Exam M Fall 2006

FINAL ANSWER KEY

Question #	*Answer*		*Question #*	*Answer*
1	A		21	A
2	D		22	D
3	B		23	C
4	E		24	D
5	C		25	B
6	B		26	A
7	D		27	E
8	E		28	C
9	D		29	E
10	D		30	A
11	A		31	C
12	D		32	D
13	A		33	E
14	C		34	B
15	C		35	C
16	E		36	D
17	C		37	B
18	B		38	B
19	A		39	E
20	B		40	D

****BEGINNING OF EXAMINATION****

1. Michael, age 45, is a professional motorcycle jumping stuntman who plans to retire in three years. He purchases a three-year term insurance policy. The policy pays 500,000 for death from a stunt accident and nothing for death from other causes. The benefit is paid at the end of the year of death.

You are given:

(i) $i = 0.08$

(ii)

x	$l_x^{(\tau)}$	$d_x^{(-s)}$	$d_x^{(s)}$
45	2500	10	4
46	2486	15	5
47	2466	20	6

where $d_x^{(s)}$ represents deaths from stunt accidents and $d_x^{(-s)}$ represents deaths from other causes.

(iii) Level annual benefit premiums are payable at the beginning of each year.

(iv) Premiums are determined using the equivalence principle.

Calculate the annual benefit premium.

(A) 920

(B) 1030

(C) 1130

(D) 1240

(E) 1350

2. You are given the survival function

$$s(x) = 1 - (0.01x)^2, \quad 0 \le x \le 100$$

Calculate $\overset{\circ}{e}_{30:\overline{50|}}$, the 50-year temporary complete expectation of life of (30).

(A) 27

(B) 30

(C) 34

(D) 37

(E) 41

3. For a fully discrete whole life insurance of 1000 on (50), you are given:

(i) $1000P_{50} = 25$

(ii) $1000A_{61} = 440$

(iii) $1000q_{60} = 20$

(iv) $i = 0.06$

Calculate $1000\,{}_{10}V_{50}$.

(A) 170

(B) 172

(C) 174

(D) 176

(E) 178

4. For a pension plan portfolio, you are given:

(i) 80 individuals with mutually independent future lifetimes are each to receive a whole life annuity-due.

(ii) $i = 0.06$

(iii)

Age	Number of annuitants	Annual annuity payment	$\ddot{a}_x$	A_x	^2A_x
65	50	2	9.8969	0.43980	0.23603
75	30	1	7.2170	0.59149	0.38681

Using the normal approximation, calculate the 95th percentile of the distribution of the present value random variable of this portfolio.

(A) 1220

(B) 1239

(C) 1258

(D) 1277

(E) 1296

5. Your company sells a product that pays the cost of nursing home care for the remaining lifetime of the insured.

(i) Insureds who enter a nursing home remain there until death.

(ii) The force of mortality, μ, for each insured who enters a nursing home is constant.

(iii) μ is uniformly distributed on the interval [0.5, 1].

(iv) The cost of nursing home care is 50,000 per year payable continuously.

(v) $\delta = 0.045$

Calculate the actuarial present value of this benefit for a randomly selected insured who has just entered a nursing home.

(A) 60,800

(B) 62,900

(C) 65,100

(D) 67,400

(E) 69,800

6. Loss amounts have the distribution function

$$F(x) = \begin{cases} (x/100)^2, & 0 \le x \le 100 \\ 1, & 100 < x \end{cases}$$

An insurance pays 80% of the amount of the loss in excess of an ordinary deductible of 20, subject to a maximum payment of 60 per loss.

Calculate the conditional expected claim payment, given that a payment has been made.

(A) 37

(B) 39

(C) 43

(D) 47

(E) 49

7. A compound Poisson claim distribution has $\lambda = 5$ and individual claim amounts distributed as follows:

x	$f_X(x)$	
5	0.6	
k	0.4	where $k > 5$

The expected cost of an aggregate stop-loss insurance subject to a deductible of 5 is 28.03.

Calculate k.

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

8. The time elapsed between claims processed is modeled such that V_k represents the time elapsed between processing the k-1$^{\text{th}}$ and k^{th} claim. ($V_1 =$ time until the first claim is processed).

You are given:

(i) $V_1, V_2, \ldots$ are mutually independent.

(ii) The pdf of each V_k is $f(t) = 0.2e^{-0.2t}$, $t > 0$, where t is measured in minutes.

Calculate the probability of at least two claims being processed in a ten minute period.

(A) 0.2

(B) 0.3

(C) 0.4

(D) 0.5

(E) 0.6

9. A casino has a game that makes payouts at a Poisson rate of 5 per hour and the payout amounts are 1, 2, 3,… without limit. The probability that any given payout is equal to i is $1/2^i$. Payouts are independent.

Calculate the probability that there are no payouts of 1, 2, or 3 in a given 20 minute period.

(A) 0.08

(B) 0.13

(C) 0.18

(D) 0.23

(E) 0.28

10. You arrive at a subway station at 6:15. Until 7:00, trains arrive at a Poisson rate of 1 train per 30 minutes. Starting at 7:00, they arrive at a Poisson rate of 2 trains per 30 minutes.

Calculate your expected waiting time until a train arrives.

(A) 24 minutes

(B) 25 minutes

(C) 26 minutes

(D) 27 minutes

(E) 28 minutes

11. For a fully discrete 20-year endowment insurance of 10,000 on (45) that has been in force for 15 years, you are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

(iii) At issue, the benefit premium was calculated using the equivalence principle.

(iv) When the insured decides to stop paying premiums after 15 years, the death benefit remains at 10,000 but the pure endowment value is reduced such that the expected prospective loss at age 60 is unchanged.

Calculate the reduced pure endowment value.

(A) 8120

(B) 8500

(C) 8880

(D) 9260

(E) 9640

12. For a whole life insurance of 1 on (x) with benefits payable at the moment of death, you are given:

(i) $$\delta_t = \begin{cases} 0.02, & t < 12 \\ 0.03, & t \geq 12 \end{cases}$$

(ii) $$\mu_x(t) = \begin{cases} 0.04, & t < 5 \\ 0.05, & t \geq 5 \end{cases}$$

Calculate the actuarial present value of this insurance.

(A) 0.59

(B) 0.61

(C) 0.64

(D) 0.66

(E) 0.68

13. For a fully continuous whole life insurance on (x), you are given:

(i) The benefit is 2000 for death by accidental means (decrement 1).

(ii) The benefit is 1000 for death by other means (decrement 2).

(iii) The initial expense at issue is 50.

(iv) Settlement expenses are 5% of the benefit, payable at the moment of death.

(v) Maintenance expenses are 3 per year, payable continuously.

(vi) The gross or contract premium is 100 per year, payable continuously.

(vii) $\mu_x^{(1)}(t) = 0.004$, $\quad t > 0$

(viii) $\mu_x^{(2)}(t) = 0.040$, $\quad t > 0$

(ix) $\delta = 0.05$

Calculate the actuarial present value at issue of the insurer's expense-augmented loss random variable for this insurance.

(A) – 446

(B) – 223

(C) 0

(D) 223

(E) 446

14. A homogeneous Markov model has three states representing the status of the members of a population.

State 1 = healthy, no benefits
State 2 = disabled, receiving Home Health Care benefits
State 3 = disabled, receiving Nursing Home benefits

The annual transition matrix is given by:

$$\begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.05 & 0.90 & 0.05 \\ 0.00 & 0.00 & 1.00 \end{pmatrix}$$

Transitions occur at the end of each year.
At the start of year 1, there are 50 members, all in state 1, healthy.

Calculate the variance of the number of those 50 members who will be receiving Nursing Home benefits during year 3.

(A) 2.3

(B) 2.7

(C) 4.4

(D) 4.5

(E) 4.6

15. A non-homogenous Markov model has:

(i) Three states: 0, 1, and 2

(ii) Annual transition matrix Q_n as follows:

$$Q_n = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{for } n = 0 \text{ and } 1, \text{ and}$$

$$Q_n = \begin{pmatrix} 0 & 0.3 & 0.7 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{for } n = 2, 3, 4, \ldots$$

An individual starts out in state 0 and transitions occur mid-year.

An insurance is provided whereby:

(i) A premium of 1 is paid at the beginning of each year that an individual is in state 0 or 1.

(ii) A benefit of 4 is paid at the end of any year that the individual is in state 1 at the end of the year.

(iii) $i = 0.1$

Calculate the actuarial present value of premiums minus the actuarial present value of benefits at the start of this insurance.

(A) – 0.17

(B) 0.00

(C) 0.34

(D) 0.50

(E) 0.66

16. You are given the following information on participants entering a special 2-year program for treatment of a disease:

(i) Only 10% survive to the end of the second year.

(ii) The force of mortality is constant within each year.

(iii) The force of mortality for year 2 is three times the force of mortality for year 1.

Calculate the probability that a participant who survives to the end of month 3 dies by the end of month 21.

(A) 0.61

(B) 0.66

(C) 0.71

(D) 0.75

(E) 0.82

17. In a population, non-smokers have a force of mortality equal to one half that of smokers.

For non-smokers, $l_x = 500(110 - x), \quad 0 \le x \le 110$.

Calculate $\mathring{e}_{20:25}$ for a smoker (20) and a non-smoker (25) with independent future lifetimes.

(A) 18.3

(B) 20.4

(C) 22.1

(D) 24.5

(E) 26.8

18. For a special fully discrete 20-year term insurance on (30):

(i) The death benefit is 1000 during the first ten years and 2000 during the next ten years.

(ii) The benefit premium, determined by the equivalence principle, is π for each of the first ten years and 2π for each of the next ten years.

(iii) $\ddot{a}_{30:\overline{20}|} = 15.0364$

(iv)

x	$\ddot{a}_{x:\overline{10}|}$	$1000A^{1}_{x:\overline{10}|}$
30	8.7201	16.66
40	8.6602	32.61

Calculate π.

(A) 2.9

(B) 3.0

(C) 3.1

(D) 3.2

(E) 3.3

19. For a fully discrete whole life insurance of 25,000 on (25), you are given:

(i) $P_{25} = 0.01128$

(ii) $P_{25:\overline{15|}}^{\;1} = 0.05107$

(iii) $P_{25:\overline{15|}} = 0.05332$

Calculate $25,000\,{}_{15}V_{25}$.

(A) 4420

(B) 4460

(C) 4500

(D) 4540

(E) 4580

20. For a special investment product, you are given:

(i) All deposits are credited with 75% of the annual equity index return, subject to a minimum guaranteed crediting rate of 3%.

(ii) The annual equity index return is normally distributed with a mean of 8% and a standard deviation of 16%.

(iii) For a random variable *X* which has a normal distribution with mean μ and standard deviation σ, you are given the following limited expected values:

$E[X \wedge 3\%]$		
	$\mu = 6\%$	$\mu = 8\%$
$\sigma = 12\%$	-0.43%	0.31%
$\sigma = 16\%$	-1.99%	-1.19%

$E[X \wedge 4\%]$		
	$\mu = 6\%$	$\mu = 8\%$
$\sigma = 12\%$	0.15%	0.95%
$\sigma = 16\%$	-1.43%	-0.58%

Calculate the expected annual crediting rate.

(A) 8.9%

(B) 9.4%

(C) 10.7%

(D) 11.0%

(E) 11.6%

21. Aggregate losses are modeled as follows:

(i) The number of losses has a Poisson distribution with $\lambda = 3$.

(ii) The amount of each loss has a Burr (Burr Type XII, Singh-Maddala) distribution with $\alpha = 3, \ \theta = 2$, and $\gamma = 1$.

(iii) The number of losses and the amounts of the losses are mutually independent.

Calculate the variance of aggregate losses.

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

22. The annual number of doctor visits for each individual in a family of 4 has a geometric distribution with mean 1.5. The annual numbers of visits for the family members are mutually independent. An insurance pays 100 per doctor visit beginning with the 4^{th} visit per family.

Calculate the expected payments per year for this family.

(A) 320

(B) 323

(C) 326

(D) 329

(E) 332

23. You are given 3 mortality assumptions:

(i) Illustrative Life Table (ILT),

(ii) Constant force model (CF), where $s(x) = e^{-\mu x}, \;\; x \geq 0$

(iii) DeMoivre model (DM), where $s(x) = 1 - \dfrac{x}{\omega}, \;\; 0 \leq x \leq \omega, \;\; \omega \geq 72$.

For the constant force and DeMoivre models, ${}_2p_{70}$ is the same as for the Illustrative Life Table.

Rank $e_{70:\overline{2|}}$ for these 3 models.

(A) ILT < CF < DM

(B) ILT < DM < CF

(C) CF < DM < ILT

(D) DM < CF < ILT

(E) DM < ILT < CF

24. A population of 1000 lives age 60 is subject to 3 decrements, death (1), disability (2), and retirement (3). You are given:

(i) The following absolute rates of decrement:

x	$q_x'^{(1)}$	$q_x'^{(2)}$	$q_x'^{(3)}$
60	0.010	0.030	0.100
61	0.013	0.050	0.200

(ii) Decrements are uniformly distributed over each year of age in the multiple decrement table.

Calculate the expected number of people who will retire before age 62.

(A) 248

(B) 254

(C) 260

(D) 266

(E) 272

25. You are given:

(i) The future lifetimes of (40) and (50) are independent.

(ii) The survival function for (40) is based on a constant force of mortality, $\mu = 0.05$.

(iii) The survival function for (50) follows DeMoivre's law with $\omega = 110$.

Calculate the probability that (50) dies within 10 years and dies before (40).

(A) 10%

(B) 13%

(C) 16%

(D) 19%

(E) 25%

26. Oil wells produce until they run dry. The survival function for a well is given by:

t (years)	$S(t)$
0	1.00
1	0.90
2	0.80
3	0.60
4	0.30
5	0.10
6	0.05
7	0.00

An oil company owns 10 wells age 3. It insures them for 1 million each against failure for two years where the loss is payable at the end of the year of failure.

You are given:

(i) R is the present-value random variable for the insurer's aggregate losses on the 10 wells.

(ii) The insurer actually experiences 3 failures in the first year and 5 in the second year.

(iii) $i = 0.10$

Calculate the ratio of the actual value of R to the expected value of R.

(A) 0.94

(B) 0.96

(C) 0.98

(D) 1.00

(E) 1.02

27. For a fully discrete 2-year term insurance of 1 on (x):

(i) $q_x = 0.1 \quad q_{x+1} = 0.2$

(ii) $v = 0.9$

(iii) ${}_1L$ is the prospective loss random variable at time 1 using the premium determined by the equivalence principle.

Calculate $\text{Var}\left({}_1L \middle| K(x) > 0\right)$.

(A) 0.05

(B) 0.07

(C) 0.09

(D) 0.11

(E) 0.13

28. For a fully continuous whole life insurance of 1 on (x):

(i) $\bar{A}_x = 1/3$

(ii) $\delta = 0.10$

(iii) L is the loss at issue random variable using the premium based on the equivalence principle.

(iv) $\text{Var}[L] = 1/5$

(v) L' is the loss at issue random variable using the premium π.

(vi) $\text{Var}[L'] = 16/45$.

Calculate π.

(A) 0.05

(B) 0.08

(C) 0.10

(D) 0.12

(E) 0.15

29. A risk has a loss amount which has a Poisson distribution with mean 3.

An insurance covers the risk with an ordinary deductible of 2. An alternative insurance replaces the deductible with coinsurance α, which is the proportion of the loss paid by the insurance, so that the expected insurance cost remains the same.

Calculate α.

(A) 0.22

(B) 0.27

(C) 0.32

(D) 0.37

(E) 0.42

30. You are the producer for the television show Actuarial Idol. Each year, 1000 actuarial clubs audition for the show. The probability of a club being accepted is 0.20.

The number of members of an accepted club has a distribution with mean 20 and variance 20. Club acceptances and the numbers of club members are mutually independent.

Your annual budget for persons appearing on the show equals 10 times the expected number of persons plus 10 times the standard deviation of the number of persons.

Calculate your annual budget for persons appearing on the show.

(A) 42,600

(B) 44,200

(C) 45,800

(D) 47,400

(E) 49,000

31. Michael is a professional stuntman who performs dangerous motorcycle jumps at extreme sports events around the world.

The annual cost of repairs to his motorcycle is modeled by a two parameter Pareto distribution with $\theta = 5000$ and $\alpha = 2$.

An insurance reimburses Michael's motorcycle repair costs subject to the following provisions:

(i) Michael pays an annual ordinary deductible of 1000 each year.

(ii) Michael pays 20% of repair costs between 1000 and 6000 each year.

(iii) Michael pays 100% of the annual repair costs above 6000 until Michael has paid 10,000 in out-of-pocket repair costs each year.

(iv) Michael pays 10% of the remaining repair costs each year.

Calculate the expected annual insurance reimbursement.

(A) 2300

(B) 2500

(C) 2700

(D) 2900

(E) 3100

32. For an aggregate loss distribution S:

(i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$.

(ii) The claim amounts are uniformly distributed on the interval (0, 8).

(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5%.

(A) 500

(B) 520

(C) 540

(D) 560

(E) 580

33. You are given:

(i) Y is the present value random variable for a continuous whole life annuity of 1 per year on (40).

(ii) Mortality follows DeMoivre's Law with $\omega = 120$.

(iii) $\delta = 0.05$

Calculate the 75th percentile of the distribution of Y.

(A) 12.6

(B) 14.0

(C) 15.3

(D) 17.7

(E) 19.0

34. For a special fully discrete 20-year endowment insurance on (40):

(i) The death benefit is 1000 for the first 10 years and 2000 thereafter. The pure endowment benefit is 2000.

(ii) The annual benefit premium, determined using the equivalence principle, is 40 for each of the first 10 years and 100 for each year thereafter.

(iii) $q_{40+k} = 0.001k + 0.001$, $k = 8, 9, \ldots 13$

(iv) $i = 0.05$

(v) $\ddot{a}_{51:\overline{9}|} = 7.1$

Calculate the 10th year terminal reserve using the benefit premiums.

(A) 490

(B) 500

(C) 530

(D) 550

(E) 560

35. For a whole life insurance of 1000 on (80), with death benefits payable at the end of the year of death, you are given:

(i) Mortality follows a select and ultimate mortality table with a one-year select period.

(ii) $q_{[80]} = 0.5 q_{80}$

(iii) $i = 0.06$

(iv) $1000A_{80} = 679.80$

(v) $1000A_{81} = 689.52$

Calculate $1000A_{[80]}$.

(A) 655

(B) 660

(C) 665

(D) 670

(E) 675

36. For a fully discrete 4-year term insurance on (40), who is subject to a double-decrement model:

(i) The benefit is 2000 for decrement 1 and 1000 for decrement 2.

(ii) The following is an extract from the double-decrement table for the last 3 years of this insurance:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
41	800	8	16
42	–	8	16
43	–	8	16

(iii) $v = 0.95$

(iv) The benefit premium, based on the equivalence principle, is 34.

Calculate ${}_2V$, the benefit reserve at the end of year 2.

(A) 8

(B) 9

(C) 10

(D) 11

(E) 12

37. You are pricing a special 3-year life annuity-due on two lives each age x, with independent future lifetimes. The annuity pays 10,000 if both persons are alive and 2000 if exactly one person is alive.

You are given:

(i) $q_{xx} = 0.04$

(ii) $q_{x+1:x+1} = 0.01$

(iii) $i = 0.05$

Calculate the actuarial present value of this annuity.

(A) 27,800

(B) 27,900

(C) 28,000

(D) 28,100

(E) 28,200

38. For a triple decrement table, you are given:

(i) Each decrement is uniformly distributed over each year of age in its associated single decrement table.

(ii) $q'^{(1)}_x = 0.200$

(iii) $q'^{(2)}_x = 0.080$

(iv) $q'^{(3)}_x = 0.125$

Calculate $q^{(1)}_x$.

(A) 0.177

(B) 0.180

(C) 0.183

(D) 0.186

(E) 0.189

39. The random variable N has a mixed distribution:

(i) With probability p, N has a binomial distribution with $q = 0.5$ and $m = 2$.

(ii) With probability $1 - p$, N has a binomial distribution with $q = 0.5$ and $m = 4$.

Which of the following is a correct expression for $\text{Prob}(N = 2)$?

(A) $0.125p^2$

(B) $0.375 + 0.125p$

(C) $0.375 + 0.125p^2$

(D) $0.375 - 0.125p^2$

(E) $0.375 - 0.125p$

40. A compound Poisson distribution has $\lambda = 5$ and claim amount distribution as follows:

x	$p(x)$
100	0.80
500	0.16
1000	0.04

Calculate the probability that aggregate claims will be exactly 600.

(A) 0.022

(B) 0.038

(C) 0.049

(D) 0.060

(E) 0.070

END OF EXAMINATION