monitor and mitigate these key risks with high exposure. Inference rules in a fuzzy logic model may help not only to identify the cause of a certain risk but also to design efficient and effective mitigation plans.

Fuzzy logic systems assist us in building knowledge of risks in two ways.

1. The systems keep risk managers and subject matter experts free from the inference part for many risks and let them focus on cause-and-effect relationships based on their knowledge.
2. Risk assessment results flow into the risk decision-making process, and the outcome of the decision can then be fed back into the system to refine the fuzzy sets, rules and understanding.

Fuzzy logic models may be used with other risk models such as decision trees and artificial neural networks to model complicated risk issues like policyholder behaviors.

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Appendix. The Use of Experience Data

In [Section 5.3](#), the role of experience data is discussed. In this section, a simple example is used to illustrate how experience data can be used to refine the model parameters or even switch to a new model when the data is sufficient. The example and assumption used may not make much sense in the business world, but it was chosen for ease of understanding.

Company ABC plans to provide personal mortgage loan services to its clients. Since it does not have any experience, it uses a fuzzy logic model based on the inputs from experienced credit analysts to make loan decisions.

Using information such as age, gender, income, current debt and job title, each loan applicant will get a credit score. The credit score is then input into the fuzzy logic model to assess the risk of default and make its loan decision. The initial model set up is given below.

Input Variable: Credit Score

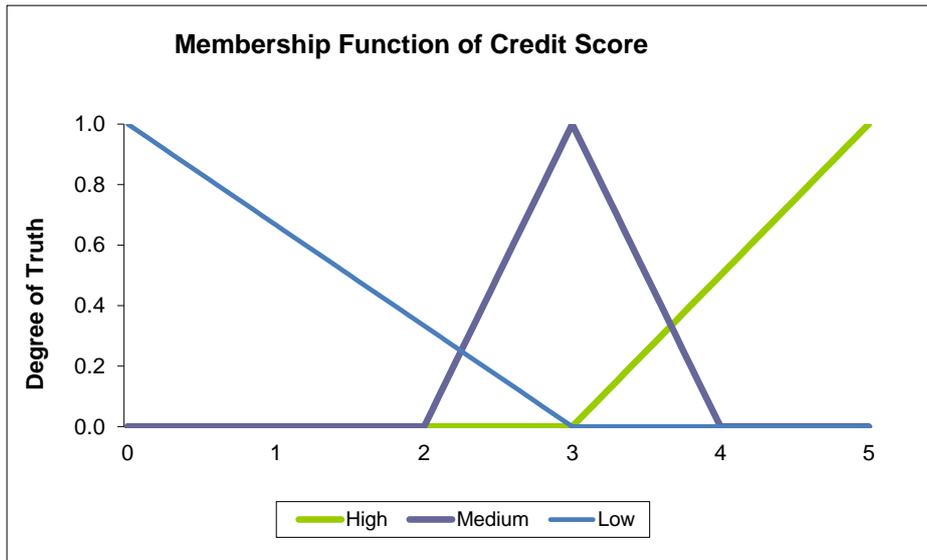
Membership functions

$$\mu^{High}(x) = \begin{cases} 0 & x \leq 3 \\ (x-3)/2 & 3 < x \leq 5 \end{cases}$$

$$\mu^{Medium}(x) = \begin{cases} 0 & x \leq 2 \\ (x-2)/1 & 2 < x \leq 3 \\ (4-x)/1 & 3 < x \leq 4 \\ 0 & x > 4 \end{cases}$$

$$\mu^{Low}(x) = \begin{cases} (3-x)/3 & 0 \leq x \leq 3 \\ 0 & x > 3 \end{cases}$$

Figure 21. Credit Score Membership Functions



Output Variable: Default Risk

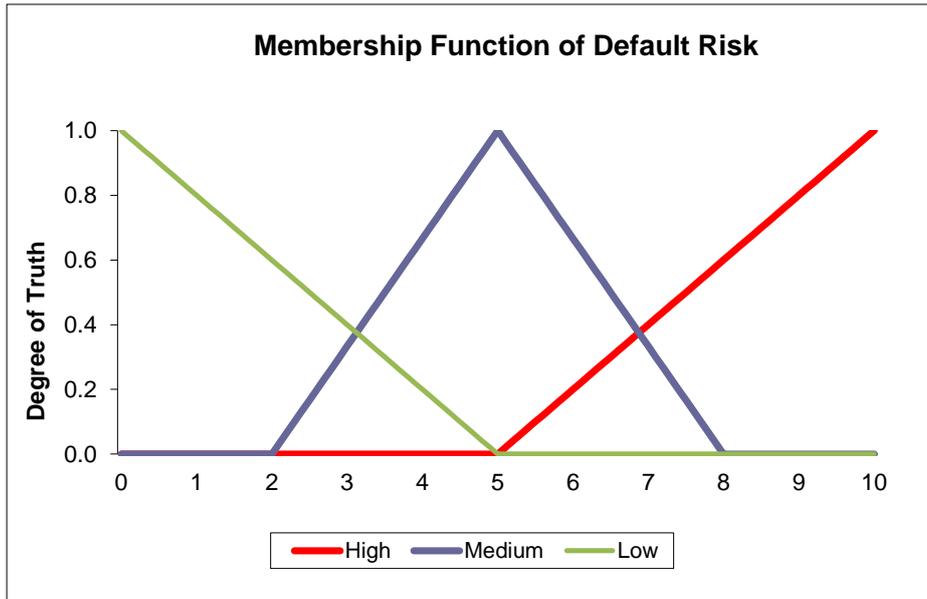
Membership functions

$$\mu^{High}(x) = \begin{cases} 0 & x \leq 5 \\ \frac{(x-5)}{5} & 5 < x \leq 10 \end{cases}$$

$$\mu^{Medium}(x) = \begin{cases} 0 & x \leq 2 \\ \frac{(x-2)}{3} & 2 < x \leq 5 \\ \frac{(8-x)}{3} & 5 < x \leq 8 \\ 0 & x > 8 \end{cases}$$

$$\mu^{Low}(x) = \begin{cases} \frac{(5-x)}{5} & 0 \leq x \leq 5 \\ 0 & x > 5 \end{cases}$$

Figure 22. Default Risk Membership Functions



Inference rule: If the credit score is high, then the default risk is low.

Defuzzification method: Average of maximum

Company ABC has issued 10 mortgage loans based on the criteria that the degree of truth that the default risk is low is greater than 50 percent. This is the equivalent of saying that an applicant with a credit score above 3 may get the loan application approved. Regarding the decisions of mortgage lending, the fuzzy logic model is not superior to a traditional model based on classical set theory. The decision is either a yes or no. However, the fuzzy logic model is useful for assessing the exposure to default risk, either individually or in aggregate. The default probability function is required by traditional models based on probability theory. With insufficient experience data, it is difficult to estimate the default probability for each specified credit score. The fuzzy logic model may get around the problem of specifying the default probability function but can still estimate the level of default risk for each loan application. In the absence of traditional models, the fuzzy logic model may be a practical alternative to assist in assessing the exposure to default risk.

The credit scores and the calculated default risk level are listed below. After one year, two of the 10 loans cannot be repaid as scheduled.

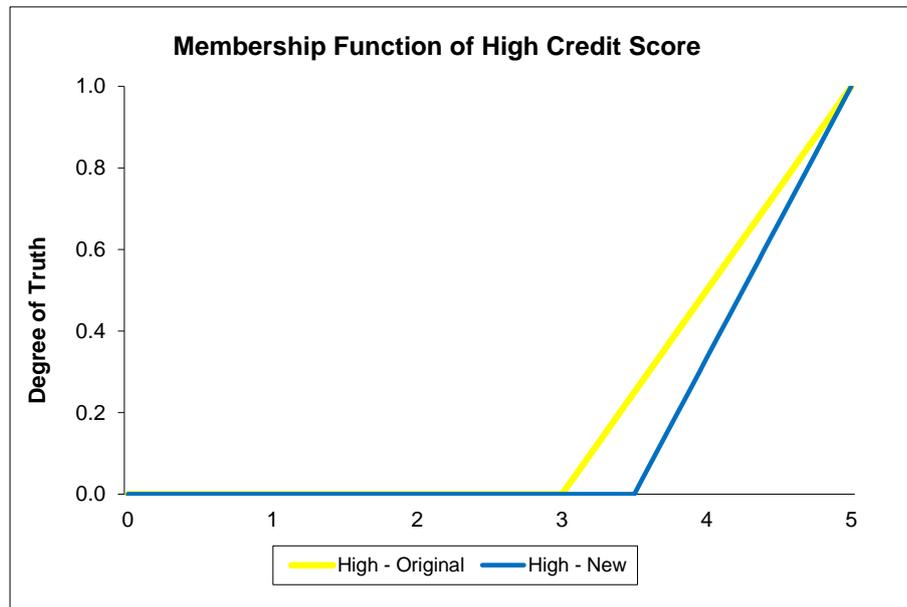
Table 13. Fuzzy Logic Model Output: Default Risk

	Input Variables	Output Variable	Low Default Risk	Experience
No.	Credit Score	Default Risk	Degree of Truth	Default?
1	4.76	0.30	94%	No
2	3.45	1.93	61%	Yes
3	3.99	1.26	75%	No
4	3.43	1.96	61%	No
5	3.54	1.82	64%	No
6	4.04	1.20	76%	No
7	4.62	0.48	91%	No
8	4.05	1.19	76%	No
9	4.02	1.23	76%	No
10	3.08	2.40	52%	Yes

A 20 percent default rate is much higher than the industry level. Clearly the degree of truth for low default risk is too high compared to that implied by the experience. Therefore, the company’s credit analysts agree to revise the model parameter. One of the options is to adjust the membership function for the fuzzy set “High Credit Score.” The membership function is shifted to the right as shown below.

$$\mu_{Original}^{High}(x) = \begin{cases} 0 & x \leq 3 \\ \frac{(x-3)}{2} & 3 < x \leq 5 \end{cases} \quad \mu_{New}^{High}(x) = \begin{cases} 0 & x \leq 4 \\ \frac{(x-4)}{1} & 4 < x \leq 5 \end{cases}$$

Figure 23. High Credit Score Membership Functions



If the fuzzy set “High Credit Score” had been defined by the new membership function, the two defaulted loans would have been denied in the first place and these two defaults would not have occurred.

Table 14. Revised Fuzzy Logic Model Output: Default Risk

	Input Variables	Output Variable	Low Default Risk	Experience
No.	Credit Score	Default Risk	Degree of Truth	Default?
1	4.76	0.40	92%	No
2	3.45	5.00	0%	Yes
3	3.99	1.68	66%	No
4	3.43	5.00	0%	No
5	3.54	2.43	51%	No
6	4.04	1.60	68%	No
7	4.62	0.63	87%	No
8	4.05	1.58	68%	No
9	4.02	1.63	67%	No
10	3.08	5.00	0%	Yes

Other options for model adjustment include adjusting the membership functions for the output set and refining the inference rules by adding a fuzzy hedge or new rules. The goal, however, should be the same in this case: reduce the degree of truth for low default risk implied by the model.

Once sufficient experience data are available, more refined models may be used for loan decision-making. For example, instead of relying on the experts' opinions about the membership functions, the models can be fully calibrated to the experience data.

Taking it a step further, models based on probability theory may be used to replace the fuzzy logic model. The default probability can be modeled as a function of the credit score, removing the process of fuzzification and defuzzification in the fuzzy logic model. The loan decision can be made based on the estimated default probability. For example, each loan with a probability of default less than 1 percent can be issued.