SOCIETY OF ACTUARIES Investment Section

Risks & Rewards

ISSUE 60 AUGUST 2012

PRICING AND HEDGING FINANCIAL AND INSURANCE PRODUCTS

PART 1: COMPLETE AND INCOMPLETE MARKETS

By Mathieu Boudreault

This paper is the first excerpt of the article: "Pricing and hedging financial and insurance products" which will be available from the Society of Actuaries' website. Comments are welcome.

Suppose your insurance company has issued a new block of equity-indexed products. To manage these policies, you use stochastic scenarios based upon your economist's best estimates regarding equity index returns and yields on investment-grade bonds. On the grounds of these assumptions, you determine that the company can spend 300 bps per year over the next five years for an equity-based guarantee. In order to manage the risk underlying this guarantee, you contact the investment bank but the required derivatives cost 700 bps! Why is this possible and what can we do about it? To make sense of it, we have to better understand the modern financial mathematics that underpins active risk management.

In the latter situation, the bank does not necessarily have a smarter or more risk-averse economist. Banks however price their derivatives to be consistent with the other instruments available, i.e. stocks, bonds and plain vanilla instruments such as swaps, futures and options. They use these instruments to hedge their positions and the price they charge is consistent with the cost of the hedging strategy. Otherwise, arbitrage opportunities could arise. Thus, the key to modern financial mathematics is no-arbitrage pricing.

The primary purpose of this paper is to explain in plain English without any cumbersome formulas (almost!) how financial mathematics applies in modern finance and in today's insurance industry. I describe how arbitrage-pricing and risk-neutral pricing are equivalent and I illustrate with simple examples how to deal with complete and incomplete markets. When possible, I try to link these concepts to traditional and equity-linked insurance.

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CHAIRPERSON'S CORNER

We have all lived through an unparalleled time of change. Yet, we will likely see more changes within the next few years than we have in our lifetimes. For example, compare the capability of smart phones to the cell phone that you had in 2002 or even in 1992. Can you imagine what types of devices you will be dependent on in 2022?

Technological advances occur at an exponential rate, not a linear one. In 2005, Aubrey de Grey incorporated this fact when we estimated that the first person to live to 1,000 will probably only be born 10 years later than the first person to live to 150. Is the first person to live to 150 already alive? How about 1,000? Is it possible to manage this "tail" risk in a cost-sensitive, competitive environment? Or would industry-wide changes be necessary?

At a minimum, the global population is aging—birth rates are decreasing while life expectancies are increasing. Given current demographics and the propensity for investors to reduce equity allocations as they age, an aging population should have a positive effect on bond prices and a negative effect on equity prices. Furthermore, as longevity increases, investors should discount future cash flows less since the trade-off between present and future should be less important. In my opinion, these assertions will inevitably lead to continuing low interest rates, poor equity returns over the next one to two decades, and ultimately a lower portion of capital raised through equity markets.

Unfortunately, we often assume capital markets will solve our problems for us (i.e., erase our debts), a notion that often leads us to hesitate changing our investment strategies or, worse yet, to price products assuming markets will "revert back to the mean." If we are slow to respond and markets perform as I suspect they will, actuaries will likely suffer severe public scrutiny. Furthermore, if governments do not address the existing social insurance problems (e.g., Medicare), their economies will continue to be weighed down and eventually overwhelmed by these programs. In order for our profession to remain relevant, we not only need to improve our solutions but also need to influence the regulatory environment to assure better solutions are possible.

In March, we held the "Long-Term Financial Planning Summit" in New York. There were 31 attendees who represented members from each of the sponsoring sections: investments, pensions (corporate and public), social insurance, long term care, and forecasting and futurism. Although there was not a consensus about the solution, I believe there was wide acceptance that our profession may need to consider some fairly dramatic changes to avoid public scrutiny and to remain relevant. We are developing two sessions for the 2012 annual meeting and a webcast to share perspectives with and to solicit feedback from a broader audience.

Although my attention has been largely focused on the aforementioned issues, the Investment Section Council has been hard at work ensuring our members receive value through various forums. The 2012 Investment Symposium had its largest attendance and great reviews across the board, especially for the new pension track. Our ALM Investment Seminars in Shanghai and Taipei were also very successful. Given section members have expressed the desire to have more webcasts and podcasts available, we are working with presenters from each of these events to develop a few webcasts and a series of podcasts that should be available by the end of the year. In the meantime, we hope you enjoy this edition of *Risks & Rewards*!

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> "… WHEN TWO PORTFOLIOS HAVE EXACTLY THE SAME PAYOFFS IN EXACTLY THE SAME SCENARIOS, THEN BOTH PORTFOLIOS SHOULD HAVE THE SAME PRICE."

ARBITRAGE-PRICING LAYS DOWN THE LAW (OF ONE PRICE)

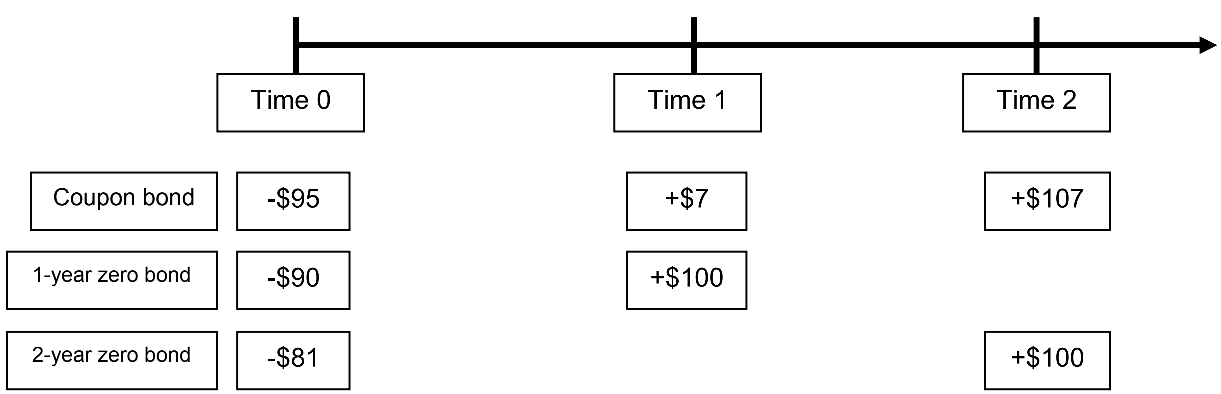
The mathematics of financial engineering mainly deals with the pricing[1] and hedging of financial assets known as derivatives. Contrarily to stocks that are priced according to their future cash flow potential (future dividends and capital gains), derivatives' pricing usually takes the dynamics of the stock as given. One of the objectives of financial engineering is to compute the price of a derivative in this context and find an appropriate risk management strategy. To meet these objectives, we need to define the most basic concepts of financial mathematics which are *arbitrage* and the *law of one price*.

There is an arbitrage opportunity when a zero-investment (net) may yield a profit, without any loss possibility. We often say there is *no free lunch* in a market where arbitrage opportunities do not exist. It is important to make sure no arbitrage opportunities arise because it would mean investors would have a much easier way to make profits without assuming risk. Thus, derivatives and other financial products are priced to avoid these. This leads to the law of one price that stipulates that when two portfolios have exactly the same payoffs in exactly the same scenarios, then both portfolios should have the same price (or cost) to avoid arbitrage opportunities.

Example 1: In a fixed-income market (see Figure 1), a two-year coupon bond with annual coupons of 7 percent trades at \$95 (face value of \$100). Moreover, a one-year zero-coupon bond trades at \$90 and a two-year zero-coupon bond trades at \$81. Given that there is no credit risk, no liquidity risk, no transaction costs, no taxes, etc., is there an arbitrage opportunity?

Solution: A two-year coupon bond can be constructed with one-year and two-year zero coupon bonds. Indeed, 0.07 unit of the one-year zero and 1.07 unit of the two-year zero-coupon bond yield exactly the same cash flows than the two-year coupon bond. The zero-coupon bond portfolio costs \$92.97 while the exactly equivalent coupon bond trades at \$95. Thus, there is an arbitrage opportunity.

Figure 1: Illustration of the cash flows of the bonds available in the market



In the previous example, we have replicated the cash flows of a coupon bond with a set of zero-coupon bonds. Replication (or hedging) is not a new concept since it can be seen as a type of immunization.

Example 1 (cont'd): a very stable insurance line of business is such that we know with almost certainty that 200 people will die in year one and 350 people will die in year two. The insurance benefit is $1,000. How can we exactly hedge the insurance cash flows with the bonds available (given there is no credit risk, no liquidity risk, no transaction costs, no taxes, etc.)?

Solution: the company has to pay $200,000 at time one and $350,000 at time two. Thus, by buying 2,000 units of the one-year zero-coupon bonds and 3,500 units of the two-year zero-coupon bonds, the cash flows are exactly hedged. The cost of this immunization strategy is 2,000 x $90 + 3,500 x $81 = $463,500.

A portfolio of assets is said to be a *replicating portfolio* if it is specifically designed and dynamically updated such that it exactly replicates the cash flows of an asset or a derivative. By the law of one price, the cost of the replicating portfolio should also represent the true and unique price of the derivative. Otherwise, arbitrage opportunities would exist.

In practice, exploiting an arbitrage involves accounting for *market frictions*, regulations and other restrictions. However, mathematical finance textbooks usually assume a frictionless market. In such a market, the following assumptions hold: no transaction costs, perfectly liquid and divisible assets, lending and borrowing interest rates are the same (thus no default from both sides of the transaction), no taxes, no restrictions on buying and selling (and short-selling), etc. None of these assumptions are observed in practice but they might be approximately true for large investment banks. Indeed, the volume of transactions for investment banks is huge, meaning that transaction costs are minimal and assets are approximately divisible (a block of 100 stocks is small with respect to their volume of transactions). Moreover, before 2008, these banks had the best credit rating possible, meaning lending and borrowing rates were very close to the risk-free rate.

In the insurance industry, public policy prevents individuals and corporations from actively trading insurance contracts. If it were the case, that would introduce an incentive to cause the covered event! Thus, even if there are many identical policies with different prices, a policyholder cannot make arbitrage profits out of these contracts (by selling the costliest, which is not even allowed) and will typically buy the cheapest available.

One might also wonder if there are arbitrage opportunities in actual markets. First, arbitrageurs, investment banks and hedge funds use a looser definition of arbitrage, being able to accept a small amount of risk when exploiting an opportunity. However, how small the risk is depends on many factors and the case of Long-Term Capital Management illustrates how difficult it can be to exploit arbitrage opportunities without any substantial risk. Nowadays, arbitrage opportunities may exist in very tiny time windows over assets that are cross-listed on different markets. These opportunities do not last long: a few thousandths of a second and are exploited by supercomputers with complex algorithms.[2]

In conclusion, the absence of arbitrage (and the law of one price in many cases) should guide how derivatives are priced, no matter what are the assumptions for the evolution of the stock price, or of the underlying market. Market frictions, regulations and other restrictions in the financial

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and insurance industry simply make it more difficult (or impossible) to exploit arbitrage opportunities. That does not invalidate the principles underlying no-arbitrage pricing.

SIMPLE CASE: COMPLETE MARKETS

Introduction and assumptions

To illustrate how we should price and hedge a claim under complete markets, we will assume that there is a financial market where only two assets are traded: a risk-free bond (also known as Treasury bond) and a risky asset (say a stock). The bond is risk-free in the sense that default does not exist in such a market so that the value of the bond grows with the risk-free rate. The initial value of the stock is observed and its price at the end of the period can only take two different values: this is the single-step binomial tree. Thus, the stock is risky in the sense that at time 0, there is uncertainty on whether the stock will go down or go up. The two terminal values are fixed and known by every market participant at inception. We further assume there are no market frictions and there are no arbitrage opportunities between the stock and the bond. Consequently, if one invests in the stock (compared to an equivalent investment in the bond), it should be possible to lose or make money out of the stock. Alternatively, the stock cannot *always* earn more (or always earn less) than a risk-free bond.

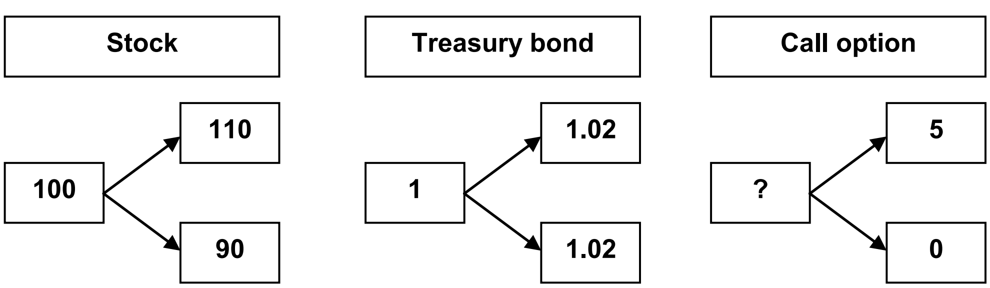
Replicating portfolio

Example 2: A stock currently trades at $100 (see Figure 2) and can take two different values at the end of the year: $110 or $90. A Treasury bond trades at $1 and will be worth $1.02 at the end of the period. According to the analysts, the probability that the stock will be worth $110 at the end of the year is 75 percent. What should be the price of a call option with strike price $105 in order to avoid arbitrage opportunities?

Solution: We will try to find a replicating portfolio that exactly replicates the cash flows of the option. If the stock trades at $110 ($90) at the end of the year, the call option is worth $5 ($0). Solving a system of two-equations with two unknowns, one gets that a portfolio that holds 0.25 unit of a stock and a loan of $22.06 exactly replicates the cash flows of the option. The cost of this portfolio is $2.94, which should be the price of the option.

To price the option in the latter example, we used the law of one price. That is, we first tried to find how to trade in the assets available at time 0 in a way that exactly replicates the cash flows of the option. Thus, to avoid arbitrage opportunities, the cost of the portfolio should correspond to the price of the option.

Figure 2: Illustration of the possible outcomes of the stock, Treasury bond and call option in the single-step binomial tree



One important conclusion can be drawn from this numerical example. In the market that we defined and its assumptions, the replicating portfolio yields the exact same payoff as the derivative, in every possible scenario. Thus, no matter how likely each scenario really is, the replicating strategy will pay off the same amount as the derivative. Hence, the probability (that will be known as *real probability* or *physical probability* later) of observing a rise in the price of the asset is not a relevant input in the price of the option (that avoids arbitrage opportunities). This is because this probability is already an important factor in determining the current price of the stock, which we take as a given when pricing derivatives. If it is felt that the current stock price is inappropriate, then the derivative will be "mispriced," but consistent with the cost of replication. Thus, the replicating strategy **only** tells you how to hedge the derivative given the current stock price and the underlying model (and its assumptions); nothing else.

Finally, in the exact previous setup, i.e., where a risk-free bond and a stock are traded, and the stock only has two possible values at the end of a period, the exact no-arbitrage price of a derivative can be found for all possible payoff values. Indeed, as long as one can find a unique solution to a system of two equations and two unknowns, there will be a unique replicating portfolio associated to this derivative. A market where each possible derivative can be replicated is known as a *complete market*. We often say that in a complete market, all risks can be replicated.

Risk-neutral pricing

In financial mathematics, there also exists another equivalent way to price a derivative, which is known as risk-neutral pricing. In the one-step binomial tree, it is straightforward to check that these two are exactly equivalent. Indeed, by algebraically writing the cost of the replicating portfolio and reorganizing the terms (see Appendix for the details), one can obtain a very interesting expression. Thus, the price of a derivative can be rewritten as the discounted (at the risk-free rate) expectation of its future cash flows, under an alternative probability measure, known as risk-neutral probability measure. The latter is known as risk-neutral because only risk-neutral agents would expect a return equivalent to the risk-free rate (no risk premium) on any risky asset.

Example 2 (cont'd): how can we find the no-arbitrage price of the call option using risk-neutral pricing? What is that price?

Solution: following the derivations in Appendix, we find that the risk-neutral probability of observing \$110 at time one is 60 percent, which is in no way related to the postulated 75 percent determined earlier by the analyst. The price of this option is thus 60 percent x \$5 discounted at 2 percent, which yields \$2.94 as well.

The fact that this expectation was rewritten from the cost of the replicating portfolio further illustrates that the risk-neutral probabilities are really not related to the true (or physical) probability of observing an increase in the price of the stock. Risk-neutral probabilities are **only** useful when the **no-arbitrage price** of a derivative needs to be found. In all other contexts such as risk management, asset management, investment decisions and stress testing, the true probability (determined by the analyst) is what matters.

In example 2, to answer the questions, "What is the probability that the option is in-the-money?" and "Is this option a good deal or a winning bet?" one should use 75 percent, which is the probability postulated by the analyst. Thus, risk-neutral probabilities can be seen as a mathematical convenience so that we can write down the price of a derivative as a simple expectation. In many contexts, it can be very helpful, but at the cost of making simple calculations unintuitive.

Moreover, we argued earlier that no matter what scenario is ultimately realized at the option maturity, the replicating strategy should be effective. Thus, no matter how risky the stock is, or no matter what our perception of risk is (risk aversion), the replicating strategy is the unique way to exactly replicate the derivative's payoffs in every scenario.

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Thus, being long (short) the derivative and short (long) the appropriate amount of stock yields a risk-free position.

Finally, risk-neutral pricing **does not** imply that investors are risk-neutral. This would be, of course, untrue as stocks entail a significant risk premium. Risk-neutral valuation is a **consequence** of using arbitrage-free pricing and replicating portfolios. The *fundamental theorem of asset pricing* links the absence of arbitrage to the existence of risk-neutral probabilities.[3]

Investment guarantees and equity-linked insurance

We illustrate in this section how basic investment guarantees can be represented in a complete market environment.

Example 3: a 65-year-old individual invests \$100 in an equity-linked insurance policy that provides an investment guarantee. The underlying stock chosen by the policyholder may take two possible values at the end of the period: \$90 or \$110. A minimum return of 3 percent is guaranteed on the policy, upon death or survival, and the risk-free rate is 2 percent. According to mortality tables, this individual has a 1 percent probability of death by the end of the period. In a frictionless market, what is the no-arbitrage premium that should be paid by the policyholder for the investment guarantee?

Solution: because the payoffs of the contract do not change if the individual survives or not, the policy can be seen as a stock and a plain vanilla put option in a complete market. When the market is frictionless, the premium paid by the policyholder for the investment guarantee is the no-arbitrage price of the put option, which has a strike price of \$103. As discussed earlier, one can use replicating portfolios or risk-neutral pricing to find the price of the option and/or the risk management strategy for this liability. Because the binomial tree is the same as in example 2, using risk-neutral pricing, we have that the price is 40 percent x \$13 discounted at 2 percent, which is \$5.10. The risk management strategy for this policy can be found with replicating portfolios. On one hand, the insurance company sells a share of a stock, which can be hedged by buying a stock as well. It also sells a put option, which can be replicated by selling 0.65 units of the stock (for proceeds of \$65) and investing what remains in the Treasury bond (the difference between the stock position and the value of the put).

In that example, the individual invests \$100 and pays an additional premium of \$5.10 at time 0 so that at maturity, the payment is either \$110 or \$103. This is because we have assumed that the premium is paid up front instead of a penalty on the return. Even though the policy has considerably reduced the volatility of the returns in the stock, it may be difficult to call this product an investment guarantee because in the down scenario, the investor loses \$2.10. In an arbitrage-free market, it is impossible to always earn more (or always less) than the Treasury bond without assuming some level of risk.

A more reasonable payment scheme for this contract could be \$104 in the up scenario and \$101 in the down scenario, which is similar to a participating policy that penalizes the upside, for a "guarantee" against the downside. It can be found that for an initial investment of \$100.78 (\$100 plus the initial premium), this price does not create any arbitrage opportunity.

Conclusion

Using no-arbitrage pricing yields two typical methods to price a basic financial derivative: finding the replicating portfolio or risk-neutral pricing. These two approaches are exactly equivalent. Moreover, risk-neutral probabilities are only relevant when one deals with finding the price of a derivative under no-arbitrage. In all other contexts, physical or real probabilities should be used.

It is obvious that representing the evolution of a stock by such a simplistic model cannot be realistic. However, a time step can be a year, month, day, hour, minute, or a second, etc. Repeating the one-step binomial tree at each period over a longer time horizon is one way to make the latter approach more realistic. It turns out that using a single-step binomial tree at every instant for the price of the stock (with appropriately chosen possible outcomes)[4] results in continuous rates of returns that are normally distributed. This is the Black-Scholes' model that will be thoroughly discussed in the next excerpt.

BEING MORE REALISTIC: INCOMPLETE MARKETS

Introduction and assumptions

In the one-step binomial tree model, we have assumed that the stock only took two possible values at the end of the period. It resulted that the market was complete, meaning that all possible payoffs of a derivative could be replicated. We will now relax this assumption through a simple market model in which we will be able to draw very valuable conclusions.

Assume that under normal market conditions, the stock can take two values at the end of the period. Under extreme circumstances (say a crash period, default of the company, etc.), the stock may take a third possible value. We will once again assume that by investing in the stock, the investor may make more or less money than by investing in the risk-free bond, so that there is no arbitrage between the stock and the bond.

Replicating portfolio

Example 2 (cont'd): Under an extreme circumstance (read: bankruptcy), the stock depicted in Example 2 might take the value 0 (or close to). Is it possible to replicate the payoffs of the call option over all possible scenarios (see Figure 3)?

Solution: To create a replicating portfolio that works under each of the three scenarios, we need to find the solution of a system having three equations and two unknowns. A unique solution does not exist in this context.

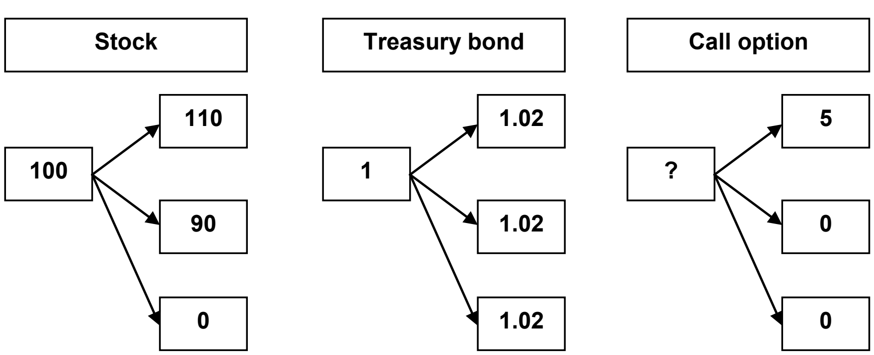
When there are two assets that are traded and three possible outcomes (that we have interpreted as normal and extreme scenarios), it is generally impossible to find a unique replicating portfolio that will work in each scenario. Such markets are known to be *incomplete markets*. In incomplete markets, some derivatives may have a unique replicating portfolio (attainable claims), but the vast majority do not. Thus, incomplete markets are truly what are observed in reality, with some risks that cannot be hedged.

What happens if one ignores the third outcome?

In fact, the replicating portfolio may work very well but once in a while, it may not work. The following example illustrates the situation.

Figure 3:

Illustration of the possible outcomes of the stock, Treasury bond and call option in the single-step trinomial tree



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"… NEW ASSETS AND FINANCIAL DERIVATIVES HELP COMPLETE THE MARKET."

Example 2 (cont'd): The risk manager analyzes the credit risk of the firm using reports from rating agencies. He figures that the probability of default (stock is worthless) is 2 percent by the end of the period. He decides to hedge the normal scenarios. Analyze the appropriateness of the strategy.

Solution: By hedging the first two scenarios, one obtains the same replicating portfolio as in the one-step binomial tree section. According to the rating agencies, that would mean that 98 percent of the time, the replicating portfolio would work and exactly replicate the payoffs of $5 or $0 when the stock is respectively worth $110 or $90. However, if the company does default, a loan still has to be repaid ($22.50 of capital and interest) with a stock that is worthless.

How does the risk manager replicate his risks in this context?

A risk manager will never leave such a possibility open without taking any risk attenuation measures. In this simple market, there are no financial assets available to exactly replicate the extreme outcome. The risk manager will have to use judgment in assessing the risk of his positions. He may choose to replicate any pair of outcomes and choose the pair that is the most appropriate. He may also pick a strategy that yields a minimal loss under each scenario.

What is the true price of a derivative?

Unfortunately, for a derivative that does not have a unique replicating portfolio, there is no unique price. Only a range (an interval) of prices makes sure that the derivative does not introduce arbitrage opportunities. The seller and buyer of the derivative will have to agree on a price in the latter range. In this case, it is very likely the buyer and seller will both assume a level of risk, as perfect replication does not exist.

Completing the markets

The introduction of new assets and financial derivatives help complete the market. In other words, those additional assets may help a risk manager attain a greater level of replication of its cash flows. The following example illustrates how financial innovation may help deal with credit risk.

Example 2 (cont'd): A second risky asset is now available in the market (see Figure 4). This product pays off $100 if the stock is worthless, and 0 otherwise. It trades at $3. How does this product affect the risk management and pricing of the call option?

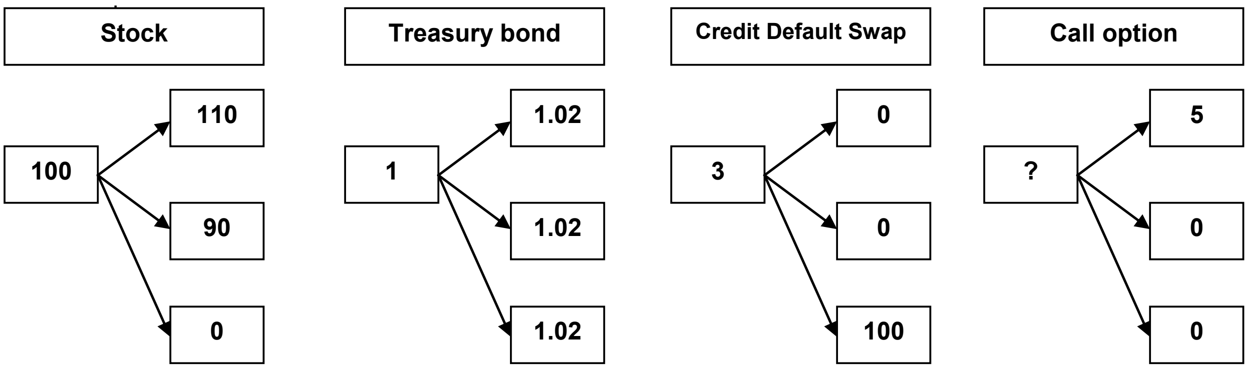
Solution: It can be seen that this asset acts as an insurance against default. This is a simplistic representation of what is known as a credit default swap (CDS). In order to replicate the three possible outcomes of the call option, we now have three assets. This yields three equations and three unknowns. We find that we need 0.25 share of the stock, a loan of $22.06 and 0.225 unit of this insurance. In case of default, we still need to repay the loan with interest, which is $22.50. The insurance will pay off only in case of default, in order to pay back the loan. The cost of the insurance is 0.225 times $3, which is 67.5¢. Because we have found a unique replicating portfolio, the unique no-arbitrage price of this derivative is $2.94+67.5¢, i.e., $3.61.

It should be noted that the CDS acts as a fundamental asset, just like the stock and the bond. In a market represented by a trinomial tree where only a stock and a bond are traded, one cannot replicate the payoffs from the CDS just like the call option could not be replicated earlier in Figure 3. In trinomial trees, one requires any combination of three assets to replicate a fourth one.[5] It is straightforward to think that to replicate the call option, one needs positions in the stock, the bond and the CDS. But if the current price of the option is known, then one could replicate the CDS payoffs with the stock, bond and call option.

Risk-neutral pricing

One can also find the price of a financial derivative in a one-step trinomial tree using risk-neutral pricing. According to the risk-neutral pricing principles, we need to find the prob-

Figure 4:
Illustration of the possible outcomes of the stock, Treasury bond, CDS and call option in the single-step trinomial tree



abilities such that we expect a return of the risk-free rate on all risky assets traded in this market. As discussed earlier, risk-neutral pricing or replicating portfolios are equivalent and are the consequence of using the absence of arbitrage to price derivatives. With two assets (risky and risk-free) and three outcomes, we have an infinite number of risk-neutral probabilities, which will also yield a range of prices (rather than a unique price) that avoid arbitrage opportunities.

When we add a third asset, as in the credit risk example, we can solve for unique risk-neutral probabilities. Relating to Example 2, we can have a real probability of default (as given by Moody's or Standard and Poor's for example) and a risk-neutral default probability, which is once again, totally unrelated to the true default probability.

Investment guarantees and equity-linked insurance
We have used credit risk as a way to interpret market incompleteness and introduced credit default swaps to complete this market. Strictly from a financial engineering viewpoint, mortality risk creates market incompleteness. As it will be seen in the following examples, traditional actuarial techniques can be used to deal with this issue.

In example 3, the payment upon death or survival was exactly the same. Thus, even though the insurance company faces mortality risks and incomplete markets, it was possible to find a unique replicating portfolio and a unique price. This is an example of an attainable claim.

Example 3 (cont'd): we now assume that the payment upon death or survival is different. Suppose that upon death, the minimum return is 1 percent whereas upon survival, the minimum return is 0 percent. In both cases, the upside is capped at 6 percent. What is the no-arbitrage price of this policy assuming frictionless markets?

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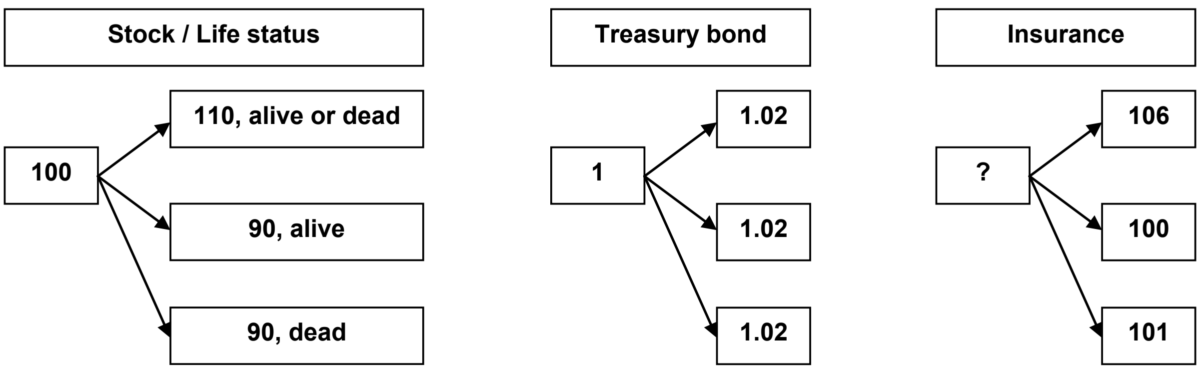
Figure 5:
Illustration of the possible outcomes of the stock, Treasury bond and the insurance in a single-step trinomial tree



Solution: this is an additional example (see Figure 5) where there are more outcomes (three) than the number of assets available in the market (two). One cannot find a unique no-arbitrage price or a unique replicating portfolio.

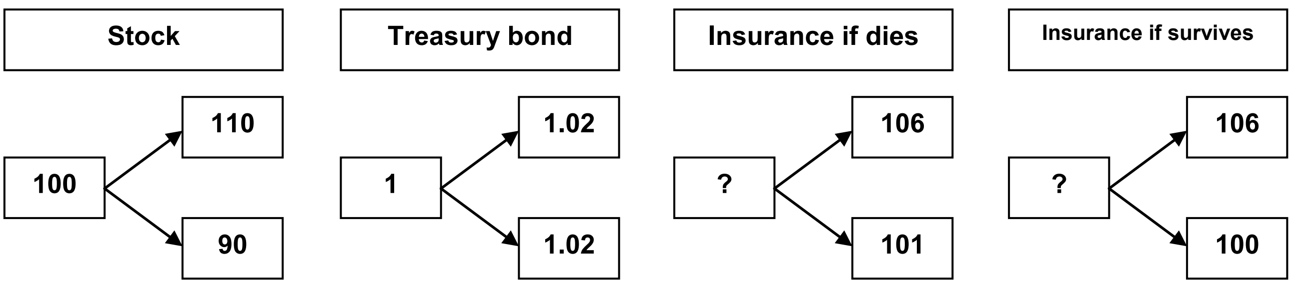
Public policy of course forbids insurance companies to monetize their policies so that we cannot complete markets as with credit risk. Thus, insuring the life of one individual is like a bet: it remains risky. However, the role of insurance companies is to pool these risks to better predict the total loss in a portfolio. Since mortality risk is generally independent[6] from one life to the other, the insurer can predict relatively well the number of deaths at each time period.

Example 3 (cont'd): assume that the insurance company insures the life of 10,000 independent individuals aged 65, each with a death probability of 1 percent (according to an appropriate mortality table). These individuals have the same risk characteristics and hold identical portfolios. How can we price and manage the previous equity-linked insurance in this context?

Solution: using the law of large numbers, it is possible to say that approximately 100 deaths will happen and 9,900 people will survive.[7] Thus, the positions of the insurance company are as follows (see Figure 6): (1) short 100 derivatives that pay \$106 in the up scenario and \$101 in the down scenario and (2) short 9,900 derivatives that pay \$106 in the up scenario and \$100 in the down scenario. Using risk-neutral pricing or replicating portfolios, we find that the (no-arbitrage, frictionless market) price of the first contract is \$101.96 while the second is \$101.57. The replicating strategy required is 0.25 (0.3) unit of stock for each of the first (second) contract. Thus, for the 10,000 lives, 100 x 0.25 + 9,900 x 0.3 = 2995 shares of stock are required. The rest of the proceeds are invested in the Treasury bonds.

When one uses risk-neutral pricing in the context of example three, one sees that two types of expectations are used. Conditional upon survival (or death), a risk-neutral expectation is applied to find the no-arbitrage price of the derivative when the individual survives (dies). However, the value of the portfolio is weighted by the true number of deaths and survivors. The weights are determined using a mortality table, which is an observed or real death probability. Overall, those are nested expectations; with the outside expectation taken with real death probabilities and the inside expectation computed with risk-neutral probabilities of observing an increase in the price of the stock.

Figure 6:
Illustration of the possible outcomes of the stock, Treasury bond and the insurance represented in binomial trees



Example 3 showed a practical example where we can manage risks in an incomplete market. However, it is important to understand that the example featured a very large set of independent and identically distributed policyholders, so that assuming 100 deaths is reasonable. In reality, policyholders have different risk characteristics (and hold different portfolios) so that the realized mortality is very likely to deviate (positively or negatively) from expectations. In that case, traditional actuarial techniques are necessary to deal with these deviations that will make the hedge portfolio imperfect.

CONCLUSION

In this paper, we have illustrated fundamental concepts of modern financial mathematics such as arbitrage pricing under complete and incomplete markets. It was shown that under absence of arbitrage, the price of a derivative should correspond to the cost of the replicating portfolio. In a complete market, the price is unique, whereas in an incomplete market, perfect replication is rarely possible, and a range of price prevents arbitrage opportunities. In incomplete markets, buyers and sellers have to assume some level of risk. In all cases, to find the no-arbitrage price of a financial derivative, the replicating portfolio or risk-neutral pricing are equivalent approaches to find such price. The risk-neutral probability is only relevant in the context of finding the price of a derivative under absence of arbitrage; in all other cases, the true probability measure matters.

We have used basic financial engineering and actuarial mathematics to deal with equity-linked insurance. In reality, insurance is very much different from investment banking. First, public policy prevents people and insurers from trading individual life insurance policies just like other basic financial assets. In this case, prices can deviate from their no-arbitrage equivalents, meaning the best an individual can do is opting for the cheapest contract. Second, insurance contracts involve asymmetry of information between the policyholder and the company; the former always knows more about its risks than the latter, requiring the company to underwrite the policy. Finally, people buy insurance and equity-linked products for family estate management and tax considerations.

One should be cautious regarding the latter three arguments. First, rational investors would already account for tax differentials between insurance and financial assets. Indeed, two assets having the same payoffs but taxed differently should have different prices. The difference would only be due to taxes to make sure there is no arbitrage between the

CONTINUED ON **PAGE 14**

> "FINANCIAL AND ACTUARIAL TOOLS USED IN THIS PAPER CAN AND SHOULD BE USED TO MANAGE THE RISK OF EQUITY-LINKED INSURANCE."

two assets. Moreover, the fact that markets cannot monetize insurance policies is a major impediment indeed. The danger however is if the contract is underpriced, even when accounting for mortality and underwriting. The rational individual could long (buy) the insurance contract and short (sell) the replicating portfolio from the financial markets, making a "sure" profit.

More importantly, financial and actuarial tools used in this paper can and should be used to manage the risk of equity-linked insurance. The actuary should keep in mind that perfect hedging in incomplete markets (which is the reality) is impossible, but neither dynamic hedging, nor traditional actuarial techniques are perfect methods. Stress-testing is the key.

In the upcoming article, we will discuss the Black-Scholes' model, its imperfections and how we can improve Black-Scholes' for financial and insurance products.

APPENDIX

In this section, we show how we can link replicating portfolios and risk-neutral pricing in the context of the single-period binomial tree. In general, the *fundamental theorem of asset pricing* is used to link the two approaches.

Suppose the assumptions regarding the one-period binomial tree hold. The current stock price is s_0 and its future possible prices are s_1^u and s_1^d in the up and down scenarios respectively. The payoffs of the derivative in the up and down scenarios are c_1^u and c_1^d. We would like to find the current price of the derivative c_0 such that there is no arbitrage opportunity. Let x be the number of stocks that we should hold in the period to exactly replicate the payoffs of the derivative, while y is the number of Treasury bonds. The value of such a bond is one at inception, and $1 + r$ at maturity.

To find the appropriate replicating portfolio, we build the system of equations that allow us to exactly replicate the payoff of the derivative in each scenario. We thus solve for a set of two equations with two unknowns:

$$xs_1^u + y(1+r) = c_1^u$$
$$xs_1^d + y(1+r) = c_1^d.$$

Subtracting the two equations allow us to easily find x which is

$$x = \frac{c_1^u - c_1^d}{s_1^u - s_1^d}.$$

One easily recognizes delta. Furthermore,

$$y = \frac{1}{1+r}\frac{c_1^d s_1^u - c_1^u s_1^d}{s_1^u - s_1^d}.$$

The cost of this portfolio at inception is simply

$$c_0 = s_0 x + 1y = s_0\frac{c_1^u - c_1^d}{s_1^u - s_1^d} + \frac{1}{1+r}\frac{c_1^d s_1^u - c_1^u s_1^d}{s_1^u - s_1^d}$$

by substituting the values of x and y. Now, we isolate $\frac{1}{1+r}$ and put everything on the same denominator. We get

$$c_0 = \frac{1}{1+r}\left((1+r)s_0\frac{c_1^u - c_1^d}{s_1^u - s_1^d} + \frac{c_1^d s_1^u - c_1^u s_1^d}{s_1^u - s_1^d}\right)$$
$$= \frac{1}{1+r}\left(\frac{c_1^u(1+r)s_0 - c_1^d(1+r)s_0 + c_1^d s_1^u - c_1^u s_1^d}{s_1^u - s_1^d}\right).$$

We now group the terms in c_1^u and c_1^d to get

$$c_0 = \frac{1}{1+r}\left(c_1^u\frac{s_0(1+r) - s_1^d}{s_1^u - s_1^d} + c_1^d\frac{s_1^u - s_0(1+r)}{s_1^u - s_1^d}\right).$$

Now, let $q = \frac{s_0(1+r)-s_1^d}{s_1^u-s_1^d}$. We find that $1-q = \frac{s_1^u-s_0(1+r)}{s_1^u-s_1^d}$. Furthermore, because there are no arbitrage opportunities between the stock and the bond, q is necessarily between zero and one (bounds excluded) since investing in the stock implies that we can make more or less money than investing in the risk-free asset. Then, the cost of the replicating portfolio can be written as

$$c_0 = \frac{1}{1+r}(q \times c_1^u + (1-q) \times c_1^d)$$

where q has the characteristics of a probability. Thus, the cost of the replicating portfolio can be rewritten as an expectation (under an alternative probability measure q) of future cash flows, discounted at the risk-free rate. To expect a rate of return equivalent to the risk-free rate is equivalent to having a universe where agents are risk-neutral. Even if we had supposed that the true probability of an increase were p (physical or real probability measure), this quantity would have been irrelevant when pricing derivatives under no-arbitrage.

ACKNOWLEDGMENTS

The author would like to thank Maciej Augustyniak, Dick Joss, Joseph D. Koltisko, Pierre-Laurence Marchand and Jean-François Renaud for their valuable comments.

END NOTES

[1] By pricing or price we mean finding the value of a tradeable security as of a given date.
[2] Some experts blame these supercomputers and their algorithms for the Flash Crash of May 2010.
[3] In a single-step binomial tree (see Appendix), if there is no arbitrage between the stock and the Treasury bond, then the risk-neutral probability is unique.
[4] Those are the Cox-Ross-Rubinstein and Jarrow & Rudd binomial trees for example.
[5] This is the case when the payoff of any of the three fundamental assets cannot be written as a linear combination of the other two. Otherwise, one of these fundamental assets would be redundant and could not be used to replicate a fourth one. Mathematically, this is the necessary condition to solve a system of three equations with three unknowns, i.e., the matrix built with the payoffs of the fundamental assets should be of full rank.
[6] In a population, wars and epidemics are factors that create dependence between lives. However, these risks are often excluded in life insurance policies. Moreover, within a couple, it is generally recognized that spouses' lives are somewhat dependent. Finally, it is often assumed that financial markets do not affect mortality experience and vice-versa.
[7] The usual formulation of the law of large numbers in that context is that the mean proportion of deaths goes to 0.01 with certainty. However, in large portfolios, it can be easily seen that the standard deviation of the number of deaths relative to the mean, will to 0. Hence, the error committed by assuming 100 deaths should be small in relative terms.

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A FEW COMMENTS ON ACADEMIC FINANCE

By Dick Joss

Over the past two years I have written four articles for Risks and Rewards and have given presentations based on these articles at conferences sponsored by the Society of Actuaries. The articles, which deal with purely mathematical issues, have been critical of some models in modern academic finance. The four articles are:

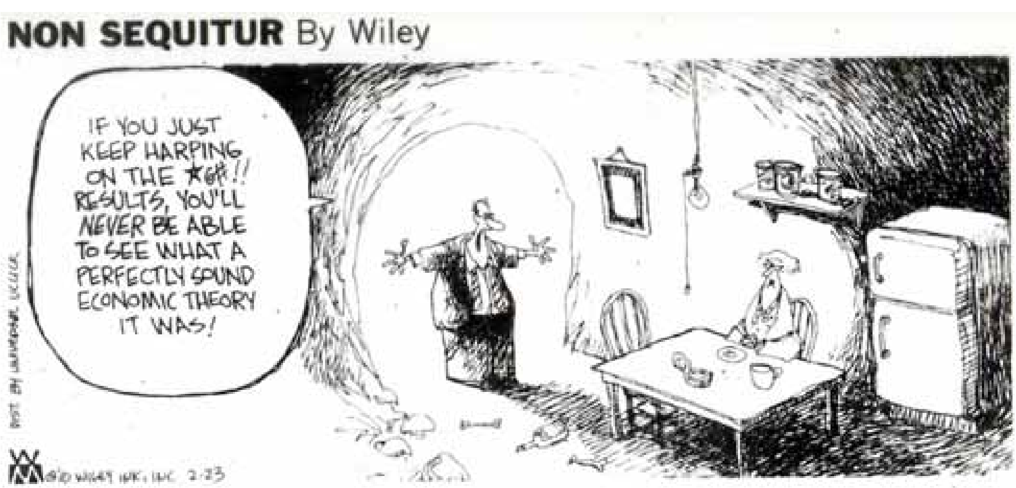
1. Those Pesky Arithmetic Means. R&R – February 2011;
2. Arbitrage and Stock Option Pricing: A Fresh Look at the Binomial Model. R&R – August 2011;
3. Those Pesky Arithmetic Means (Part 2). R&R – February 2012; and
4. A Fresh Look at Lognormal Forecasting. R&R – February 2012.

I am not the only person raising concerns. Articles in such general business publications as *Fortune, Forbes, Business Week,* and *The Economist* have all raised questions about the reliability of the mathematical models that are currently being used. Dr. Craig Barrett, who at the time was the CEO of Intel, wrote an April 23, 2003, op-ed for *The Wall Street Journal* stating that he was uncomfortable signing off on Intel's annual report because of concerns about the Black-Scholes stock option pricing model. In the same vein Warren Buffet went to great lengths in the 2008 Berkshire Hathaway annual report to demonstrate that the Black-Scholes formula could not possibly be right.

Even some academics have raised red flags. Dr. David S. Bates in his paper, "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in the Deutsche Mark Options," included the following sentence near the end of the paper: "The ultimate research agenda may therefore be to identify those omitted 'fundamentals' that are showing up as parameter shifts in current option pricing models." The problems highlighted in the four *R&R* articles would be just such omitted "fundamentals."

The collapses of Long-Term Capital Management, Bear Stearns, and Lehman Brothers Holdings all add to the level of concern. Even more recently, the $2 billion loss reported by JP Morgan Chase on its derivative investments has generated a new call for more regulation. But if the root cause of the problems are omitted fundamentals, then new regulation is unlikely to provide any relief.

Finally, even the comic strips have gotten in to the act. Below is a *Non-Sequitur* panel which highlights a very valid concern with respect to the academic finance mathematical models.



In short, maybe the "perfectly sound" economic theory contains a few holes.

These issues are all extremely important to actuaries. Like most professional advisors, actuaries are subject to the potential for malpractice litigation. Actuaries do not have the luxury of relying upon theories that may have mathematical problems. As actuaries, we need to study any concerns (such as those presented in the four R&R articles) and make sure that advice provided to our clients or employers is based on a solid mathematical foundation.

MATHIEU BOUDREAULT RESPONSE

In response to the above mentioned articles, Mathieu Boudreault, assistant Professor at UQAM, agreed to write two articles concerning the complex issues in the mathematics of financial engineering and their impact on actuarial science. The first of these articles appears in this issue of R&R, and the second is scheduled to be published next February.

First and foremost, I want to thank Dr. Boudreault for writing a most interesting article. He uses clear illustrations and well written explanations to highlight the current approaches to financial engineering. I encourage all actuaries to take the time to read and thoroughly understand the article. I agree completely with Dr. Boudreault that actuaries need to become more familiar with these complex issues.

My concerns with modern academic finance are not so much theoretical as they are practical. In some cases, the theory may make complete sense, but the application of the theory in practice may be difficult or even impossible. Perhaps, it is these very real practical difficulties that are creating the concern expressed by the general business community.

In addition, many of the key conclusions of academic finance are based on the assumption that observed historical investment returns may be treated as independent and identically distributed (i.i.d.) data. However, Dr. Boudreault and many other leading members of the academic finance community have now conceded that this assumption is not true. And while many leading academics have focused their attention on an investment return assumption that features jumps and stochastic volatility, perhaps even greater scrutiny of how historical investment returns are analyzed is warranted.

AN INTERESTING ILLUSTRATION

As noted above, the Black-Scholes stock option pricing model is perhaps the academic finance model that is creating the greatest concern in the general business community. It is easy to see where the concern comes from by looking at a simple illustration. Although the illustration will involve a longer-duration option, which is relevant for the expensing of employee stock options, the basic issues are just as critical for the pricing of shorter-term listed options. As for the longer-duration option illustration, consider the case of a stock that is currently selling for $100 a share, the strike price is also $100, the risk-free rate is 2 percent, the option term is 10 years, and the volatility is a high (but not uncommon) 120 percent. Using these inputs, the Black-Scholes formula says that the put option price should be $77 and the call option price should be $95.

To many people, these prices are just too high to even be considered as possible prices for the respective option contracts. Let's look at the put option price first. At the $77 price, the purchaser of the option needs to hope that the share price of the stock decreases 77 percent just to get his or her money back, and the absolute best that the purchaser of the option could do is hope that the company goes completely out of business and the share price drops to zero. In this case, the investor will earn an average return of 2.6 percent per year over the 10-year period. All other possibilities generate lower returns, and many possibilities result in

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the investor losing his or her entire $77 investment. Given that the investor has a choice of where to invest his or her $77, the possibility of spending it to purchase this put option contract makes no economic sense. If the investor's outlook for the stock was this poor, rather than purchasing the put option as "insurance" against the potential that the share price will drop, the investor should just sell the stock, and invest the $100 proceeds elsewhere.

As for the call option, the investor has the choice of using his or her $95 to buy the call option or buy .95 share of the stock. When these two choices are compared, the purchase of the stock always turns out to be the better investment, unless the stock averages an annual rate of return in excess of 35 percent per year over the entire 10-year period. Again, as in the case of the put option contract, it seems hard to imagine that an investor would willingly choose to invest in this particular call option contract. The possibility of investing in the underlying security makes so much more financial sense.

While many people in the general public think that these prices seem wrong, even actuaries engaged in investment hedging have also expressed concern. At the Chicago Board Options Exchange Risk Management Conference held March 11 – 13, 2012, several insurance company actuaries mentioned to me the "high cost of volatility reduction." In other words, while it is possible to use stock options to reduce portfolio volatility, the price paid for such a strategy in terms of reduced investment return seems high. Clearly, if the options were bought or sold at a different price, the cost of volatility reduction could be reduced.

The complaint of these insurance company actuaries seemed to be supported with an unusual The New York Times article on March 14, 2012, the day after the Risk Management Conference was over. In the article Greg Smith suggested that some investment banks may be putting the bank's profitability ahead of the needs of its clients. He noted that in some inner circle communications, bank employees may even refer to their clients as "muppets" for their willingness to engage in transactions that have very little potential to generate a reasonable return for their investment. In short, there seems to be some real sense that stock options are not priced correctly from both the buyer and seller perspectives.

WHERE A PROBLEM OCCURS

The source of the above noted seemingly high prices for both the put and call options can be traced back to the exact mechanics of how the assumed lognormal distribution of possible returns on the underlying security is used to price option contracts. Following along with the above illustrations the Black-Scholes model assumes that over the next 10 years there is the possibility that due purely to random chance all of the historically observed very low returns could occur together. This puts significant upward pressure on the calculated Black-Scholes put option price. The Black-Scholes model also assumes that over the next 10 years there is the possibility that due purely to random chance all of the historically observed high returns could occur together. This puts significant upward pressure on the calculated Black-Scholes call option price. Both of these assumptions about possible future return scenarios fail to take into account that in actual markets the high and low returns often cancel each other out.

This topic was discussed in R&R article 4) mentioned above, where it was suggested that observed investment return data was better described using conditional probabilities than independent probabilities. When this one change is made, the $77 put option price gets lowered to $31 and the $95 call option price gets lowered to $49. To many people, these lower prices stand a much better chance of attracting willing buyers than do their original Black-Scholes counter parts.

Furthermore, there is no alteration in the basic Black-Scholes theory to make this change. This is merely a theoretical change in how observed historical investment return

> "…HISTORICAL INVESTMENT RETURN DATA IS NOT INDEPENDENT AND IDENTICALLY DISTRIBUTED."

data is to be factored into the calculation process. In short, this difference is due solely to an "assumption" about the nature of the historical data. Clearly this one assumption has a very large impact. And given that leading academics no longer assume that historical data is i.i.d., review of this critical assumption takes on greater importance.

In addition, based on the risk-neutral and arbitrage-free theories of option pricing, if the $49 option price was not correct and the true call option price needed to be $95, then according to the law of one price and the Black-Scholes theory there should be a significant arbitrage opportunity generated if the option were actually on the market at the lower call option price of $49. But the theory that yields such an arbitrage opportunity assumes very specific growth patterns for possible returns on the underlying security. These very specific random chance growth patterns are currently a key part of the basic Black-Scholes theoretical development.

But the change in assumption about the nature of historical data also yields new and different specific growth patterns for the underlying stock. The law of one price and the Black-Scholes theory coupled with these new assumed growth patterns would support the $49 call option price. In short, once the assumption change is made, the arbitrage possibility would appear to occur for prices other than $49, not prices other than $95. Hence, the traditional Black-Scholes theory will not help at all to try and resolve the difference between the $49 call option price and the $95 call option price.

It is also important to note that neither of these theoretical approaches which entail very specific possible stock growth patterns reflects the fact that in real markets there is no chance that either of these stock growth patterns will actually play out. Hence, in real markets as generated by the buy/sell decisions of actual investors the supposed arbitrage opportunity simply does not exist. There is no way to take either of the $95 call option price or the $49 call option price and create a hedging strategy that guarantees a return to the investor no matter what happens to the underlying stock. It is always possible that the actual stock growth pattern will produce a loss from any specific hedging strategy.

This one change is not only significant for stock option pricing, but plays a role in the funding of defined benefit pension plans and the advice provided to 401(k) participants as well. A change to reflect the conditional nature of historically observed investment return data would be significant. I would hope that the issue could be discussed and debated within the academic and practitioner communities and be fully resolved.

It is now widely acknowledged in the academic finance community that historical investment return data is not i.i.d. Given the importance of these issues to actuaries, I trust that the involvement of the actuarial academic community will lead to a more full and complete discussion of these two questions: Are historical investment return data conditional data? And what consequences does this paradigm shift have for actuaries?

FINAL SUMMARY

Clearly, actuaries have a huge social responsibility. We help ensure the solvency of insurance companies and the adequacy of employee benefit plans. We need to be extra cautious of all our techniques and methods to be sure that our clients or employers receive advice that is in keeping with this social responsibility.

John Stuart Mill's classic book On Liberty was first published in 1859. In the book, Mill discusses a variety of philosophical issues on human interaction, and the development of a rational society. Of particular interest, at least to mathematicians, is the following quotation:

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"The peculiarity of the evidence of mathematical truths is that all the argument is on one side. There are no objections, and no answers to objections. But on every subject on which difference of opinion is possible, the truth depends on a balance to be struck between two sets of conflicting reasons."

Whether or not observed historical investment returns are independent or conditional data is purely a mathematical question. Actuaries need to be sure that their work is as complete and accurate as possible. Before using historical investment returns in any meaningful way, they should verify whether or not this data is conditional in nature. To get the answer wrong could lead to financial insecurity in insurance companies or inadequate funding in benefit plans.

Dick Joss, FSA, is retired. He can be contacted at rrjoss@comcast.net.

NON-TRADITIONAL ACTUARY: CARL HESS

By Risks & Rewards

Traditionally, actuaries have tended to follow one of two career paths. Either they worked for insurance companies helping to design insurance and annuity products that met the needs of policy holders, or they worked for consulting firms which helped companies design and administer their own employee benefit plans.

But increasingly, actuaries are finding some "non-traditional" sources of employment. To help shine a spotlight on some of these careers, from time to time R&R plans to include a featured interview with an actuary working in a non-traditional area. The interview for this issue is with Carl Hess, the New York-based head of Towers Watson's global investment business. The interview was completed in May 2012.

HOW DID YOU FIRST LEARN ABOUT THE ACTUARIAL PROFESSION?

Mathematics was an interest of mine in high school. I first learned about the actuarial profession when I sat for the national mathematics exam sponsored by the Society of Actuaries. I scored well enough on that exam to win an invitation to a dinner put on by the Chicago Actuarial Society. After that (or despite of that!), the actuarial profession was on my radar screen.

WHERE, AND WHEN, DID YOU GO TO COLLEGE? WHAT DEGREE OR DEGREES DID YOU EARN?

I graduated from Yale University in 1983 with a Bachelor's degree in logic. My primary academic areas of study included mathematics, philosophy and linguistics.

WHAT WAS YOUR FIRST JOB OUT OF COLLEGE?

My first job after graduating from college turned out to be pizza delivery! This is not to say that pizza delivery was designed to be a permanent job. But not long afterward (you can only survive on slices of pizza for so long) I worked with a professional recruiter who suggested that I pursue a position either in computer programming or that I should consider becoming an actuary. At this time, the primary career choices were either working with an insurance company or taking a position with a consulting actuarial firm. My dinner with the Chicago Actuarial Society paid off. Because of that dinner I actually knew what an actuary was. The idea of becoming a consulting actuary also sounded sexy, or at least sexier than working for an insurance company. The professional recruiter set up a round of interviews with consulting firms, and fairly soon I landed a job as an actuarial assistant with the consulting firm of A. S. Hansen. A. S. Hansen was later acquired by Mercer Consulting.

WHERE THERE ANY OTHER JOBS BETWEEN YOUR FIRST JOB AND YOUR CURRENT POSITION?

I left A. S. Hansen in order to take a position as the in-house actuary for Amalgamated Life Insurance Company, a third party administrator for multiemployer plans. It was at Amalgamated where I met the future Mrs. Hess. We both agreed that it would be best if one of us would find a job outside of Amalgamated Life Insurance Company. It turned out that the person who would be seeking a new position would be me. I took a job with Mercer Consulting in 1987. This position lasted two years and from there I went to The Wyatt Company to accept a position as pension actuary. This initial position later led to new positions involving fulltime work in asset liability modeling and the company's growing investment consulting business. I have stayed with The Wyatt Company through a number of mergers and acquisitions. The last of these was the 2010 merger with Towers Perrin to form Towers Watson.

WHAT ARE SOME OF THE MORE FREQUENT TASKS WITH YOUR CURRENT POSITION?

There is actually quite an interesting mix of tasks. Of course there is my day job as the head of the global investment consulting business and the various managerial responsibilities that come with this position. But even with the managerial responsibilities, I continue to work with clients. This is great as I really enjoy the clients, and it always helps me to see the issues from their perspective. Lastly, I spend a fair amount of time with public speaking, working with journalists, and writing articles for publication.

WHAT ARE SOME OF THE TASKS THAT YOU FIND MOST ENJOYABLE?

Given the global financial developments over the last four or five years, I enjoy the intellectual challenge of addressing risk, and not just investment risk. Client portfolios are so much more complicated, and we are dealing with not just stocks and bonds, but private equity, hedge funds, commodities, hedging strategies—the list goes on and on. The regulatory environment is much more challenging. I also like building and testing models, including ALM models, but there's not much time for that now. I also enjoy expressing and defending ideas that surface as part of our thought leadership efforts.

WHAT KIND OF SKILLS HELPED YOU MOST IN THE FINANCIAL CRISIS? TECHNICAL EXPERTISE? COMMON SENSE?

Technical skills were important but perhaps not as much as common sense, as we were all in uncharted waters. Communication was another essential skill in helping clients who were sometimes panicked and needed to know what to do—and what that was might change quickly. It was also important to synthesize a lot of information—something that good actuaries do routinely, I might add.

ARE THERE SOME TASKS ASSOCIATED WITH YOUR CURRENT POSITION THAT YOU FIND LESS APPEALING?

My current responsibilities naturally lead to participation in a large number of meetings. To be quite honest, I do not particularly enjoy meetings—especially all-day meetings. I'm not really good with these types of events—I get itchy!

ARE THERE ANY CONCERNS THAT YOU SEE WORKING IN A "NON-TRADITIONAL" AREA, GIVEN THAT YOU ARE WORKING FROM FOR A FIRM THAT HAS GENERALLY BEEN CONSIDERED TO BE TYPICAL ACTUARIAL EMPLOYER?

Towers Watson is one of the world's largest actuarial consultants. In addition, it happens to own one of the world's largest investment consultancies, so you might say we're working in an actuary-friendly environment. For the investment side of our business, we find that asset/liability management (most asset pools we work with are managed against some sort of liability or desired set of outflows) demands actuarial talent working in combination with experienced asset managers, economists and other professionals. Hence, for me, the types of concerns that might impact another actuary working in a "non-traditional" role have just not been an issue at all.

ARE THERE SOME CREATIVE SOLUTIONS TO PROBLEMS THAT YOU HAVE HANDLED IN YOUR CURRENT POSITION?

Many of our solutions may be more dynamic, not just creative. I'm reminded of the quote attributed to Keynes, "When the facts change, I change my mind. What do you do, sir?" So we keep a close eye on the myriad financial and economic factors that will affect our clients and attempt to be proactive in helping them deal with changing investment conditions. For example, we have a research team—the Thinking Ahead Group—that works hard to live up to its name. Our ability to anticipate risks and change is part of our strength as a business. We have introduced a number of different elements into our clients' investment strategies, including new asset classes and better benchmarks.

It also happens that the investment consulting business faces enormous competition for talent, so we have worked hard to create an intellectual challenge for our employees and a collegial work environment.

DO YOU TRAVEL MUCH? ARE THERE ANY CITIES THAT YOU PARTICULARLY ENJOY?

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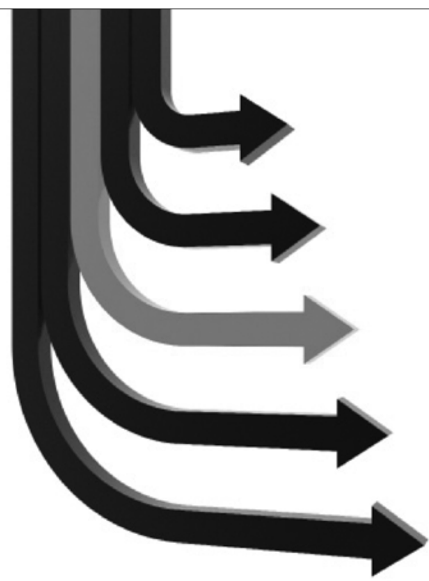
Given my global responsibilities, I travel a lot—probably 250,000+ miles a year—so it is a good thing I like a variety of environments and cultures. I love to run, so I prefer interesting cities that also have interesting runs. San Francisco is great. London has a lot of good places to run, say along the Thames or the Regents Canal. Denver's Cherry Creek trail is also fun. And few places beat the physical challenge of running in Hong Kong (heat, pollution and the Peak).

DO YOU HAVE ANY "WORDS OF WISDOM" THAT YOU MIGHT OFFER TO ACTUARIES WHO MIGHT BE CONSIDERING A CAREER OUTSIDE THE TRADITIONAL INSURANCE COMPANY OR CONSULTING FIRM?

I have two thoughts. First, I encourage actuaries to work on "soft" skills. Effective communication is the most important one that I can think of. Learn to speak and write well. Listen very well. Second, recognize that actuarial skill sets are important outside the actuarial reservation. These skills are valuable to society and will be most effective when they are used as a part of an open model that includes the talents of other professionals.



BE KIND TO YOUR RETIREMENT DECUMULATION PLAN—GIVE IT A BENCHMARK

By Daniel Cassidy, Michael Peskin, Laurence Siegel and Stephen Sexauer

Target-date funds have become one of the most popular vehicles, if not the single dominant one, for individual investing. But we've barely begun to apply institutional-quality technology to benchmarking these funds, measuring their performance, and otherwise treating them as we would any other investment. What problems are caused by this lack of attention and how can the problems be fixed?

In the crash year of 2008, for example, a sample of six funds with the "target 2015" label, intended for people retiring in about seven years, had returns ranging from -43 percent to -8 percent. Is this good or bad? One cannot tell without a benchmark. We constructed a simple, 35/65 U.S. equity-bond benchmark and found that the 2008 benchmark return was -9.54 percent, so the range of actual returns was terrible, with the exception of the fund that returned -8 percent. Such low returns could only have been earned with heavy equity exposures that are likely to be inappropriate for many investors at an age near retirement. Fiduciaries, investors, and others concerned with the investment process need to have access to benchmarks and benchmark returns so they can make informed decisions.

The principle that good investing requires benchmarks can be applied to retirement decumulation portfolios. These portfolios are unlike accumulation portfolios in several important ways. This essay focuses on the importance of benchmarks and benchmarking in the decumulation phase of lifecycle investing.

AN INSTITUTIONAL-CLASS SOLUTION

When an actuarial firm takes on an institutional mandate, its first task is to determine the schedule of retirement-income promises made to the employees by the company (or by a government or industry scheme). The objective is to fund this schedule by managing the assets matched to it. The liability schedule itself forms a benchmark, in the sense that the return on the liability can be calculated using market interest rates and other data, and the return on assets held for the purpose of paying the liability can be compared to this benchmark return.

If this is such a good idea—and we believe that it is—why don't we do something similar for individuals in the decumulation or post-retirement spending-down phase of life?

Individuals also have a liability schedule—their retirement income goal, or planned spending. Many of the characteristics of this liability schedule are common to all of us in the decumulation phase: we all need income, we all gain from longevity pooling, we all need inflation protection, and almost all of us put a high value on liquidity. A retirement decumulation strategy is highly desirable if it accomplishes all these things. Since the purpose of a benchmark is to capture the overall goals and characteristics of an investment strategy while avoiding active bets and other difficult decisions, we can ask: what is the appropriate benchmark that does all these things? And after deciding on a benchmark, we have additional questions: Can we invest directly in the benchmark, in an approach akin to indexing? Can investors beat the benchmark?

INTRODUCING THE DCDB™ BENCHMARK

In general, finance provides a rich theoretical basis for deciding what the benchmark should be in most situations. The most common example is a U.S. equity portfolio. As we noted earlier, the natural benchmark for such a portfolio is a capitalization-weighted combination of all of the liquid, publicly traded stocks in the U.S. market, because such a benchmark is (1) macroconsistent (everyone could hold it if they chose to); (2) self-rebalancing, so that there are no transaction costs caused by ordinary price changes, only by index reconstitution; and (3) mean-variance efficient according to the capital asset pricing model. A cap-weighted benchmark is also risk-minimizing in the sense of having no alpha risk (that is, no risk other than that presented by the asset class itself).

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There is, however, no theory saying what the benchmark should be for a given client in decumulation. Or it might be more accurate to say that we're still debating what the right theory is. A conversation on this topic could easily migrate among the following benchmark concepts:

- LDI—liability-driven investing is, choosing assets to match the cash flows in the liability;
- A conventional asset-class portfolio benchmark, of which 60/40 (equities/bonds) is the simplest example;
- 100 percent in U.S. Treasury inflation-protected securities (TIPS);
- A benchmark based on nominal or real annuity payouts; and
- One of the several benchmarks for target-date funds, as discussed above.

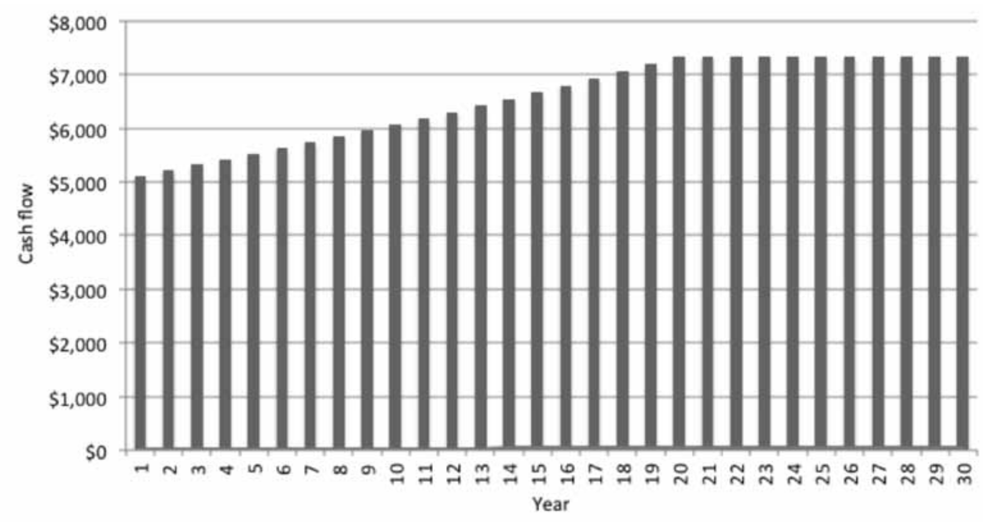
The benchmark for decumulation should be the benchmark that minimizes the four dominant decumulation risks: longevity, investment (including inflation), counterparty, and liquidity. It should also be an executable and indexable portfolio. One benchmark that does this is the DCDB benchmark, described below.

First introduced by three of us (Sexauer, Peskin and Cassidy), in a January/February 2012 *Financial Analysts Journal* article titled, "Making Retirement Income Last a Lifetime," this benchmark consists of only two assets:

1. A self-liquidating, laddered portfolio of TIPS with maturities up to 20 years, providing retirement income from ages 65 to 85; and
2. A deferred, inflation-adjusted (real) life annuity, with payments starting at age 85, and scaled so that the first deferred annuity payment is expected to be the same, in real terms, as the last cash flow from the TIPS portfolio.

(These ages are only examples. A benchmark can be constructed along these principles for any retirement age and any annuity deferral period. Thus, this benchmark is properly viewed as a family of benchmarks, one for each retirement age, gender, and so forth.)

Exhibit 1
Expected annual cash flows per $100,000 invested in DCDB benchmark portfolio



Because of the long wait to receive the deferred annuity payments, and because mortality after age 85 is high, the cost of the deferred annuity is surprisingly small, leaving most of the portfolio in liquid TIPS. For a 65-year-old male in the United States in 2010, the portfolio weights were 88 percent in the laddered TIPS portfolio and 12 percent in the deferred annuity at the time the strategy is initiated (that is, at age 65).

Exhibit 1 illustrates the year-by-year income (cash flow to the investor) generated by the DCDB benchmark portfolio, per $100,000 invested. The first 20 years' cash flows grow with the inflation rate. Starting in year 21, there are no more inflation adjustments. (The DCDB design does not include inflation-indexed deferred annuities because they are not currently available; insurance companies cannot defease the risk of issuing them because the TIPS market has no depth beyond 20 years, the same reason we cannot hedge inflation risk after the 20th year directly.)

We call the family of benchmarks that use this structure "DCDB," for "defined-contribution decumulation benchmark," but the acronym is also supposed to connote "DC to DB," defined-contribution to defined-benefit, reflecting our conviction that a well-engineered DC plan should be experienced by the participant much like a DB plan, providing predictable retirement income and having very little risk.

This benchmark has minimal risk. It provides inflation protection through age 85, does not contain any equity risk or fixed income duration-mismatch risk, and only the deferred-annuity cash flows starting at age 85 have any credit risk. To further reduce inflation risk would require annuitizing the whole investment balance in a real (inflating) life annuity, but this would expose the whole portfolio, instead of just 12 percent of it, to credit risk, and would be unacceptable to most investors because of the liquidity loss.

USES OF THE DCDB BENCHMARK

By purchasing the laddered portfolio of TIPS and the deferred life annuity, investors can invest directly in the benchmark, akin to indexing. We are aware that counterparty or credit risk in the deferred annuity component is a problem. Some investors simply will not pursue the strategy because of this risk, which cannot be eliminated by diversifying among annuity issuers because defaults are correlated. However, the gains from longevity risk pooling are so large, comprising about one-third of one's whole retirement assets according to some estimates we believe investors are foolhardy not to invest at least a modest amount in annuity-based products.

Alternatively, investors can try to beat the benchmark. Many of the millions of retirees may find greater utility in a different portfolio, say, one that contains equities or one that contains income guarantees. But these investors need a way of measuring the success of their portfolio, and the DCDB benchmark provides such a way, by revealing the cash flows that can be generated each year per $100,000 invested, without taking any equity risk and while also taking advantage of longevity risk pooling from age 85 onward (which are the years when the pooling has the largest payoff).

Investors hunger for a way to hedge longevity risk, but with traditional immediate annuities they cannot do so without sacrificing the liquidity and flexibility that they prize. This is why immediate annuities are so unpopular. The DCDB benchmark combines the best aspects of traditional low-risk investing and insurance.

SUMMARY

It is the responsibility of plan sponsors to choose an appropriate glidepath and risk profile for their plan participants,

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and also to choose the associated benchmark that represents the overall goals of the investment strategy being pursued.

Plan sponsors, consultants, advisors, and participants can use a benchmark to define, evaluate, and judge QDIA target-date portfolios. By doing so, they will know why a particular glidepath was chosen, and what its attendant risks are. They will have access to the relevant risk and return performance metrics. As now required by the U.S. Department of Labor, they will know how much retirement income their target-date portfolio can generate.

Until now, decumulation investors have been flying blind, having no benchmark with which to judge their progress. The DCDB benchmark can be used for this purpose.

Be kind to your retirement decumulation plan. Give it a benchmark.

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THE SEC'S FORM PF: ORSA FOR HEDGE FUNDS

By James Ramenda

Hedge funds have long enjoyed being one of the least regulated sectors of the financial service industry. However, SEC/CFTC rules adopted in late 2011 pursuant to Dodd-Frank have brought significant risk-related regulation to large hedge fund managers. Beginning with 2012 second-quarter end-data, the very largest hedge fund managers (over $5 billion in regulatory assets under management—RAUM) will need to file a new form each quarter, Form PF, which requires a large volume of information including exposures, counterparty risk, liquidity risk, durations, market risk factor sensitivities, and other risk measures, potentially including value at risk. Managers with RAUM of $1.5 billion will need to begin to file this form quarterly beginning with year-end 2012 data. Managers with $150 million or more will have to file Form PF annually beginning with year-end 2012, but are not required to file the same degree of risk-related information.

Conceptually, Form PF bears a resemblance to insurers' Own Risk and Solvency Assessment (ORSA) in that certain assumptions and calculations are left to the filing entity. For example, fund managers are allowed to use their own assumptions and own models for calculating VaR and other risk measures. In fact, if a fund does not calculate VaR regularly, then it doesn't need to be calculated for the filing. There is similar leeway in the filing instructions for the calculation of sensitivities to pre-specified shocks to market risk factors (The factors are equity prices, the risk-free interest rate, credit spreads, currency rates, commodity prices, implied option volatilities, and default rates for ABS, corporates and CDS.).

However, while there is leeway, there is also a catch-all for risk measures. The fund must include any risk measures that it reports either internally or to its investors. And some risk measures are definitely required, particularly durations (or alternatively, weighted average tenor or 10-year bond equivalents), segmented into 22 specified asset classes, for both longs and shorts, calculated for each month-end. The duration calculations apply to the aggregated funds managed as well as specific funds.

Form PF also applies to private equity funds and so-called liquidity funds, however, the risk-related requirements are not nearly as extensive as those described above.

A LOT OF DEVIL IN THE DETAILS

It's fair to say that the speed of adoption and breadth of information required by Form PF have come as a surprise to many managers. Many fund managers are unprepared and do not realize the extent of the calculations required for Form PF. There are also some critical details that serve to widen the scope of the reporting while also making certain risk requirements quite granular.

- The threshold for filing Form PF is regulatory assets under management, which is essentially equal to gross GAAP assets. Managers and the industry, though, typically think of their fund size in terms of net assets, i.e., long positions net of short positions. There are funds with less than $1 billion in net assets that leverage up to well over the $5 billion in RAUM threshold for early filers.

- Some of Form PF's requirements are for the aggregate of funds managed, but some measures, especially the risk requirements described above, must be applied at the individual fund level. So while some observers simplistically characterize the regulation as data aggregation, it is actually a mix of aggregation, disaggregation, and then re-aggregation into specified buckets—but with some complex risk calculations sprinkled throughout these processes. Many compliance professionals do not have the background to appreciate the complexity imposed by the risk calculations.

- The requirement that any risk measure reported internally or to investors must be included in the filing

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creates a catch-all requirement for which the SEC at this writing has yet to provide definitive details. Distinguishing risk measures from portfolio valuation and asset selections tools is subjective. For example, are CAPM parameters risk metrics? Greeks? Fundamentals like price-to-book value or price-to-earnings? Technical analyses?

- While the use of VaR and other risk measures reported internally or to investors certainly captures the spirit of using one's own risk assessment, it certainly falls short of the insistence on VaR-based approaches that is present in other financial services regulation, e.g., Basel Accords, Solvency II, RBC C-3 Phase II, etc.

LOOKING AHEAD

On a different level, this regulation is a watershed event. Up until now hedge funds have been left completely to their own devices regarding risk. Standardization of hedge fund risk has now begun and it is likely that fund investors, potential investors, and intermediaries will soon be tailoring their risk inquiries to include Form PF data. Even though funds are under no obligation to disclose the information other than to the regulators, market pressure likely will force at least some of this data to be released. There are already private sector initiatives to accomplish exactly this on a voluntary basis, such as OPERA (Open Protocol Enabling Risk Aggregation).

Investors will find this information useful in several respects when assessing a fund.

1. What types of risk is the fund willing to undertake, e.g., long-short duration mismatch, market factor risk, concentrations with respect to asset class, geography, counterparties, or illiquid assets?
2. How levered is the fund?
3. What type of off-strategy investments does the fund typically hold?
4. What does the time series data for the fund's risk measures indicate about how, and how often, the fund changes its risk preference?
5. What type of tail risk does the fund's strategy create?

Of course, hedge funds are not always enthusiastic about answering questions such as these, far less so with the specificity Form PF requires. Fund managers often feel that standard metrics do not properly reflect the way in which they make investments decisions and, in any case, they do not wish to make it easier for anyone to reverse-engineer their strategies.

However, what these newly-risk-regulated fund managers may only be beginning to appreciate is that questions like those above are not simply due diligence questions, but are of interest to investors in aggregating their own investment risks for their own governance and regulatory purposes. The very existence of this data virtually assures that many institutional investors will make it a condition of their investing to receive the information in some form. This in turn may cause funds to consider how their investment strategies will play out in Form PF.

As a result, Form PF and additional measures that may follow from financial reform will probably have the effect of shaping risk in addition to reporting on it, much in the way Solvency II and IFRS (and U.S. analogs of these) will shape insurance company product offerings and investment strategies.

While Form PF can be compared to ORSA conceptually in terms of risk disclosure, the comparison falls short with respect to solvency. Measures suggestive of tail risk and the possibility of systemic risk are certainly included, but there is no solvency standard, per se. Rather, solvency is only covered implicitly in the collateral and margin requirements that underlie the fund's holdings.

> "STANDARDIZATION OF HEDGE FUND RISK HAS NOW BEGUN. …"

How much more is to come? There are estimates that Dodd-Frank ultimately will spawn 400 rules, only about a quarter of which have been promulgated to date. Not all of these expected rules will affect hedge funds directly, though many will certainly have an indirect impact through the effects on other financial institutions which invest in hedge funds. But certainly, the early shock waves have been significant for fund managers.

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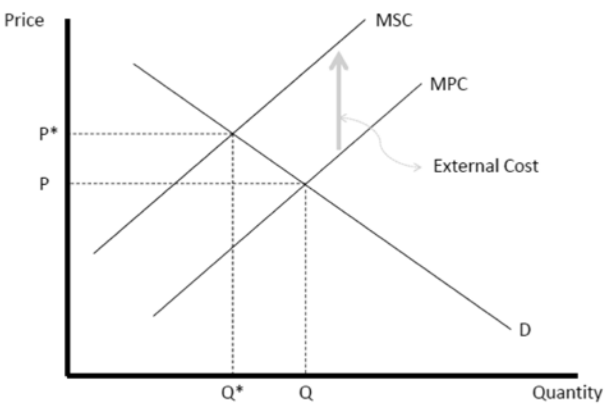
NEGATIVE EXTERNALITY: A FRAMEWORK FOR CONTEMPLATING SYSTEMIC RISK

By Rick Gorvett

The problem of firms or industries that engage in activities that ultimately have harmful effects on other organizations or people is a difficult and longstanding economic issue. Even before Ronald Coase published his Nobel Prize-worthy insight more than half a century ago (Coase, 1960), many well-known economists had analyzed and opined on this question. It is a measure of the difficulty, importance, and applicability of this problem that significant debate continues to this day.

The classic example of this type of activity and the resulting harm it imposes on otherwise innocent parties—a situation commonly referred to as a *negative externality*—is that of a polluting industrial firm and its consequent impact on the nearby surrounding community. However, another example that can be interpreted and analyzed within the framework of negative externality, is the *systemic risk* associated with a potential broad-based failure of the financial or economic system. The recent financial crisis exemplifies how important are such risks and considerations for global economic health and prosperity.

Negative Externality



This article briefly examines systemic risk in the context of an economic analysis of a negative externality. First, the characteristics of negative externalities are described, and then insights for the analysis and management of systemic risk are examined.

WHAT IS A NEGATIVE EXTERNALITY?

A negative externality occurs when an organization undertakes an activity that causes harm or costs to one or more third parties—for example, to society. In particular, when an operational decision is made, a negative externality exists if the total cost associated with that operational decision is not borne entirely by the firm, but rather is borne in part by another party. The classic example of a negative externality is pollution, in which the impact of a firm's industrial activity causes harm to those geographically proximate to the polluting firm (which is why negative externalities are sometimes called local or neighborhood costs).

In the context of traditional neoclassical economic theory, a simple supply-demand diagram can help to understand the distorting effect that a negative externality can have on a market:

Here, demand curve D represents marginal benefit, and supply curve MPC represents marginal private cost. Based on these curves, the market will find an equilibrium at (Q,P). However, when actions are taken to recognize and reflect the social costs, the supply curve is more realistically expressed as MSC, marginal social cost, which includes both private costs and the negative external costs. This "corrected" analysis would find an equilibrium at (Q*,P*). Thus, the negative externality leads to too high an equilibrium quantity. This leads to a level of social gain that is lower than it could be.

At the time Coase published his paper, this problem of "social cost" was well-known amongst economists. Probably the most influential thinking on this issue at that time stemmed from Arthur Pigou's *The Economics of Welfare* (Pigou, 1920), in which he considered, for instance in the polluting-firm example, two categories of costs/benefits: private and social. This led to accepted remedies such as direct governmental regulation, or taxing the polluter (either to recompense the social costs incurred from the firm's operations, or perhaps to discourage the cost-producing activity itself). This latter approach has come to be known as a Pigou, or Pigovian, tax. Conceptually, if the firm is liable for the tax, and the tax is based on the marginal social cost associated with the damages produced by the firm's activity, the firm is forced to recognize and internalize the true total (private plus social) cost of the activity.

While this was the common wisdom at the time, Coase's 1960 paper presented a new conceptual framework for negative externalities. To quote from that paper:

> "The traditional approach has tended to obscure the nature of the choice that has to be made. The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to B, would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm."

Within this framework, Coase analysis ultimately results in the following prescriptions and observations regarding situations involving negative externalities:

- Traditional approaches—e.g., Pigou taxes—may not be either appropriate or desirable.
- Under the assumption of no transaction costs, private negotiation amongst parties will produce an efficient solution, regardless of the liability promulgated by the legal system.
- When transactions costs are recognized, the best and most appropriate solution must be determined on a case-by-case basis, and is heavily dependent upon the specifics of the legal system.
- The "problem has to be looked at in total *and* at the margin."

Thus, when it comes to identifying potential remedies for negative externalities, there are traditional interventionist policies such as direct regulation and Pigou taxes, but there are also market-based remedies involving negotiations and bargaining between and amongst the parties and society. All such prescriptions should be considered, especially when realities such as the existence of transaction costs are taken into account.

SYSTEMIC RISK AS A NEGATIVE EXTERNALITY

The recent (and, to some degree, continuing) financial crisis has led to consideration of a variety of proposed interventionist remedies, designed to signal and help prevent potential systemic risk problems. *Systemic risk* is the possibility of a significant impairment to the overall economic and financial system. Due to the ever-increasing interconnectedness and interdependence of economies and financial markets, the failure or collapse of one or more financial intermediaries at the "micro" level can lead to broad-based market instability at the "macro" level, as a result of undercapitalization, liquidity, and flight-to-quality issues.

One can readily imagine that an individual financial firm, whether because of regulatory requirements or internal risk management procedures, might make decisions or take actions to protect *itself* from major impairment, but that

CONTINUED ON **PAGE 34**

"… SYSTEMIC RISK CAN BE VIEWED AS A NEGATIVE EXTERNALITY."

those decisions or actions might not serve, and might not be in the best interest of, the health of the financial system *overall*. In fact, individual firm activity might act *against* the common good by serving to *decrease* the solvency of the system. Such perverse incentives can easily result from non-risk-based regulation, or from regulation that focuses on the status of individual firms rather than their potential contribution to overall system health. Thus, systemic financial risk can be viewed as a negative externality: actions of an individual firm, while justifiable and beneficial to the firm on a stand-alone basis, may produce external costs on the overall financial system and society.

As mentioned in the prior section, there are several types of potential remedies for situations involving negative externalities. In considering possible responses to systemic risk, it is important to remember that, as Coase advocated, all options should be considered, and each situation should be assessed on an individual-case basis. For our purposes, in evaluating systemic risk as a negative externality, we can categorize each of the remedies as one of two types: interventionist policies, and market-based solutions.

Interventionist policies include, for example, direct regulation, and Pigovian taxes.

Direct regulation: With direct regulatory control, the main issue in addressing systemic risk would be how to manage overall systemic risk through regulations and requirements directed at individual firms. Part of that issue would be practical: structuring regulations such that they have the desired effect both on the individual company and on the financial system and society at large. Another part of the issue would be quantitative: identifying and measuring the marginal impact of an individual firm's actions and decisions on the overall system and determining the marginal cost to society of adding an additional unit of systemic risk to a firm's operations. It is important here to understand and model the financial market as a complex system, with multi-firm interconnectedness and interdependencies. Such financial risk modeling and analysis would be an area where actuaries could provide significant value.

Pigovian taxes: By taxing the firm whose activity produces the external societal cost, an incentive is provided for the firm to reduce or even avoid the activity. This basic approach has much intuitive appeal and familiarity. By basing the level of the tax (at least conceptually) on the marginal social cost—the marginal increase in systemic risk—of the firm's activity, the true cost of the externality is explicitly recognized and is allocated to the appropriate party. A Pigovian systemic risk tax could be risk-based (determined as a function of an individual financial intermediary's specific characteristics—its financial attributes, liquidity situation, and modeled contribution to macro risk), pre-assessed (so that the tax is paid by all firms, including and especially those firms most likely to fail and thus to impose macro costs on the overall markets), and collected for the purpose of partially offsetting future systemic loss costs. As Coase pointed out, however, a Pigou tax may not produce an appropriate or desirable (or particularly efficient) societal outcome.

As an alternative to interventionist approaches, a market-based solution might be, for example:

Tradable permits: As has been proposed with respect to carbon-producing activities, a tradable systemic risk permit system would involve the identification of an overall "permissible" level of systemic risk, the initial allocation of that total level of risk to those firms that contribute to the overall risk of the financial system, and the potential for trading such permits amongst firms. The market would provide a basis for appropriate and socially-efficient pricing of these transactions. Of course, a big issue in such a scheme would involve how the level of "acceptable" overall systemic risk would be determined (and then distributed or allocated to the various financial firms). Once that is determined, however, this approach largely becomes an optimization problem: how to optimize societal benefits (or minimize societal costs) within specified risk-level constraints. Again, actuarial and modeling skills could provide techniques of significant value to such a process.

SUMMARY

It is becoming increasingly clear that systemic risk can be viewed as a negative externality. The key question now is the best approach to dealing with systemic risk in this light. Although most of the current public policy approaches tend to be interventionist, additional attention to actuarial-based modeling of potential market-based solutions may suggest and encourage viable, or even preferable, alternatives.

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SOA RESEARCH IN PROGRESS: INTEREST RATE SWAPS EXPOSED

By Paul G. Ferrara and Seyed Ali Nezzamoddini

The SOA's Research Department coordinates a broad range of useful analysis and exploration of topics relevant for investment actuaries. Check it out at *http://www.soa.org/content.aspx?id=3429*. One project in the works explores the inherent risk in interest rate swaps. This work will be completed in 2012, so watch for it! The contact for the project oversight group is Steven Siegel (*ssiegel@soa.org)* at the Society of Actuaries.

The project abstract runs as follows:

Vanilla interest rate (IR) swaps may be viewed as very simple interest rate derivatives, but the implications of entering into such contracts may not be so readily apparent. Specifically, investment managers and asset/liability managers in the insurance industry are often presented with such contracts from investment banks as hedging solutions; however, the potential downside of such deals is not always clarified in the corresponding proposals. In this article, we will take a look at plain-vanilla IR swaps under various interest rate regimes and analyze the potential exposure to losses on such swaps as a result of the swaps being used for speculative purposes, or upon counterparty default. We also discuss some issues surrounding the use of IR swaps in hedging. To perform this analysis we use stochastic yield-curve simulation via the Black-Karasinski model. Significant time is spent discussing both the theory and implementation of such stochastic IR scenario generators. The potential exposure to counterparty default will be explored by calculating both the concepts of expected future exposure (EFE) and potential future exposure (PFE). Further, similar stochastic techniques will be used to illustrate the exposure upon using such swaps for speculative purposes, as in the notorious cases of Proctor & Gamble, and the Alabama public schools.

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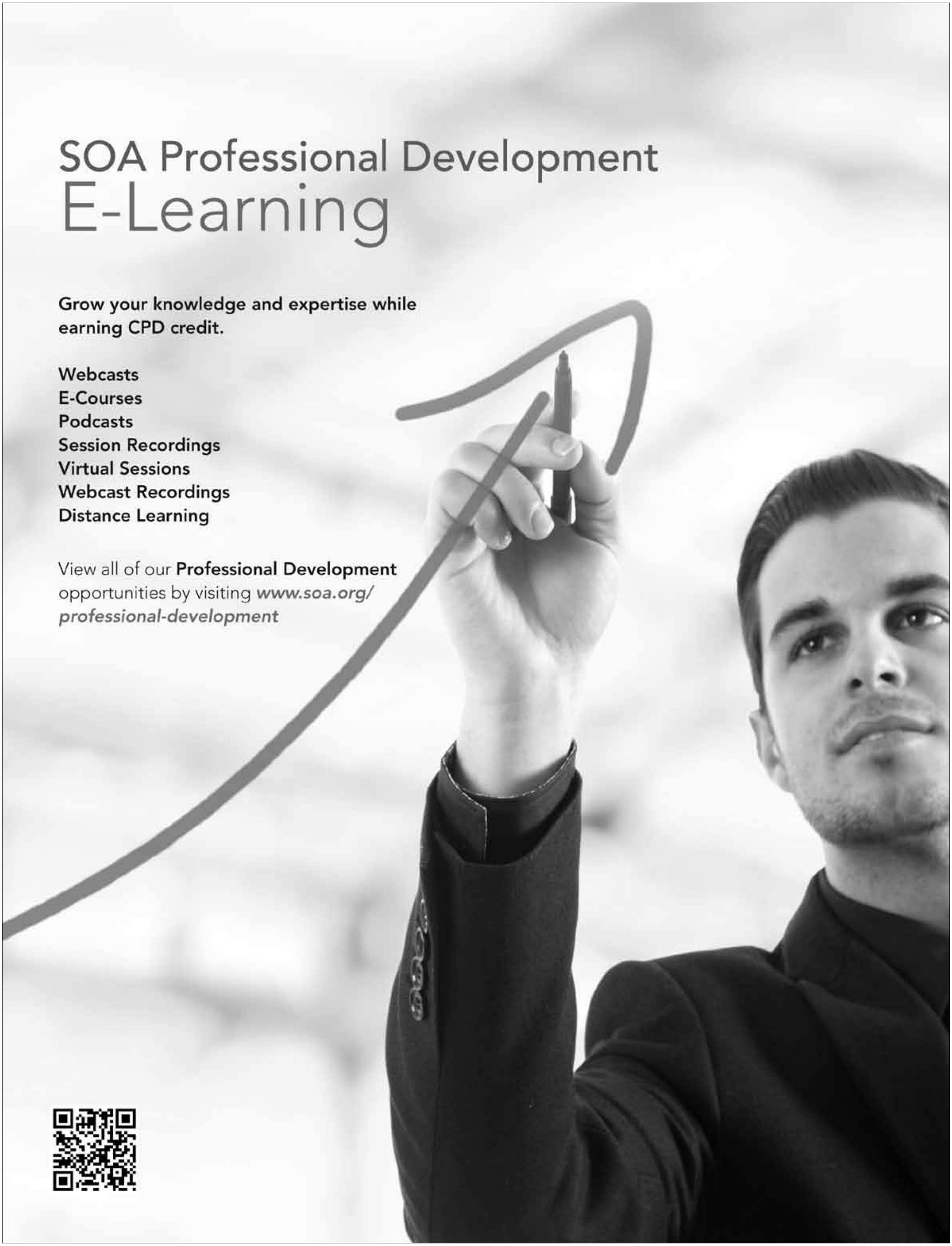
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