Session 22: Actuarial Applications of Survival Models
SESSION 22: ACTUARIAL APPLICATIONS OF SURVIVAL ANALYSIS

JOHN MCGARRY
Life & Annuity Symposium
May 20th, 2019
AGENDA

- Introduction
- Methods
- Survival Time
- Standard Error
- Risk Factors
INTRODUCTION

- Survival analysis quantifies the risk of decrement from a population for one or more events
  - Hazard ratio, relative risk to base
  - Survival time, average time to event
  - Survival rate, probability of survival to time $t$
- Used across multiple disciplines
  - Survival – Clinical Medicine
  - Reliability/Failure - Engineering
  - Event History – Sociology
  - Duration – Economics
- Insurance
  - Persistency
  - Claim Continuation
EXAMPLES

- Clinical, Actuarial
  - Time to death
  - Time to incidence
  - Time to recovery or relapse

- Engineering
  - Time to failure

- Operational
  - Time to new business decline, issue or completion
  - Time to claim decline, settlement, closure
IMPLICIT CONCEPTS

- State + Event
  - Clinical : Cancer Diagnosis + Cancer Death
  - Insurance : Insured Life + Death, Lapse
  - Engineering : Operating + Failure
  - Insurance : Application in UW + Issue, Decline, Complete
  - Insurance : Claim in Process + Settlement, Decline, Closure

- Time Origin: Entry into State

- Time Scale: Days, Months, Quarters, Years
OTHER EVENTS

- Exit from Population due to events other than specified event – Right Censored
  - Cancer: drop outs, non-cancer deaths
  - Mortality: lapse
  - Lapse: death
- Independent or non-informative if they are not related to the study event
- Dependent or informative if related, e.g.
  - Sick patients drop out of cancer study
  - Healthy lives lapse policies
- Censored events assumed independent
STUDY PERIOD

- Study defined by
  - Calendar period: Start Date to End Date
    - 1\textsuperscript{st} Jan 2013 – 31\textsuperscript{st} Dec 2015
  - Duration: Period from entry
    - 1\textsuperscript{st} Year/12 Months in State

- Right Censored
  - Period after study end, non-informative

- Left Truncated
  - For lives entering state before study start

- Right Truncated
  - Only lives exiting state due to decrement before study end
METHODS

- Simple
- Actuarial
- Kaplan Meier
SIMPLE METHOD

- No truncation or censoring
  - All lives enter during study period
  - No drop outs/withdrawals
  - Study ends when last life suffers decrement

- Probability of decrement before $t_i$
  - $i q_0 = i D_0 / N$
  - $i D_0$ = Decrements before time $t_i$
  - $N$ = Number of lives in study

- Survival rate, i.e. the probability of surviving to time $t_i$
  - $S(t_i) = i p_0 = (1 - i D_0 / N)$
ACTUARIAL METHOD 1

- Exposure defined over durational period divided into equal time periods: \( t_j; j = 1, n \)
- Probability of decrement in current period \( t_j \)
  - \( q_j = D_j/E_j \)
- \( D_j = \) Decrments over period \( t_j \)
- \( E_j = N_j - 0.5W_j = \) Exposure over period \( t_j \)
  - Independent or single decrement rate
  - \( E_j = N_j \) for dependent, multiple decrement rate
- \( N_j = \) Number of lives at start of period \( t_j \)
- \( W_j = \) Withdrawals over period \( t_j \)
- Force, instantaneous failure rate, hazard - \( \mu, \lambda \)
  - \( h_{j+0.5} = D_j/(E_j - 0.5D_j), h_j = (D_{j-1} + D_j)/(2N_j) \)
ACTUARIAL METHOD 2

- Probability of surviving the current period, or conditional survival given survival to end of prior period, $p_j = 1p_j$
  - $p_j = 1 - q_j = 1 - D_j/E_j$
- Survival rate, i.e. the probability of surviving past time $t_i$ from entry.
  - $S(t_i) = i_p_0 = \Pi_j (1 - D_j/E_j)$
    - $S(t_1) = (1 - D_1/E_1)$
    - $S(t_2) = (1 - D_1/E_1)(1 - D_2/E_2)$
- Life Table or Actuarial Estimator
ACTUARIAL EXAMPLE

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<th>t</th>
<th>Nt</th>
<th>Dt</th>
<th>Wt</th>
<th>Et</th>
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KAPLAN MEIER METHOD

- Take increasingly small intervals of time - Product Limit Estimator
- Each event occurs at time \( t_k; k = 1, m \), giving irregular time intervals, \((t_{k-1}, t_k)\) and for tied events in \( t_k \), assume withdrawals last
- Survival rate
  - \( S(t) = \prod_{k: t_k<t} (1 - D_k/E_k) \)
- \( D_k = \) number of decrements at time \( t_k \)
- \( E_k = N_k = \) lives at risk at time \( t_k \)
- Hazard, force of decrement, instantaneous or age specific failure rate - \( \lambda, \mu \)
  - \( h_k = D_k/E_k \)
- Monthly actuarial method (dependent rates) is equivalent to monthly KM for large sample
KAPLAN MEIER EXAMPLE

- KM curve is discontinuous, stepping down at each decrement time with censored events plotted on curve.
- Useful for small samples to see decrements, less useful for large samples.
EXAMPLES

- Persistency
- Claim Closure
PERSISTENCY EXAMPLE 1

- Annual persistency rates
- 12 Years

<table>
<thead>
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<td>1,729</td>
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<td>1,267</td>
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<td>1,433</td>
<td>171</td>
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PERSISTENCY EXAMPLE 2

- By Termination Reason

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<td>Y 4</td>
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<td>67%</td>
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<tr>
<td>Y 5</td>
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<td>63%</td>
<td>61%</td>
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<td>Y 6</td>
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<td>60%</td>
<td>58%</td>
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<tr>
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## PERSISTENCY EXAMPLE 3

### By Issue Year

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<tr>
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<td></td>
<td>49%</td>
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<tr>
<td>Y 11</td>
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<td>47%</td>
</tr>
<tr>
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*Left Truncated*  
*Right Censored*
PERSISTENCY EXAMPLE 4

- Monthly persistency rates
- 1 Year

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<th>Et</th>
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Chart showing the monthly persistency rates for 1 year.
CLAIM CLOSURE EXAMPLE 1

- Operational– turn-around time
- RBNS - pending claims with “haircut” for declines
- IBNS - stable development pattern

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<td>5,816</td>
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<td>1,911</td>
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<td>560</td>
<td>243</td>
<td>2.02%</td>
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<td>11</td>
<td>296</td>
<td>218</td>
<td>0.53%</td>
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<td>12</td>
<td>66</td>
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CLAIM CLOSURE EXAMPLE 2

- By Closed Reason

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<td>81%</td>
<td>79%</td>
</tr>
<tr>
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<td>45%</td>
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<td>3</td>
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<td>33%</td>
<td>29%</td>
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<tr>
<td>4</td>
<td>86%</td>
<td>25%</td>
<td>21%</td>
</tr>
<tr>
<td>5</td>
<td>84%</td>
<td>19%</td>
<td>15%</td>
</tr>
<tr>
<td>6</td>
<td>81%</td>
<td>14%</td>
<td>11%</td>
</tr>
<tr>
<td>7</td>
<td>77%</td>
<td>10%</td>
<td>7%</td>
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<td>8</td>
<td>75%</td>
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<td>5%</td>
</tr>
<tr>
<td>9</td>
<td>72%</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>10</td>
<td>67%</td>
<td>3%</td>
<td>2%</td>
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CLAIM CLOSURE EXAMPLE 3

- By Opened Month
- 2015 Q3TD Monthly

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PROBABILITY FUNCTIONS
RELATED FUNCTIONS

- Survival Function: Probability of surviving up to time $t$
  \[ S(t) = Pr(T \geq t) \]
- Cumulative Distribution: Probability of event occurring before time $t$
  \[ F(t) = Pr(T < t) = 1 - S(t) \]
- Density: Probability of event occurring at time $t$
  \[ f(t) = Pr(T = t) \]
- Hazard: Probability of event given survival to at time $t$
  \[ h(t) = Pr(T = t | T \geq t) = f(t)/S(t) \]
- Cumulative Hazard:
  \[ H(t) = \sum_{u<t} h(u) \approx -\log(S(t)) = -\sum_{u<t} \log(1 - h(u)) \]
  (KM)
FUNCTIONS EXAMPLE

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SURVIVAL TIME

- Mean
- Median
MEAN SURVIVAL TIME

• Expectation or average duration at event
  • \( e = \sum_i S(t_i) = \sum_i i p_0 \)
  • Area under survival curve

• Undiscounted annuity
  ▪ \( a = \sum_i i p_0 \cdot v^i \)
  ▪ With payments at times \( t_i \) then \( a, e \) are a measure of lifetime value
    ▪ DI Claims – total cost of claim
    ▪ Payout Annuities – total cost of benefit
    ▪ Life – total value of premium
  ▪ May be more meaningful than average decrement rates
## MEAN TIME EXAMPLE 1

### Time to Claim Closure

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MEAN TIME EXAMPLE 2

- Closure By Closed Reason

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MEAN TIME EXAMPLE 3

- Persistency by Termination Reason

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<td>Y 12</td>
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MEDIAN SURVIVAL TIME

- Quartiles
  - Upper Quartile $\tau_{0.75}$: $S(\tau_{0.75}) = 0.75$
  - Median $\tau_{0.5}$: $S(\tau_{0.5}) = 0.5$
  - Lower Quartile $\tau_{0.25}$: $S(\tau_{0.25}) = 0.25$

- Comments
  - Summarizes survival curve at key points
  - Where ultimate survival time not complete allows comparison at key points
  - May be more useful than mean – e.g. time at which 75% of claims are closed
MEDIAN TIME EXAMPLE 1

- Time to Claim Closure
STANDARD ERROR

- Crude: \( \sqrt{\frac{i p_0 (1 - i p_0)}{N_1}} \)
- Greenwood’s Formula
  - \( se(S(t_i)) = S(t_i) \sqrt{\sum_j \frac{D_j}{(E_j - D_j)E_j}} \)
- Raw 95% Confidence Interval
  - \( S(t_i) \pm 1.96 se(S(t_i)) \) - may be < 0 or > 1
- Log-log approach
  - \( L(t_i) = \log \left( -\log(S(t_i)) \right) \)
  - \( se(L(t_i)) = \frac{1}{\log(S(t_i))} \sqrt{\sum_j \frac{D_j}{(E_j - D_j)E_j}} \)
  - \( S(t_i)e^{\pm 1.96 se(L(t_i))} \)
STANDARD ERROR EXAMPLE 1

- Time to Claim Closure
- Small sample
- 117 claims

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STANDARD ERROR EXAMPLE 2

- Time to Claim Closure
- Full sample
- 22,549 claims

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RISK FACTORS

- Review
- Single Factor Test - Log Rank
- Multiple Factor Test – Cox Proportional Hazards
RISK FACTORS

- The risk of decrement may be affected by a number of factors within the population.
- Risk factors for mortality are gender, smoker status, underwriting class and occupation as well as age.
- The survival behavior for a population is based on the mix of risk factors within the population.
- Comparing two different populations or the same population at different times may be misleading if there are different mixes of risk factors.
- Estimating future populations requires that the mix of factors used to estimate survival behavior matches the base population to which it is applied.
- Identifying risk factors requires comparing survival behavior for each risk factor against the population.
- Risk factors divide a population with heterogeneous risk into subpopulations with homogeneous risk.
- Number of risk factors depends on purpose of analysis.
RISK FACTOR EXAMPLES
SINGLE FACTOR TEST

- Log Rank: calculate $\chi^2$ with $M - 1$ degrees of freedom for $M$ subpopulations
- $H_0$: subpopulations $i = 1, M$ have the same survival from the total population $All = \sum_i$

\[ \chi^2 = \sum_i \frac{(Obs_i - Exp_i)^2}{Exp_i} = \sum_i \frac{\left(\sum_j D_{j,i} - \sum_j (E_{j,i} \cdot D_{j,All} / E_{j,All})\right)^2}{\sum_j (E_{j,i} \cdot D_{j,All} / E_{j,All})} \]

- $Exp_i$ calculated at each time $t_j$
- $Obs_i = \sum_j D_{j,i} = D_i$ - decrements for subpopulation $i$
- For 1 DoF at 5% reject $H_0$ if $\chi^2 > 3.84$
- Only tests significance, does not quantify
SINGLE FACTOR EXAMPLE 1

<table>
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<th>Rel</th>
</tr>
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<tbody>
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<th>Et B</th>
<th>Et</th>
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<th>Dt B</th>
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<th>Exp B</th>
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\[ \chi^2 = \frac{(10 - 11.62)^2}{11.62} + \frac{(26 - 24.38)^2}{24.38} = 0.33 \]
SINGLE FACTOR EXAMPLE 2

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<th>Pop</th>
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<td>97%</td>
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<td>Tot</td>
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<table>
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<tr>
<th>t</th>
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<th>Et B</th>
<th>Et</th>
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\[ \chi^2 = \frac{(6,827 - 7,137)^2}{7,137} + \frac{(13,765 - 13,455)^2}{13,455} = 20.57 \]
HAZARD RATIO

- Hazard Ratio Estimate
  \[ HR = \frac{Obs_A/Exp_A}{Obs_B/Exp_B} \]

- Examples
  - Single Factor Ex 1: \( \frac{10/11.62}{26/24.38} = 81\% \)
    - Risk of closure is 19% lower for A compared to B but is not credible
  - Single Factor Ex 2: \( \frac{6,827/7,137}{13,765/13,455} = 94\% \)
    - Risk of closure is 6% lower for A compared to B and is credible
COX PROPORTIONAL HAZARDS

- Multiple risk factor test
- Quantifies survival behavior
- Regression:
  \[ h(t) = h_0(t)e^{(\sum_i \beta_iZ_i)} \]
  \[ \log(h(t)/h_0(t)) = \sum_i \beta_iZ_i \]
- Assumes
  - Hazards are proportional
  - Hazard ratio is constant
- If not proportional for a variable, define as separate populations or strata and investigate separately for each
Testing Proportional Hazards

Plot $\log \left( -\log(S(t)) \right)$ against $\log(t)$

<table>
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<tr>
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<th>LogH A</th>
<th>LogH B</th>
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CLOSING

Reference
Chapter 19: Survival Models
Predictive Modeling Applications in Actuarial Science Vol I
Frees, Derrig, Meyers, 2014, Cambridge

Contact
jmcgarry@insightdecision.com
Actuarial Applications of Survival Models

Ian Duncan FSA FIA FCIA FCA CSPA MAAA
May 21st 2019
SOCIETY OF ACTUARIES
Antitrust Compliance Guidelines

Active participation in the Society of Actuaries is an important aspect of membership. While the positive contributions of professional societies and associations are well-recognized and encouraged, association activities are vulnerable to close antitrust scrutiny. By their very nature, associations bring together industry competitors and other market participants.

The United States antitrust laws aim to protect consumers by preserving the free economy and prohibiting anti-competitive business practices; they promote competition. There are both state and federal antitrust laws, although state antitrust laws closely follow federal law. The Sherman Act, is the primary U.S. antitrust law pertaining to association activities. The Sherman Act prohibits every contract, combination or conspiracy that places an unreasonable restraint on trade. There are, however, some activities that are illegal under all circumstances, such as price fixing, market allocation and collusive bidding.

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- Do not discuss prices for services or products or anything else that might affect prices
- Do not discuss what you or other entities plan to do in a particular geographic or product markets or with particular customers.
- Do not speak on behalf of the SOA or any of its committees unless specifically authorized to do so.
- Do leave a meeting where any anticompetitive pricing or market allocation discussion occurs.
- Do alert SOA staff and/or legal counsel to any concerning discussions
- Do consult with legal counsel before raising any matter or making a statement that may involve competitively sensitive information.

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Ian Duncan, FSA, FIA, FCIA, FCA, CSPA, MAAA

- Health actuary since 1982; consultant since 1989.
- Founder and former president of PM company, Solucia Consulting (now SCIO Health Analytics) 1998.
- Founder and president, Santa Barbara Actuaries Inc.
- Professor of Actuarial Statistics, University of California Santa Barbara;
- Author of several books and over 60 peer-reviewed studies on healthcare management and predictive modeling.
- SOA Board of Directors 2012-5; Massachusetts Health Insurance Exchange board 2007-14.
AGENDA

Two Applications to show the potential for applying Survival Modeling in Health Actuarial work:

1. Modeling termination rates of Permanently Disabled Workers’ Compensation Claimants.
2. Modeling life expectancy of patients in hospice.
Actuaries and Predictive Modeling
Actuaries have practiced PM forever....

Actuaries have been using predictive methods since the profession began.
Actuaries have practiced PM forever....

Actuaries have been using predictive methods since the profession began. Examples:

- Mortality Rates (survival functions = parametric predictive models).

Gompertz

Makeham
Workers’ Compensation Claimant Termination
Forthcoming Paper

Study is forthcoming in Variance, the CAS peer-reviewed journal.

Using Survival Analysis to Predict Workers' Compensation Termination
Ian Duncan FSA FIA FCIA FCA CSIA MAAA1 2 Nhan Huynh MS3 Janet Duncan FCAS FSA MAAA1 and Roberto Molinari PhD1

Abstract
The standard method for calculating reserves for permanently injured worker benefits (indemnity and medical) is a combination of adjuster-estimated case reserves and reserves for incurred but not reported claims (IBNR) using a triangle method. There has been some interest in other reserving methodologies based on a calculation of future payments for the expected lifetime of the injured worker using a table of mortality rates. This method [1] is required by the State of California for estimating future medical reserves on permanently disabled workers under self-insured plans, using the most recent U.S. Life Tables as the basis. We examined the experience of an excess insurance pool using different methods to determine the appropriateness of the standard table as an estimator of claim termination. The estimated pool termination rates were significantly higher than the standard table for most ages. We also calculated termination hazard rates using both Kaplan Meier and Cox proportional hazards models and found that the modelled termination hazard was significantly higher than the standard table mortality rates. Finally, because life expectancy is only one component of the State of California reserve formula we cannot conclude that the formula results in over-reserving for future medical claims. If this approach is to continue to be used, a more appropriate method for calculating termination rates should be considered.

Background
Workers’ compensation insurance covers all work-related injuries and illnesses with medical care, wage replacement, and death benefits. In California private and public employers are required to have workers’ compensation insurance for their employees. Most public entities self-insure their exposures below a Self-Insured Retention (SIR) and insure their exposure above the SIR through an excess workers’ compensation insurance policy. The SIR is the amount specified in the insurance policy that must be paid by the insured before the excess insurance policy will respond to a loss. Public employers may purchase

1 University of California Santa Barbara; 2 Pennsylvania State University.
3 The authors acknowledge with gratitude the collaboration of CSAC-435 and its chief Actuary, John Alltop FCAS MAAA in the preparation of this study as well as UCSB students Shannon Najmcoli MS, Elizabeth Riedeli BS and Jerrick Zhang BS who performed the initial analysis.
Objective

• Analyze CA Office of Self-Insured Plans’ (OSIP) prescribed reserving methodology

  Future Medical Reserves
  = Average(past 3 years medical payments) * Future Life Expectancy

• Uses healthy life table: U.S. Life Table 2011 (Provided by CDC/NCHS National Vital Statistics System)

• Hypothesis: OSIP’s method often overestimates reserves

Objectives

  1. Test the applicability of the U.S. Life Tables to the Workers’ Comp experience
  2. Determine an appropriate Life Table for Workers’ Comp Reserves
     a. Fit models to data using only age as a predictor, like the 2011 Life Table
     b. Develop a model using covariates to predict duration of claim
Source Data

Initially

• 1,124,473 data records
• 121,110 unique claims
• 126 columns describing each claim
• Columns have information ranging from personal information to claim type
• Claim history from 1985 through 2016

We extracted an analytical file from the source data

• To include only permanent disability claims and covariates for modeling
Creation of Analytical File

• Developed an algorithm to identify the permanent disability (PD) claims.
• Recoded variable ‘Entity Group’ and ‘Cause of Loss’ to be more accurate.
• Completed ‘Body Part Code’ based on existing code and body part descriptions.
• Created Severity variable:

\[ \text{Severity} = \frac{\text{Incurred Medical Expenses}}{\text{Duration of Claim}} \]
Methodology

• Used ‘Date Closed,’ when available, to determine duration for those that closed.
• Created Variable ‘Status’ to tell if the claim is observed (closed) or censored (still open).
• Derived $\hat{q}_x$ and fitted models to the estimates.
• Used Imputation Methods to estimate the missing gender values.
• Conducted survival analysis using Cox Proportional Hazard models
  • Checked proportional hazard assumptions.
Estimates

• Life Table Estimation ($q_x$) for our data
  \[ \hat{q}_x = \frac{d_x}{l_x - 0.5w_x} \]

  \( \hat{q}_x := \text{Rate of Mortality in Interval } x \text{ to } x + 1 \)
  \( d_x := \text{Number of Deaths in Interval } x \text{ to } x + 1 \)
  \( l_x := \text{Number Alive in Interval } x \text{ to } x + 1 \)
  \( w_x := \text{Number of Terminations} + \text{Number Censored in Interval } x \text{ to } x + 1 \)

• Estimates allow us to compare our data with the 2011 U.S. Life Table.
  • Provided by CDC/NCHS National Vital Statistics System
Comparison of 2011 Life Table and Estimates of $q_x$: Males

- **2011 Life Table $q_x$: Males**
- **Estimates $q_x$: Males**
- **95% Confidence Interval**

Probability of Claim Closure vs. Age (in Years)
Model-fitting to $\hat{q}_x$ Estimates

• Gompertz Fitted Model: $\hat{q}_x = e^{-7.19859 + 0.08404x}$

• Polynomial Fitted Model

$$\hat{q}_x = 0.04148 + 1.06074x - 0.15914x^2 + 0.46304x^3 - 0.21410x^4 + 0.40654x^5 + 0.02539x^6 + 0.15662x^7$$

• Quadratic Fitted Model

$$\hat{q}_x = 0.07755 + 0.72185x + 0.35528x^2$$

• Models selection based on lowest AIC ($\text{AIC} = 2k - 2\ln(\hat{L})$)

  • Where $k =$ no. parameters and $L$ is the maximum likelihood.
Fitted Models vs. $q_x$ Estimates

- Gompertz Model
- Polynomial Model
- Quadratic Model
2011 U.S. Life Table vs. Fitted Models

- Probability of Closure: $q_X$
- Age (in Years)

- Red: 2011 U.S. Life Table
- Green: Gompertz Model
- Purple: Polynomial Model
- Blue: Quadratic Model
Cox Proportional Hazard Model

- Incorporates covariates in order to better predict survival.
- Predicts the hazard rate ($\mu = \text{force of mortality}$).
- Reminder: (Exam MLC)
- R Routine for estimation.

Proportional Hazard Assumptions:
- Hazard functions of each covariate’s categories (i.e. their possible values) are proportional with time
- Can be tested by examining Log-log plots and Schoenfeld Residuals

$$tp_x = \exp \left\{ - \int_0^t \mu_{x+s} ds \right\}$$

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<tr>
<td>Years Employed at DOL</td>
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<tr>
<td>Body Part Code</td>
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<tr>
<td>Cause of Loss Code</td>
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<tr>
<td>Severity</td>
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<tr>
<td>Gender</td>
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Data Imputation

~19% of ‘Gender’ Values were “Unknown/Other”
  • Use MICE package in R to impute missing values
    (MICE = Multivariate Imputation by Chained Equations)
    https://cran.r-project.org/web/packages/mice/mice.pdf

How it Works:
MICE uses observed values to predict missing values on a variable by variable basis using logistic regression.
  • Compare models with and without the imputed data

Interpretation from: Analytics Vidhya https://www.analyticsvidhya.com/blog/2016/03/tutorial-powerful-packages-imputing-missing-values/
Evaluation: Testing PH Assumptions

![Log-Log Plot of Imputed Gender](image)
Evaluation: Testing PH Assumptions

Schoenfeld Residuals

BetaHat for Imputed Gender

Duration

0.77 2 2.8 3.6 4.7 6.2 8.8 18
Cox Proportional Hazard Models

• \(h_{\text{Imputed}}(t)\)
  \[ = e^{\beta(\text{Entity Group}) - .000319(\text{Severity})} \]
  \[ \times e^{-0.0871(\text{Years Employed at DOL}) - .532(\text{Gender})} \]
  \[ \times e^{-0.0144(\text{Body Part Code}) - .007(\text{Age at DOL})} \]
  \[ \times e^{-0.000294(\text{Severity*Years Employed at DOL}) - .000502(\text{Severity*Gender})} \]
  \[ \times e^{.00883(\text{Gender*Age at DOL})} \]

• Under \(H_0: \text{KM model is more accurate than Cox PH model}\), the likelihood ratio test returns a p-value of zero. Therefore, we reject \(H_0\).
Concluding Remarks

• Based on our data, the 2011 U.S. Life Table underestimates the probability of closure ($q_x$) and would therefore overestimate the Future Medical Reserves.

• Our best model, $h_{No Imputation}(t)$, better represents probability of closure based on the covariates.

• With the new model, CSAC-EIA could better estimate their Future Medical Reserves.
Predicting Life Expectancy of Patients in Hospice
In the prior (Workers’ Compensation) model we saw how we could model survival with a single variable (age; $x$) as well as the dependence of survival on multiple co-variates (Cox PH model).

One issue with the standard Cox model is that the values of the co-variates are fixed at entry; this is fine for (say) sex or admitting diagnosis; it may not be appropriate when values vary over time.

In this (hospice patient survival) model we extend the co-variates by allowing their values to vary with time.
• Data for this study represent patients admitted to hospice; to be eligible for the CMS hospice benefit, a patient must be certified by two clinicians to have \( \leq 6 \) months’ life expectancy.

• Data include demographics, diagnoses and a full prescribing record (both palliative and therapeutic).
Data

Age at Death (2015 Admissions)

Histogram of Age at Death in 2015 Enrollments

Status as-of July 2017
### Table 1  Prevalence of Four (CMS) Drug Types: 2015 Admissions

<table>
<thead>
<tr>
<th></th>
<th>Analgesic</th>
<th>Anti-cholinergic</th>
<th>Anti-nausea</th>
<th>Anxiolytic</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients with at least one of 4 drug types</td>
<td>69,335</td>
<td>54,984</td>
<td>25,971</td>
<td>59,669</td>
<td>209,959</td>
</tr>
<tr>
<td>Number of patients without any of 4 drug types (2.6%)</td>
<td>73,068</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>93.4%</td>
<td>74.0%</td>
<td>35.0%</td>
<td>80.3%</td>
<td>282.7%</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>74,275</td>
</tr>
</tbody>
</table>
Distribution of Other (potentially-ineffective) Drugs

<table>
<thead>
<tr>
<th>Drug classes</th>
<th>Cancer</th>
<th>Heart</th>
<th>Lung</th>
<th>Dementia</th>
<th>Liver</th>
<th>Gastro-intestinal</th>
<th>Kidney</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antibiotics</td>
<td>38.5%</td>
<td>39.7%</td>
<td>52.1%</td>
<td>61.7%</td>
<td>37.8%</td>
<td>53.2%</td>
<td>33.2%</td>
</tr>
<tr>
<td>Antihypertensives</td>
<td>23.0%</td>
<td>69.1%</td>
<td>31.9%</td>
<td>33.3%</td>
<td>29.5%</td>
<td>27.6%</td>
<td>61.8%</td>
</tr>
<tr>
<td>Proton pump inhibitors</td>
<td>48.9%</td>
<td>29.5%</td>
<td>37.3%</td>
<td>25.9%</td>
<td>57.9%</td>
<td>36.2%</td>
<td>35.1%</td>
</tr>
<tr>
<td>Inhaler</td>
<td>19.6%</td>
<td>15.9%</td>
<td>48.5%</td>
<td>6.6%</td>
<td>16.7%</td>
<td>7.4%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Anticoagulants</td>
<td>6.6%</td>
<td>17.8%</td>
<td>6.5%</td>
<td>3.4%</td>
<td>0.7%</td>
<td>6.5%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Antifungals</td>
<td>5.9%</td>
<td>4.4%</td>
<td>5.1%</td>
<td>5.6%</td>
<td>4.6%</td>
<td>5.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Diabetes medications</td>
<td>3.5%</td>
<td>3.4%</td>
<td>2.5%</td>
<td>2.3%</td>
<td>4.6%</td>
<td>1.8%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Antivirals</td>
<td>1.3%</td>
<td>0.7%</td>
<td>0.9%</td>
<td>1.4%</td>
<td>0.7%</td>
<td>0.3%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Statins (HMG CoA reductase inhibitors)</td>
<td>0.2%</td>
<td>1.0%</td>
<td>0.4%</td>
<td>0.3%</td>
<td>0.5%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Dementia medications</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>1.4%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Appetite stimulant</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Vitamins &amp; supplements</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Column total</td>
<td>18,619</td>
<td>7,856</td>
<td>6,879</td>
<td>4,809</td>
<td>668</td>
<td>469</td>
<td>402</td>
</tr>
<tr>
<td>Total patients</td>
<td>12,612</td>
<td>4,326</td>
<td>3,714</td>
<td>3,384</td>
<td>437</td>
<td>340</td>
<td>262</td>
</tr>
<tr>
<td>Prescription per class</td>
<td>1.48</td>
<td>1.82</td>
<td>1.85</td>
<td>1.42</td>
<td>1.53</td>
<td>1.38</td>
<td>1.53</td>
</tr>
</tbody>
</table>
• There is considerable literature on the use of other (potentially ineffective) drugs in hospice; particularly about the rate at which patients are weaned from these drugs as use of other (palliative) drugs increases.

• Analysis shows the rate at which patients with different diagnoses cease to use classes of therapeutic drugs.
Another application of Survival: Use of other (PI) Drugs

- Kaplan-Meier approach estimates the (survival functions for) the duration of patients taking additional drugs during hospice admission.
- y-axis: the survival probability; x-axis: duration of drug prescription (days).
Relative Drug Strength vs. Diagnosis Categories

Analgesic Drugs

Mean

Median

D = Death date

Analgesic dose

(-\infty, D-360)
[D-380, D-240)
[D-240, D-180)
[D-180, D-120)
[D-120, D-90)
[D-90, D-60)
[D-60, D-30)
[D-30, D-19)
[D-19, D-7)
[D-7, D-3)
[D-3, D]
Relative Drug Strength vs. Diagnosis Categories

Analgesic Drug Strength by Diagnosis

- cancer
- lung
- psych
- heart
- kidney

D = Death date

Analgesic dose

-360, D-240, D-180, D-120, D-90, D-60, D-30, D-15, D-7, D-3, D
Similar Pattern in other drug classes

**Anti-cholinergic**

**Anxiolytic**

**Anti-nausea**
\[ \lambda(t; \{D_i(v_1), v_1 \in [o,t], N_i^{Lax}(v_2), v_2 \in [o,t], N_i^{Antinau}(v_3), v_3 \in [o,t], N_i^{Anxio}(v_4), v_4 \in [o,t]\}) \]

\[ = \lambda_0(t)e^{\beta X + \alpha D_i(t) + \delta N_i^{Lax}(t) + \phi N_i^{Antinau}(t) + \rho N_i^{Anxio}(t)} \]

where

\[ D_i(v_1), v_1 \in [o,t], N_i^{Lax}(v_2), v_2 \in [o,t], N_i^{Antinau}(v_3), v_3 \in [o,t], N_i^{Anxio}(v_4), v_4 \in [o,t] \]
### Time-dependent Cox Model

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Coefficient</th>
<th>Exp(coef) (Hazard ratio)</th>
<th>P-value</th>
<th>Lower 95 % (of HR)</th>
<th>Upper 95% (of HR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender Male</td>
<td>0.117</td>
<td>1.124</td>
<td>0</td>
<td>1.117</td>
<td>1.132</td>
</tr>
<tr>
<td>Level Of Care HC (Home Care)</td>
<td>0.265</td>
<td>1.303</td>
<td>0</td>
<td>1.273</td>
<td>1.334</td>
</tr>
<tr>
<td>Level Of Care IPU (In-Patient Unit)</td>
<td>1.454</td>
<td>4.28</td>
<td>0</td>
<td>4.152</td>
<td>4.411</td>
</tr>
<tr>
<td>Level Of Care LTC (Long-Term care)</td>
<td>0.397</td>
<td>1.488</td>
<td>0</td>
<td>1.453</td>
<td>1.524</td>
</tr>
<tr>
<td>Age</td>
<td>0.002</td>
<td>1.002</td>
<td>0</td>
<td>1.002</td>
<td>1.002</td>
</tr>
<tr>
<td>Infection</td>
<td>0.323</td>
<td>1.381</td>
<td>0</td>
<td>1.342</td>
<td>1.421</td>
</tr>
<tr>
<td>Cancer</td>
<td>0.23</td>
<td>1.259</td>
<td>0</td>
<td>1.246</td>
<td>1.272</td>
</tr>
<tr>
<td>Diabetes</td>
<td>-0.003</td>
<td>0.997</td>
<td>0.7</td>
<td>0.984</td>
<td>1.011</td>
</tr>
<tr>
<td>Gastrointestinal</td>
<td>-0.031</td>
<td>0.97</td>
<td>0.001</td>
<td>0.952</td>
<td>0.988</td>
</tr>
<tr>
<td>Liver</td>
<td>0.249</td>
<td>1.283</td>
<td>0</td>
<td>1.251</td>
<td>1.316</td>
</tr>
<tr>
<td>Musculoskeletal</td>
<td>0.07</td>
<td>1.073</td>
<td>0</td>
<td>1.045</td>
<td>1.101</td>
</tr>
<tr>
<td>Psych</td>
<td>-0.224</td>
<td>0.799</td>
<td>0</td>
<td>0.791</td>
<td>0.806</td>
</tr>
<tr>
<td>Lung</td>
<td>-0.073</td>
<td>0.93</td>
<td>0</td>
<td>0.921</td>
<td>0.939</td>
</tr>
<tr>
<td>Heart</td>
<td>0.043</td>
<td>1.044</td>
<td>0</td>
<td>1.035</td>
<td>1.054</td>
</tr>
<tr>
<td>Kidney</td>
<td>0.213</td>
<td>1.238</td>
<td>0</td>
<td>1.219</td>
<td>1.257</td>
</tr>
<tr>
<td>Injury</td>
<td>0.199</td>
<td>1.221</td>
<td>0</td>
<td>1.096</td>
<td>1.359</td>
</tr>
<tr>
<td>HCCs risk score</td>
<td>0.028</td>
<td>1.028</td>
<td>0</td>
<td>1.022</td>
<td>1.034</td>
</tr>
<tr>
<td>Analgesic dose (time-dependent)</td>
<td>0.068</td>
<td>1.07</td>
<td>0</td>
<td>1.07</td>
<td>1.071</td>
</tr>
<tr>
<td>Number of laxative prescriptions (time-dependent)</td>
<td>0.002</td>
<td>1.002</td>
<td>0.008</td>
<td>1.001</td>
<td>1.004</td>
</tr>
<tr>
<td>Number of anti-nausea prescriptions (time-dependent)</td>
<td>0.005</td>
<td>1.005</td>
<td>0.004</td>
<td>1.002</td>
<td>1.009</td>
</tr>
<tr>
<td>Number of anxiolytic (time-dependent)</td>
<td>0.116</td>
<td>1.123</td>
<td>0</td>
<td>1.122</td>
<td>1.125</td>
</tr>
</tbody>
</table>
Time-dependent Log-normal Accelerated Failure Time Model

\[ S(t; \{X, N_i(u), u \in [0,t], D_i(v), v \in [0,t]\}) = 1 - \Phi \left( \frac{\log(t) - (BX + \gamma N_i(t) + \alpha D_i(t))}{\sigma} \right) \]

Where \( N_i(u) \) is the # of prescriptions

and \( D_i(v) \) is the prescription dosage
### Time-dependent Log-normal Accelerated Failure Time Model

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Coefficient</th>
<th>S.E</th>
<th>Exp(coef) (Acceleration Factor)</th>
<th>Lower 95% (of AF)</th>
<th>Upper 95% (of AF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meanlog (Intercept)</td>
<td>4.375</td>
<td>0.026</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sdlog ((\sigma))</td>
<td>1.466</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender Male</td>
<td>-0.166</td>
<td>0.005</td>
<td>0.847</td>
<td>0.838</td>
<td>0.856</td>
</tr>
<tr>
<td>Level Of Care HC (Home Care)</td>
<td>-0.411</td>
<td>0.017</td>
<td>0.663</td>
<td>0.641</td>
<td>0.685</td>
</tr>
<tr>
<td>Level Of Care IPU (In-Patient Unit)</td>
<td>-1.854</td>
<td>0.022</td>
<td>0.157</td>
<td>0.150</td>
<td>0.164</td>
</tr>
<tr>
<td>Level Of Care LTC (Long-Term care)</td>
<td>-0.607</td>
<td>0.017</td>
<td>0.545</td>
<td>0.527</td>
<td>0.563</td>
</tr>
<tr>
<td>Age</td>
<td>0.001</td>
<td>0.000</td>
<td>1.001</td>
<td>1.000</td>
<td>1.001</td>
</tr>
<tr>
<td>Infection</td>
<td>-0.517</td>
<td>0.021</td>
<td>0.596</td>
<td>0.572</td>
<td>0.622</td>
</tr>
<tr>
<td>Cancer</td>
<td>-0.238</td>
<td>0.008</td>
<td>0.788</td>
<td>0.776</td>
<td>0.800</td>
</tr>
<tr>
<td>Diabetes</td>
<td>0.026</td>
<td>0.010</td>
<td>1.026</td>
<td>1.006</td>
<td>1.046</td>
</tr>
<tr>
<td>Gastrointestinal</td>
<td>0.048</td>
<td>0.014</td>
<td>1.049</td>
<td>1.021</td>
<td>1.078</td>
</tr>
<tr>
<td>Liver</td>
<td>-0.332</td>
<td>0.019</td>
<td>0.718</td>
<td>0.691</td>
<td>0.745</td>
</tr>
<tr>
<td>Musculoskeletal</td>
<td>-0.086</td>
<td>0.020</td>
<td>0.918</td>
<td>0.883</td>
<td>0.954</td>
</tr>
<tr>
<td>Psych</td>
<td>0.358</td>
<td>0.007</td>
<td>1.430</td>
<td>1.411</td>
<td>1.450</td>
</tr>
<tr>
<td>Lung</td>
<td>0.071</td>
<td>0.007</td>
<td>1.074</td>
<td>1.058</td>
<td>1.090</td>
</tr>
<tr>
<td>Heart</td>
<td>-0.049</td>
<td>0.007</td>
<td>0.952</td>
<td>0.939</td>
<td>0.965</td>
</tr>
<tr>
<td>Kidney</td>
<td>-0.314</td>
<td>0.011</td>
<td>0.731</td>
<td>0.715</td>
<td>0.747</td>
</tr>
<tr>
<td>Injury</td>
<td>-0.297</td>
<td>0.081</td>
<td>0.743</td>
<td>0.634</td>
<td>0.871</td>
</tr>
<tr>
<td>HCCs risk score</td>
<td>-0.020</td>
<td>0.005</td>
<td>0.980</td>
<td>0.972</td>
<td>0.989</td>
</tr>
<tr>
<td>Analgesic dose (time-dependent)</td>
<td>-0.098</td>
<td>0.001</td>
<td>0.907</td>
<td>0.906</td>
<td>0.908</td>
</tr>
<tr>
<td>Number of laxative prescriptions (time-dependent)</td>
<td>-0.004</td>
<td>0.001</td>
<td>0.996</td>
<td>0.993</td>
<td>0.999</td>
</tr>
<tr>
<td>Number of anti-nausea prescriptions (time-dependent)</td>
<td>0.002</td>
<td>0.003</td>
<td>1.002</td>
<td>0.996</td>
<td>1.008</td>
</tr>
<tr>
<td>Number of anxiolytic (time-dependent)</td>
<td>-0.215</td>
<td>0.002</td>
<td>0.807</td>
<td>0.804</td>
<td>0.810</td>
</tr>
</tbody>
</table>
Prediction Errors: Time-dependent AFT model
### Example of Use of the Model

<table>
<thead>
<tr>
<th>CphPatient ID</th>
<th>t</th>
<th>Dose</th>
<th>Laxative</th>
<th>Anti-nausea</th>
<th>Anxiolytic</th>
<th>Actual LOS</th>
<th>Remaining expected life time</th>
<th>Remaining LOS</th>
<th>Absolute difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient 3:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>xxxx545</td>
<td>0</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>21.682</td>
<td>6.0</td>
<td>15.682</td>
</tr>
<tr>
<td>xxxx545</td>
<td>1</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>21.643</td>
<td>5.0</td>
<td>16.643</td>
</tr>
<tr>
<td>xxxx545</td>
<td>2</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>21.605</td>
<td>4.0</td>
<td>17.605</td>
</tr>
<tr>
<td>xxxx545</td>
<td>3</td>
<td>0.000</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>5.296</td>
<td>3.0</td>
<td>2.296</td>
</tr>
<tr>
<td>xxxx545</td>
<td>4</td>
<td>2.571</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>4.187</td>
<td>2.0</td>
<td>2.187</td>
</tr>
<tr>
<td>xxxx545</td>
<td>5</td>
<td>2.571</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>4.186</td>
<td>1.0</td>
<td>3.186</td>
</tr>
<tr>
<td>xxxx545</td>
<td>6</td>
<td>2.571</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>7</td>
<td>4.184</td>
<td>0.0</td>
<td>4.184</td>
</tr>
</tbody>
</table>
Q&A