An Ex-Chair's Corner by S. David Promislow

Editor's note: This column originally ran in the June 1997 issue of Expanding Horizons. With the upcoming changes in the SoA syllabus, I feel that discussing the topic of actuarial notation ties in well with related discussions concerning the syllabus, in particular, and how the actuary fits in with other financial professionals, in general.

While teaching my actuarial mathematics course, I consistently find that the major difficulty for students is not so much the ideas or concepts but rather, the notation. Even professional mathematicians from other fields, who are interested in learning the subject, find the material relatively easy but complain about the excessive and involved notation. This raises a number of questions. Does our system of international actuarial notation serve its intended purposes? What are those purposes anyway? Does it interfere with the learning and advancement of the subject? Should there be modifications?

I tell my students that learning the notation is like learning a new language. I caution them to be patient, inform them that it will all soon be second nature and state that it's all part of the training of an actuary. They listen but usually remain unconvinced. It is as if I told them that I was planning to teach the courses in Russian, and they had to master that first.

There is no doubt that notation plays a major role in every branch of scientific endeavor. Some suggest that the reason for the failure of the Romans to make any progress in mathematics, as compared to the Greeks, was because of their cumbersome notation for numbers, namely Roman numerals. Serious mathematical historians dispute this, claiming that it was the Romans' penchant for the practical, like road building, that led them to eschew abstraction–but that is a subject for another article. In any event, the example of Roman numerals illustrates that notational systems that were once in vogue outlive their usefulness and give way to others.

Is this true for our system? Recall that it was developed in a strictly deterministic setting. The modern approach to the subject is a stochastic one, and the attempt to blend the traditional actuarial notation with the standard symbols of probability theory is sometimes awkward and confusing. As an example, one of our main objects of study is the random variable $T(x)$, the future lifetime of a person aged x. Yet, we have no standard symbol for the probability density function, and we often express it in a cumbersome way in terms of other symbols. Curiously enough, the more complicated conditional density, the hazard rate function, does have its own symbol because it arose naturally in the deterministic model as the force of mortality.

A good system of notation should help convey ideas or guide one to the proper algorithms. Leibniz' notation in calculus is a prime example of the latter. I am not so sure that the existing actuarial notation accomplishes these goals. Here is one of dozens of examples that I have noted. Earlier this year, I gave my class the following question, which appears on the Course 3 (former Course 150) sample exams.

For a ten-year select-and-ultimate table, you are given that

$$l_{[x]+t} = \begin{cases} \frac{\sqrt{60}}{9}\left(1 - \frac{t}{100}\right), & 0 \le t < 10, \\ \frac{\sqrt{70-t}}{10}, & 10 \le t < 70. \end{cases}$$

Find $\mathring{e}_{30}$.

Students had more trouble with this relatively simple question than they should have. The key idea is that one obtains an expectation of a non-negative random variable by integrating the survival function. This is something they seemed to know from their probability theory course or, if not, learned in the present course. Yet, for many, the somewhat involved notation for select mortality prevented them from noticing that they were staring directly at a survival function.

Many will defend our system of notation. It is true that it serves as a standard. We can write a formula, and actuaries all over the world know exactly what we mean. This by itself is not sufficient reason to be complacent. Any system of notation could accomplish the same result. Standards are not always good standards, and they are subject to change.

Others will defend the system on the grounds that it is logical and precise. We have a well-defined set of rules that enables us to tell exactly what a particular symbol means. Well, almost always. There are exceptions, and like any other precise set of rules, the exceptions cause great difficulties to the uninitiated. Besides this, being logical and precise does not necessarily imply being useful. The system of Roman numerals is very precise but not very useful. Of course, some dispute this latter point. I recently read an article by a devotee of Roman numerals who attempted to show that it was really very easy to multiply, and even divide, with these–once you knew the technique. The last phrase is the telling one. Almost anything is easy to do once you know how. The problems occur during the learning stage.

I am not enough of a revolutionary to suggest that we suddenly scrap the entire system of international actuarial notation. Actuarial educators, however, should be aware of some of its shortcomings, appreciate the difficulties it causes students, and perhaps slowly work toward some modification of the system.

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