Equity-Based Insurance Guarantees Conference

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Chicago, IL

Dynamic vs Static Replication

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Dynamic vs Static Replication

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Milliman Financial Risk Management LLC
November 12, 2019 (1330 - 1430 hours)
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Overview of Options and Replicating Strategies
The World of Options

• Exposures can go far beyond vanilla calls/puts
  • Exercise types: European, American, Bermudan, Asian, ...
  • Underlying: single name, basket, chooser, rainbow, spread, variance/correlation, ...
  • Payoff profile: binary, power, range, ...
  • Boundary conditions: barrier, knock-in/out, ...
  • Exotic terms: cancelable, extendable, amortizing, ...
  • Exotic strikes: lookback, cliquet, ...

• Frequently insurance liabilities (particularly life) are long dated and impacted by policyholder behavior
Dynamic vs Static Replication

• Delta hedge alone?
• Include market options to lower trade rebalancing?
• Not a yes/no choice, more of a continuum
• Even (relatively) model independent static replication of variance swaps requires continuous delta hedging
Hedge Effectiveness

• Accuracy is constrained
• Limited market instruments (e.g. long dated)
• Liquidity, transaction costs, slippage
• Model/data dependency
• Policyholder behavior
Costs of Trade Rebalancing

• Execution commission: roughly $1.00/contract
• Exchange commission: CBOE charges up to $0.80/contract ($0.35-$0.55/contract on SPX)
• Clearing commission: roughly $1.00/contract
• Clearing fees: OCC charges up to $0.055/contract
• Options Regulatory Fee: $0.0388/contract
• Bid/ask spread: at least a few basis points
Hedge Ratio Model Dependency

Call Delta as of 2019-10-15
Expiration Date 2020-09-18 AM; Payoff Date 2020-09-21

- Black Scholes
- Practioners’ Black Scholes
- Heston
- Heston (minimum variance)
Opportunities

• Dynamic replication can harvest implied volatility premium/term structure
• Can earn spread via cash management
• Optimized ALM, statistical hedge replication
• Maintain adaptability/flexibility/control
Example: “Static” 1-Year S&P 500 Volatility Management

Calendar Year 2018, Target Volatility = 15%

<table>
<thead>
<tr>
<th>S&amp;P 500 Price Return 1/2/2018-12/31/2018</th>
<th>Static 15% VM Option Portfolio</th>
<th>Dynamic 15% VM Account (no leverage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; -14.73%</td>
<td>Outperform</td>
<td>?</td>
</tr>
<tr>
<td>-14.73% - 0%</td>
<td>Underperform</td>
<td>?</td>
</tr>
<tr>
<td>0%</td>
<td>-4.23%</td>
<td>?</td>
</tr>
<tr>
<td>0% - +7.69%</td>
<td>Underperform</td>
<td>?</td>
</tr>
<tr>
<td>&gt; +7.69%</td>
<td>Outperform</td>
<td>?</td>
</tr>
</tbody>
</table>

Realized: -7.01% -10.21% -5.87%

• S&P 500 price return between -14.73% - +7.69  
  (S&P 500 total return including dividends = -5.18%)

• Outcome: “static” 15% VM underperforms

RESULTS BASED ON SIMULATED OR HYPOTHETICAL PERFORMANCE RESULTS HAVE CERTAIN INHERENT LIMITATIONS. UNLIKE THE RESULTS SHOWN IN AN ACTUAL PERFORMANCE RECORD, THESE RESULTS DO NOT REPRESENT ACTUAL TRADING. ALSO, BECAUSE THESE TRADES HAVE NOT ACTUALLY BEEN EXECUTED, THESE RESULTS MAY HAVE UNDER-OR OVER-COMPENSATED FOR THE IMPACT, IF ANY, OF CERTAIN MARKET FACTORS, SUCH AS LACK OF LIQUIDITY. SIMULATED OR HYPOTHETICAL TRADING PROGRAMS IN GENERAL ARE ALSO SUBJECT TO THE FACT THAT THEY ARE DESIGNED WITH THE BENEFIT OF HINDSIGHT. NO REPRESENTATION IS BEING MADE THAT ANY ACCOUNT WILL OR IS LIKELY TO ACHIEVE PROFITS OR LOSSES SIMILAR TO THESE BEING SHOWN. MILLMAN DOES NOT MANAGED THE UNDERLYING FUND.
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Dynamic vs Static Hedging
Annualized Savings from CBOE Options Replication

- In this example, we replicated listed CBOE call options with futures contracts on the S&P 500.
- Savings are annualized and reflect the difference between the implied volatility of the call option versus the realized volatility of the replicated position.
- The assumed volatility used to calculate option delta for replication was based on the MGI value for the remaining time-to-maturity of the option.
- Longest option tenors were three years, but we modeled multiple tenors here.
- All savings figures were annualized for comparability.
Replicating 6Y Option – Stylized Example

- Used OTC Dealer Implied Data
- Only Data From 2009 – Present is Available
- 5 Options Replicated (annually, starting at 1/9/2009, with 6-year maturity)
- Volatility Used for Delta Target for Replication Taken From MGI
- Bond Portfolio is ZCB with Maturity Matching Time to Expiry of Synthetic Option

Average Annualized Savings Via Replication: 1.62%
Milliman Dynamic Hedged Equity Strategy (MDHE)

MDHE has been a component of the Milliman Managed Risk Strategy since MMRS was incepted.

- MDHE is a long-dated constant maturity put replication that uses a delta adjustment to the equity exposure
  - Similar to a protective put, or synthetic long call
  - Implemented with futures contracts
- Seeks to reduce the downside exposure of the portfolio during significant and sustained market declines by:
  - Capturing gains after favorable returns on the portfolio’s underlying holdings
  - Harvesting gains from the portfolio’s offsetting positions after severe market downturns
- Can augment the strategy with options which may reduce capital in shock scenarios and increase downside protection

For illustrative purposes only, does not represent the performance of any actual investment or portfolio, and should not be viewed as a recommendation to buy/sell.
Milliman Dynamic Hedged Equity Strategy (MDHE)

- The addition of the options is designed to further enhance the tail risk protection provided by the capital protection strategy.

- Long S&P 500 put options are incorporated into Milliman Dynamic Hedged Equity (MDHE) strategy in a delta-neutral manner by utilizing long equity futures to maintain consistent net equity exposure with the core MDHE strategy.

- The amount of equity options to be held by the fund is established by evaluating a 5% equity shock scenario on a daily basis and the resulting projected trading requirements that are generated by the shock in order to restore the fund to its volatility and target equity range.

- The options strategy is designed to hold options that will provide 80% of these projected equity trading requirements.

- The put options purchased are ~6 month (180-day) to expiration and are sold when there is ~35 days to expiration.
Growth of $100mm

Dynamic Hedged Equity: All Equity Portfolio

SEE SLIDE 23 FOR ADDITIONAL INFORMATION

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Growth of $100mm
Dynamic Hedged Equity + Options: All Equity Portfolio

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Growth of $100mm
Dynamic Hedged Equity + Options: All Equity Portfolio

Backtested Results

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## Backtested Performance Analysis: All Equity Portfolio

### Analytics and Standardized Performance (2000-2018)

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<tr>
<th>Time Period</th>
<th>Non-Mgd</th>
<th>MDHE</th>
<th>MDHE + Options</th>
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<tr>
<td><strong>Full Period</strong></td>
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<tr>
<td>Return</td>
<td>4.03%</td>
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<td>Volatility</td>
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<td>Drawdown</td>
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<td>-34.53%</td>
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<td>Max Volatility</td>
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<td>Return/Risk</td>
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<tr>
<td><strong>Max Drawdown (Net Equity) (Avg.)</strong></td>
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<td>3 YR</td>
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<td>5 YR</td>
<td>-18.88%</td>
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### Ann. Returns

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<td>1 YR</td>
<td>-8.95%</td>
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<td>-7.91%</td>
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<tr>
<td>3 YR</td>
<td>7.20%</td>
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<td>5 YR</td>
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### Volatility

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<td>1 YR</td>
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<td>3 YR</td>
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<td>5 YR</td>
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### Max Volatility

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<tr>
<td>1 YR</td>
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</tr>
<tr>
<td>3 YR</td>
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<tr>
<td>5 YR</td>
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### AVG SII CHARGE

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<tr>
<td>1 YR</td>
<td>37.95%</td>
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<tr>
<td>3 YR</td>
<td>0.11</td>
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### RETURN/CAPITAL

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<tr>
<td>1 YR</td>
<td>0.11</td>
<td>0.22</td>
<td>0.47</td>
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</tbody>
</table>

See Slide 23 for additional information.

Results based on simulated or hypothetical performance results have certain inherent limitations. Unlike the results shown in an actual performance record, these results do not represent actual trading. Also, because these trades have not actually been executed, these results may have under-or over-compensated for the impact, if any, of certain market factors, such as lack of liquidity. Simulated or hypothetical trading programs in general are also subject to the fact that they are designed with the benefit of hindsight. No representation is being made that any account will or is likely to achieve profits or losses similar to these being shown.
### Backtested Performance Analysis: All Equity Portfolio

<table>
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<tr>
<th>Year</th>
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<th>MDHE</th>
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<td>-17.1%</td>
<td>96.3%</td>
<td>96.3%</td>
</tr>
</tbody>
</table>

SEE SLIDE 23 FOR ADDITIONAL INFORMATION

RESULTS BASED ON SIMULATED OR HYPOTHETICAL PERFORMANCE RESULTS HAVE CERTAIN INHERENT LIMITATIONS. UNLIKE THE RESULTS SHOWN IN AN ACTUAL PERFORMANCE RECORD, THESE RESULTS DO NOT REPRESENT ACTUAL TRADING. ALSO, BECAUSE THESE TRADES HAVE NOT ACTUALLY BEEN EXECUTED, THESE RESULTS MAY HAVE UNDER-OR OVER-COMPENSATED FOR THE IMPACT, IF ANY, OF CERTAIN MARKET FACTORS, SUCH AS LACK OF LIQUIDITY, SIMULATED OR HYPOTHETICAL TRADING PROGRAMS IN GENERAL ARE ALSO SUBJECT TO THE FACT THAT THEY ARE DESIGNED WITH THE BENEFIT OF HINDSIGHT. NO REPRESENTATION IS BEING MADE THAT ANY ACCOUNT WILL OR IS LIKELY TO ACHIEVE PROFITS OR LOSSES SIMILAR TO THESE BEING SHOWN.

SEE ADDITIONAL INFORMATION ON PAGE 23.
Additional information

• **Net Equity:** represents the net effective equity exposure of the Managed Risk Portfolio, including the baseline exposure from the underlying holdings, and the effect of the hedging strategy, which dials up or down the equity exposure in response to risk signals

• **Dynamic Hedged Equity:** All Equity Portfolio (slides 17, 18, 21, 22)
  - **Non-Managed Risk:** represents the hypothetical historical performance of a 100% MSCI ACWI Index portfolio, assuming that they did not employ an active risk management strategy
  - **Managed Risk:** represents the hypothetical historical performance of the non-managed risk investment portfolio, assuming they employed the Milliman Dynamic Hedge Equity (MDHE) Strategy over the entire time period

• **Dynamic Hedged Equity + Options:** All Equity Portfolio (slides 19-20)
  - **Non-Managed Risk:** represents the hypothetical historical performance of a 100% MSCI ACWI Index portfolio, assuming that they did not employ an active risk management strategy
  - **Managed Risk:** represents the hypothetical historical performance of the non-managed risk investment portfolio, assuming they employed the Milliman Dynamic Hedge Equity + Options (MDHE + Options) Strategy over the entire time period
Deep Reinforcement Learning Application
Overview of Deep Reinforcement Learning

What is Reinforcement Learning?
• Training an intelligent agent by allowing it to interact with a given environment and learning from trial and error within that environment

Why in Portfolio Management?
- Rather less mathematical constraints / assumptions for modeling – model-free
- Proven to work well to capture complex / non-linear patterns
- Neural net based deep learning can alleviate curse of dimensionality – enabling large scale portfolio management

Examples of Different Reinforcement Agents
• Deep Q-Network (DQN): Generic Q-network with deep learning overlay
• Policy Search Based
  ✓ Policy Gradient (PG)
  ✓ Generic Actor-Critic
  ✓ Proximal Policy Optimization (PPO): Surrogate objective function
Deep Reinforcement Learning – Deep Hedging (1)

Objective of Deep Hedging

- Given market signal information up to time t, liabilities, and a number of hedging instruments in a pre-defined asset universe, it is trying to determine the most optimized holdings across different hedging instruments while meeting some of the key constraints (i.e. liquidity limit, trading costs, etc) within their risk appetite (i.e. convex risk measure such as CTE)

- And, this is done through the Deep Reinforcement Learning (DRL) by providing “proper” rewards within the environment for certain actions (how to allocate holdings across different assets to hedge liabilities) that the agent takes
Deep Reinforcement Learning – Deep Hedging (2)

Overall Set-up

- Discrete time and market with friction
- \{l_0, ..., l_k\} is a set of market signals up to time k (t_k) that forms the filtration up to t_k
- Z is a \(F_T\) measurable random variable indicating liabilities (or, contingent claims)
- \(\delta_k^i\) is ith asset holdings at time \(t_k\)
- \(H_k\) is a set of constraints that \(\delta_k\) is subject to at \(t_k\)
- \((\delta \ast S)_T\) is defined as \(\sum \delta_k \ast (S_{k+1} - S_k)\)
- \(C_T(\delta)\) is defined as \(\sum C_k (\delta_{k+1} - \delta_k)\), where \(C_k\) can take fixed, proportional, and rather complicated cross-asset cost functional forms
- \(P_0\) is defined as cash injection or extraction
- \(\rho\) is a convex risk measure meeting the following three properties:
  - Monotone decreasing: if \(x_1 \geq x_2\), then \(\rho(x_1) \leq \rho(x_2)\); in words, this means more favorable positions require less cash injection
  - Convex: \(\rho(\alpha x_1 + (1 - \alpha)x_2) \leq \alpha \rho(x_1) + (1 - \alpha)\rho(x_2)\); in words, diversification works
  - Cash-invariant: \(\rho(x + c) = \rho(x) - c\), where \(c \in \mathbb{R}\); in words, adding cash to a position reduces the need for more by that amount
Overall Set-up Cont.

- The problem we are trying to solve here becomes solving the following convex objective function:

\[
\pi(-Z) := \inf_{\delta \in H} \rho(-Z + P_0 + (\delta \ast S)_T - C_T(\delta))
\]

- where \( \inf \) indicates the greatest lower bound

- In the neural net sense, what we are after using that objective function here is getting the most optimized holdings across different hedging instruments while meeting some of the key constraints (i.e. liquidity limit, trading costs, etc.) within certain risk appetite (i.e. convex risk measure such as CTE)

- \( \delta^0_k := f^\theta(I_k, \delta^0_{k-1}) \)

- where \( \theta \) indicates a set of parameters for the trained neural net

- \( f \) is a composite functional form of neural nets (i.e. \( f(g(h(x)))) \)

- \( \delta^0_{k-1} \) indicates the recurrent nature of the neural nets

- (a. k. a. past information from the previous time steps cascade forward to form time dependencies)

- And, we do this rebalancing exercise on a daily basis

  ✓ The key structure of training an artificial agent is achieved through the DRL environment
Deep Reinforcement Learning – Equity Allocation Example (1)

Overall Environment Set-up and Assumptions

- Portfolio: M risk assets (in my case, plain equities) + 1 risk-free asset (cash); in this exercise, M == 5
- State: state space indicates the “market condition” at a specific point in time, such as closing prices, mid prices, volume, PE ratio, PB ratio, etc. – can stem from one of those “signal processed” data by Natural Language Processing (NLP)
- In this study, somehow we use closing and high prices and we claimed that the combination of these two produced rather “better” results
- Fixed window for a time-series training == 10
- Action: action space is defined here as the “proper” or “desirable” allocating weights. Obviously $\sum_{i=1}^{M+1} a_{i,t} = 1$
- Reallocation is assumed to be done once a day in this exercise
- Reward: fluctuation of wealth minus transaction cost. Just for the experiment purpose, the transaction cost was assumed to be 0.25%
- Train the agent in a way that it maximizes the reward (i.e. Profit or Sharpe ratio in this example)
Deep Reinforcement Learning – Equity Allocation Example (2)

Sample Example Testing Results

- Agent 0: Deep Reinforced Agent
- Agent 1, 2, and 3: Other Sub-optimal Allocation Strategies
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