1. **Learning Objectives:**
1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
(1k) Estimate written, earned and unearned premiums.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies.*

**Solution:**
(a) Calculate the written premiums for 2011.

<table>
<thead>
<tr>
<th>Policy Number</th>
<th>Policy Premium</th>
<th>Policy Effective Date</th>
<th>Policy Expiration Date</th>
<th>Written in CY 2011</th>
<th>WP in CY 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>1,000</td>
<td>July 1, 2009</td>
<td>June 30, 2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>102</td>
<td>1,200</td>
<td>April 1, 2010</td>
<td>March 31, 2012</td>
<td>50%</td>
<td>600</td>
</tr>
<tr>
<td>103</td>
<td>2,400</td>
<td>January 1, 2010</td>
<td>December 31, 2014</td>
<td>20%</td>
<td>480</td>
</tr>
<tr>
<td>104</td>
<td>800</td>
<td>April 1, 2011</td>
<td>March 31, 2013</td>
<td>50%</td>
<td>400</td>
</tr>
<tr>
<td>105</td>
<td>1,400</td>
<td>October 1, 2011</td>
<td>September 30, 2013</td>
<td>50%</td>
<td>700</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2,180</td>
</tr>
</tbody>
</table>

Example: policy #103 is a five-year policy. Since written premiums are divided into equivalent annual values and recorded on the anniversary of the effective date, 20% of the policy premium is considered written in calendar year 2011.
1. Continued

(b) Calculate the earned premiums for 2011.

<table>
<thead>
<tr>
<th>Policy Number</th>
<th>Policy Number</th>
<th>Policy Premium</th>
<th>Policy Effective Date</th>
<th>Policy Expiration Date</th>
<th>Earned in CY 2011 %</th>
<th>EP in CY 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>100</td>
<td>1,000</td>
<td>July 1, 2009</td>
<td>June 30, 2011</td>
<td>25.0%</td>
<td>250</td>
</tr>
<tr>
<td>102</td>
<td>1,200</td>
<td>1,200</td>
<td>April 1, 2010</td>
<td>March 31, 2012</td>
<td>50.0%</td>
<td>600</td>
</tr>
<tr>
<td>103</td>
<td>2,400</td>
<td>2,400</td>
<td>January 1, 2010</td>
<td>December 31, 2014</td>
<td>20.0%</td>
<td>480</td>
</tr>
<tr>
<td>104</td>
<td>800</td>
<td>800</td>
<td>April 1, 2011</td>
<td>March 31, 2013</td>
<td>37.5%</td>
<td>300</td>
</tr>
<tr>
<td>105</td>
<td>1,400</td>
<td>1,400</td>
<td>October 1, 2011</td>
<td>September 30, 2013</td>
<td>12.5%</td>
<td>175</td>
</tr>
</tbody>
</table>

Total: 1,805

Example: policy #104 is a two-year policy. Nine months of this policy are earned in calendar year 2011, or 9/24 = 37.5%. Earned premiums are therefore 37.5%×800 = 300.

(c) Calculate the unearned premiums as of December 31, 2011.

Solve directly by using the number of months unearned from each annual premium as of December 31, 2011.

<table>
<thead>
<tr>
<th>Policy Number</th>
<th>Equivalent Annual Premium</th>
<th>Written Date in 2011</th>
<th># Months Unearned at Dec 31, 2011</th>
<th>UEP at Dec 31, 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>500</td>
<td>n/a</td>
<td>n/a</td>
<td>0</td>
</tr>
<tr>
<td>102</td>
<td>600</td>
<td>April 1, 2011</td>
<td>3</td>
<td>150</td>
</tr>
<tr>
<td>103</td>
<td>480</td>
<td>January 1, 2011</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>104</td>
<td>400</td>
<td>April 1, 2011</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>105</td>
<td>700</td>
<td>October 1, 2011</td>
<td>9</td>
<td>525</td>
</tr>
</tbody>
</table>

Total: 775

1. Continued

(d) Describe two examples of coverages for which premium is not earned evenly throughout the year.

Coverage: Property catastrophe coverage for hurricanes or hail coverage. Description: the exposure to claims is not spread evenly throughout the year, but instead concentrated over specific months.

Coverage: Aggregate stop loss coverage. Description: Given the nature of the coverage, the exposure to claims is much greater near the end of the policy term rather than during the initial months of coverage.
2. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:

Commentary on Question:
This question tests the candidate’s ability to calculate the IBNR reserves using the development method, the Bornhuetter Ferguson method, and the Benktander method. This question also tests the candidate’s ability to determine if experience is better or worse than expected.

Solution:
(a) Estimate accident year 2012 IBNR reserves as of December 31, 2014 using:

(i) the Development method,

(ii) the Bornhuetter Ferguson method, and

(iii) one iteration of the Benktander method.

(i) Development method:

36 months to ultimate development factor:
\[ = 1.250 \times 1.100 \times 1.040 = 1.430 \]

2012 ultimate claims
\[ = \text{Reported claims} \times 36 \text{ months to ultimate development} \]
\[ = (330,000 + 400,000) \times 1.430 = 1,043,900 \]

2012 IBNR = 2012 Ultimate – 2012 Reported Claims
\[ = 1,043,900 – 730,000 = 313,900 \]
2. Continued

(ii) Bornhuetter Ferguson method:

Expected ultimate claims from expected method
\[ = \text{Earned premiums} \times \text{Expected claim ratio} \]
\[ = 1,600,000 \times 0.68 = 1,088,000 \]

\[ \text{IBNR} = \text{Ultimate claims} \times \left[ 1 - \frac{1}{36 \text{ months to ultimate development factor}} \right] \]
\[ = 1,088,000 \times \left[ 1 - \frac{1}{1.430} \right] = 327,161 \]

(iii) One iteration of Benktander method:

The ultimate claims from the Bornhuetter Ferguson method is used for the input to the Benktander method.

Bornhuetter Ferguson ultimate claims = Reported + IBNR
\[ = 730,000 + 327,161 = 1,057,161 \]

Benktander IBNR = \[1,057,161 \times \left[ 1 - \frac{1}{1.430} \right] = 317,888 \]

(b) Explain if this business is performing better or worse than expected using the methods above.

Commentary on Question:
There are two approaches that can be used. Either approach is acceptable.

Approach 1: Compare claim ratios for all three methods.
- Development method: \[\frac{1,043,900}{1,600,000} = 65.2\%\]
- Bornhuetter Ferguson method: \[\frac{1,057,161}{1,600,000} = 66.1\%\]
- Benktander method: \[\frac{(730,000 + 317,888)}{1,600,000} = 65.5\%\]
Since all methods have a lower claim ratio than the expected claim ratio (68%), the business is performing better than expected.

Approach 2: Compare expected reported claims to actual reported claims at 36 months development.
- Expected reported claims at 36 months development:
\[ = 1,600,000 \times 0.68 \times \frac{1}{1.430} = 760,839 \]
- Actual reported claims at 36 months development = 730,000
Since actual claims are less than expected claims, the business is performing better than expected.
2. **Continued**

(c) Identify one weakness of the Benktander method.

One weakness of the Benktander method is that there is no clear guidance with respect to the appropriate number of iterations to perform.
3. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
*This question tests the calculation of ultimate claims using the Generalized Cape Cod method.*

**Solution:**
(a) Calculate the total used-up on-level earned premiums.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>On-Level Earned Premiums</th>
<th>Cumulative Reported Development Factors</th>
<th>Expected % Developed</th>
<th>Used-up On-Level Earned Premiums</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>15,000</td>
<td>1.5</td>
<td>66.7%</td>
<td>10,000</td>
</tr>
<tr>
<td>2014</td>
<td>12,000</td>
<td>3.0</td>
<td>33.3%</td>
<td>4,000</td>
</tr>
<tr>
<td>Total</td>
<td>27,000</td>
<td></td>
<td></td>
<td>14,000</td>
</tr>
</tbody>
</table>

(b) Calculate the total adjusted reported claims.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Reported Claims</th>
<th>Trend</th>
<th>Adjusted Reported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>6,000</td>
<td>1.04</td>
<td>6,240</td>
</tr>
<tr>
<td>2014</td>
<td>4,000</td>
<td>1.00</td>
<td>4,000</td>
</tr>
<tr>
<td>Total</td>
<td>10,000</td>
<td></td>
<td>10,240</td>
</tr>
</tbody>
</table>

Note: trend using the annual claim trend of 4% to 2014 claim costs (i.e., 1 year for 2013 and 0 years for 2014).
3. Continued

(c) Calculate the total expected claims based on reported data.

\[
\begin{align*}
(8) &= \frac{(7)}{(4)} \\
(9) &= (4)(9) \\
(10) &= (4)(10) \\
(11) &= (4)(9) \\
(12) &= (4)(10) \\
(13) &= \frac{(1)(13)}{(6)} \\
(14) &= (1)(13)/(6)
\end{align*}
\]

80% decay factors
Used-up On-Level
Earned Premiums ×
Decay

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Claim Ratios</th>
<th>2014</th>
<th>2013</th>
<th>2014</th>
<th>2013</th>
<th>Expected Claim Ratios</th>
<th>Expected Claims Based on Reported</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>62.4%</td>
<td>80%</td>
<td>100%</td>
<td>8,000</td>
<td>10,000</td>
<td>71.52%</td>
<td>10,315</td>
</tr>
<tr>
<td>2014</td>
<td>100.0%</td>
<td>100%</td>
<td>80%</td>
<td>4,000</td>
<td>3,200</td>
<td>74.93%</td>
<td>8,992</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19,307</td>
</tr>
</tbody>
</table>

Column (13): for 2013, 71.52% = sumproduct of columns (8) and (12) divided by the sum of column (12), or \((62.4\% \times 10,000 + 100\% \times 3,200) / (10,000 + 3,200)\).

(d) Calculate the total projected ultimate claims.

\[
\begin{align*}
(15) &= 1 - (3) \\
(16) &= (14)(15) \\
(17) &= (16)+(5)
\end{align*}
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Expected % Undeveloped</th>
<th>Expected Claims Unreported</th>
<th>Projected Ultimate Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>33.3%</td>
<td>3,438</td>
<td>9,438</td>
</tr>
<tr>
<td>2014</td>
<td>66.7%</td>
<td>5,995</td>
<td>9,995</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>9,433</td>
<td>19,433</td>
</tr>
</tbody>
</table>

(e) Describe two differences between the Cape Cod method and the Generalized Cape Cod method.

Any two of the following are acceptable:
- The Generalized Cape Cod uses a judgmentally selected decay factor to assign different weights to each year in the experience period.
- In the Cape Cod method, expected claims for each year in the experience period are derived from the same expected claim ratio. In the Generalized Cape Cod method, a distinct expected claim ratio is obtained for each year in the experience period.
- The Generalized Cape Cod method takes into account the relationship between the variance and trending, which if not considered could cause excessive weight to be given to years that are out of date.
4. **Learning Objectives:**

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(4a) Identify the time periods associated with trending procedures.

(4c) Choose trend rates and calculate trend factors for claims.

(5g) Calculate risk classification changes and territorial changes.

**Sources:**


**Commentary on Question:**

*This question tests basic general insurance risk classification.*

**Solution:**

(a) Calculate the trended ultimate pure premium for rating class A.

Past pure premium trend = $1.02 \times 1.05 - 1 = 7.10\%$

Legislative reform is expected to reduce frequency trend to 2%.

Future pure premium trend = $1.02 \times 1.02 - 1 = 4.04\%$

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>(1) Average Accident Date</th>
<th>(2) Effective Date of Reform</th>
<th>(3) Average Accident Date Forecast Period</th>
<th>(4) Past Trend Months [(1) to (2)]</th>
<th>(5) Future Trend Months [(2) to (3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>1-Jul-12</td>
<td>1-Jan-16</td>
<td>1-May-17</td>
<td>42</td>
<td>16</td>
</tr>
<tr>
<td>2013</td>
<td>1-Jul-13</td>
<td>1-Jan-16</td>
<td>1-May-17</td>
<td>30</td>
<td>16</td>
</tr>
<tr>
<td>2014</td>
<td>1-Jul-14</td>
<td>1-Jan-16</td>
<td>1-May-17</td>
<td>18</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>(6) Past Pure Premium Trend @7.10%</th>
<th>(7) Future Pure Premium Trend @4.04%</th>
<th>(8) Trend Factor</th>
<th>(9) Trended Ultimate Claims</th>
<th>(10) Trended Ultimate Pure Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>1.271</td>
<td>1.054</td>
<td>1.340</td>
<td>296,140</td>
<td>247</td>
</tr>
<tr>
<td>2013</td>
<td>1.187</td>
<td>1.054</td>
<td>1.251</td>
<td>328,388</td>
<td>261</td>
</tr>
<tr>
<td>2014</td>
<td>1.108</td>
<td>1.054</td>
<td>1.168</td>
<td>348,473</td>
<td>274</td>
</tr>
</tbody>
</table>
4. Continued

Notes:  
(6) = 1.071^{(4)/12}
(7) = 1.0404^{(5)/12}
(9) = (Reported Claims at Dec. 31, 2014)×(Cumulative Development Factor)×(8)
(10) = (9)÷(Earned Exposures)

Trended ultimate pure premium = (0.2×247) + (0.3×261) + (0.5×274) = 265.

(b) Explain why written exposures should be used instead of earned exposures in a pure premium single variable analysis.

Commentary on Question:
*It is important to note that written exposures are a better reflection of the distribution to be expected in the future rating period, and not just note that written exposures recognize the distribution quicker.*

The use of written exposures is assumed to be the most representative of the distribution of exposures during the period for which the rates will be in effect.

(c) Calculate the credibility-weighted pure premium rating class relativities.

Commentary on Question:
*The industry relativities need to be rebalanced in order to have columns (3) and (7) on the same basis.*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1,300</td>
<td>265</td>
<td>1.081</td>
<td>450</td>
<td>75.0%</td>
<td>1.15</td>
<td>1.076</td>
<td>1.080</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>900</td>
<td>200</td>
<td>0.816</td>
<td>288</td>
<td>60.0%</td>
<td>0.85</td>
<td>0.795</td>
<td>0.808</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>700</td>
<td>266</td>
<td>1.085</td>
<td>200</td>
<td>50.0%</td>
<td>1.20</td>
<td>1.123</td>
<td>1.104</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2,900</td>
<td>245.07</td>
<td>1.000</td>
<td>938</td>
<td>1.069</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:  
(2)_{\text{Total}} = \text{sumproduct}[(1),(2)] / \text{sum}(1)
(3)_i = (2)_i / (2)_{\text{Total}}
(5) = \text{square root}[(4) / 800]
(6)_{\text{Total}} = \text{weighted average using } (1)
(7) = (6)_i / (6)_{\text{Total}}
(8) = (3)(5) + (7)[1 – (5)]
5. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5k) Calculate rates for claims-made coverage.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:
This question tests the understanding of claims-made ratemaking.

Solution:
(a) Calculate the total pure premium based on the coverage described above.

Total pure premium = nose pure premium + 2016 occurrence pure premium + 2017 occurrence pure premium + 2018 occurrence pure premium

The nose pure premium =
(12,000/3)×(0.8 + 0.8 + 0.82) = 8,960
Total pure premium = 8,960 + 12,000×(1 + 1.25 + 1.252) = 54,710.

(b) Calculate the level premium that NIC proposes to offer AJ.

Premium = (pure premium + expense fee + policy fee)
Premium = (54,710 + 3,000 + 190) = 57,900
Level premium = Premium/3 = 57,900/3 = 19,300.

(c) Explain how each of the following might or might not affect the frequency and severity of claims experience:

(i) The existence of an effective risk management program

(ii) AJ’s planned retirement at the end of 2018

Commentary on Question:
Other answers that stated an effect and gave a reason were also considered.

(i) The effective risk management program could reduce the frequency of claims because there could be fewer incidents giving rise to claims and could also reduce the severity if proactive risk management steps deal with the claim before it increases over time.
5.  Continued

(ii) The retirement could increase the number of claims because claimants feel it is the last opportunity. Also, pre-retirement activity could represent a phase-out thereby leading to a reduced number of claims. Morale hazard could lead to an increase in severity, for example, if AJ uses less care.
6. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5b) Calculate expenses used in ratemaking analyses including expense trending procedures.

Sources:

Commentary on Question:
This question tests the expense provisions that are used in ratemaking.

Solution:
(a) Explain whether you would consider each of the following expenses as either fixed or variable for ratemaking purposes:

(i) Policyholder dividends

(ii) Reinsurance provisions

(i) Policyholder dividends:
Variable as it varies as a function of premiums. The CAS Statement of Principles Ratemaking (Casualty Actuarial Society, 1988) describes policyholder dividends as a non-guaranteed return of premiums charged to operations as an expense.

(ii) Reinsurance provisions:
Variable as it varies as a function of premiums. If a provision for reinsurance is included, actuaries should consider the amount to be paid to the reinsurer; ceding commissions or allowances; expected reinsurance recoveries; and other relevant information specifically relating to cost, such as a retrospective profit-sharing agreement and reinstatement premiums between the reinsured and the reinsurer.

(b) Calculate the acquisition expense ratios.

Commentary on Question:
Candidates need to use the correct denominators for determining expense ratios:
- Fixed expense ratios use trended earned premium at current rate level
- Variable expense ratios use written premium
6. Continued

Fixed Acquisition Expense Ratio:

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Trended Earned Premiums at Current Rate Level</th>
<th>Acquisition Expenses</th>
<th>Fixed Acquisition Expenses</th>
<th>Trending Period (years)</th>
<th>Trend Factors</th>
<th>Trended Fixed Acquisition Expenses</th>
<th>Trended Fixed Acquisition Expense Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>11,245</td>
<td>1,722</td>
<td>574.0</td>
<td>3.75</td>
<td>1.027</td>
<td>589.2</td>
<td>5.2%</td>
</tr>
<tr>
<td>2013</td>
<td>11,828</td>
<td>1,802</td>
<td>600.7</td>
<td>2.75</td>
<td>1.019</td>
<td>612.3</td>
<td>5.2%</td>
</tr>
<tr>
<td>2014</td>
<td>13,006</td>
<td>1,933</td>
<td>644.3</td>
<td>1.75</td>
<td>1.012</td>
<td>652.2</td>
<td>5.0%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.1%</td>
</tr>
</tbody>
</table>

Notes: (4) Trending period from average earned date in experience period (July 1 each year), to average earned date in forecast period (April 1, 2016). (5) Trend factors = 1.007(4) (7) Total is the average.

Variable Acquisition Expense Ratio:

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Written Premiums</th>
<th>Variable Acquisition Expenses</th>
<th>Variable Acquisition Expense Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>11,880</td>
<td>1148.0</td>
<td>9.7%</td>
</tr>
<tr>
<td>2013</td>
<td>12,348</td>
<td>1201.3</td>
<td>9.7%</td>
</tr>
<tr>
<td>2014</td>
<td>13,521</td>
<td>1288.7</td>
<td>9.5%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>9.6%</td>
</tr>
</tbody>
</table>

Note: Total is the average.

(c) Calculate the taxes and licenses expense ratio.

**Commentary on Question:**

*Candidates need to use the correct denominator (written premium) for determining taxes and licenses expense ratio.*

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Written Premiums</th>
<th>Taxes and Licenses</th>
<th>Taxes and Licenses Expense Ratios</th>
<th>Taxes and Licenses Expense Ratios With 20% Adjustment in 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>11,880</td>
<td>290</td>
<td>2.4%</td>
<td>2.9%</td>
</tr>
<tr>
<td>2013</td>
<td>12,348</td>
<td>345</td>
<td>2.8%</td>
<td>2.8%</td>
</tr>
<tr>
<td>2014</td>
<td>13,521</td>
<td>425</td>
<td>3.1%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>3.0%</td>
<td></td>
</tr>
</tbody>
</table>
6. Continued

(d) Explain how such an approach could lead to a mismatch between actual expenses and premium provisions for salaries and rent.

**Commentary on Question:**
*Other explanations are possible.*

On an aggregate basis, using an all-variable expense approach will cause the fixed expense provision to be overstated/(understated) when the pricing indicates an increase/(decrease) in the rates.
7. Learning Objectives:

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:

Commentary on Question:
This question tests the candidate’s ability to adjust the average case estimate triangle for a large claim, as well as to estimate IBNR with the Berquist-Sherman adjustments when there has been an adjustment in case reserves.

Solution:

(a) Calculate the average case estimate triangle, adjusted to eliminate the large accident year 2013 claim.

1. Determine the adjusted reported claims triangle, by reducing the accident year 2013 reported claims for the large claim:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Reported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>3,850</td>
</tr>
<tr>
<td>2013</td>
<td>4,326</td>
</tr>
<tr>
<td>2014</td>
<td>5,045</td>
</tr>
</tbody>
</table>

2. Adjust the paid claims for the large accident year 2013 claim paid in calendar year 2014 (i.e., 24 months development):

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Paid Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>2,200</td>
</tr>
<tr>
<td>2013</td>
<td>2,472</td>
</tr>
<tr>
<td>2014</td>
<td>2,461</td>
</tr>
</tbody>
</table>
7. Continued

3. Adjust the open counts for the large claim open count removed in accident year 2013 12 months development (note: the claim was closed in 2014 and therefore no adjustment to open counts is required at 24 months development):

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Open Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>900</td>
</tr>
<tr>
<td>2013</td>
<td>989</td>
</tr>
<tr>
<td>2014</td>
<td>960</td>
</tr>
</tbody>
</table>

4. Calculate the adjusted average case estimate triangle:

\[
\text{Average Case Estimate} = \frac{\text{Adjusted Reported Claims} - \text{Adjusted Paid Claims}}{\text{Adjusted Open Counts}}
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Average Case Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>1.833</td>
</tr>
<tr>
<td>2013</td>
<td>1.875</td>
</tr>
<tr>
<td>2014</td>
<td>2.692</td>
</tr>
</tbody>
</table>

(b) Explain why the adjusted average case estimate triangle indicates decreasing, increasing or stable case reserve adequacy.

The average case estimates are increasing in the last calendar year (i.e., the latest diagonal). This suggests an increase in case reserve adequacy.
7. Continued

(c) Calculate the indicated IBNR using the reported development method, with a Berquist-Sherman adjustment.

1. Adjusted average case triangle (use latest diagonal from part (a) adjusted back with 0% trend).

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Case Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>2.692</td>
</tr>
<tr>
<td>2013</td>
<td>2.692</td>
</tr>
<tr>
<td>2014</td>
<td>2.692</td>
</tr>
</tbody>
</table>

2. Adjusted reported claims triangle:
   \[
   = (\text{Adjusted Case Estimates})(\text{Adjusted Open Counts}) + \text{Adjusted Paid Claims}
   \]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Reported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2012</td>
<td>4,623</td>
</tr>
<tr>
<td>2013</td>
<td>5,134</td>
</tr>
<tr>
<td>2014</td>
<td>5,045</td>
</tr>
</tbody>
</table>

3. Determine development factors using the reported development method:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Age-to-Age Development Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12-24</td>
</tr>
<tr>
<td>2012</td>
<td>1.173</td>
</tr>
<tr>
<td>2013</td>
<td>1.180</td>
</tr>
<tr>
<td>Avg</td>
<td>1.176</td>
</tr>
</tbody>
</table>

4. Calculate IBNR:
   \[
   (1) \quad (2) \quad (3) \quad (4) = (3) - (1)
   \]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Reported Claims</th>
<th>Age-to-Ultimate Development Factors</th>
<th>Ultimate Claims</th>
<th>IBNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>5,533</td>
<td>1.000</td>
<td>5,533</td>
<td>0</td>
</tr>
<tr>
<td>2013</td>
<td>6,056</td>
<td>1.020</td>
<td>6,179</td>
<td>123</td>
</tr>
<tr>
<td>2014</td>
<td>5,045</td>
<td>1.020 x 1.176</td>
<td>6,055</td>
<td>1,010</td>
</tr>
</tbody>
</table>

| Total         | 16,634                   | 17,767                              | 1,133           |
8. **Learning Objectives:**
   7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**
(7b) Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding of various risk management strategies for catastrophe losses.*

**Solution:**
(a) Describe key risk management strategies that are employed with regard to catastrophe losses by each of the following private sector stakeholders:
   
   **Residential property owners**
   Residential property owners rarely take steps with regard to construction. They tend to rely exclusively on insurance, but they may not even do that.

(b) **Commercial property owners**
Commercial property owners are more concerned with taking loss control steps and tend to buy insurance.

(c) **Insurers**
Insurers limit coverage through policy terms and reduced concentration risk. They will purchase reinsurance and will use models to understand the risk.

(d) **Reinsurers**
Reinsurers can reduce concentration risk through working with multiple insurers who cover different territories and/or different types of catastrophe risk.
9. **Learning Objectives:**
4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**
(4a) Identify the time periods associated with trending procedures.

(4c) Choose trend rates and calculate trend factors for claims.

**Sources:**

**Commentary on Question:**
*This question tests the calculation of pure premium trend, as well as considerations when selecting data points to include in trending procedures.*

**Solution:**
(a) State four considerations in the selection of which data points to include in trending procedures.

Any four of the following are acceptable:

- Balance the need for stability with the need for responsiveness to the most recent experience.
- Assign greater weight to the most recent experience for short-tail lines of business.
- Have a sufficient number of data points in the experience period to determine a pattern for the annual change.
- Consider both long-term and short-term trend indications for long-tail lines of business.
- Consider the effect of changes in coverage, economic, regulatory and legal environments over time.
- The experience of the most recent data points may be too immature for long-tail lines of business.
- Consider excluding outliers.
(b) Calculate the pure premium trend factors for accident year 2013.

Semi-annual severity trend = \( e^{0.0123} - 1 = 0.0124 \)
Annual severity trend = \( (1 + 0.0124)^2 - 1 = 2.5\% \)

Semi-annual frequency trend = \( e^{0.0101} - 1 = 0.01015 \)
Annual frequency trend = \( (1 + 0.01015)^2 - 1 = 2.0\% \)

Pure premium trend = \( (1 + 2.5\%) \times (1 + 2.0\%) - 1 = 4.6\% \)

The average accident date for the accident year 2013 experience period is the average accident date for accident year 2013, which is July 1, 2013.

The average accident date for the forecast period for six-month policies is June 1, 2016. The average accident date for the forecast period for twelve-month policies is September 1, 2016. The average accident date for the forecast period assuming 50% are six-month policies and 50% are twelve-month policies is July 15, 2016.

Trend period = July 1, 2013 to July 15, 2016 = 36.5 months.

Pure premium trend factor for accident year 2013 = \( 1.046^{(36.5/12)} = 1.147 \).

(c) Explain the circular trending process when projecting ultimate claims using the Cape Cod method.

Projections from the development method applied to reported claims and counts are input to the initial trending procedures. Selected trend rates are used for the Cape Cod method to project ultimate claims.
10. Learning Objectives:
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3f) Evaluate premium liabilities.

Sources:

Commentary on Question:
This question tests the determination of premium liabilities.

Solution:
(a) Calculate the gross and net unearned premium.

Earned premiums in quarter 4 are 20%, so 80% are unearned.
Gross unearned premium = 50,000×0.8 = 40,000

Ceded quota share = Gross × 25% = 40,000 × 0.25 = 10,000
Net unearned premium = Gross – Ceded quota share = 40,000 – 10,000 = 30,000.

(b) Calculate the equity in the gross and net unearned premiums.

<table>
<thead>
<tr>
<th></th>
<th>Gross</th>
<th>Quota Share</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident Year 2015 Expected Claims (60% of unearned premium)</td>
<td>24,000</td>
<td>–6,000</td>
<td>18,000</td>
</tr>
<tr>
<td>ULAE (5% of gross claims)</td>
<td>1,200</td>
<td>1,200</td>
<td></td>
</tr>
<tr>
<td>Policy Administration Costs (4% of gross written premium, 9 months unearned at December 31, 2014)</td>
<td>1,500</td>
<td>1,500</td>
<td></td>
</tr>
<tr>
<td>Premium Liabilities (sum of expected claims, ULAE and policy administration costs)</td>
<td>26,700</td>
<td>20,700</td>
<td></td>
</tr>
<tr>
<td>Equity (unearned premium – premium liabilities)</td>
<td>13,300</td>
<td>9,300</td>
<td></td>
</tr>
</tbody>
</table>
10. **Continued**

Recalculate the equity in the gross and net unearned premiums.

<table>
<thead>
<tr>
<th>Description</th>
<th>Gross</th>
<th>Quota Share</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accident Year 2015 Expected Claims (80% of unearned premium)</td>
<td>32,000</td>
<td>–8,000</td>
<td>24,000</td>
</tr>
<tr>
<td>ULAE (5% of gross claims)</td>
<td>1,600</td>
<td></td>
<td>1,600</td>
</tr>
<tr>
<td>Policy Administration Costs (4% of gross written premium, 9 months unearned at December 31, 2014)</td>
<td>1,500</td>
<td></td>
<td>1,500</td>
</tr>
<tr>
<td>Premium Liabilities (sum of expected claims, ULAE and policy administration costs)</td>
<td>35,100</td>
<td></td>
<td>27,100</td>
</tr>
<tr>
<td>Equity (unearned premium – premium liabilities)</td>
<td>4,900</td>
<td></td>
<td>2,900</td>
</tr>
</tbody>
</table>
11. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5i) Calculate rates for large accounts.

**Sources:**

**Commentary on Question:**
*This question tests the calculation of an experience rating modification.*

**Solution:**
(a) Calculate the total reported claims subject to experience rating limited by the basic limit and the MSL.

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>Claim ID</th>
<th>Claims at April 1, 2015</th>
<th>Reported Indemnity &amp; ALAE at April 1, 2015 Limited by Basic Limits and MSL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Indemnity</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Limits</td>
<td>Basic Limits</td>
</tr>
<tr>
<td>July 1, 2012 – June 30, 2013</td>
<td>1</td>
<td>18,000</td>
<td>18,000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>25,000</td>
<td>25,000</td>
</tr>
<tr>
<td>July 1, 2013 – June 30, 2014</td>
<td>3</td>
<td>45,000</td>
<td>30,000</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate the expected unreported claims and ALAE at April 1, 2015.

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>(1) Basic Premium</th>
<th>(2) % Unreported</th>
<th>(3) AELR</th>
<th>(4) = (1)(2)(3)</th>
<th>Unreported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 2012 – June 30, 2013</td>
<td>76,200</td>
<td>10%</td>
<td>0.63</td>
<td></td>
<td>4,801</td>
</tr>
<tr>
<td>July 1, 2013 – June 30, 2014</td>
<td>80,000</td>
<td>30%</td>
<td>0.63</td>
<td></td>
<td>15,120</td>
</tr>
<tr>
<td>Total</td>
<td>156,200</td>
<td></td>
<td>0.63</td>
<td></td>
<td>19,921</td>
</tr>
</tbody>
</table>
11. Continued

(c) Calculate the experience modification.

Actual Loss Ratio (ALR) = \frac{124,000 + 19,921}{156,200} = 0.9214

Experience modification = \frac{ALR - AELR}{AELR} \times Z = \frac{0.9214 - 0.63}{0.63} \times 0.35 = 0.162

(d) Explain how you would further adjust premium to reflect this loss reduction program in the following policy periods:

(i) Next year’s policy period

(ii) The policy period five years from now

(i) In the first year, the premium should be adjusted using a schedule rating credit.

(ii) In five years, the experience should reflect any decrease in losses resulting from the loss reduction program, and therefore the program would implicitly be accounted for by the experience rating modification. No further schedule rating credit should be applied.
12. Learning Objectives:
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

Sources:

Commentary on Question:
This question tests the understanding and the mechanics of estimating unpaid unallocated loss adjustment expenses using a paid-to-paid method.

Solution:
(a) Estimate unpaid ULAE as of December 31, 2014 using a paid-to-paid method.

Commentary on Question:
Paid ULAE in calendar year 2013 should be adjusted to remove the 1,500 of unusual expenses before calculating ratios. Alternatively, calendar year 2013 data could be removed from the ratio analysis.

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Adjusted Paid ULAE</th>
<th>Claims</th>
<th>Paid-to-Paid ULAE Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>7,200</td>
<td>72,000</td>
<td>10.0%</td>
</tr>
<tr>
<td>2012</td>
<td>7,700</td>
<td>75,000</td>
<td>10.3%</td>
</tr>
<tr>
<td>2013</td>
<td>7,700</td>
<td>76,000</td>
<td>10.1%</td>
</tr>
<tr>
<td>2014</td>
<td>7,400</td>
<td>73,000</td>
<td>10.1%</td>
</tr>
<tr>
<td>Total</td>
<td>30,000</td>
<td>296,000</td>
<td>10.1%</td>
</tr>
</tbody>
</table>

Selected ratio = 10.1%.

<table>
<thead>
<tr>
<th></th>
<th>As of Dec. 31, 2014</th>
<th>Expense Multiplier</th>
<th>Unpaid ULAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case Reserves</td>
<td>20,000</td>
<td>75%</td>
<td>1,515</td>
</tr>
<tr>
<td>IBNER</td>
<td>5,000</td>
<td>75%</td>
<td>379</td>
</tr>
<tr>
<td>IBNYR</td>
<td>10,000</td>
<td>100%</td>
<td>1,010</td>
</tr>
<tr>
<td>Total</td>
<td>35,000</td>
<td></td>
<td>2,904</td>
</tr>
</tbody>
</table>

Unpaid ULAE = (Selected Ratio)(Reserve)(Expense Multiplier)
12. Continued

(b) Identify the weakness in the classical paid-to-paid method according to Kittel.

The classical paid-to-paid method overestimates the unpaid ULAE for a growing company in an inflationary environment.

(c) Explain why the weakness identified in part (b) occurs.

The weakness in part (b) occurs because the numerator of the paid-to-paid ratio is more reactive to the increasing exposure than the denominator.

(d) Explain these two major drawbacks.

1. Ratio-based methods do not recognize the fact that the amount of ULAE does not depend solely on the magnitude of total claims in the portfolio.
2. Unpaid ULAE determined using ratio-based methods will fluctuate in response to changes in the estimates of the unpaid claims.
13. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
This question tests the candidate’s ability to project ultimate claims using the development-based frequency-severity method.

**Solution:**

(a) Calculate the trended frequency at the 2014 cost level for each accident year.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Projected Ultimate Counts</th>
<th>Earned Exposures</th>
<th>Frequency</th>
<th>Trend</th>
<th>Trended Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>900</td>
<td>41,800</td>
<td>0.02153</td>
<td>0.9703</td>
<td>0.02089</td>
</tr>
<tr>
<td>2012</td>
<td>880</td>
<td>41,000</td>
<td>0.02146</td>
<td>0.9801</td>
<td>0.02104</td>
</tr>
<tr>
<td>2013</td>
<td>850</td>
<td>40,800</td>
<td>0.02083</td>
<td>0.9900</td>
<td>0.02063</td>
</tr>
<tr>
<td>2014</td>
<td>790</td>
<td>40,900</td>
<td>0.01932</td>
<td>1.0000</td>
<td>0.01932</td>
</tr>
</tbody>
</table>

Example: \( (4)_{2011} = (1 - 0.01)^3 \)

(b) Select the 2014 cost level frequency. Justify your selection.

Average (2011-2013) = 0.02085
Selected frequency = 0.021
One year is only one data point so a two- or three-year average should be considered. The most recent year is too uncertain so should not be used in selection of the 2014 cost level frequency.
13. Continued

(c) Calculate the trended severity at the 2014 cost level for each accident year.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Severity</th>
<th>Trend</th>
<th>Trended Severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>14,800</td>
<td>1.093</td>
<td>16,172</td>
</tr>
<tr>
<td>2012</td>
<td>14,600</td>
<td>1.061</td>
<td>15,489</td>
</tr>
<tr>
<td>2013</td>
<td>15,400</td>
<td>1.030</td>
<td>15,862</td>
</tr>
<tr>
<td>2014</td>
<td>14,200</td>
<td>1.000</td>
<td>14,200</td>
</tr>
</tbody>
</table>

Example: \((2)_{2011} = (1 + 0.03)^3\)

(d) Select the 2014 cost level severity. Justify your selection.

Average (2011-2013) = 15,841
Selected severity = 15,841
One year is only one data point so a two- or three-year average should be considered. The most recent year is too uncertain so should not be used in selection of the 2014 cost level severity.

(e) Calculate the accident year 2014 ultimate claims.

The estimated ultimate count for 2014 is the selected frequency from part (b) of 0.021 multiplied by the exposure of 40,900.
\[0.021 \times 40,900 = 859.\]

The ultimate claims are equal to the selected count of 859 multiplied by the selected severity of 15,841.
\[859 \times 15,841 = 13,607,419.\]

(f) Explain how the result from part (e) differs from an estimate based on the development method.

The unadjusted estimated 2014 claims would be determined by multiplying the projected count by the projected severity: \(790 \times 14,200 = 11,218,000\)

The difference is due to the use of trended prior data instead of a single current data point. It can be seen that the trended average frequency produces a higher count of 859, and the trended average severity produces a higher severity of 15,841.
14. **Learning Objectives:**
   1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
1. Adjust historical earned premiums to current rate levels.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s ability to adjust premium to current rate levels.*

**Solution:**
(a) Calculate the weighted average rate level relative value for each of 2011 and 2012, using a rate level relative value of 1 at the beginning of 2011.

<table>
<thead>
<tr>
<th>Rate Level</th>
<th>Rate Level Relative Value</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>87.5%</td>
<td>12.5%</td>
</tr>
<tr>
<td>B</td>
<td>1.06</td>
<td>12.5%</td>
<td>84.375%</td>
</tr>
<tr>
<td>C</td>
<td>1.06×1.1 = 1.166</td>
<td>3.125%</td>
<td></td>
</tr>
<tr>
<td><strong>Weighted average rate level</strong></td>
<td><strong>1.0075</strong></td>
<td><strong>1.0558</strong></td>
<td></td>
</tr>
</tbody>
</table>

e.g., Area at rate level A in CY 2012 = (1/2)(6/12)(6/12)
Weighted average rate level for CY 2011 = (1.0)(0.875) + (1.06)(0.125) = 1.0075

(b) Calculate the premium on-level factor for 2012 for the purpose of:

(i) Projecting ultimate claims as of December 31, 2015

(ii) Ratemaking analysis
14. Continued

<table>
<thead>
<tr>
<th>Rate Level</th>
<th>Rate Level Relative Value</th>
<th>2011</th>
<th>2012</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>87.5%</td>
<td>12.5%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.06</td>
<td>12.5%</td>
<td>84.375%</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.166</td>
<td></td>
<td>3.125%</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1.2593</td>
<td></td>
<td>12.5%</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1.1082</td>
<td></td>
<td>75.0%</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>1.1968</td>
<td></td>
<td>12.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Weighted average rate level</td>
<td>1.0075</td>
<td>1.0558</td>
<td>1.1381</td>
</tr>
</tbody>
</table>

(i) For projecting ultimate claims as of December 31, 2015, need to adjust to average rate level in CY 2015:

On-level factor for 2012 $= \frac{1.1381}{1.0558} = 1.078.$

(ii) For ratemaking analysis, need to adjust to current rate level:

On-level factor for 2012 $= \frac{1.1968}{1.0558} = 1.134.$

(c) Calculate the weighted average rate level for 2012.

Diagram for CY 12 reflecting mandatory rate reduction:
14. Continued

<table>
<thead>
<tr>
<th>Rate Level</th>
<th>Value</th>
<th>% at Rate Level in 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>12.5%</td>
</tr>
<tr>
<td>B</td>
<td>1.06</td>
<td>54.167%</td>
</tr>
<tr>
<td>C</td>
<td>1.06×0.85 = 0.901</td>
<td>30.208%</td>
</tr>
<tr>
<td>D</td>
<td>0.901×1.1 = 0.9911</td>
<td>3.125%</td>
</tr>
</tbody>
</table>

Weighted average rate level 1.0023
15. **Learning Objectives:**

1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**

(1b) Identify different types of data used for actuarial analysis.

(1j) Create a claims development triangle from claims transaction data.

**Sources:**


**Commentary on Question:**

*This question tests the fundamental understanding and determination of IBNR reserves, incurred claims, and incremental paid and reported claims.*

**Solution:**

(a) Determine the IBNR reserves recorded in the December 31, 2013 balance sheet.

\[
\text{IBNR} = \text{Ultimate} - (\text{paid to date} + \text{case}) = 100,000 - (10,000 + 60,000) = 30,000.
\]

(b) Determine the IBNR reserves recorded in the December 31, 2014 balance sheet.

\[
\text{IBNR} = 90,000 - (40,000 + 25,000) = 25,000.
\]

(c) Determine the calendar year incurred claims recorded in the 2014 income statement.

**Commentary on Question:**

*There are two approaches that can be used.*

1. Use formula 3.8 in text:

\[
\text{Incurred}(2014) = \text{liability}(2014) - \text{liability}(2013) + \text{paid}(2014)
\]

\[
\begin{align*}
\text{liability}(2014) &= 25,000 + 25,000 = 50,000 \\
\text{liability}(2013) &= 30,000 + 60,000 = 90,000 \\
\text{paid in 2014} &= 40,000 - 10,000 = 30,000 \\
\text{therefore, incurred}(2014) &= 50,000 - 90,000 + 30,000 = -10,000
\end{align*}
\]

2. Incurred(2014) = ultimate @ Dec. 31, 2014 – ultimate @ Dec. 31, 2013

\[
= 90,000 - 100,000 = -10,000.
\]
15. **Continued**

(d) Calculate the incremental claims paid (including ALAE) and incremental reported claims at the end of calendar years 2013, 2014 and 2015 for this claim.

<table>
<thead>
<tr>
<th>CY</th>
<th>Paid in CY</th>
<th>Change in Case in CY</th>
<th>(1)</th>
<th>(2)</th>
<th>(3) = (1) + (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>50</td>
<td>+250</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>325</td>
<td>–225</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>125</td>
<td>–25</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(e) Identify a situation where the reported claims may decrease over time.

Examples include: salvage, recoveries, and conservative initial estimates.
16. **Learning Objectives:**
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.
   6. The candidate will understand the need for monitoring results.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(6b) Analyze actual claims experience relative to expectations.

**Sources:**

**Commentary on Question:**
*This question tests the fundamental understanding of estimating ultimate claims using the development method. Candidates also need to be able to estimate expected reported claims for an interim period between actuarial analyses using the approach in Friedland Chapter 36.*

**Solution:**
(a) Describe two considerations in selecting development factors.

Any two of the following are acceptable:

- Volume of experience in the development triangles and the credibility of the insurer’s experience
- Stability/variability of individual factors at each maturity interval as well as the averages
- Any discernible trends
- The number of recent age-to-age factors in each maturity interval that are greater than or less than the various average values
- Factors preceding and following the particular maturity age interval
- Effect of known changes in the internal or external environments that could influence future development
- Influence of large claims
- Relevance of other data, benchmarks, etc.
- Selected factors from prior actuarial work
16. Continued

(b) Calculate ultimate claims for all accident years using the development method and a simple all-year average of the age-to-age development factors.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Age to Age Development Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12-24</td>
</tr>
<tr>
<td>2011</td>
<td>2.350</td>
</tr>
<tr>
<td>2012</td>
<td>2.400</td>
</tr>
<tr>
<td>2013</td>
<td>2.100</td>
</tr>
<tr>
<td>2014</td>
<td></td>
</tr>
<tr>
<td>Simple All</td>
<td>2.283</td>
</tr>
</tbody>
</table>

Age to Ultimate: 3.586 1.571 1.122 1.020

(1) (2) (3) = (1)(2)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Reported Claims at Dec. 31, 2014</th>
<th>Age-to-Ult Dev Factor</th>
<th>Ultimate Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>3,102</td>
<td>1.020</td>
<td>3,164</td>
</tr>
<tr>
<td>2012</td>
<td>3,456</td>
<td>1.122</td>
<td>3,878</td>
</tr>
<tr>
<td>2013</td>
<td>2,520</td>
<td>1.571</td>
<td>3,959</td>
</tr>
<tr>
<td>2014</td>
<td>1,300</td>
<td>3.586</td>
<td>4,662</td>
</tr>
</tbody>
</table>

(c) Calculate the expected reported claims as of March 31, 2015 for accident years 2012, 2013 and 2014, using a linear interpolation of the expected percentage reported derived from the cumulative development factors.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Selected Ultimate Claims</th>
<th>Claims Reported as of Dec. 31, 2014</th>
<th>% Reported Dec. 31, 2014</th>
<th>% Expected Reported at Mar. 31, 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>3,164</td>
<td>3,102</td>
<td>98.0%</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>3,878</td>
<td>3,456</td>
<td>89.1%</td>
<td>91.4%</td>
</tr>
<tr>
<td>2013</td>
<td>3,959</td>
<td>2,520</td>
<td>63.7%</td>
<td>70.0%</td>
</tr>
<tr>
<td>2014</td>
<td>4,662</td>
<td>1,300</td>
<td>27.9%</td>
<td>36.8%</td>
</tr>
</tbody>
</table>

Example: (4)\textsubscript{2014} = 0.25\times63.7\% + 0.75\times27.9\%
16. Continued

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Expected Reported as of Mar. 31, 2015</th>
<th>Actual Reported as of Mar. 31, 2015</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>3,542</td>
<td>3,659</td>
<td>117</td>
</tr>
<tr>
<td>2012</td>
<td>2,772</td>
<td>2,950</td>
<td>178</td>
</tr>
<tr>
<td>2013</td>
<td>1,717</td>
<td>1,983</td>
<td>266</td>
</tr>
</tbody>
</table>

(d) Describe two possible explanations for the differences between the actual reported claims as of March 31, 2015 and the results from part (c).

Any two of the following are acceptable (other explanations are possible):

- It is possible that prior selections of ultimate claims may be inadequate
- It is possible that there were larger than expected paid claims in the first quarter
- Possible legal decision that affected all years

(e) Recommend an action to either resolve or investigate each explanation identified in part (d).

Commentary on Question:

Recommended actions in part (e) need to align with the corresponding answer to part (d).

Any two of the following are acceptable (other explanations are possible):

- Look at paid analysis to see if it has similar pattern or not
- Check with claims department to see if first quarter had larger than usual claims activity
- Investigate if such legal action and whether it would explain the increase
17. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5d) Calculate loadings for catastrophes and large claims.

**Sources:**

**Commentary on Question:**
This question tests the understanding of large claim loadings for ratemaking.

**Solution:**
(a) Explain one advantage of applying a large claim loading to limited claims instead of using total limits claims.

Applying a loading to capped claims can provide stability to the ratemaking analysis, as claims over the cap can be erratic.

(b) Calculate the loading for large claims for each accident year, adjusted to the future rating period.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Trend Period (Months)</th>
<th>Severity Trend Factor at 500,000 Limit</th>
<th>Total Limits</th>
<th>Trended Ultimate Claims</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>45</td>
<td>1.201</td>
<td>1.244</td>
<td>7,446</td>
<td>9,206</td>
</tr>
<tr>
<td>2013</td>
<td>33</td>
<td>1.144</td>
<td>1.174</td>
<td>7,207</td>
<td>8,570</td>
</tr>
<tr>
<td>2014</td>
<td>21</td>
<td>1.089</td>
<td>1.107</td>
<td>6,970</td>
<td>9,742</td>
</tr>
</tbody>
</table>

Notes:
(2) = 1.05^{(1)/(12)}
(3) = 1.06^{(1)/(12)}
(4) = (Selected Ultimate Claims at 500,000 Limit)×(2)
(5) = (Selected Ultimate Claims at Total Limits)×(3)
17. Continued

(c) Select a loading for large claims based on the calculations in part (b). Justify your selection.

Average = 1.274. The factors are a bit erratic, so the average is reasonable.

(d) Calculate the relative severity trend rate for the 500,000 to total limits layer.

Relative trend rate = \[\frac{1 + \text{severity trend at total limits}}{1 + \text{severity trend at 500,000 limit}} - 1 = \frac{1.06}{1.05} - 1 = 0.952\%\].

(e) Calculate the indicated ultimate claims for accident year 2013 by applying the loading for large claims selected in part (c).

Trend factor for loading: \[1.00952^{(33/12)} = 1.026\]

Loading for large claims adjusted to AY 2013 cost level = \[1.274 / 1.026 = 1.242\]

Indicated ultimate claims at total limits = \[6,300 \times 1.242 = 7,825\]
18. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

**Sources:**

**Commentary on Question:**
This question tests the candidate’s understanding of claim size distributions.

**Solution:**
(a) Illustrate this concept using a graph with labels to show how the graph relates to the equation above.

The graph should show horizontal and vertical strips similar to what is shown on page 53 of Lee. These should be shown below a horizontal line at 200, representing the policy limit.
18. Continued

(b) Calculate the expected loss size on Policy A after one year and after two years, assuming an annual loss trend of 10%.

\[
\int_{0}^{200} e^{-x/110} dx = 110 \left( 1 - e^{-200/110} \right) = 92.14
\]

\[
\int_{0}^{200} e^{-x/121} dx = 121 \left( 1 - e^{-200/121} \right) = 97.83
\]

(c) Explain why the expected loss size on Policy A increases by less than 10% per year.

Losses are capped at the policy limit, which mutes the effect of the trend.

(d) Explain why the percentage increase in the expected loss size on Policy A during the second year is smaller than the percentage increase during the first year.

With each succeeding year of trend, the policy limit becomes lower in the loss distribution, which increases the muting effect.
19. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2d) Explain the effect of changing conditions on the projection methods cited in (2b).

Sources:

Commentary on Question:
This question tests the understanding of how various changing conditions affect the estimates of ultimate claims.

Solution:
(a) Explain what effect the claim ratio deterioration will have on reported claim development factors.

If all other assumptions are steady-state, then deterioration in the claim ratio will not impact the patterns in the claim development triangle.

(b) Explain which of the following two methods is likely to produce a more accurate estimate of ultimate claims under this scenario:
(i) the reported development method, or
(ii) the reported Bornhuetter Ferguson method.

Since the development factors are not impacted by the claim deterioration, the development method will produce a better estimate than the Bornhuetter Ferguson method which will be understated because the expected claim ratio is not picking up the unexpected deterioration.

(c) Explain what effect the change in mix of business will have on the reported claim development factors.

If all other assumptions are steady-state, then the mix change will increase the reported development factors as the mix shifts to have more weight on the longer-tailed liability coverage.
19. Continued

(d) Explain which of the following two methods is likely to produce a more accurate estimate of ultimate claims under this scenario:

(i) the reported development method, or

(ii) the reported Cape Cod method.

The Cape Cod method will likely be more accurate than the development method. Both methods will reflect similar increases from the higher development factors, but the development method will lag in reflecting the increasing claims ratio because of the smaller proportion of claims being reported at earlier stages of development.

(e) Explain what effect the legislative change will have on the reported claim development factors.

If all other assumptions are steady-state, then the legislative change will increase the latest diagonal of the triangle although the change may be difficult to see in eighteen months.

(f) Explain which of the following two methods is likely to produce a more accurate estimate of ultimate claims under this scenario:

(i) the reported Bornhuetter Ferguson method, or

(ii) the reported Cape Cod method.

If the a priori claim ratio in the Bornhuetter Ferguson method is not adjusted for the legislative change, then the Cape Cod method will be more responsive to actual changes in the experience.
20. **Learning Objectives:**
4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**
(4a) Identify the time periods associated with trending procedures.

(4e) Choose trend rates and calculate trend factors for exposures.

**Sources:**

**Commentary on Question:**
*This question tests trend adjustments to premium.*

**Solution:**
(a) Calculate and select the annual premium trend to account for changes in the proportion of automobile policyholders with the 5% discount.

<table>
<thead>
<tr>
<th>Experience Period</th>
<th>Proportion of Policyholders with 5% Discount</th>
<th>Average Discount Factor</th>
<th>Annual Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>15.0%</td>
<td>0.993</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>21.0%</td>
<td>0.990</td>
<td>–0.30%</td>
</tr>
<tr>
<td>2013</td>
<td>23.0%</td>
<td>0.989</td>
<td>–0.10%</td>
</tr>
<tr>
<td>2014</td>
<td>25.0%</td>
<td>0.988</td>
<td>–0.10%</td>
</tr>
</tbody>
</table>

e.g., 2011 Average discount factor = \( (15\%) \times (1-5\%) + (1-15\%) \times 1 = 0.993 \)

Selected trend due to the shift in loyalty discount = –0.10%
2012 is an outlier and does not reflect recent experience. Therefore, –0.10% is selected.
20.  Continued

(b) Calculate the trend factor to be used for 2012 using written premiums for the trending analysis and the annual trend selected in part (a).

- Average written date of premiums earned in experience period = January 1, 2012
- Average written date of premiums earned in first steady-state year = January 1, 2014
- Average written date of the forecast period = April 1, 2016
- Trending period in years up to first steady-state year = January 1, 2012 to January 1, 2014 = 2 years
- Trending period in years post steady-state = January 1, 2014 to April 1, 2016 = 2.25 years
- Premium trend factor = \((1–0.1\%)^2(1+0\%)^{2.25} = 0.998\)

(c) Explain an advantage of using written premiums instead of earned premiums for premium trend analyses.

Written premiums are considered to be the preferred source for premium trend analyses, as written premiums reflect shifts in the mix of exposures more quickly than earned premiums.


21. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**

**Commentary on Question:**

This question tests the calculation of overall rate indications using a pure premium approach.

**Solution:**

(a) Describe two considerations appropriate for self-insured entities in choosing either the pure premium or the claim ratio approach for a ratemaking analysis.

The pure premium approach is usually the preferred approach for self-insured entities. Self-insured entities do not maintain earned premiums in the same manner as insurers.

Funding contributions are similar to earned premiums but are often influenced by management actions with respect to other objectives of the risk management program in any particular year. Thus, funding contributions from year to year would not necessarily form a consistent basis for ratemaking purposes; the use of ratemaking exposures provides a more stable basis for funding analyses.

(b) Recommend the number of years to include in the weighted average pure premium for the ratemaking analysis. Justify your recommendation.

Four years gets to the full credibility standard (balance of responsiveness and stability).
21. Continued

(c) Calculate the weighted average pure premium.

The weighted average is calculated over the most recent four years of experience, using earned exposures as the weights.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Earned Vehicles</th>
<th>Ultimate Counts</th>
<th>Trended Ultimate Pure Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>82,800</td>
<td>1,517</td>
<td>184</td>
</tr>
<tr>
<td>2012</td>
<td>78,300</td>
<td>1,174</td>
<td>172</td>
</tr>
<tr>
<td>2013</td>
<td>78,400</td>
<td>1,114</td>
<td>161</td>
</tr>
<tr>
<td>2014</td>
<td>81,100</td>
<td>1,135</td>
<td>178</td>
</tr>
<tr>
<td>Total</td>
<td>320,600</td>
<td></td>
<td>173.93</td>
</tr>
</tbody>
</table>

(d) Calculate the indicated rate.

First need to determine $1 - V - Q$ using formula 31.5: 

$$PCR = \frac{1 - V - Q}{1 + G}$$

$$1 - V - Q = PCR(1 + G) = 0.80 \times 1.07 = 0.856$$

Using formula 31.1: 

$$R_i = \frac{PP + F}{1 - V - Q} = \frac{173.93 + 35}{0.856} = 244.08$$

(e) Calculate the experience claim ratio.

Use formula 31.4: 

$$ECR = \frac{C}{E \times R_c}$$

$$C = (82,800 \times 184) + (78,300 \times 172) + (78,400 \times 161) + (81,100 \times 178) = 55,761,000$$

$$ECR = \frac{C}{E \times R_c} = \frac{55,761,000}{320,600 \times 250} = 69.57\%$$